Primordial magnetic fields induced by cosmological particle creation

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We study the primordial magnetic field generated by stochastic currents produced by scalar charged particles created at the beginning of the radiation dominated epoch. We find that, for the mass range 10^{-6} GeV$\lesssim m \lesssim 10^{2}$ GeV, a field of sufficient intensity to seed different mechanisms of galactic magnetic field generation, while still consistent with observational and theoretical constraints, is created coherently over a galactic scale. [S0556-2821(98)03412-2]

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I. INTRODUCTION

At present there exists huge observational evidence about the presence of magnetic fields throughout the Universe: our own galaxy is endowed with a homogeneous magnetic field $B \simeq 3\times 10^{-6}$ Ga and similar field intensity is detected in high redshift galaxies [1] and in damped Lyman alpha clouds [2].

The origin of these fields is still unclear. Research is performed mainly along the idea of a cosmological mechanism: a seed primordial field that would be further amplified by protogalactic collapse and differential rotation or a nonlinear dynamo [3]. Several mechanisms have been proposed to explain the origin of the seed field. It has been suggested that a primordial field may be produced during the inflationary period if conformal invariance is broken [4,5]. In string-inspired models, the coupling between the electromagnetic field and the dilaton breaks conformal invariance and may produce the seed field [6]. Gauge invariant couplings between the electromagnetic field and the space-time curvature also break conformal invariance but produce, in general, an uninterestingly small seed field [7]. Other mechanisms are based on a first order cosmological phase transition [8] and on the existence of topological defects [9].

In this paper we propose a new mechanism for primordial field generation in the early Universe, based on stochastic currents generated by particle creation of scalar charged species [10]. We assume the presence during inflation of a charged, minimally coupled, scalar field in its invariant vacuum state [11]. When the transition to a radiation dominated Universe takes place, quantum creation of charged particles occurs. We assume that the field mass is smaller than the vacuum energy density during inflation, H, and therefore can consider the transition from that period to radiation dominance as instantaneous.

As the number of positive charged species is the same as the number of negative charged ones (there is no physical reason why it should not be so), the mean electric current is zero. Nevertheless, quantum fluctuations around the mean give rise to a non-vanishing current. We compute the rms amplitude of these fluctuations and use it as the source term in the equation for the magnetic field. We must stress the fact that the field must be a scalar minimally coupled one: it is straightforward to check that with a massive conformally coupled scalar field very few particles are created and therefore a very weak magnetic field is created. For spinorial fields we show in the Appendix that, due to the conformal invariance of massless fermions, the number of particles created is very small and consequently the magnetic field produced is extremely weak.

II. CALCULATION OF THE MAGNETIC FIELD

As the process of magnetic field generation that we are studying takes place after inflation, there is no loss of generality if we work in a spatially flat Universe. In conformal time, $d\tau = dt/a(t)$, we have $g_{\mu\nu} = a^2(\tau)\eta_{\mu\nu}$, $\eta_{\mu\nu}$ being the Minkowskian metric tensor. The canonical scalar field, which we assume to be massive and minimally coupled, is written as $\varphi = \phi/a$.

Part of this process takes place before the electroweak transition. Then part of the created photons are a combination of the isospin and hypercharge bosons, where the coefficients are respectively the sine and cosine of the Weinberg angle θ_w [6,12]. We will consider the magnetic field generated by only the hypercharge sector and in this sense the figures obtained constitute a lower bound to the effective intensity of the field. The amplitude of the electromagnetic field would be smaller than the pure $U(1)$ boson by a factor of $\cos\theta_w$, but recalling that the electroweak coupling constant is $g = q/\cos\theta_w$, we obtain the same amplitude for the created electromagnetic field. The magnetic field is then defined from the spatial components of the field tensor, $B_i = (1/2)\varepsilon_{ijk}F_{jk}$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, A_μ being the vector potential in the Lorentz gauge $A^{\mu}_{;\mu} = 0$.

