Characterization of unstable particles

Decio Cocolicchio
Dipartimento di Matematica, Università della Basilicata, Potenza, Via N. Sauro 85, 85100 Potenza, Italy and Istituto Nazionale di Fisica Nucleare, Sezione di Milano, Via G. Celoria 16, 20133 Milano, Italy

(Received 14 October 1997; published 2 June 1998)

The properties of the unstable particles are analyzed relativistically in a spectral form similar to the solvable Friedrichs-Lee model of the nonrelativistic theory. Singular threshold effects are considered. The approach is then extended to a renormalizable quantum field theory that includes unstable particles. Their dynamical behavior is then investigated by examining the renormalization effects for the propagators. The connection with the Källen-Lehmann spectral representation is established and some phenomenological implications are discussed. [S0556-2821(98)05812-3]

PACS number(s): 03.65.Db, 11.10.Ef, 11.10.Gh

I. INTRODUCTION

The problem of decaying particles, scattering resonances, and generic metastable states in quantum physics continues to be of current interest. Recently, there has been considerable discussions concerning the definition of unstable states [1], which becomes an acute problem in models based on scattering theory [2]. This problem is not only of mathematical interest since many confusing issues affect the understanding of the production and the decay of large width unstable heavy fundamental particles such as top quark, gauge bosons, and eventually Higgs bosons [3], which make questionable how they are to be studied. In particular, the W and Z gauge bosons both have a sizable width and the same might be true for the top quark and the Higgs boson. Despite the impressive successes of the standard model of the electroweak interactions, the analytical structure of the resonant dynamics governing these particles will play a particularly relevant new role influencing sensibly the planning accuracy of the next generation of experiments at the forthcoming colliders, such as the second phase of the Large Electron Positron (LEP2) collider and the Large Hadron Collider (LHC) at CERN, as the Tevatron at Fermilab.

The intrinsic dissipative nature of the unstable system and its decay in quantum mechanics [1], in particle physics [4] and in statistical mechanics [5], faced with the problem of the complex eigenvalues for the Hamiltonian, and therefore with the extension from the usual Hilbert space (a space of square integrable functions) to a rigged Hilbert space (a space of distributions), in order to maintain the Hermiticity of the Hamiltonian. This procedure is not unique and different distribution spaces can be defined which are based on different test function spaces. If we choose Φ_- as the test function space, generated by the eigenfunctions of the energy $\mathcal{E}$ which are analytic in the lower complex halfplane, when the real variable $\mathcal{E}$ is promoted to a complex variable z (precisely Hardy class functions), we obtain the dual space $\Phi_-^\times$, which is the required extension of the Hilbert space $\mathcal{H}$. The corresponding Gel'fand triplet is then

$$\Phi_- \subset \mathcal{H} \subset \Phi_-^\times . \tag{1}$$

If the same procedure is performed in the upper complex plane, the resulting triplet reads

$$\Phi_+ \subset \mathcal{H} \subset \Phi_+^\times . \tag{2}$$

The first of these choices, hence the space $\Phi_-^\times$, corresponds to unstable decaying states while the second one, namely $\Phi_+^\times$, corresponds to unstable growing states. In fact, the complex poles of the transition matrix are related, as it is well known, with unstable physical states. These poles can then be transformed into complex eigenvalues z_n of the Hamiltonian. According to this method, a pair of dual spaces is necessary in order to separately represent the future-decaying and future-growing (past-decaying) states. The essence of the proposal of a rigged Hilbert space is clearly devoted to make rigorous the decay formalism and to retain the dynamical semigroup composition law in the evolution of unstable quantum states with Hermitian Hamiltonian. The eigenvalues z_n of a Hermitian operator are not real anymore in this extended space. If $\mathrm{Im}\, z_n > 0$ then a growing prefactor appears in the time evolution of the corresponding eigenvector $|\psi_{n+}\rangle$, giving rise to a growing state belonging to the rigged Hilbert space $\Phi_+^\times$. On the contrary, if $\mathrm{Im}\, z_n < 0$ the prefactor is a decaying one, the corresponding state $|\psi_{n-}\rangle$ is decaying and belongs to another rigged Hilbert space, $\Phi_-^\times$. Finally, $\mathrm{Im}\, z_n = 0$ corresponds to an ordinary stable state belonging to the ordinary Hilbert space $\mathcal{H} = \Phi_+^\times \cap \Phi_-^\times$ (more general models contain both, growing and decaying states [1]). The choice between Φ_- or Φ_+ is irrelevant, since these two objects are identical (namely one can be obtained from the other by a mathematical symmetry transformation), and therefore they are not distinguishable. Only the names *past* and *future* or *decaying* and *growing* will change but the physics is the same. The hints to double the phase space degrees of freedom are intimately connected with the problem of the quantization of dissipative open systems [6]. In these open systems, the doubled degrees of freedom play the role of the inclusion of an effective coupled bath adopted to take into account the dissipative effects. The description of the original dissipative system is then recovered by eliminating those bath variables which are not relevant by means of an appropriate averaging procedure. The strategy to include additional bath variables yields formally an isolated configuration (bath plus system), which can, of course, be studied in the canonical quantization scheme. The application of this method to problems with large quantum numbers is well known in many fields of physics. For instance, such an effective approach is extensively adopted in the general framework of statistical and thermal field theories where usually the applications of coherent-state condensates and the functional formalism of path integrals are especially useful in integrating out the subset of plethoric dynamical field variables and to yield effective dynamics at a more phenomenological level. Anyway, these results seem to reflect the well known requirements of additional complementarity relations, which occur at the classical level, to make the equations of motion for dissipative systems derivable from a variational principle [7].

On the other side, instead of extending the representation space, an alternative technique consists in doubling the time integration contour of the path integral representation of quantum systems with infinite degrees of freedom. This closed time-path Green-function formalism was introduced early into the many-body theories [8] in order to describe both equilibrium and nonequilibrium systems in a unified framework.

Anyway, it is clear that the main difficulties arise because it is not easy to deal with unstable particles in the realm of the ordinary quantum theory, as they cannot be represented by asymptotic states. The standard perturbation expansion breaks down in the absence of the observable asymptotic states for unstable particles. The controversial issue consists in defining properly the characteristic space-time dependence of the survival probability of any metastable state which can deviate from a pure exponential decay law at very short or very long times, as compared to the lifetime of the unstable particle, and in dependence of the structure of the prepared initial state. More recently, such a problem has been discussed extensively in the context of the quantum field theory [9], and in order to show that the features of unstability are a manifestation of the fact that an unstable system cannot be considered isolated [10]. This intrinsic dissipative nature is peculiar of open systems and faced with the problem of complex eigenvalues for the Hamiltonian. This approach is addressed to decompose a total closed system into a relevant subsystem, with a character of elementarity, and the remaining environmental degrees of freedom which must be integrated out to yield an effective formulation at a more phenomenological level. We already mentioned that this intrinsic nature of unstable systems can be afforded rigorously with a doubling of the path of the functional integration [8], or with the doubling of the ordinary Hilbert space [11]. However, in these rigorous approaches, we are led to consider matrix valued Green's functions (propagators), which contain spurious information, and that complicate their use in a perturbation expansion.

