# Neutrino oscillation constraints on neutrinoless double-beta decay

S. M. Bilenky

Joint Institute for Nuclear Research, Dubna, Russia and Institute for Advanced Study, Princeton, New Jersey 08540

C. Giunti

INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Università di Torino, Via P. Giuria 1, I-10125 Torino, Italy

C. W. Kim

Department of Physics and Astronomy, The Johns Hopkins University, Baltimore, Maryland 21218 and School of Physics, Korea Institute for Advanced Study, Seoul 130-012, Korea

M. Monteno

INFN, Sezione di Torino, and Dipartimento di Fisica Sperimentale, Università di Torino, Via P. Giuria 1, I–10125 Torino, Italy (Received 21 November 1997; published 7 May 1998)

We have studied the constraints imposed by the results of neutrino oscillation experiments on the effective Majorana mass  $|\langle m \rangle|$  that characterizes the contribution of Majorana neutrino masses to the matrix element of neutrinoless double-beta decay. We have shown that in a general scheme with three Majorana neutrinos and a hierarchy of neutrino masses (which corresponds to the standard seesaw mechanism) the results of neutrino oscillation experiments imply rather strong constraints on the parameter  $|\langle m \rangle|$ . From the results of the first reactor long-baseline experiment CHOOZ and the Bugey experiment it follows that  $|\langle m \rangle| \leq 3 \times 10^{-2}$  eV if  $\Delta m^2 \leq 2$  eV<sup>2</sup> ( $\Delta m^2$  is the largest mass-squared difference). Hence, we conclude that the observation of neutrinoless double-beta decay with a probability that corresponds to  $|\langle m \rangle| \approx 10^{-1}$  eV would be a signal for a nonhierarchical neutrino mass spectrum and/or nonstandard mechanisms of lepton number violation. [S0556-2821(98)04811-5]

PACS number(s): 14.60.Pq, 23.40.-s

### I. INTRODUCTION

The investigation of the fundamental properties of neutrinos [neutrino masses and neutrino mixing, the nature of massive neutrinos (Dirac or Majorana?), neutrino magnetic moments, etc.] is the most important problem of today's neutrino physics. This investigation is one of the major directions of the search for physics beyond the standard model.

At present, there are three experimental indications in favor of neutrino oscillations. The first indication comes from solar neutrino experiments (Homestake [1], Kamiokande [2], GALLEX [3], SAGE [4], and Super-Kamiokande [5]). The second indication was found in the Kamiokande [6], IMB [7], Soudan [8], and Super-Kamiokande [9] atmospheric neutrino experiments. The third indication in favor of neutrino oscillations was obtained by the Liquid Scintillation Neutrino Detector (LSND) Collaboration [10,11]. On the other hand, in many short-baseline (SBL) reactor and accelerator experiments (see the reviews in Ref. [12]) and in the recent long-baseline (LBL) reactor experiment CHOOZ [13] no indications in favor of neutrino oscillations were found.

Neutrino oscillation experiments cannot provide an answer to the question: what type of particles are massive neutrinos, Dirac or Majorana? (see Ref. [14]). The answer to this question, which is of fundamental importance, could be obtained from experiments on the investigation of processes in which the total lepton number  $L=L_e+L_{\mu}+L_{\tau}$  is not conserved. The classical process of this type is neutrinoless double- $\beta$  decay  $[(\beta\beta)_{0\nu}]$ 

$$(A,Z) \to (A,Z+2) + e^{-} + e^{-}.$$
 (1.1)

The neutrinoless double- $\beta$  decay of different nuclei has been searched for in many experiments (see, for example, Ref. [15]). No positive signal was found up to now. The most stringent limits on the half-lives for  $(\beta\beta)_{0\nu}$  decay were found in <sup>76</sup>Ge and <sup>136</sup>Xe experiments. In the experiments of the Heidelberg-Moscow [16,17] and Caltech-Neuchatel-PSI [18] Collaborations it was found that

$$T_{1/2}(^{76}\text{Ge}) > 1.2 \times 10^{25} \text{ y} \quad (90\% \text{ C.L.})$$
  