The equation for $\vec{B}$ reads

$$\left[\frac{\partial^2}{\partial\tau^2} - \nabla^2 + \sigma(\tau)\frac{\partial}{\partial\tau}\right]\vec{B} = \vec{\nabla}\times\vec{j} \tag{1}$$

where $\sigma(\tau)$ is a time-dependent conductivity of the plasma

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and $\vec{j}$ represents the spacelike components of the electric current which for the scalar field is $j_\mu = iq(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*) + 2q^2 A_\mu \phi^* \phi$.

The basic point in our analysis is that while the expectation value of the current vanishes, there are quantum and statistical fluctuations which build up a non-vanishing root mean square value. The source in these equations can be phenomenologically substituted by a stochastic, Gaussian current [13]. Our goal is to estimate the magnetic field produced by this stochastic source. Under the assumption that the source is Gaussian, its statistics are fixed once the two point source-source correlation function is given. Since the radiated field will be weak in any case, as a first approximation we can compute this correlation function in the absence of a macroscopic electromagnetic (e.m.) field.

As $\langle \vec{j} \rangle = 0$, the only contribution to the electric current will be due to the quantum fluctuations of the fields. In order to study if the quantum fluctuations of the massive fields produce a non-zero electric current we calculate the two point correlation function

$$N_{ii'}[(\tau,\vec{r}),(\tau',\vec{r}\,')] \equiv \langle \{ (\vec{\nabla}\times\vec{j})_i(\tau,\vec{r}), (\vec{\nabla}\times\vec{j})_{i'}(\tau',\vec{r}\,') \} \rangle. \tag{2}$$

The scalar field can be written in terms of two real fields according to $\phi(\tau,\vec{r}) = \{\phi_1(\tau,\vec{r}) + i\phi_2(\tau,\vec{r})\}/\sqrt{2}$ so that the spacelike part of the current j_μ, at vanishing e.m. field reads $\vec{j} = q\{\phi_2 \vec{\nabla}\phi_1 - \phi_1 \vec{\nabla}\phi_2\}$ and

$$\vec{\nabla}\times\vec{j} = 2q(\vec{\nabla}\phi_2)\times(\vec{\nabla}\phi_1). \tag{3}$$

If we consider that ϕ_1 and ϕ_2 commute, we can write

$$N_{ii'}[(\tau,\vec{r}),(\tau',\vec{r}\,')] = 4q^2 \epsilon_{ijl}\epsilon_{i'j'l'} \{ \partial^2_{jj'} G^+ \partial^2_{ll'} G^+ + \partial^2_{jj'} G^- \partial^2_{ll'} G^- \} \tag{4}$$

where we have introduced positive and negative frequency propagators $G^+ \equiv \langle \phi_i(\tau,\vec{r}) \phi_i(\tau',\vec{r}\,') \rangle$ and $G^- \equiv \langle \phi_i(\tau',\vec{r}\,') \phi_i(\tau,\vec{r}) \rangle$. Writing the propagators in terms of their Fourier components we get

$$N_{ii'}[(\tau,\vec{r}),(\tau',\vec{r}\,')] = 4q^2 \int \frac{d\vec{\kappa}\, d\vec{\kappa}'}{(2\pi)^3} (\vec{\kappa}\times\vec{\kappa}')_i (\vec{\kappa}\times\vec{\kappa}')_{i'} \times e^{i(\vec{\kappa}+\vec{\kappa}')\cdot(\vec{r}-\vec{r}\,')} G^+_\kappa(\tau,\tau') G^+_{\kappa'}(\tau,\tau') + (\tau,\vec{r} \leftrightarrow \tau',\vec{r}\,'). \tag{5}$$

Expanding the fields as $\phi = (2\pi)^{-3/2} \int d^3\kappa\, \phi_\kappa(\tau) e^{i\vec{\kappa}\cdot\vec{r}} + \text{H.c.}$ the Fourier transform of the propagators is given by $G^+_\kappa(\tau,\tau') = \phi_\kappa(\tau)\phi^*_\kappa(\tau') = G^-_\kappa(\tau',\tau)$.