In principle, we may define the properties of unstable particles investigating globally a reaction involving the initial production, the intermediate propagation and its final decay. The essential content of this investigation consists in analyzing the possible existence of singularities in the multiple sheets of the Riemann surface into which the Fourier transform of the propagator can be continued analytically. Such a propagator's method has the great advantage to appear natural and indeed independent of various production and decay mechanisms, although it is not immediate to have a model and to solve the ambiguities connected to its complex analytical structure. However, in several situations, the instability properties seem to be incorporated consistently by adopting the averaging process of radiative corrections. This process yields complex singularities in the propagator and suffers in general from the presence of divergences which imposes the renormalization of the parameters in the Hamiltonian. This path of renormalization has unraveled a large number of tantalizing possibilities, but unfortunately without accommodating the significant features associated with the gauge invariance of the theory. A number of approaches has been developed to understand the singularities connected to unstable particles in perturbation theory. Nevertheless, some sort of resummation of the perturbation is required to introduce an absorptive part into the renormalized propagator which can account for the Breit-Wigner resonance shape. Indeed, in the context of renormalizable gauge theories, many pathologies seem to affect the discussion about the correct form of the resummed propagator of gauge vector unstable bosons in the resonance region. Recently, the accuracy attained in LEP experiments regarding the measurements of the fundamental parameters, the mass and width of the Z^0 gauge boson, has raised the question whether the extracted value of the renormalized on-shell Z^0 mass is gauge dependent in higher orders of perturbation theory. This last point is not only of academic interest as gauge breaking terms are often much larger numerically than the gauge invariant result. Thus, to circumvent the problem, there are several recent theoretical proposals. On this side, a decisive impulse to obtain the gauge invariance of the result at any order of perturbation theory is assured by a Laurent expansion around the complex pole [12] with the supplement of a suitable renormalization scheme to define properly the regularized mass [13]. Evidently, to obtain a more realistic description, it is necessary to incorporate also the additional contributions induced at threshold. This problem is handsomely overtaken if we are willing to comply also with other underlying characteristics of an unstable particle. Unstable intermediate states are associated with poles in their invariant momentum lying off the physical sheet below the real axis. The associated residues can be used to define generalized matrix elements for processes with unstable particles as external states which satisfy unitarity sum rules that are analogous to those for stable particles but continued off the real axis. An explanation of these shortcomings is the fact that the basic dynamics is given in terms of Heisenberg (or interacting) fields whereas the physically relevant quantities are given by expectation values of observables expressed in terms of asymptotic in- or out-fields, also called physical or free fields. In the formalism of quantum field theories the in- and out-fields are obtained by the weak limit of the Heisenberg fields in asymptotic regions of space and time where the interaction is negligible. The meaning of the weak limit is that the realization of basic dynamics in terms of the in- and outfields is not unique so that the limit in the asymptotic region, is representation dependent. This representation dependence of the asymptotic limit arises from the existence of infinitely many unitarily nonequivalent representations of the canonical (anti-) commutation relations. Of course, since observables are described in terms of asymptotic fields, unitarily inequivalent representations describe different, i.e., physically inequivalent, situations. It is therefore of crucial importance, in order to get physically meaningful results, to investigate with much care the mapping among Heisenberg or interacting fields and free fields. Such a mapping is usually called the Haag expansion or the dynamical map and it is based on the concept of interpolating field for a composite particle, which was introduced independently by Nishijima, Haag, and Zimmermann [14]. Only in a very rude and naive approximation we may assume that interacting fields and free fields share the same vacuum state and the same Fock space representation [15]. These remarks obviously apply only to quantum field theories, namely to systems with an infinite number of degrees of freedom. In quantum mechanics, the von Neumann theorem ensures that the representations of the canonical commutation relations are each other unitarily equivalent and no problem arises with uniqueness of the asymptotic limit. In quantum field theories, however, the von Neumann theorem does not hold and much more careful attention is required when considering any mapping among interacting and free fields. With this warning, the evolution of unstable states deserves a careful analysis. In the in-out formalism of quantum field theory, instead of extending the representation space, the evolution is connected with the averaging process of the quantum corrections. The introduction of the dynamical map which relates bare fields and the radiatively corrected asymptotic fields, specifies, among many representations of the canonical commutation relation, one representation suitable for the description of the decay system. From this point of view, the dynamical map gives an ensemble of representations among a statistical average of dynamics, each of which has a deterministic evolution. In this sense, the analytical features of the dynamical map yield one-to-many correspondence instead of one-to-one. It is purpose of this paper to discuss the essential problems of the structure of unstable particles in a covariant formulation. Apart from mathematical complexities, the essential task is to spell out the proper physical interpretation.

First, we discuss the decay formalism in the framework of ordinary time dependent canonical formalism. In this section we elucidate several unstable particle ideas including those of second-sheet poles, discrete energy dissolved into the continuum, and unitary time evolution with deviations from exponential decay. The covariant generalization of the decay problem with relativistic kinematics is discussed in Sec. III. This relativistic covariant version illustrates how the mass change and the decay width are generated by the interaction in a generalized invariant proper time representation. In Sec. IV, we analyze the effects of the inclusion of quantum corrections into renormalizable field theories which lead to catastrophic results, with the appearance not only of complex poles, or ghosts, in the analytic continuation of the propagators into second Riemann sheets but also of branch lines corresponding to resonant intermediate states or to anomalous composite structures. In fact, the character of branch lines is based on the analytic properties of the corresponding dispersive and absorptive parts. We explore the use of these properties for the description of resonant states and we discuss their application into the treatment of singularity structure related or to possible resonant new ordinary states or to anomalous structure thresholds which describe effects due to the possibility that a given particle can be considered as a composite system of other particles. In the case of quantum electrodynamics (QED), the appearance of a ghost in the one loop corrections to the photon propagator, the so-called Landau ghost, is not taken as a serious drawback of the theory. This is because the momentum scale at which the ghost appears is far from measurable and, at this scale, QED should probably be modified to include the effects of other electroweak effects. It is probably an indication of the sickness of QED as a fundamental theory at high energies [16]. In renormalizable field theories, a multitude of techniques (such as structure functions, exponential and running coupling constants) have been developed to control the structure of the ultra-violet, infrared and collinear singularities in a relatively easy way, or to improve the convergence of the perturbation expansion, or reordering the expansion. In the case of hadronic theories in the presence of confinement, the existence of ghost poles is related to the short distance behavior of the model interactions and can be cured employing a subnuclear quark structure with new degrees of freedom which provide a reasonable description of the property of asymptotic freedom [17]. In this paper we will endeavor to show the peculiarities of this singular behavior which causes the trouble and fosters the belief that must be armed with a deeper knowledge to come with it.

II. THE DECAY FORMALISM

Unstable quantum mechanical states ought to be represented by generalized eigenvectors corresponding to complex eigenvalues of the Hamiltonian, the so-called Gamow vectors, which appeared in the early studies about the α decay of atomic nuclei [18]. Energy eigenvectors with complex eigenvalues appear simple and useful, but were considered just as heuristic approximations, since they are excluded from ordinary quantum mechanics, in which the energy operator H is requested self-adjoint, with consequent meaningful real eigenvalues. Then the decay formalism becomes rigorous only within an extension of Hilbert space, namely in a rigged Hilbert space [11]. In spite of its undeniable shortcomings, the theory of unstable nonrelativistic systems was systematically settled out by Weisskopf and Wigner [4] in their work on the spectral linewidth for atomic radiation. In this theory, the unstable system is represented by a wave function ψ which is supposed to be an eigenfunction of an unperturbed Hamiltonian H_0. The action of the full perturbed Hamiltonian $H = H_0 + H_{int}$ then causes a nontrivial evolution of the wave function, $\psi(t) = e^{-iHt}\psi$, and induces transition to a new bound state or to a single continuum (i.e., with energy as the only quantum number). Such a simple model is good enough to give an account of the peculiar effects of decaying systems.