Heidelberg-Moscow, (1.2)

$$T_{1/2}(^{136}\text{Xe}) > 4.2 \times 10^{23} \text{ y} \quad (90\% \text{ C.L.})$$
  
Caltech-Neuchatel-PSI. (1.3)

The standard mechanism of  $(\beta\beta)_{0\nu}$  decay is the mechanism of the mixing of neutrinos with Majorana masses. In accordance with the hypothesis of neutrino mixing (see Refs. [19–21]), the left-handed flavor neutrino fields  $\nu_{\ell L}$  are combinations of fields of neutrinos with definite masses:

$$\nu_{\ell L} = \sum_{i} U_{\ell i} \nu_{iL} \quad (\ell = e, \mu, \tau), \tag{1.4}$$

where  $\nu_i$  is the field of neutrinos with mass  $m_i$  and U is the unitary mixing matrix. If massive neutrinos are Majorana particles, the fields  $\nu_i$  satisfy the Majorana condition  $\nu_i$ 

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 $=\nu_i^c \equiv C \bar{\nu}_i^T$  (*C* is the charge conjugation matrix), the total lepton number is not conserved, and  $(\beta\beta)_{0\nu}$  decay is possible. In the framework of neutrino mixing the process (1.1) is a process of the second order in the charged current (CC) weak interaction Hamiltonian

$$\mathcal{H}_{I} = \frac{G_{F}}{\sqrt{2}} 2 \sum_{\ell=e,\mu,\tau} \overline{\ell}_{L} \gamma^{\alpha} \nu_{\ell L} j_{\alpha}^{\mathrm{CC}} + \mathrm{H.c.}, \qquad (1.5)$$

with a virtual neutrino. In Eq. (1.5)  $G_F$  is the Fermi constant and  $j_{\alpha}^{CC}$  is the standard hadronic charged current. The matrix element of  $(\beta\beta)_{0\nu}$  decay is proportional to the effective Majorana neutrino mass (see, for example, Refs. [19–21]):

$$\langle m \rangle = \sum_{i} U_{ei}^{2} m_{i} \,. \tag{1.6}$$

The negative results of the experiments searching for  $(\beta\beta)_{0\nu}$  decay imply upper bounds for the parameter  $|\langle m \rangle|$ . The numerical values of the upper bounds depend on the model that is used for the calculation of the nuclear matrix elements. From the results of the <sup>76</sup>Ge and <sup>136</sup>Xe experiments the following upper bounds were obtained:

$$|\langle m \rangle| < (0.5 - 1.1) \text{ eV} (^{76}\text{Ge}[16, 17, 22]), (1.7)$$

$$|\langle m \rangle| < (2.3 - 2.7) \text{ eV} (^{136}\text{Xe}[18]).$$
 (1.8)

Significant progress in the search of neutrinoless double- $\beta$  decay is expected in the future. Several collaborations are planning to reach a sensitivity of 0.1–0.3 eV for  $|\langle m \rangle|$  [16,17,23].

Contributions to the matrix element of  $(\beta\beta)_{0\nu}$  decay of different nonstandard mechanisms for violation of the lepton number (right-handed currents [21,24,25], supersymmetry with violation of *R* parity [24,26–28], and others [29,30]) have recently been considered in the literature. At present, it is not possible to distinguish different mechanisms. It is obvious that it is important to obtain independent information about the contribution to the matrix element of the  $(\beta\beta)_{0\nu}$  decay of Majorana neutrino masses and mixing, given by the effective Majorana neutrino mass  $|\langle m \rangle|$ .