Our next task is the evaluation of the scalar field modes. In the absence of electromagnetic fields, the scalar field satisfies the Klein-Gordon equation

$$\left[\frac{\partial^2}{\partial\tau^2} + \kappa^2 + m^2 a^2(\tau) - \frac{\ddot{a}(\tau)}{a(\tau)} \right] \phi_\kappa(\tau) = 0. \tag{6}$$

We assume instantaneous reheating at $\tau = 0$,

$$a(\tau) = \begin{cases} \dfrac{1}{H(\tau_0 - \tau)} & \text{inflation,} \\[2ex] \dfrac{1}{H\tau_0}\left(1 + \dfrac{\tau}{\tau_0}\right) & \text{radiation,} \end{cases} \tag{7}$$

and normalize the scale factor by taking $H\tau_0 = 1$. In terms of the dimensionless variables $y_0 = H\tau$ and $k = \kappa/H$ the modes in the inflationary epoch read

$$\phi^I_k(y_0) = \frac{\sqrt{\pi}}{2} \sqrt{1-y_0}\, H^{(1)}_\nu[k(1-y_0)], \quad \nu = \frac{3}{2}\sqrt{1 - \frac{4}{9}\frac{m^2}{H^2}}. \tag{8}$$

For the radiation dominated era we write

$$\phi^{WKB}_k(y_0) = \alpha_k f^0_k + \beta_k f^{0*}_k \tag{9}$$

where f^0_k is the WKB solution,

$$f^0_k(y_0) \sim \frac{e^{-i\Omega_k(y_0)}}{\sqrt{2\omega_k(y_0)}}$$

where $\omega_k(y_0) = \sqrt{k^2 + (m^2/H^2) a(y_0)^2}$, $\Omega_k(y_0) = \int^{y_0} dy'_0 \omega_k(y'_0)$. α_k and β_k are the Bogolubov coefficients connecting the modes (8) with the WKB basis (9) at $y_0 = 0$. In the limit $k \ll m/H \ll 1$ they are given by

$$\beta_k \simeq -\alpha_k \simeq -i \frac{\Gamma(3/2)}{4} \sqrt{\frac{\pi H}{m}} \frac{1}{k^{3/2}}. \tag{10}$$

The non-zero value of β_k reflects particle creation from the gravitational field, rather than from the decay of the inflaton field. Indeed the occupation number for long wavelength modes diverges as k^{-3}, much in excess of the k^{-1} Rayleigh-Jeans tail of the thermal spectrum produced by the reheating process. This excess noise results in an enhancement of the magnetic field over and above the equilibrium fluctuations. When the given scale enters the horizon, particle-antiparticle annihilation becomes efficient, and the extra noise disappears. It is worth mentioning that the logarithmic divergence in the total number of particles created up to $k_{\max}$ can be removed with a suitable infrared cutoff, say the mode corresponding to the horizon $\sim 10^{-26}$. The energy density associated with these particles, $\rho = mH^3 \int^{k_{\max}} d^3k |\beta_k|^2$, after a cutoff is imposed is $\sim 10^{-16}$ smaller than the one of the cosmic microwave background radiation (CMBR).

To give a quantitative estimate of the magnetic field amplitude, let us come back to the evaluation of the two point correlation function (2), which can be written as $N_{ii'} = N^0_{ii'} + N^1_{ii'} + N^2_{ii'}$. The first term is the noise that would be present in the absence of particle creation during reheating and does not concern us here. The other terms are the contribution due to the particles created at the beginning of the radiation era and are given by

$$N^1_{ii'}[(\tau,\vec{r}),(\tau',\vec{r}')]\to 4q^2H^8\int\frac{d\vec{k}d\vec{k}'}{(2\pi)^3}(\vec{k}\times\vec{k}')_i(\vec{k}\times\vec{k}')_{i'}e^{i(\vec{k}+\vec{k}')\cdot(\vec{y}-\vec{y}')}G^0_{1k}(y_0,y_0')\,\delta G_{1k'}(y_0,y_0') \tag{11}$$