The wave function can be expanded as

$$|\psi(t)\rangle = \alpha(t)|\psi_0\rangle + \int_0^\infty \beta(E,t)|\psi_E\rangle dt \qquad (3)$$

with α and β the probability amplitudes of the system in the state $|\psi_0\rangle$ (corresponding to the bound state of energy $E_0 < 0$) and the positive continuum spectrum (which we supposed to start at $E=0$). The time evolution, predicted by the Wigner-Weisskopf theory, can be understood more easily by expressing the evolution operator by means of the Cauchy representation with the contour integration of the exact Green's function (the propagator) performed around the spectrum of H:

$$U(t)=\frac{1}{2\pi i}\int_C dz\, e^{-izt}G(z),$$
$$G(z)=[z-H]^{-1}. \tag{4}$$

This formalism has a rich bibliography [19]. As usual, we now introduce the partition of the Hilbert space of the composed system (a single bound state and a simple continuum) by means of the following projection operators:

$$\mathcal{P}=|\psi_0\rangle\langle\psi_0|,$$
$$\mathcal{Q}=\mathcal{I}-\mathcal{P}. \tag{5}$$

The amplitudes of interest are then determined by the reduced propagators

$$\mathcal{P}G(z)\mathcal{P} \text{ and } \mathcal{Q}G(z)\mathcal{P}. \tag{6}$$

Setting $\mathcal{I}=\mathcal{P}+\mathcal{Q}$ in the identity $\mathcal{I}(z-H)G(z)\mathcal{I}=\mathcal{I}$ one obtains, after some operator algebra,

$$\mathcal{P}G(z)\mathcal{P}=[z-H_0-\mathcal{P}\mathcal{R}(z)\mathcal{P}]^{-1},$$
$$\mathcal{Q}G(z)\mathcal{P}=\mathcal{Q}[z-\mathcal{Q}H\mathcal{Q}]^{-1}\mathcal{Q}H_{int}\mathcal{P}[z-H_0-\mathcal{P}\mathcal{R}(z)\mathcal{P}]^{-1}, \tag{7}$$

with the level shift $\mathcal{R}(z)$ defined as

$$\mathcal{R}(z)=H_{int}+H_{int}\mathcal{Q}[z-\mathcal{Q}H\mathcal{Q}]^{-1}\mathcal{Q}H_{int},$$
$$R(z)=\langle\psi_0|\mathcal{R}(z)|\psi_0\rangle. \tag{8}$$

The nondecay amplitude $\alpha(t)$ for survival of the initial state is then given by

$$\alpha(t)=\frac{1}{2\pi i}\int_C dz\, e^{-izt}[z-E_0-R(z)]^{-1}, \tag{9}$$

where C is a contour that depends on the nature of the spectrum of H and we assume that at $t=0$ the system is in the initial state $|\psi_0\rangle$ with eigenenergy E_0 and that the interaction can be switched on instantaneously.

The probability amplitude $\alpha(t)$ to find the initial state ''undecayed'' after a time t can be calculated by closing the integration contour in the lower half plane and using the method of residua. Usually, it can be written in a closed form if we restrict the Hilbert space of possible states with a sort of a superselection rule for which the level shift $R(z)$ reduces exactly to a second-order formula:

$$R(z)=\int_0^\infty dE\,\frac{|\langle\psi_0|H_{int}|\psi_E\rangle|^2}{z-E}. \tag{10}$$

This simplification follows from the essential feature of the model. In fact, it was reasonably assumed that the unstable quantum state has only projections on continuum states in which it decays, implying, physically, that there are no ''final state'' interactions. This is often a reliable physical approximation in many decaying systems, for which we can neglect the rescattering of the decay particles. In general, the decay products have several channels available and no unique prescription exists if several resonant states contribute.

Although the function $R(z)$ might show a branch cut and the propagator $\mathcal{P}G(z)\mathcal{P}$ may additionally have isolated poles on the second Riemann sheet and, in the case of the so-called ''virtual bound states,'' on the real axis to the left of the branch point [19], usually, the integral over the cut is negligible, the real pole is absent and among the possible poles on the second sheet, we can argue that the dominant contribution derives from that located at $z=E_0+R(E_0)$ for slowly varying $R(z)$. Thus we obtain an exponential decay of the initial state:

$$\alpha(t)=\exp\{-i(E_0+R(E_0))t\} \tag{11}$$

so that, with $R(E_0)=\Delta_0-(i/2)\Gamma_0$, we get the survival probability:

$$|\alpha(t)|^2=\exp(-\Gamma_0 t). \tag{12}$$

The constants Δ_0 and Γ_0 can obviously be interpreted as the induced shift and the induced width of the state $|\psi_0\rangle$. Such an approximation, called the pole approximation (or Weisskopf-Wigner approximation), usually works except for very short times when other poles, lying further from the real axis, may become important and for very long times when the cut contribution (decaying as a power function) exceeds the exponent.

In the case of the threshold region, for instance, the relative significance of the cut term increases; $R(E)$ may in some range be a rapidly varying function so that the dominant pole on the second sheet can approach the cut and possibly the real axis. Furthermore, this pole could disappear from the inside of the contour and a new resonant pole E_b could appear on the real axis to the left of the branch point. The integral over the cut can be replaced by an integral over an half-line. Finally, one obtains

$$\alpha(t)=\frac{e^{-iE_bT}}{[1-R'(E_b)]}+\frac{1}{2\pi}\int_0^\infty dE\, e^{-iEt}$$
$$\times\frac{\Gamma(E)}{[E-E_0-\Delta(E)]^2+\frac{1}{4}[\Gamma(E)]^2}, \tag{13}$$

with $R(E)=\Delta(E)-(i/2)\Gamma(E)$ where

$$\Gamma(E)=2\pi|\langle\psi_0|H_{int}|\psi_E\rangle|^2,$$
$$\Delta(E)=P\int_0^\infty dE'\,\frac{|\langle\psi_0|H_{int}|\psi'_E\rangle|^2}{E-E'}. \tag{14}$$

The first term is the residue of the integrand at E_b, a real solution of the equation

$$E_b = E_0 + R(E_b). \tag{15}$$

The pole of the propagator at $z = E_b$ implies the existence of a bound state $|\psi_{E_b}\rangle$ of energy E_b of the dressed system, eigenstate of the total Hamiltonian at the moment of switching on the interaction. High above the threshold, the term including E_b is absent because there no solutions to the previous equation. Moreover, the functions $\Gamma(E)$ and $\Delta(E)$ are usually slowly varying, so the spectrum is, to a good approximation, given by a Lorentzian profile. The peak is located at $(E_0 + \Delta_0)$ where the constant Δ_0 represents the shift in the initial state and $\Gamma \simeq \Gamma_0$ represents the linewidth. Of course, in the threshold region, the solution E_b appears and $\Gamma(E)$ and $\Delta(E)$ may vary rapidly; they cannot be interpreted simply as width and shift and, as the spectrum is defined for positive energies only, the curve is cut off at $E = 0$. The form of this cutoff depends essentially on the properties of the quantum system considered. The E_b term gives the dynamics and represents, physically, the importance of the dressed system (eigenstate of the initial Hamiltonian) which has been created at the moment of switching on the interaction. As long as the interaction is present, the initial state can be considered ''trapped'' in this state $|\psi_{E_b}\rangle$ with a probability $|\langle \psi_{E_b} | \psi_0 \rangle|^2 = [1 - R'(E_b)]^{-1}$. At the moment it is switched off, the dressed bound state ceases to exist and it is partially transferred to the bare continuum.