In this paper, we will show that the existing neutrino oscillation data imply rather strong constraints on the effective Majorana mass  $|\langle m \rangle|$  under the general assumption of a neutrino mass hierarchy. The first estimates of the parameter  $|\langle m \rangle|$  obtained from the data of SBL reactor experiments were given in Ref. [31] and a more detailed analysis, including the results of the Krasnoyarsk [32] and Bugey [33] experiments and the first results of the LSND experiment [10] was presented in Ref. [34]. Since these analyses have been carried out, new results of the LSND experiment have been published [11] and the results of the first LBL reactor experiment CHOOZ appeared [13]. We will use all these data and the results of the Kamiokande [6] and Super-Kamiokande [9] atmospheric neutrino experiments in order to obtain new bounds on the effective Majorana mass  $|\langle m \rangle|$ . In Secs. III and IV we will see that these data imply rather strong constraints on this parameter. In Sec. V we present some remarks on nonhierarchical neutrino mass spectra.

# **II. THREE NEUTRINOS WITH A MASS HIERARCHY**

The results of the experiments at the CERN  $e^+e^-$  collider LEP on the measurement of the invisible width of the Z boson imply that only three flavor neutrinos exist in nature (see Ref. [35]). The number of light massive Majorana neutrinos is equal to 3 in the case of a left-handed Majorana mass term and can be more than 3 in the general case of a Dirac and Majorana mass term (see, for example, Refs. [19– 21]). Let us notice that the result of LEP measurements does not exclude this last possibility. If the number of light massive Majorana neutrinos is more than 3, sterile neutrinos must exist. The sterile fields do not enter in the standard neutral current and their effect is not seen in LEP experiments.

We will consider here the simplest case of three light Majorana neutrinos.<sup>1</sup> As is well known, a general characteristic feature of the mass spectra of leptons and quarks is the hierarchy of the masses of the particles of different generations. What about neutrinos? Different possibilities for the mass spectrum of three neutrinos were considered in the literature (see Refs. [34,36–38]). We assume that the neutrino masses  $m_1$ ,  $m_2$ ,  $m_3$ , as the masses of quarks and leptons, satisfy the hierarchy<sup>2</sup>

$$m_1 \ll m_2 \ll m_3$$
. (2.1)

Such a spectrum corresponds to the seesaw mechanism for neutrino mass generation [39] which is the only known mechanism that explains naturally the smallness of neutrino masses with respect to the masses of other fundamental fermions. We do not assume, however, any specific (quadratic or linear) seesaw relation between neutrino masses. We will use only the results of neutrino oscillation experiments in the general framework of a hierarchy (2.1) of neutrino masses.

In all solar neutrino experiments (Homestake [1], Kamiokande [2], GALLEX [3], SAGE [4], and Super-Kamiokande [5]) the detected event rates are significantly smaller than the event rates predicted by the existing standard solar models (SSMs) [40]. Moreover, a phenomenological analysis of the data of the different solar neutrino experiments, in which the values of the neutrino fluxes predicted by the SSMs are not used, strongly suggest that the solar neutrino problem is real [41]. In order to take into account the results of solar neutrino experiments in the framework of a hierarchy of neutrino masses, it is necessary to assume that  $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ is relevant for the suppression of the flux of solar  $\nu_e$ 's. In this case, the results of the solar neutrino experiments and the predictions of the SSMs can be reconciled if

$$\Delta m_{21}^2 \sim (0.3 - 1.2) \times 10^{-5} \text{ eV}^2 \text{ or } \Delta m_{21}^2 \sim 10^{-10} \text{ eV}^2,$$
(2.2)

in the case of Mikheyev-Smirnov-Wolfenstein (MSW) resonant transitions [42] and just-so vacuum oscillations [43], respectively.

<sup>&</sup>lt;sup>1</sup>Some remarks about the case of four neutrinos are presented in Sec. V.

<sup>&</sup>lt;sup>2</sup>Another possible mass spectrum of three neutrino is discussed in Sec. V.