$$N^2_{ii'}[(\tau,\vec{r}),(\tau',\vec{r}')]\to q^2H^8\int\frac{d\vec{k}d\vec{k}'}{(2\pi)^3}(\vec{k}\times\vec{k}')_i(\vec{k}\times\vec{k}')_{i'}e^{i(\vec{k}+\vec{k}')\cdot(\vec{y}-\vec{y}')}\delta G_{1k}(y_0,y_0')\,\delta G_{1k'}(y_0,y_0') \tag{12}$$

where

$$G^0_{1k}=G^{+0}_k+G^{-0}_k=\frac{\cos\Omega_k(y_0,y_0')}{\sqrt{\omega_k(y_0)\omega_k(y_0')}} \tag{13}$$

with $\Omega_\kappa(y_0,y_0')=\Omega_\kappa(y_0)-\Omega_\kappa(y_0')$,

$$\delta G_{1k}(y_0,y_0')=2\alpha_k\beta^*_k f^0_k(y_0)f^0_\kappa(y_0')+2\alpha^*_k\beta_k f^{0*}_k(y_0)f^{0*}_\kappa(y_0')+2|\beta_k|^2G^0_{1k}(y_0,y_0'). \tag{14}$$

The energy density of the magnetic field can be calculated from the two point correlation function generated by the stochastic current:

$$\langle B(\vec{x})B(\vec{x}')\rangle=H^4\int\frac{dy_0d\vec{y}}{(2\pi)^2}\frac{dy_0'd\vec{y}'}{(2\pi)^2}\times G^{ret}(y_0,\vec{y};x_0,\vec{x})G^{ret}(y_0',\vec{y}';x_0,\vec{x}')\times\{\overline{N}^1_{ii}[(y_0,\vec{y}),(y_0',\vec{y}')]+\overline{N}^2_{ii}[(y_0,\vec{y}),(y_0',\vec{y}')]\} \tag{15}$$

where G^{ret} are the causal propagators for the Maxwell field, obtained from the homogeneous solutions to Eq. (1). By Fourier transforming the causal propagators, the spatial integrals can be readily performed by virtue of the simple spatial dependence of the propagators. We have

$$\langle B(\vec{x})B(\vec{x}')\rangle=H^4\int dy_0dy_0'\int\frac{d\vec{k}d\vec{k}'}{(2\pi)^3}|\vec{k}\times\vec{k}'|^2\{4q^2\overline{G}^0_{1k}(y_0,y_0')\,\delta\overline{G}_{1k'}(y_0,y_0')+q^2\,\delta\overline{G}_{1k}(y_0,y_0')\,\delta\overline{G}_{1k'}(y_0,y_0')\}\times e^{i(\vec{k}+\vec{k}')\cdot(\vec{x}-\vec{x}')}G^{ret}_{|k+k'|}(y_0,x_0)G^{ret}_{|k+k'|}(y_0',x_0). \tag{16}$$

The energy density of the magnetic field coherent over a given dimensionless spatial scale λ is given by $E_\lambda=(1/a^4V^2_\lambda)\int d\vec{x}\int d\vec{x}'\langle B(\vec{x})B(\vec{x}')\rangle$ which amounts to inserting the window function $\mathcal{W}_{kk'}(\lambda)\equiv V_\lambda^{-1}\int_{V_\lambda}d\vec{x}e^{i(\vec{k}+\vec{k}')\cdot\vec{x}}$. The energy density reads $\rho_B=a^{-4}\langle B^2_\lambda\rangle$, where

$$\langle B^2_\lambda\rangle=H^4\int dy_0dy_0'\int\frac{d\vec{k}d\vec{k}'}{(2\pi)^3}\mathcal{W}^2_{kk'}(\lambda)|\vec{k}\times\vec{k}'|^2\times G^{ret}_{|k+k'|}(y_0,x_0)G^{ret}_{|k+k'|}(y_0',x_0)\times\{4q^2\overline{G}^0_{1k}(y_0,y_0')\,\delta\overline{G}_{1k'}(y_0,y_0')+q^2\,\delta\overline{G}_{1k}(y_0,y_0')\,\delta\overline{G}_{1k'}(y_0,y_0')\}. \tag{17}$$