The generalizations of the exponential decay law are discussed extensively in the literature about unstable quantum mechanical systems [1]. In particular, a lot of attention has been given to discussions on the nonexponential decay in a spontaneous radiation emission for an excited atom. The exact solution of such a system was proposed by Friedrichs [20] and in an elegant covariant form by Lee [21] and many others [22]. For very long times, these general properties of the singularities characterizing unstable particles become effective and essential to deal, for instance, with the case of proton decay whose predicted lifetime appears longer than the present age of the universe or for the very rare double-beta decay processes. Hence, according to very general assumptions, a deviation of the exponential law for the decay of present-day protons is not excluded. The Friedrichs-Lee model has been very useful for the study of the properties of unstable systems and provides a framework for the analytic study of decay. This model originally motivated the construction of the generalized Gamow states with exact exponential decay which belong to a rigged Hilbert space [11]. Such states have found application in the theory of iterated maps [23] and play an important role in the study of irreversible processes [5]. The Friedrichs-Lee model is completely soluble and provides a closed analytic form for what is called the reduced resolvent $\mathcal{G}(z)$. In the theory of linear operators in Hilbert space and, therefore, in more general quantum theory, we can introduce the concept of the resolvent $\mathcal{G}(\lambda, \mathcal{H})$ of an operator $\mathcal{H}$,

$$\mathcal{G}(\lambda, \mathcal{H}) = [\lambda \mathcal{I} - \mathcal{H}]^{-1} = \int_0^\infty e^{-\lambda t} \exp(t\mathcal{H}) dt, \tag{16}$$

with the assumptions that the spectrum was bounded from below and the ground state (vacuum) normalized to have zero energy eigenvalue. The survival amplitude can then be rewritten in this spectral formalism by means of the Laplace transform

$$\alpha(t) = \frac{1}{2\pi i} \int_C d\lambda \, \mathcal{G}(\lambda) e^{-i\lambda t}. \tag{17}$$

However, the general quantum theory of unstable systems says little about the non-exponential corrections to the pole approximation. As a matter of fact, these corrections would come from singularities of $\mathcal{G}(\lambda)$ in the complex λ plane but they are indeed not properly known. Consequently, the influence of singularities responsible for any violation of the pure exponential decay law cannot be resolved clearly. Furthermore, the nonanalycity of the real and the imaginary part of the level shift operator could be reflected in the appearance of nonanalytic structures (cusp) in the spectrum, the so-called Wigner combs [24]. On the other side, it is important to have a relativistic model for the description of the particle decay involving a real change in the total particle mass of the system. An unstable particle generally decays into a final state of two or more particles in a process for which the total energy is conserved, but the total mass is not. The equivalence between mass and energy, which enters quantitatively in the kinematical description of such a process, is a fundamentally relativistic relation. It is of interest, therefore, to describe unstable systems, or dissipative systems in general, by extending these considerations to quantum field theory. There is, however, an even more compelling reason for using an explicitly relativistic description to deal with unstable particles, namely, the fact that for a system described in a Galilean invariant form (as opposed to a relativistic form) there are phase ambiguities that arise when one considers the combination of states with different masses [25]. Indeed in the relativistic quantum theory, the kinematical characterization of unstable particles may find difficulties since it is connected with a complex rest mass eigenvalue of a representation of the Poincaré group [26].

III. THE RELATIVISTIC GENERALIZATION

A first approach to achieve a relativistic generalization of the Friedrichs-Lee model useful to deal with the transition from the initial boson state V to the final scalar N_1 and N_2 decay states can then be provided by using a Poincaré invariant parameter τ.

The continuous quantum mechanical evolution of the wave function for a one-particle system,

$$\Psi_\tau = e^{-iK\tau} \Psi_0, \tag{18}$$

is influenced by a generalized invariant Hamiltonian which in the rest frame can be split into the relativistic form [21,22].

$$K = K_0 + K_I, \tag{19}$$

where K_0 determines the particle spectrum and K_I governs the interactions according to the following expressions:

$$K_0=\int d^4p\,\frac{p^2}{2M_V}\,b^\dagger(p)b(p)+\int d^4p\,\frac{p^2}{2M_1}\,a_1^\dagger(p)a_1(p)+\int d^4k\,\frac{k^2}{2M_2}\,a_2^\dagger(k)a_2(k),$$

$$K_I=\int d^4k\,d^4p\{f(k)b^\dagger(p)a_1(p-k)a_2(k)+f(k)^*a_1^\dagger(p-k)a_2^\dagger(k)b(p)\}. \tag{20}$$

Here, $b(p)$ is the annihilation operator for the V particle, and $a_1(p)$, $a_2(k)$, for the N_1 and N_2, respectively. This generalized Hamiltonian assures that there is nontrivial interaction only in the decay sector which is determined by the vertex factor $f(k)$. Clearly, the second term K_I represents the continuous spectrum of the decay channel and it is determined by the support properties of the coupling function $f(k)$. If the initial particle V is represented by the normalized state

$$\Psi_0=\int g(p)b^\dagger(p)d^4p|0\rangle \tag{21}$$

the evolved state Ψ_τ, as a function of the invariant time τ, is then given by

$$\Psi_\tau=\int d^4p\,A(p,\tau)b^\dagger(p)|0\rangle+\int d^4p\,d^4kB(p,k,\tau)a_1^\dagger(p)a_2^\dagger(k)|0\rangle, \tag{22}$$

where we have chosen for the initial condition

$$A(p,0)=g(p),\qquad B(p,k,0)=0. \tag{23}$$

Following precisely parallel to the analysis of the previous section, we can formulate this relativistic generalization by means of a spectral approach. Considering the Hilbert space as a direct sum space over the absolutely conserved total energy momentum of the system, we can decompose the generalized Hamiltonian for each four-momenta p^μ of the V particle:

$$K_p=\mathcal{M}(p)\mathcal{P}_0+\overline{K}_p+K_I, \tag{24}$$

where $\mathcal{P}_0$ represents the generally covariant projector chosen to select out single-particle intermediate states and characterizing the decay system whereas K_I is supposed with matrix elements only between the continuum spectrum and the discrete state ϕ_p with eigenvalue $\mathcal{M}(p)$. At this point, the problem has been reduced to solve the relativistic resolvent operator. Denoting the term $\overline{K}_p$ corresponding to the continuous spectrum by means of a spectral $\omega(k)$ function, for each value of p^2, we may write

$$\overline{K}_p=\int w(k)|k\rangle\langle k|d^4k, \tag{25}$$

or introducing a continuous label λ to identify the continuum eigenstates $\{|\psi_\lambda\rangle\}$

$$\overline{K}_p=\int w(\lambda)|\lambda\rangle\langle\lambda|d\lambda, \tag{26}$$

we derive the generalized form of the reduced resolvent at each p

$$\mathcal{G}(z)=i\int d^4p\,\frac{|g(p)|^2}{z-\mathcal{M}-R(z,p)}, \tag{27}$$

where

$$R(z,p)=\int d\lambda\,\frac{|\langle\psi_\lambda|K_I|\phi_p\rangle|^2}{z-\omega(\lambda)}=\int d^4k\,\frac{|f(k)|^2}{z-\omega(k)}. \tag{28}$$

A real pole is found for a stable V particle. However, if unstable the pole becomes complex and if the system results even more sophisticated, essential complexities are not excluded. The proper-time relativistic analogue of the survival amplitude is then given by

$$\alpha(\tau)=\frac{1}{2\pi i}\int_C e^{-iz\tau}\mathcal{G}(z)dz, \tag{29}$$

where C is a contour which depends on the spectrum. The complex poles which dominate the decay law, in this proper time evolution, can then be investigated in a way similar to the discussion of the nonrelativistic form. It is worth noting that, for each value of p^2, there is a shift of the unperturbated eigenvalue [for $f(k)=0$] from $\mathcal{M}(p)$ to a complex pole in the second Riemann sheet determined by the vertex function $f(k)$. This can be interpreted as the acquisition of a complex part for the total energy momentum of the system and corresponds to unstable particle states. The analytic structure of the resolvent operator becomes a powerful tool in the quantum description. Its relevance is due to the direct connection among its singularities and physically significant properties. In general, the singularities will be assumed as only poles on the real axis corresponding to stable states or poles in the analytical continuation of the unphysical second Riemann sheet corresponding to unstable particles. Instead, the appearance of singularities bound in the continuum reveals an unexpected interest. In this case, the singularity will be a branch line and will correspond to multiparticle states. They are related to possible intermediate states which mediate the decay (ordinary absorptive thresholds), or they are anomalous structure thresholds which describe effects due to the possibility that a given particle can be considered as a composite system of other particles. They appear in the physical sheet only if a loosely condensate bound system is involved, otherwise they remain in a secondary Riemann sheet. Anyway, there are a large variety of covariant models which may be extracted in accord with the form of the spectrum. From this point on we can buttress directly the analogy of the Friedrichs-Lee model. Of course these complexities can be better realized by means of an extension of the Hilbert space introducing complex distributions, as we have already mentioned.