Under the assumption of a neutrino mass hierarchy, the effective Majorana mass  $|\langle m \rangle|$  is given by [31]

$$|\langle m \rangle| \simeq |U_{e3}|^2 m_3 \simeq |U_{e3}|^2 \sqrt{\Delta m^2},$$
 (2.3)

with  $\Delta m^2 = m_3^2 - m_1^2$ .

# III. CONSTRAINTS FROM REACTOR NEUTRINO EXPERIMENTS AND THE LSND EXPERIMENT

In order to obtain information on  $|U_{e3}|^2$  and the effective Majorana mass  $|\langle m \rangle|$  from the results of reactor oscillation experiments, we will follow the method presented in Ref. [34] (see also Ref. [31]).

In the case of a small  $\Delta m_{21}^2$  and a neutrino mass hierarchy, the probability of the transitions  $\nu_{\ell} \rightarrow \nu_{\ell'}$  of terrestrial neutrinos is given by

$$P_{\nu_{\ell'} \to \nu_{\ell'}} = |\delta_{\ell'\ell'} + U_{\ell'3} U^*_{\ell'3} (e^{-i(\Delta m^2 L/2p)} - 1)|^2.$$
(3.1)

Here *L* is the distance between the neutrino source and the detector and *p* is the neutrino momentum. In Eq. (3.1) we used the unitarity of the mixing matrix and we took into account the fact that for the distances and energies of neutrinos in terrestrial experiments  $\Delta m_{21}^2 L/2p \ll 1$ . For the  $\nu_{\ell}$  ( $\bar{\nu}_{\ell}$ ) survival probability, from Eq. (3.1) we have (see Ref. [34])

$$P_{\nu_{\ell} \to \nu_{\ell}} = P_{\bar{\nu}_{\ell} \to \bar{\nu}_{\ell}} = 1 - \frac{1}{2} B_{\ell;\ell} \left( 1 - \cos \frac{\Delta m^2 L}{2p} \right), \quad (3.2)$$

where the oscillation amplitudes  $B_{\ell;\ell}$  are given by

$$B_{\ell;\ell} = 4 |U_{\ell,3}|^2 (1 - |U_{\ell,3}|^2).$$
(3.3)

Several oscillation experiments with reactor  $\bar{\nu}_e$ 's have been performed in the last few years (see the review in Ref. [12] and Ref. [13]). No indications in favor of neutrino oscillations were found in these experiments.

We will consider the square of the largest neutrino mass  $m_3^2 \approx \Delta m^2$  as a parameter and we will consider values of this parameter in the wide range of sensitivity of SBL and LBL reactor neutrino experiments

$$10^{-3} \text{ eV}^2 \le \Delta m^2 \le 10^3 \text{ eV}^2.$$
 (3.4)

From the negative results of reactor neutrino experiments, at any fixed value of  $\Delta m^2$  in the range (3.4) for the amplitude  $B_{e:e}$  of  $\bar{\nu}_e \rightarrow \bar{\nu}_e$  transitions we have the upper bound

$$B_{e;e} \leq B_{e;e}^0, \tag{3.5}$$

where the quantity  $B_{e;e}^0$  is the ordinate of the point (at the corresponding  $\Delta m^2$ ) of the exclusion curve of a reactor neutrino oscillation experiment in the  $\sin^2 2\theta - \Delta m^2$  plane ( $B_{e;e}$  corresponds to  $\sin^2 2\theta$  in the two-neutrino scheme). In our numerical calculations, we have used the 90% C.L. exclusion plots of the SBL Bugey [33] experiment and of the first LBL neutrino reactor experiment CHOOZ [13] (the inclusion of the results of the Krasnoyarsk [32] experiment in the analysis





FIG. 1. The  $\Delta m^2$  versus  $a_e^0$  plot obtained from the 90% C.L. exclusion plots of the Bugey [33] and CHOOZ [13] reactor neutrino oscillation experiments [see Eq. (3.8)].

does not add any new constraint). These experimental results provide the most stringent limits for the neutrino oscillation amplitude  $B_{e:e}$ .