As the window function satisfies $\mathcal{W}_{kk'}(\lambda)\approx 1$ for $|\vec{k}+\vec{k}'|\lambda\lesssim 1$ and $\mathcal{W}_{kk'}(\lambda)\approx 0$ for $|\vec{k}+\vec{k}'|\lambda\gtrsim 1$, we can take as the upper limit in the momentum integrals $k_{\max}\simeq 1/\lambda$ and $\mathcal{W}_{kk'}=1$ in the interval $(0,k_{\max})$. All the cosmological interesting scales are such that $k\ll 1$: For example for a galaxy we have that its physical scale today is (if it had not collapsed) $\lambda_G\simeq 1$ Mpc$\simeq 10^{38}$ GeV$^{-1}=\lambda_c T_{rh}/T_{today}\simeq 10^{28}\lambda_c$ ($T_{rh}\simeq 10^{15}$GeV is the temperature of the Universe at the end of reheating and $T_{today}\simeq 10^{-13}$ GeV is the present temperature of the microwave background). The comoving wavelength is $\lambda_c\simeq 10^{10}$ GeV^{-1}, taking $H=10^{11}$ GeV, the dimensionless scale $\lambda=H\lambda_c\simeq 10^{21}$ and the corresponding dimensionless momentum $k\simeq 10^{-21}$.

The retarded propagators are constructed with the homogeneous solutions of Eq. (1). If we take the conductivity (cf. Refs. [4,8]) $\sigma\simeq Te^{-2}=T_{rh}e^{-2}(1+y_0)^{-1}=\sigma_0\ (1+y_0)^{-1}$, $\sigma_0\simeq e^{-2}T_{rh}=e^{-2}\sqrt{Hm_{\rm Pl}}=He^{-2}\sqrt{m_{\rm Pl}/H}$, $\overline{\sigma}_0\equiv\sigma_0/H=e^{-2}\sqrt{m_{\rm Pl}/H}$ being the dimensionless conductivity, the homogeneous equation (1) reads (for scales larger than the horizon)

$$\frac{\partial^2}{\partial y_0^2}B_k+\frac{\overline{\sigma}_0}{(1+y_0)}\frac{\partial}{\partial y_0}B_k=0 \tag{18}$$

where we have used $B_k(y_0)=\int d^3ye^{-i\vec{k}\cdot\vec{y}}B(\vec{y},y_0)$. The causal propagator for this equation is given by

$$G^{ret}_{|k+k'|}=-\frac{1+y_0}{\overline{\sigma}_0-1}\left[1-\left(\frac{1+y_0}{1+x_0}\right)^{\overline{\sigma}_0-1}\right]. \tag{19}$$

If $x_0 \gg y_0$, the final expression for the propagator is $G^{ret}_{|k+k'|} \simeq (1+y_0)/\overline{\sigma}_0$.

In spite of the fact that the scalar field is not in thermal equilibrium with the background radiation, the electromagnetic interaction with it causes a correction to the value of the field mass, proportional to the temperature of the bath; i.e., we have $m^2 = m_0^2 + eT^2 = m_0^2 + eT_{rh}^2/(1+y_0)^2$. We will consider this correction in the frequencies $\omega_k(y_0)$ but not in the Bogolubov coefficients because we assume instantaneous particle creation. In fact, when the transition from inflation to radiation dominance occurs, large numbers of of particles are created very quickly by parametric resonance. Thermalization is a process that takes place afterwards and during a longer period of time [14]. As for the ratio m/H, if we consider, for example, the Higgs boson mass ($m \simeq 10^2$ GeV), we have $m/H \simeq 10^{-9}$. We can therefore neglect the terms k^2 in front of those m^2/H^2. When we perform the products $\overline{G}^0_{1k}(y_0,y_0')\delta\overline{G}_{1k'}(y_0,y_0')$ and $\delta\overline{G}_{1k}(y_0,y_0')\delta\overline{G}_{1k'}(y_0,y_0')$ in Eq. (17) we will have terms where the exponentials cancel and terms where this does not occur. The latter oscillate and hence can be neglected because they will give a smaller contribution than the first ones when the time integrations are performed. These integrations are performed between 0 and Υ, where Υ is the dimensionless time when a scale k^{-1} reenters the horizon; after this time, the current quickly relaxes to its equilibrium value through particle-antiparticle annihilation. From the expression of the Bogolubov coefficients, Eq. (10), we can see that the contribution from the term quartic in β_k overwhelms the quadratic one. The leading contribution to Eq. (17) reads