IV. UNSTABLE PARTICLES IN QUANTUM FIELD THEORY

The further generalization of the decay formalism within the framework of a renormalizable theory encounters considerable difficulties on the definition of unstable particles which is often rather complicated in a model based on scattering theory. Instead of extending the representation space, another appropriate formalism emerges for describing unstable particles. In S-matrix theory, the real poles of the scattering amplitude correspond to stable intermediate particles whereas the instability of an unstable particle consists in shifting the pole location to a complex value in the different sheets of the Riemann surface into which the corresponding propagator can be analytically continued. The poles of the scattering amplitude are the zeros of the inverse propagators. The mass of an unstable particle is usually defined as the real part of the pole. This pole position is a physical quantity: it has meaning independent of any theoretical framework, and of any scattering process. It is settled that the definition of an unstable particle can be introduced without recourse to the S-matrix formalism, by considering simply the subtleties related to the mass renormalization of the propagator.

For definiteness, the space time evolution of a bare particle is governed by the causal propagator which is essentially described in momentum space by

$$\Delta(q^2)=\frac{1}{q^2-M_0^2}, \tag{30}$$

where M_0 denotes the bare mass and we neglect, for the moment, the transverse part in the case of a massive vector field and the projector operators for fermions.

If we consider the effects of the interactions and if we limit ourselves to perturbative field theories, after the Dyson summation of one-particle-irreducible two-point Green's functions (which may eventually include mixing with other particles and tadpole contributions), the dressed renormalized propagator can be written as

$$\Delta_R(q^2)=\frac{1}{q^2-M_0^2+\Pi(q^2)}, \tag{31}$$

where the vacuum polarization $\Pi(q^2)$ represents the collection of all irreducible proper self-energy (bubble) diagrams. Assuming, tacitly, that the real part of $\Pi(q^2)$ is analytic near the on-shell renormalized point $q^2=M_R^2$, we can perform the following Taylor expansion:

$$\begin{aligned}\mathrm{Re}\ \Pi(q^2)=&\mathrm{Re}\ \Pi(M_R^2)\\&+\mathrm{Re}\ \Pi'(M_R^2)(q^2-M_R^2)+\cdots\end{aligned} \tag{32}$$

in order to define the on-shell renormalized mass parameter

$$M_R^2\simeq M_0^2-\mathrm{Re}\ \Pi(M_R^2), \tag{33}$$

and the wave-function renormalized constant

$$Z^{-1}\simeq 1+\mathrm{Re}\ \Pi'(M_R^2). \tag{34}$$

Then, we may cast the propagator into the resonant Breit-Wigner form

$$\Delta_R(q^2)\simeq\frac{Z^{-1}}{q^2-M_R^2+iZ\ \mathrm{Im}\ \Pi(q^2)}. \tag{35}$$

This form for the renormalized propagator will run into problems not only with the gauge invariance but also to define properly the effective pole position of the scattering amplitude. In fact, the imaginary part of the two point function is related by unitarity to the sum of the squares of the truncated Green's function connecting the particle to various final states. The Im $\Pi(M_R^2)$ is related to the particle width via unitarity. For a stable particle Im $\Pi(q^2)=0$ and the parameter M_R is then the particle's mass. For an unstable particle Im $\Pi(q^2)\neq 0$ and the pole position is complex. The Dyson summation of quantum corrections in the propagator leads to a finite width, and it is clearly only needed when the unstable particle can kinematically be on its mass shell. Indeed for an unstable particle with spacelike momentum, the imaginary part of the self-energy is zero; hence no finite width should be used. It is a pure kinematical problem which particles should be given a finite width and which not. The essential variable here is the virtuality q^2 of the unstable particles. In the on-mass-shell renormalization scheme the pole position of the propagators coincide with their physical masses and the residues of the pole is normalized to unity.

In the case of the unstable particles, this is somewhat a subtle question, since the ''mass'' lies in the continuum created by open decay channels in the sense that the singularity is not a simple pole but it may coincide with a branch cut or another pole. The problem together with its solution has been recently rediscovered in connection with the precision measurements of the electroweak neutral gauge boson mass and width [3] and it can be correlated to the attempts to constrain the effective form factor characterizing the decays of heavy mesons [27]. To make this approach clearer, it is worth noting that the renormalization effects on the form of the particle's propagator in the neighborhood of the one-particle pole and the intertwined issue of the correct wave-function normalization in the Källen-Lehmann dispersion representation for the vacuum polarization require exactly one subtraction: by definition, the self-energy has to vanish at the observable mass at which the propagator has a pole. However, the structure singularities can be understood independently of the perturbation methods, on the basis of analyticity and unitarity. The inclusion of quantum corrections into the propagator imposes deep care to keep the matrix element gauge invariant [3]. This last point is not only of academic interest, as gauge breaking terms are often much larger numerically than the gauge invariant result.

On the other side, within the framework of a renormalizable theory, as it is well known [28], $\Pi(q^2)$ has a branch point on the real axis when q^2 is at a threshold. Therefore, the failure of the Taylor expansion about a real renormalized mass of the vacuum polarization function $\Pi(q^2)$ in the threshold region suggests retrieving the definition of the physical mass and width of a particle in terms of the position q_0^2 of the complex pole in the particle's propagator

$$q_0^2=\left(M_{phys}-\frac{i}{2}\,\Gamma_{phys}\right)^2, \tag{36}$$

where

$$\Delta_R^{-1}(q_0^2)=[\Delta^{-1}(q_0^2)+\Pi(q_0^2)]$$
$$=q_0^2-M_0^2+\Pi(q_0^2)=0, \tag{37}$$

so that the mass is defined as the real part of the pole in the energy plane. The relation between the physical mass and the renormalized mass can be recovered from the analytic structure of $\Pi(q^2)$. Anyway, to solve Eq. (37), we stress that the Taylor expansion of $\Pi(q^2)$ may not converge in the threshold region. In the absence, though, of any theoretical model, the vacuum polarization function Π can be summarized according to the characteristic structure of its singularities. In general, if the singularities are complex poles, they are directly related to the intermediate states which introduce absorptive singularities. On the other side, thresholds are branch lines which are related to the peculiarities of the model under examination. They describe physical effects due to the possibility that a given particle can be considered as a composite system of other particles or they are manifestations of the resonant mixing of the particle with a bound state. They appear in the physical sheet of the propagator only if a loosely bound composite system is involved, otherwise, they remain in a secondary Riemann sheet. However, by proceeding from the more theoretical to the more physical aspects of the problem, the issue becomes critical only when we begin the expansion using just a real renormalized mass and the threshold branch points lie on the real axis. The pole positions of the propagators coincide with their physical masses in the on-mass shell renormalization scheme, whereas the definition of the width based on the complex pole position is rather different in the threshold region [3]. Following the procedure outlined in the first paper of Ref. [3] and in Ref. [29], the inverse propagator can be written as

$$\Delta_R^{-1}(q^2)=(q^2-q_0^2)-[\Pi(q^2)-\Pi(q_0^2)]$$
$$=(q^2-q_0^2)Z_R^{-1}(q^2,q_0^2), \tag{38}$$

where

$$Z_R^{-1}(q^2,q_0^2)=\left[1-\frac{\Pi(q^2)-\Pi(q_0^2)}{q^2-q_0^2}\right]. \tag{39}$$

