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Do experiments suggest a hierarchy problem?

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The hierarchy problem of the scalar sector of the standard model is reformulated, emphasizing the role of experimental facts that may suggest the existence of a new physics large mass scale, for instance, indications of the instability of matter or indications in favor of massive neutrinos. In the seesaw model for the neutrino masses a hierarchy problem arises if the mass of the right-handed neutrinos is larger than approximately 10^7 GeV: this problem, and its possible solutions, are discussed. [S0556-2821(98)01611-7]

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We speak of a hierarchy problem when two largely different energy scales are present in the theory, but there is no symmetry that stabilizes the light scale from corrections coming from the large scale [1].

This problem is commonly invoked to argue against the simple structure of the Higgs potential of the standard model, since the massive parameter μ^2 appearing as $-\mu^2|H|^2$ in the potential (the light scale) can, in principle, receive corrections from any larger scale. To which kind of mass does the problem pertains? It can be formulated in terms of the renormalized mass, let us say in the modified minimal subtraction ($\overline{\text{MS}}$) scheme, noticing that at external momenta above a heavy threshold scale M_{heavy} the parameter μ^2 will acquire loop contributions of the order of M^2_{heavy} times the coupling of the heavy particle. In this case, the renormalization group flow in the standard model is unnatural in the sense that the initial conditions at some large scales have to be extremely fine-tuned to reproduce a Higgs boson mass below the TeV scale, if the coupling of the Higgs particle with the particle of mass M_{heavy} is not very small. From another point of view, it was remarked that the bare scalar mass receives quadratic corrections, if the theory is regulated with a cutoff in the momenta [2]. This aspect may be considered less relevant, since the standard model is a renormalizable theory, and there is no way to give sense to bare parameters in this context; the cutoff can be thought of as a technical device, and in a last analysis, other regulators can be chosen.

Note that to speak of a ‘‘problem’’ one is taking a theoretical point of view: One does not like to assume, without motivation, that a hypothetical fundamental theory that should explain the observed quantities and the various parameters of the standard model should be forced to have a fine-tuning such as the one discussed above. This principle can be used to select possible extensions of the standard model, after having stated a quantitative criterion of naturalness (such a program was formulated in [3]).

Once this principle is accepted, the discussion about its actual relevance is reduced to two experimental terms. The first is, if a fundamental Higgs particle exists. Assuming that it exists, we face the other aspect: before speaking of a hierarchy problem, one has to understand if there are signals of physics beyond the standard model, that, in turn, point to the existence of larger energy scales.

We will not rely on the Planck mass scale in the following discussion, since in our opinion the formulation of a quantum theory of the gravity is in a preliminary stage, and the experimental perspectives are unclear [4]. Instead we want to discuss the relevance of signals of violations of the global symmetries of the standard model, the baryon and the lepton numbers B and L, paying attention to the experimental perspectives that we can foresee at present.

Let us start discussing possible signals of matter instability. If discovered, they would strongly suggest the existence of a large mass scale, most probably related to a deeper layer of gauge unification (the alternative hypothesis of light mediators of matter instability, very weakly coupled with the matter, should be seriously considered if nucleon decay modes that do not conserve $B-L$, for instance those which conserve $B+L$, would be positively observed [5,6]). Suppose that proton decay signals would be within reach, say at Superkamiokande. To be concrete, let us imagine the case in which the decay channels involving strange mesons are the dominating ones. This may indicate that the physics responsible for the proton decay and the origin of the (family hierarchical) fermion masses is the same. Assuming that the couplings involved in the decay are of the order of a typical Yukawa coupling $m_s/v\approx 10^{-3}$, a sufficient suppression of the nucleon lifetime can be obtained only if the mass of the mediator M_X is close to 10^{12} GeV (we assumed: $\Gamma_p\sim M_X^{-4}$). Therefore, μ^2 receives the contribution $\delta\mu^2\approx y^2M_X^2/(4\pi)^2$ that is much larger than $1\ \text{TeV}^2$, unless the effective coupling y of the light Higgs with the heavy particle is very small, approximatively $y<10^{-8}$. It is easy to understand that for a typical theoretical scheme (in which y can appear at one-loop or even at tree level) the contributions to $\delta\mu^2$ can be very large. In conclusion, this scenario would probably make us reflect about the hierarchy problem and its solution.

It is remarkable that the supersymmetric extensions of the standard model, with masses of the supersymmetric particles around the electroweak scale, are able to offer a way out from the hierarchy problem due to the nonrenormalization theorem [7] and at the same time are compatible with the hypothesis of a minimal SU(5) unification group structure at an energy scale around 2×10^{16} GeV [8,9]. This may be regarded as *the* solution [10], but in the present stage of development it is not clear if a gauge hierarchy problem has to be addressed, since no signal of matter instability has been found yet. In this connection, it is important to remark that