$$\langle B_\lambda^2 \rangle \sim \frac{4}{3} q^2 \left(\frac{H}{m_0}\right)^6 H^4 k_{\max}^4 \left(\frac{1}{\overline{\sigma}_0} \sqrt{\frac{m_0^2(1+y_0)^2}{H^2} + \frac{eT_{rh}^2}{H^2}} \Bigg|_0^{\Upsilon} \right)^2. \tag{20}$$

For the scale considered, $k_{\max} \simeq 10^{-21}$, for the electroweak mass scale, $m_0 \simeq 10^2$ GeV, $\overline{\sigma}_0 \simeq e^{-2} 10^4$, $T_{rh} \simeq 10^{15}$ GeV, $q^2 = e^2 = 1/137$ and recalling $\Upsilon \sim \pi/(2k_{\max})$ we first note that, by simply replacing these figures in Eq. (20), the term proportional to $m_0^2(1+y_0)^2$ overwhelms the one proportional to T_{rh}^2 in the square root by a factor of $\sim 10^{20}$. Therefore we can take

$$\langle B_\lambda^2 \rangle \sim \frac{4}{3} q^2 \left(\frac{H}{m_0}\right)^6 H^4 k_{\max}^4 \frac{1}{\overline{\sigma}_0^2} \frac{m_0^2 \Upsilon^2}{H^2} = \frac{4}{3} q^2 \left(\frac{H}{m_0}\right)^4 H^4 k_{\max}^2 \frac{1}{\overline{\sigma}_0^2} \tag{21}$$

$$\langle B_\lambda^2 \rangle \sim 10^{24} \ \text{GeV}^4 \rightarrow r \equiv \frac{\rho_B}{\rho_{bck}} \sim 10^{-36} \tag{22}$$

where $\rho_{bck} = T_{rh}^4$ is the energy density of the CMBR. These values correspond to a comoving field of strength $B_\lambda \sim 10^{32}$ G. The physical field $B_{ph} = a^{-2} B_\lambda$, such that $\rho_B \equiv B_{ph}^2$, gives a present value of

$$B_{ph} \sim 10^{-24} \ \text{G}. \tag{23}$$

This value satisfies the constraints imposed by the anisotropy in the microwave background [15] and big bang nucleosynthesis [16] and (marginally) suffices to seed the galactic dynamo [3,8,17]. Stronger fields are obtained by diminishing the value of the field mass. We can estimate the lower bound to the mass by demanding that today $B_{ph} \sim 10^{-9}$ G, according to the mentioned constraints. We obtain $m \gtrsim 10^{-6}$ GeV. Considering the value of the mass used throughout the paper as the upper bound we find the range 10^{-6} GeV $\lesssim m \lesssim 10^2$ GeV. Fields outside of this range will contradict observational and/or theoretical constraints.