In general $Z_R^{-1}(q^2,q_0^2)$ is divergent. We assume that one subtraction is sufficient to make it finite. In fact, if the gauge invariant expansion of the resonant part is about the complex pole q_0^2 of Eq. (37), we can decompose the space-time dressed propagator according to the following expression:

$$\Delta_R(x'-x)$$
$$=\int\frac{d^4q}{(2\pi)^4}\,e^{-iq\cdot(x'-x)}\left[\frac{F(q_0^2)}{q^2-q_0^2}+\frac{F(q^2)-F(q_0^2)}{q^2-q_0^2}\right], \tag{40}$$

with

$$F(q^2)=\frac{q-q_0^2}{q^2-M_0+\Pi(q^2)}=\left[1+\frac{\Pi(q^2)-\Pi(q_0^2)}{q^2-q_0^2}\right]^{-1}, \tag{41}$$

where according to l'Hôpital's rule we have

$$F(q_0^2)=[1+\Pi'(q_0^2)]^{-1}. \tag{42}$$

This characterization of an unstable particle represents a deeper insight into any resonant scattering process. The first term in the space-time propagator is connected to the finite space-time propagation whereas the second is a contact term. It is worth noting that they are both produced by the same field so that the simultaneous presence of these two parts avoids a direct correspondence between the field and the particle. This is, in fact, Schwinger's point of view of a field theory [30]. A field is not supposed to be defined in the case of an unstable particle just because of the absence of its asymptotic states. Rather one defines a more fundamental field which describes a localized excitation, and it is different from the conventional one in which each particle is assigned a field. In other words, the question as to what is the lifetime of an unstable particle depends on the manner in which the particle was prepared in the sense that it is only meaningful to consider the lifetime of an unstable particle with a definite four-momentum. Therefore, unstable particles are understood in the context of field theories only in association with the Green functions which can really describe the propagation from their production to their decay space-time positions. The form of these propagators determines the time evolution of the decay probabilities. Nevertheless, the threshold singularities merit a particular attention. At the threshold for on shell production of two scalar particles, the analytic form of the vacuum polarization is given by

$$\Pi_{1\ loop}(q^2)=\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m_1^2)[(q-k)^2-m_2^2]}$$
$$=B_0(m_1^2,m_2^2;q^2). \tag{43}$$

Without considering the details of the singularities determined by the Landau equations, after a subtracted regularization to cure the divergences, the analytic expression of the scalar two points function B_0 [31], let us write

$$4\pi[\Pi(s)-\Pi(s_1)]=\left\{\frac{\lambda^{1/2}}{2s}\log\left[\frac{\sqrt{(m_1+m_2)^2-s}+\sqrt{(m_1-m_2)^2-s}}{\sqrt{(m_1+m_2)^2-s}-\sqrt{(m_1-m_2)^2-s}}\right]-\frac{m_1^2-m_2^2}{2s}\log\frac{m_1^2}{m_2^2}\right\}-\{s\to s_1\}, \tag{44}$$

with

$$\lambda=\lambda(s,m_1^2,m_2^2)=[s-(m_1+m_2)^2][s-(m_1-m_2)^2], \tag{45}$$

where we replace $q^2=s$. In this form, it is clear that $s=0$, $s=(m_1-m_2)^2$ were irrelevant singularities. In the physical Riemann sheet, for real q^2 near threshold, the reflection property, let us write

$$\mathrm{Im}\;\Pi(s)=\frac{\lambda^{1/2}}{4\pi s}\,\Theta[s-(m_1+m_2)^2]. \tag{46}$$

The same result can be obtained by the once subtracted dispersion relation, i.e., by a Cauchy formula along an integral contour (often named the african shield) encircling the branch cut. In particular, the self-energy function satisfies the once subtracted dispersion relation

$$[\mathrm{Re}\;\Pi(q^2)-\mathrm{Re}\;\Pi(q_1^2)]$$

$$=\frac{(q^2-q_1^2)}{\pi}\int_{(m_1+m_2)^2}^{\infty}ds'\frac{\mathrm{Im}\;\Pi(s')}{(s'-q_1^2)(s'-q^2)}. \tag{47}$$

The standard procedure to express the Feynman integrals on their cut consists of performing a Taylor expansion in external momenta, whose coefficients make relatively easier the remaining calculation in the whole complex plane. Sometimes, it appears worthwhile to develop and extend the method of the Taylor expansion to make it applicable to the various kinetical situations. For example, in the case of the multitude of methods to compute the two loop self-energies [32], the Taylor expansion does not seem to exist. In fact, there is no threshold solution of the relative Landau equations because of the factorized logarithmic singularities. In our case, the subtracted dispersion relations are applied by assuming that the dominant contribution comes from the lowest pole corresponding to the lightest intermediate state which can be produced. The existence of an anomalous structure threshold (due for instance to a new lighter weakly bound state), causes the failure of the application of the dispersion relations. In this case, a discontinuity is crossed in the loop integration because the intermediate particle can go on shell for unphysical values of the momentum. When the new lighter bound is included as the lowest singularity, using the appropriate contour, the dispersion relations cease to fail. However, in general, the precise knowledge of $\Pi(q^2)$ can be achieved at various levels of sophistication by means of the details of the experimental situation and it often requires some kind of analytic continuation to result sensibly. In fact, sometimes, the nearest singularity which characterizes the radius of convergence of the Taylor series is a branch point associated with the nearest threshold. To make this point clearer, it is worth expressing the full propagator in the Kallen-Lehmann representation

$$\Delta_F(s)=\int\frac{\rho(\mu)d\mu}{s-\mu}, \tag{48}$$

where the range of the integration extends from threshold to infinity below the poles of the integrand. At this level of accuracy, the lower limit may recede to infinity and the cut contributions can be ignored. Whatever the spectral function $\rho(\mu)$ is supposed to be non-negative, it is simple to show that the propagator $\Delta_F(s)$ has no poles or zeros off the real s-axis [33]. This is known as the Herglotz property. Furthermore, if $\Delta_F(s)$ possesses the Herglotz property, then so does the inverse

$$\Delta_F^{-1}(s)=[s-M_0^2-\mathcal{R}(s)], \tag{49}$$

where the integral

$$\mathcal{R}(s)=\int\frac{\sigma(\mu)d\mu}{s-\mu}, \tag{50}$$

in general is divergent and therefore needs regularization. The spectral functions are obviously related: $\sigma(s)=|\Delta_F^{-1}(s)|^2\rho(s)$. The usual mass and wave function renormalizations are performed by imposing the condition that the renormalized propagator has a pole at the physical mass M with unit residue. This condition implies that the renormalized propagator is given by

$$\Delta_R^{-1}=(s-M^2)\left[1-(s-M^2)\int d\mu\,\frac{\sigma_R(\mu)}{(\mu-M^2)^2(s-\mu)}\right], \tag{51}$$

with $\sigma_R(\mu)=\mathcal{Z}\sigma(\mu)$. Notice that we have absorbed all renormalization effects into the function $\mathcal{Z}$ which represents the residue of the full propagator at the mass pole and in terms of renormalized quantities it is given by the following relation:

$$\mathcal{Z}=\left[1-\int d\mu\,\frac{\sigma_R(\mu)}{(\mu-M)^2}\right]=\left[\int d\mu\,\rho_R(\mu)\right]^{-1}. \tag{52}$$