From Eqs. (3.3) and (3.5), it follows that  $|U_{e3}|^2$  must satisfy one of two inequalities:

$$|U_{e3}|^2 \le a_e^0 \tag{3.6}$$

or

where

$$|U_{e3}|^2 \ge 1 - a_e^0, \tag{3.7}$$

 $a_e^0 = \frac{1}{2} (1 - \sqrt{1 - B_{e,e}^0}). \tag{3.8}$ 

In Fig. 1, we have plotted the values of the parameter  $a_e^0$  obtained from the 90% C.L. exclusion plots of the Bugey and CHOOZ experiments. Figure 1 shows that  $a_e^0$  is small for  $\Delta m^2$  in the range (3.4). Thus, the results of the reactor oscillation experiments imply that  $|U_{e3}|^2$  can only be small or large (close to 1).

The results of the solar neutrino experiments exclude the possibility of a large value of  $|U_{e3}|^2$ . The argument goes as follows. The averaged probability  $P_{\nu_e \to \nu_e}^{\text{sun}}(E)$  for solar  $\nu_e$ 's to survive, in the case of a neutrino mass hierarchy with  $\Delta m_{21}^2$  relevant for the oscillations of solar neutrinos, is given by (see Ref. [44])

$$P_{\nu_e \to \nu_e}^{\text{sun}}(E) = (1 - |U_{e3}|^2)^2 P_{\nu_e \to \nu_e}^{(1,2)}(E) + |U_{e3}|^4, \quad (3.9)$$



FIG. 2. The lower bound  $(P_{\nu_e \to \nu_e}^{sun})_{min}$  for the probability of solar  $\nu_e$ 's to survive in the case of a large value of the parameter  $|U_{e3}|^2$  ( $\geq 1 - a_e^0$ ). The values of  $a_e^0$  are obtained from the 90% C.L. exclusion plots of the Bugey [33] and CHOOZ [13] reactor neutrino oscillation experiments.

where  $P_{\nu_e \to \nu_e}^{(1,2)}(E)$  is the  $\nu_e$  survival probability due to the mixing of  $\nu_1$  and  $\nu_2$  and E is the neutrino energy. If  $|U_{e3}|^2$  satisfies the inequality (3.7), we have  $P_{\nu_e \to \nu_e}^{\text{sun}}(E) \ge (1$  $(-a_e^0)^2 \equiv (P_{\nu_e \to \nu_e}^{\text{sun}})_{\text{min}}$ . In Fig. 2 we have plotted the values of  $(P_{\nu_{a} \rightarrow \nu_{a}}^{sun})_{min}$  obtained from the exclusion plots of the Bugey and CHOOZ experiments. It can be seen that  $(P_{\nu_e \rightarrow \nu_e}^{sun})_{min}$  $\approx 0.9$  for  $\Delta m^2 \gtrsim 2 \times 10^{-3}$  eV<sup>2</sup>. Furthermore, Eq.  $(3.9)^{\circ}$  implies that the maximal variation of  $P_{\nu_e \to \nu_e}^{sun}(E)$  as a function of neutrino energy is given by  $(1 - |U_{e3}|^2)^2$ . If  $|U_{e3}|^2$  satisfies the inequality (3.7), we have  $(1 - |U_{e3}|^2)^2 \le (a_e^0)^2$ , which is a very small quantity [from Fig. 1 one can see that  $(1-|U_{e3}|^2)^2 \leq 9 \times 10^{-2}$  for  $\Delta m^2$  in the range (3.4) and  $(1-|U_{e3}|^2)^2 \leq 4 \times 10^{-3}$  for  $\Delta m^2 \geq 2 \times 10^{-3}$  eV<sup>2</sup>]. Thus, if  $|U_{e3}|^2$  is large,  $P_{\nu_e \to \nu_e}^{\text{sun}}(E)$  is practically constant. The large lower bound for the survival probability  $P_{\nu_e \rightarrow \nu_e}^{sun}$  and its practical independence of the neutrino energy are not compatible with the data of the solar neutrino experiments (see Refs. [45,46]). Therefore, from the two possibilities for the element  $|U_{e3}|^2$ , small [see Eq. (3.6)] or large [see Eq. (3.7)], the results of solar neutrino experiments allow us to choose only one:  $|U_{e3}|^2$  must be small and satisfies the inequality (3.6).