III. CONCLUSIONS

To conclude, we have presented a new mechanism for the generation of a primordial magnetic field, based on the breaking of conformal invariance. In previous works [4–6], conformal invariance was broken in order to amplify quantum fluctuations of $\vec{B}$. The amplification was in general very small. Here scalar, massive charged particles minimally coupled to gravity and coupled to the electromagnetic field produce stochastic fluctuations in the source of Maxwell equations which in turn generate an astrophysically relevant primordial field. For conformally coupled massive scalar field, the Bogolubov coefficient scale with the momenta as $k^{-1/2}$ which means that the number of created particles is very small and consequently the magnetic field generated will be too weak to be astrophysically interesting ($\sim 10^{-80}$ times smaller than in the minimally coupled case, for the physical parameters used in this paper). The fact that spinor fields are conformal invariant if massless manifests itself in that the proposed mechanism does not generate strong enough magnetic fields, as is shown in the Appendix. Finally we must say that the value of the field quoted in Eq. (23) is to be considered as a rough estimate; a more precise evaluation will require a more detailed consideration of the whole electroweak gauge theory and of the non-equilibrium evolution of current and field.

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APPENDIX

Here we show that the magnetic field generated by an electric current originated by the creation of fermionic particles is indeed negligible. The Dirac equation for spinors in a Friedman-Robertson-Walker (FRW) universe reads (in conformal time) [18,19]

$$[i\gamma^\mu \partial_\mu - ma(y_0)]\chi(y_0,\vec{y}) = 0 \tag{A1}$$

where $\chi(y_0,\vec{y}) = a^{3/2}(y_0)\psi(y_0,\vec{y})$. y_0 and $\vec{y}$ are the dimensionless time and space coordinates defined in the paper and in this Appendix m is to be understood as a dimensionless mass, i.e. $m \rightarrow m/H$. The electric current reads

$$j^\mu = e\overline{\chi}(y_0,\vec{y})\gamma^\mu \chi(y_0,\vec{y})$$

where $\overline{\chi}(y_0,\vec{y}) = \chi^\dagger(y_0,\vec{y})\gamma^0$, i.e. the Dirac adjoint. The positive and negative energy spinors u_{ks} and v_{ks} read

$$u_{ks} = \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{k}}{k^2}\left(ma(y_0) + i\frac{\dot{f}_k(y_0)}{f_k(y_0)}\right)C_s \\ C_s \end{pmatrix} f_k(y_0)e^{i\vec{k}\cdot\vec{y}} \tag{A2}$$

$$v_{ks} = \begin{pmatrix} C_s \\ -\frac{\vec{\sigma}\cdot\vec{k}}{k^2}\left(ma(y_0) - i\frac{\dot{f}^*_k(y_0)}{f^*_k(y_0)}\right)C_s \end{pmatrix} f^*_k(y_0)e^{-i\vec{k}\cdot\vec{y}} \tag{A3}$$

where $f_k(y_0)$ is a solution to the equation

$$\left[\frac{\partial^2}{\partial y_0^2} + k^2 + m^2a^2(y_0) - im\frac{\partial a(y_0)}{\partial y_0}\right]f_k = 0. \tag{A4}$$

The spinors are normalized according to the (time independent) product $\int_\Sigma d\Sigma \overline{\chi}\gamma^0\chi$, Σ being a spacelike hypersurface.

The noise kernel (2) due to this field reads (after a rather long calculation)

$$N_{ii}(y_0,y_0') = 4q^2\int d\vec{k}\int d\vec{p}\frac{f_k(y_0')f_p(y_0')f^*_k(y_0)f^*_p(y_0)}{N_kN_pN_kN_p} \times\{\mathcal{M}^{(1)}_{k,p}(y_0,y_0') + \mathcal{M}^{(2)}_{k,p}(y_0,y_0') + |\vec{k}+\vec{p}|^2\} \tag{A5}$$

where

$$\mathcal{M}^{(1)}_{k,p}(y_0,y_0') = \frac{|\vec{k}+\vec{p}|^2}{k^2p^2}\left[ma(y_0') + i\frac{\dot{f}_k(y_0')}{f_k(y_0')}\right]\left[ma(y_0') + i\frac{\dot{f}_p(y_0')}{f_p(y_0')}\right]\left[ma(y_0) - i\frac{\dot{f}^*_k(y_0)}{f^*_k(y_0)}\right]\left[ma(y_0) - i\frac{\dot{f}^*_p(y_0)}{f^*_p(y_0)}\right] \tag{A6}$$