It follows that the spectral representation of the propagator becomes

$$\Delta_R(s) = \int \frac{\rho_R(\mu) d\mu}{s-\mu}, \tag{53}$$

where $\rho_R(\mu) = \rho(\mu)/\mathcal{Z}$. Incidentally, we can say that another way to look at the issue of the normalization is to examine the form of the propagator in the vicinity of the one-particle pole. Relaxing the requirement that the residue of this pole should be unity, and assuming that the residue of the propagator takes different values, in general, we may express the spectral densities as

$$\sigma_R(s) = \int d\,\mu K(s,\mu)\rho_R(\mu)$$

and

$$\rho_R(s) \simeq \delta(s-M^2) + |\Delta_R^{-1}(s)|^{-2}\sigma_R(s), \tag{54}$$

for $s = s(1+i\epsilon)$ and with the kernel $K(s,\mu)$ depending on the theory. In the case of new physical effects, new poles or cuts can lie in the neighborhood of the pole $s = M^2$ (which is fixed by the renormalization procedure). Actually, the main difficulty arises from a negative sign of the $\mathcal{Z}$, since it destroys the Herglotz property of Δ_R^{-1}. The spectral function $\rho_R(s)$ can be negative for some value of s. This eventuality introduces a discontinuity in the integrand of Δ_R^{-1}, whose real part (principal value integral) has a logarithmic singularity [28]. Negative $\rho_R(s)$ represents presumably the presence of negative metric states, sometimes also referred to as ghosts. For instance, in the case of QED, the interaction kernel gives rise to a negative spectral function [34]. Nevertheless, theories with massless gauge vector bosons can be reformulated in terms of an indefinite metric [35], and the positivity of the spectral functions is not a necessary requirement. Anyway, in massive theories, the negativity of ρ_R reflects the inadequacy of the approximations, or the inconsistency of the theory. It is worth noting that the introduction of a cutoff does not automatically remove the ghost problem. In the local relativistic field theory, the origin of the problem lies in the negative value of $\mathcal{Z}$. Physically, all that means that a reasonable description eliminates these complex and singular sensitivities or inelegantly modifies its analytical structure by means of some artificious approximations or with frustration employs substructural degrees of freedom in the interactions.

V. PHENOMENOLOGICAL IMPLICATIONS AND CONCLUSIONS

At present there are no practical implications between the renormalized mass and the real part of the propagator. In the case of hadron resonances [36], which in principle are thought to be derived by QCD, the mass is regarded as a Breit-Wigner resonance parameter extracted experimentally in the resonant cross section. This approach is adopted by the suspicious Particle Data Group with the relevant exception of the $S^*/f_0(975)$ meson. In this case, the assumption of the q^2 independence of the self-energy is more problematic [37]. In general, hadron dynamics is described by an effective low-energy Lagrangian, constructed so as to be compatible with QCD. Usually one adopts an effective chiral Lagrangian, obtainable via the Coleman-Callan-Wess-Zumino construction [38]. Such a Lagrangian, involving terms of arbitrary high order in derivatives, will produce momentum dependence in all observables. Such q^2 dependence could also be shown to be consistent with a number of other approaches as well as QCD sum rules and chiral perturbation theory techniques and with the vector meson dominance approach. It is thus appropriate to revisit and generalize the usual understanding of meson spectroscopy mainly in the search of gluonium or hybrid states.

The physical effects due to the location of complex pole singularities has been stressed recently for the resonant shape of the Z^0 neutral gauge boson, in the context of providing gauge and process independent definition of the Z^0 mass and width in the electroweak standard model [3]. Analogous and more interesting is the case of the heavy top quark [39], the charged gauge boson [40], and the forthcoming neutral Higgs boson [41]. Clearly, although any effect is small, however, it is not expected entirely out of range for the future precision measurements of the partial width of the Z^0.

Any attempts to determine the correct treatment of unstable particles were faced with the problem of selecting gauge independent observables. Now the Breit-Wigner form is not enough to preserve gauge invariance, due to the artifact that if we want a gauge invariant property valid order by order in perturbation theory, we must invoke some Ward-Takahashi identities, so we are forced to consider higher order corrections [42], although it is reasonable to expect that the Breit-Wigner ansatz will contain the biggest contribution of the absorptive part. Until the dynamics of unstable particles has been described in terms of initial and final asymptotic states, the results are unitary and causal. The unitarity of the S matrix even in the presence of unstable particles has been considered by Veltman in the paper of Ref. [2]. Nevertheless, the use of on-shell particles configurations becomes misleading if the resummation of the self-energy graphs takes into account higher order corrections [42]. Although most of the investigations have been concerned with the resonant enhancements in the scattering cross section due to the position of ordinary poles, threshold effects, mainly in e^+e^- collisions, are claimed to be of controversial interest [43]. The long time during the decayed particles stays close together, allowing strong interactions to build up rich structures of bound states and resonances. This picture becomes relevant in the resonant region where the narrow width approximation of the Weisskopf-Wigner method will be insufficient for many purposes. To obtain a more realistic description it is necessary to consider the additional threshold contributions whose resummation could produce an exponent with the Fermi-Watson singularities from the infra-red pole.

In conclusion, in this paper we propose a full-fledged discussion of the properties of the unstable particles. The investigation ranges from the solvable Friedrichs-Lee model of the quantum nonrelativistic theory to the renormalizable quantum field theory. In particular, the threshold singularities are considered to elucidate several often forgotten renormalization effects for the propagator. The connection with the Källen-Lehmann spectral representation is then established and some phenomenological implications are discussed.

[1] For a review see L. Fonda, G. C. Ghirardi, and A. Rimini, Rep. Prog. Phys. **41**, 587 (1978); E. C. G. Sudarshan and C. B. Chiu, Phys. Rev. D **47**, 2602 (1993); H. Nakazato, M. Namiki, and S. Pascazio, Int. J. Mod. Phys. B **10**, 247 (1996).

[2] P. T. Matthews and A. Salam, Phys. Rev. **112**, 283 (1958); **115**, 1079 (1959); R. Jacob and R. G. Sachs, *ibid.* **121**, 350 (1961); M. Veltman, Physica (Amsterdam) **29**, 186 (1963); R. Eden, P. Landshoff, D. Olive, and J. Polkinghorne, *The Analytic S-matrix* (Cambridge University Press, Cambridge, England, 1966).

[3] For an update see R. Stuart, in *Perspective for Electroweak Interactions in e^+e^- Collisions*, Proceedings of the Ringberg Workshop, Ringberg Castle, Germany, 1995, edited by B. A. Kniehl (World Scientific, Singapore, 1995); A. Aeppli, G. J. van Oldenborgh, and D. Wyler, Nucl. Phys. **B428**, 126 (1994); J. Papavassiliou and A. Pilaftsis, Phys. Rev. Lett. **75**, 3060 (1995); Phys. Rev. D **53**, 2128 (1996).

[4] V. Weisskopf and E. P. Wigner, Z. Phys. **63**, 54 (1930); **65**, 18 (1930).

[5] I. Antoniou and I. Prigogine, Physica A **192**, 443 (1993).

[6] H. Feshbach and Y. Tikochinsky, Trans. NY Acad. Sci. **38** (Ser. II), 44 (1997); E. Celeghini, M. Rasetti, and G. Vitiello, Ann. Phys. (N.Y.) **215**, 156 (1992).

[7] H. Bateman, Phys. Rev. **38**, 815 (1931); H. Dekker, Phys. Rep. **80**, 1 (1980).