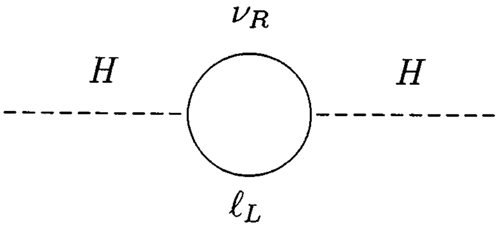

FIG. 1. The Feynman diagram originating the corrections in Eq. (1); ν_R denotes the right-handed neutrino of mass M_R, $\ell_L = (\nu_L, e_L)$ the leptonic and H the Higgs doublets.

supersymmetric grand unified models that predict that nucleon decay signal may be within reach (in the near future) have been indeed proposed [11]. However, one should not forget that some supersymmetric grand unified models can already be excluded by present experimental information on matter stability [12] or, on the extreme opposite, that some models entail an essentially stable nucleon [13]. Even if somewhat disappointing, it may be fair to say that this is due to the fact that the ‘‘supersymmetric grand unification’’ is still not a completely defined program. Coming back to the main focus of the present work, we conclude that (despite the theoretical promises) the experimental studies of matter stability do not permit us at present to infer the existence of a hierarchy problem.

There is, however, an independent way of arguing a hierarchy problem in certain extensions of the standard model. This argumentation is based on the presence of nonzero neutrino masses, that could imply the solution of long standing problems with solar neutrino flux, and may be confirmed by the next round of experiments.

It is, in principle, also possible that the neutrino masses are related to a new gauge structure manifesting itself at higher scales; if this is true, we would again face a gauge hierarchy problem [14]. However, we want to be conservative in the assumptions. So, instead of jumping to conclusions, we address the question: What can we learn, using the indications of nonzero neutrino masses, about the structure of the theory that should extend the standard model?

Let us consider the seesaw model for neutrino masses [15]. The heavy right-handed neutrinos, with mass M_R, couple with the Yukawa coupling y_ν to the left-handed neutrinos, and give them a mass $m_\nu = (y_\nu v)^2/M_R$ ($v = 174$ GeV). In nonsupersymmetric theories the renormalized mass μ^2 will receive corrections of the order of

$$\delta\mu^2 \approx \frac{y_\nu^2}{(2\pi)^2} M_R^2 \ln(q/M_R), \tag{1}$$

for momenta q larger than M_R (see Fig. 1). We can rewrite these corrections as

$$\delta\mu^2 \approx \frac{m_\nu M_R^3}{(2\pi v)^2} \ln(q/M_R). \tag{2}$$

Equation (2) points to the hierarchy problem that is inherent to the seesaw models for neutrino masses.

Let us specify Eq. (2) in two concrete cases, considering neutrinos that may be relevant to the solution of the solar neutrino problem and may serve as the hot dark matter (HDM) candidate, respectively. Assuming small mixing, the contribution to μ^2 will not exceed 1 TeV^2 if the following upper bounds hold true:

$$m_\nu(\text{solar}) = 3\times 10^{-3}\ \text{eV} \Rightarrow M_R \lesssim 7.4\times 10^{7}\ \text{GeV},$$

$$m_\nu(\text{HDM}) = 6\ \text{eV} \Rightarrow M_R \lesssim 5.8\times 10^{6}\ \text{GeV}. \tag{3}$$

In the previous estimation we assumed the logarithm of order unity (in other terms, we used the criterion of naturalness: $d\mu^2/d \ln q \lesssim 1$ TeV^2). Let us stress that the figures in Eq. (3) should be taken as indicative, since we assumed that the mixing angles and the phases in the lepton matrices are small; their presence can modify to a certain extent the relation between the masses of light and heavy neutrinos. However, for given values of the left- and right-handed neutrino masses, the radiative contribution to μ^2 tends to increase in the presence of mixing and phases.

Under the same assumptions, the conditions (3) on M_R are equivalent to upper bounds on the Yukawa couplings:

$$m_\nu(\text{solar}) = 3\times 10^{-3}\ \text{eV} \Rightarrow y_\nu \lesssim 8.5\times 10^{-5},$$

$$m_\nu(\text{HDM}) = 6\ \text{eV} \Rightarrow y_\nu \lesssim 1.1\times 10^{-3}. \tag{4}$$

For comparison, note that if $M_R \approx 1$ TeV (of interest for search at accelerators) the Yukawa couplings are $y_\nu \approx 3.1\times 10^{-7}$, 1.4×10^{-5} in the two cases considered.

Therefore, to be able to assess the presence of a hierarchy problem, we still lack the information on the scale of the Majorana neutrinos M_R, or on the size of the Yukawa couplings. A recent discussion [16] on the structure of the right-handed mass matrix in the seesaw model suggests masses larger than those in Eq. (3). Notice however, that the underlying assumption is the unification of the Yukawa couplings of the neutrinos and of the up-type quarks; for smaller neutrino Yukawa couplings, lighter M_R's are needed. For instance, this is what happens if neutrinos are Dirac particles, that is, when $M_R \ll m_\nu$ (and there is no direct Majorana mass term); the neutrino mass reduces to $y_\nu v$, and the Yukawa couplings are very small ($y_\nu = 1.7\times 10^{-14}$ for solar neutrinos and $y_\nu = 3.4\times 10^{-11}$ for HDM component neutrinos).