$$\mathcal{M}^{(2)}_{k,p}(y_0,y_0') = \frac{[\vec{k}\cdot(\vec{k}+\vec{p})][(\vec{k}+\vec{p})\cdot\vec{p}]}{k^2p^2} \times\left\{\left[ma(y_0') + i\frac{\dot{f}_k(y_0')}{f_k(y_0')}\right]\left[ma(y_0') + i\frac{\dot{f}_p(y_0')}{f_p(y_0')}\right] + \left[ma(y_0) - i\frac{\dot{f}^*_k(y_0)}{f^*_k(y_0)}\right]\left[ma(y_0) - i\frac{\dot{f}^*_p(y_0)}{f^*_p(y_0)}\right]\right\}. \tag{A7}$$

$N_{k,p}$ are the normalization factors which we calculate for inflation. The solution to the field equation (A4) for the inflationary period that corresponds to positive frequency for $y_0\to-\infty$ reads

$$f_k(y_0) = (1-y_0)^{1/2}H^{(1)}_\nu[k(1-y_0)], \quad \nu = \frac{1}{2} + im \tag{A8}$$

and therefore

$$N_k = N_k = \frac{2}{\sqrt{\pi}}\frac{1}{k^{1/2}}. \tag{A9}$$

For the radiation dominated period and for $k/m \ll 1$ we write the mode functions as [20]

$$f_k = \alpha_k e^{im(1+y_0)^2/2} + \beta_k e^{im(1+y_0)^2/2}\int_{z(y_0)}^\infty e^{-s^2/2}ds \tag{A10}$$

where $z(y_0) = (1+i)\sqrt{m}(1+y_0)$ and α_k and β_k are the Bogolubov coefficients obtained by matching the modes and their time derivatives at $y_0 = 0$. In the limit $k/m \ll 1$ they read

$$\alpha_k = -\frac{i}{\pi}\Gamma\left(\frac{1}{2} + im\right)e^{im/2}\left(\frac{2}{k}\right)^{1/2+im} \tag{A11}$$

$$\beta_k = -\frac{i}{2\pi}\Gamma\left(-\frac{1}{2} + im\right)e^{im/2}\left(\frac{2}{k}\right)^{1/2+im}k^2. \tag{A12}$$

We see that in this case there is no divergence in the number of quanta created and therefore the expected electric current will be very small. We replace the modes (A10) in Eqs. (A6) and (A7) and with the obtained expressions we evaluate

$$\langle B^2\rangle = H^4\int\frac{dy_0}{(2\pi)^2}\frac{dy_0'}{(2\pi)^2} \times G^{ret}(y_0,x_0)G^{ret}(y_0',x_0)N_{ii}(y_0,y_0')\mathcal{W}^2_{kp}(\lambda) \tag{A13}$$

with the same considerations that were made for the scalar field. The main contribution comes from the first two terms and they are respectively

$$\langle B^2\rangle^{(1)} \simeq q^2H^4\frac{8e^{2im(1+T_{\max})^2}}{m^2\overline{\sigma}_0^2\left[\frac{1}{4} + m^2\right]^2\cosh^2[\pi m]}k^{10}_{\max} \tag{A14}$$

$$\langle B^2\rangle^{(2)} \simeq q^2H^4\frac{64}{m^2\overline{\sigma}_0^2\left[\frac{1}{4} + m^2\right]\cosh^2[\pi m]}k^8_{\max}. \tag{A15}$$

For example, we have that for a mass $m = 10^{-11}$ (i.e. physical mass ~ 1 GeV in units of $H = 10^{11}$ GeV), $k_{\max}$ and $\overline{\sigma}_0$ the same as the ones used in the paper, $\langle B^2\rangle^{(1)} \sim 10^{-160}$ GeV^4 and $\langle B^2\rangle^{(2)} \sim 10^{-116}$ GeV^4, values completely negligible in comparison with the one quoted in Eq. (22).

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