[8] J. Schwinger, J. Math. Phys. **2**, 407 (1961); R. P. Feynman and F. L. Vernon, Ann. Phys. (N.Y.) **24**, 118 (1963); L. V. Keldysh, Sov. Phys. JETP **20**, 1018 (1965); A. O. Caldeira and A. J. Legget, Physica A **121**, 587 (1983); K. Chou, Z. Su, B. Hao, and L. Yu, Phys. Rep. **118**, 1 (1985).

[9] L. Maiani and M. Testa, CERN Report No. CERN-TH/97-246.

[10] M. Yoshimura, Tohoku Univ. Report No. TU/97/523 (1997); I. Joichi, Sh. Matsumoto, and M. Yoshimura, Tohoku Univ. Report No. TU/97/524 (1997).

[11] G. Parravicini, V. Gorini, and E. C. G. Sudarshan, J. Math. Phys. **21**, 2208 (1980); A. Bohm, *ibid.* **22**, 2813 (1981); M. Gadella, *ibid.* **24**, 1462 (1983); **24**, 2142 (1983); **25**, 2481 (1984).

[12] R. Stuart, Phys. Lett. B **262**, 113 (1991).

[13] R. Stuart, Phys. Rev. Lett. **70**, 3193 (1993).

[14] K. Nishijima, Phys. Rev. **111**, 995 (1958); R. Haag, *ibid.* **112**, 669 (1958); W. Zimmermann, Nuovo Cimento **10**, 597 (1958).

[15] H. Umezawa, *Advanced Field Theory—Micro, Macro, and Thermal Physics* (AIP Press, New York, 1993).

[16] R. L. Jaffe and P. L. Mende, Nucl. Phys. **B369**, 189 (1992); R. Tarrach, Univ. Barcelona Report No. UB-ECM-PF-38-94 (unpublished).

[17] R. Oehme, πN-Newsletter No. **7**, 1 (1992); Fermi Institute Report EFI 92-17; Mod. Phys. Lett. A **8**, 1533 (1993); Int. J. Mod. Phys. A **10**, 1995 (1995).

[18] G. A. Gamow, Z. Phys. **51**, 204 (1928); **52**, 510 (1928).

[19] See, for example, M. L. Goldberger and K. M. Watson, *Collisions Theory* (Wiley, New York, 1964), Chap. 8, pp. 437–452; L. Mower, Phys. Rev. **142**, 799 (1966); Phys. Rev. A **22**, 882 (1980).

[20] K. O. Friedrichs, Commun. Pure Appl. Math. **1**, 361 (1948).

[21] T. D. Lee, Phys. Rev. **95**, 1329 (1954).

[22] R. E. Peierls, *Proceedings of the 1954 Glashow Conference on Nuclear and Meson Physics* (Pergamon Press, New York, 1954), p. 296; L. Van Hove, Physica (Amsterdam) **22**, 343 (1956); V. Glasser and G. Källen, Nucl. Phys. **2**, 706 (1956); M. Levy, Nuovo Cimento **13**, 115 (1959); **14**, 612 (1959); F. Yndurain, J. Math. Phys. **7**, 1133 (1966); N. Nakanishi, Prog. Theor. Phys. **19**, 607 (1968); C. L. Hammer and T. A. Weber, Phys. Rev. D **5**, 3087 (1972); J. Math. Phys. **10**, 2067 (1969); D. N. Williams, Nucl. Phys. **B264**, 423 (1986); L. P. Horwitz, Found. Phys. **25**, 39 (1995).

[23] I. Antoniou and S. Tasaki, J. Phys. A **26**, 73 (1993).

[24] E. P. Wigner, Phys. Rev. **73**, 1002 (1948).

[25] V. Bargmann, Ann. Math. **59**, 1 (1954); E. Henley and W. Thirring, *Elementary Quantum Field Theory* (McGraw-Hill, New York, 1963).

[26] D. N. Williams, Commun. Math. Phys. **21**, 314 (1971); P. Exner, Phys. Rev. D **28**, 2621 (1983); M. G. Fuda, *ibid.* **41**, 534 (1990).

[27] E. de Rafael and J. Taron, Phys. Rev. D **50**, 373 (1994) and references therein.

[28] T. Battacharaya and S. Willenbrok, Phys. Rev. D **47**, 4022 (1993); K. Hagiwara, S. Matsumoto, D. Haidt, and C. S. Kim, Z. Phys. C **68**, 559 (1994); **68**, 352(E) (1995).

[29] R. Stuart, Nucl. Phys. **B498**, 28 (1997).

[30] J. Schwinger, Ann. Phys. (N.Y.) **9**, 169 (1960).

[31] G. 't Hooft and M. Veltman, Nucl. Phys. **B153**, 365 (1979); S. Bertolini and A. Sirlin, *ibid.* **B248**, 589 (1984); G. Degrassi and A. Sirlin, Phys. Rev. D **46**, 3104 (1994).

[32] D. J. Broadhurst *et al.*, Z. Phys. C **60**, 287 (1993); F. V. Tkachov, Int. J. Mod. Phys. A **8**, 2047 (1993); L. Durand *et al.*, Phys. Rev. D **48**, 1061 (1993); J. Fleisher and O. V. Tarasov, Z. Phys. C **64**, 413 (1994); S. Bauberger and M. Bohm, Nucl. Phys. **B445**, 25 (1995); F. A. Berends *et al.*, *ibid.* **B439**, 536 (1995); A. Czarnecki and V. A. Smirnov, Phys. Lett. B **394**, 211 (1997).

[33] G. Barton, *Introduction to Dispersion Techniques in Field Theory* (Benjamin, New York, 1965); W. D. Brown, R. D. Puff, and L. Wilets, Phys. Rev. C **2**, 331 (1970); L. Mizrachi, Phys. Rev. D **13**, 2891 (1976).

[34] M. Gell-Mann and F. E. Low, Phys. Rev. **95**, 1300 (1954).

[35] S. W. Gupta, Proc. Phys. Soc. London, Sect. A **63**, 681 (1950); K. Bleuler, Helv. Phys. Acta **23**, 567 (1950).

[36] D. Morgan and M. R. Pennington, Phys. Rev. Lett. **59**, 2818 (1987); Phys. Lett. B **258**, 444 (1991).

[37] S. Coleman and H. J. Schnitzer, Phys. Rev. **134**, B863 (1964); R. G. Sachs and J. F. Willemsen, Phys. Rev. D **2**, 133 (1970); H. B. O'Connel *et al.*, Phys. Lett. B **336**, 1 (1994); J. Pestieau *et al.*, Phys. Rev. D **50**, 4454 (1994); Nucl. Phys. **B440**, 237 (1995).

[38] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2239 (1969); C. G. Callan, S. Coleman, J. Wess, and B. Zumino, *ibid.* **177**, 2247 (1969).

[39] A. Pilaftsis, Z. Phys. C **47**, 95 (1990); Jiang Liu, Phys. Rev. D **47**, R1741 (1993); Univ. Pennsylvania Report No. UPR-0558T (unpublished).

[40] G. Lopez Castro, J. L Lucio, and J. Pestieau, Mod. Phys. Lett. B **259**, 3679 (1991); Int. J. Mod. Phys. A **11**, 563 (1996); M. Nowakowski and A. Pilaftsis, Z. Phys. C **60**, 121 (1993).

[41] G. Valencia and S. Willenbrock, Phys. Rev. D **46**, 2247 (1992).

[42] A. Borrelli, M. Consoli, L. Maiani, and L. Sisto, Nucl. Phys. **B333**, 357 (1990).

[43] F. J. Yndurain, Phys. Lett. B **321**, 400 (1994); M. C. Gonzalez-Garcia, F. Halzen, and R. A. Vázquez, *ibid.* **322**, 233 (1994); B. A. Kniehl and A. Sirlin, Phys. Rev. D **51**, 3803 (1995).