We can obtain interesting information on the Yukawa couplings assuming the Fukugita-Yanagida scenario for baryogenesis [17] (see also [18–20]). In this scenario the decay of the lightest right-handed neutrino, of mass M_{Rl}, originates a lepton asymmetry that, in a second stage, can be converted in the presently observed baryon asymmetry. This scenario can be realized if the Yukawa couplings provide sufficient mixing with a heavier neutrino of mass M_{Rh}:

$$\frac{M_{Rl}}{M_{Rh}} \frac{\text{Im}[(Y_\nu{}^\dagger Y_\nu)_{hl}^2]}{(Y_\nu{}^\dagger Y_\nu)_{ll}} \approx 10^{-5}, \tag{5}$$

in the case of *hierarchical* masses of right-handed neutrinos, as discussed in [19]. Considering the inequality: $|(Y_\nu{}^\dagger Y_\nu)_{hl}|^2 \leqslant (Y_\nu{}^\dagger Y_\nu)_{hh} (Y_\nu{}^\dagger Y_\nu)_{ll}$, that follows from the non-negativity of the matrix $Y_\nu{}^\dagger Y_\nu$, we obtain

$$10^{-5} \lesssim (Y_\nu{}^\dagger Y_\nu)_{hh}. \tag{6}$$

Comparing with Eq. (4), we come to the conclusion that the corrections to μ^2 exceed TeV^2; in other terms, Eq. (6) suggests the vicinity of a hierarchy problem.

This conclusion is related to a conjectural mechanism for baryogenesis, that, however, is quite natural once the existence of right-handed neutrinos has been assumed. For this reason, it is of interest to search for a loophole in the above argument. Therefore let us abandon the hypothesis of hierarchical right-handed neutrinos, and contemplate the case in which these particles are nearly degenerate; it turns out that the estimation (5) is no longer correct. In fact, the lepton asymmetry produced in the decay is dominated by the ''wave function'' contribution [19,20], that increases for smaller mass splitting, and eventually reaches its maximum when the splitting is comparable to the decay widths of the right-handed neutrinos [20]. This makes it possible to reproduce the observed baryon number with smaller Yukawa couplings rather than with those implied by Eq. (6), and gives a possibility to avoid the hierarchy problem in the minimal framework we are considering. We will not address the question of the theoretical likelihood of this very constrained scenario for neutrino masses. However, it is important to stress again that even in this framework the right-handed neutrinos would be relatively light [Eq. (3)].

Finally, we discuss possible solutions of the hierarchy problem that arises if the seesaw model is the true theory of the neutrino masses, and the right-handed masses are large in comparison with Eq. (3) [as suggested by Eq. (6), modulo the *caveats* above]. In this case, one could advocate for supersymmetry at low energy on the basis of the criterion of naturalness. We recall the argument: The quadratic corrections to the massive parameters of the Higgs potential entail in supersymmetric theories $M_R^2 - \widetilde{M}_R^2$, the mass splitting of the right-handed neutrinos and their scalar partners instead of M_R^2 [compare with Eq. (1)]; the natural expectation is that $M_R^2 - \widetilde{M}_R^2 \lesssim 1$ TeV^2, due, for instance, to a relation of this mass splitting and the splitting between the charged leptons and their scalar counterparts. As a result the presence of the large mass scale $M_R^2 \gg 1$ TeV^2 does not imply any hierarchy problem.

In this supersymmetric context, we remark that the mass splitting $M_R^2 - \widetilde{M}_R^2$ could affect, via one-loop corrections, the value of the lightest Higgs mass, in close analogy with what happens due to the top-quark–top-squark corrections [21]. In fact, these loop corrections are of the same nature of the corrections to μ^2 discussed in Eq. (1).

Of course, the argument for supersymmetry is far reaching, and does not apply only to the seesaw model. In fact, once the low-energy supersymmetry hypothesis is accepted, the light scales are ''protected'' against the presence of the heavy scales, and the theoretical speculations involving very high-energy scales do not meet these types of problems [22]. The importance of the remarks above rests in the consideration that the strongest indications in favor of physics beyond the standard model come from neutrino physics.

If the model of the neutrino masses is not the seesaw model we have other possibilities to elude the hierarchy problem: We can assume that the scale, at which the neutrino masses are generated, is not far from the electroweak one. This can happen in the models in which the smallness of the neutrino masses is related to loop effects [23]. Even in the context of minimal supersymmetric models (in particular without right-handed neutrinos) other mechanisms for the generation of the neutrino masses are possible. We are referring to the R-parity breaking models, in which *a priori* large violations of the lepton number may be present [24,25]. Again, the crucial remark is that in these models no large scale (besides the scale of the supersymmetric particles) is present. Can we distinguish this possibility? If the neutrino masses originate in these kinds of models, the expectation is that other signals of R-parity breaking should show up [26].

To summarize, massive neutrinos point to a hierarchy problem in possible extensions of the standard model, independently from the assumption of grand unification. We discussed how this remark may result in an argument in favor of certain theoretical models.

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