The limit (3.6) for  $|U_{e3}|^2$  implies the following upper bound for the effective Majorana mass  $|\langle m \rangle|$ :

$$|\langle m \rangle| \leq a_e^0 \sqrt{\Delta m^2}. \tag{3.10}$$



FIG. 3. Upper bounds for the effective Majorana mass  $|\langle m \rangle|$  obtained from the 90% C.L. exclusion plots of the Bugey (solid line) and CHOOZ (dashed line) neutrino reactor experiments. The shadowed region corresponds to the range of  $\Delta m^2$  allowed at 90% C.L. by the results of the LSND experiment, taking into account the results of all the other SBL experiments. The thick solid line represents the unitarity upper bound  $|\langle m \rangle| \leq \sqrt{\Delta m^2}$ .

The upper bounds obtained with Eq. (3.10) from the 90% C.L. exclusion plots of the Bugey [33] and CHOOZ [13] experiments for  $10^{-4} \text{ eV}^2 \leq \Delta m^2 \leq 10^3 \text{ eV}^2$  are presented in Fig. 3 (the solid and dashed lines, respectively). The thick solid line in Fig. 3 represents the unitarity upper bound  $|\langle m \rangle| \leq \sqrt{\Delta m^2}$ .

As can be seen from Fig. 3, the upper bound for the effective Majorana mass  $|\langle m \rangle|$  depends rather strongly on the value of  $\Delta m^2$  (whose square root is equal to the heaviest mass  $m_3$ ). From Fig. 3 one can see that if  $\Delta m^2$  is less than 10 eV<sup>2</sup>, the effective Majorana mass  $|\langle m \rangle|$  is smaller than  $10^{-1}$  eV. Figure 3 also shows that if  $\Delta m^2$  is less than 2 eV<sup>2</sup>, from exclusion plots of the Bugey and CHOOZ experiments it follows that  $|\langle m \rangle| \leq 3 \times 10^{-2}$  eV.

Up to now we have considered only the data of reactor neutrino experiments. Let us now also take into account the results of the LSND experiment [11]. The data of this experiment fix an allowed region of  $\Delta m^2$ . Combined with the negative results of the Bugey [33] and BNL E776 [47] experiments, the allowed plot of the LSND experiment imply that  $\Delta m^2$  lies in the range

0.3 
$$eV^2 \le \Delta m^2 \le 2.2 eV^2$$
. (3.11)

The corresponding region of allowed values of  $|\langle m \rangle|$  is represented by the shadowed region in Fig. 3. One can see that the results of the LSND experiment, together with the nega-

tive results of other SBL experiments, imply that the value of  $|\langle m \rangle| \le 3 \times 10^{-2}$  eV.

Thus, we conclude that if massive neutrinos are Majorana particles and if there is a hierarchy of neutrino masses, the existing data of reactor neutrino experiments imply a strong constraint on the parameter  $|\langle m \rangle| : |\langle m \rangle| \leq 10^{-1}$  eV for  $\Delta m^2 \leq 10$  eV<sup>2</sup>. Let us stress that the value  $|\langle m \rangle| \sim 10^{-1}$  eV corresponds to the sensitivity of the next generation of  $(\beta\beta)_{0\nu}$  decay experiments [16,17,23].

If the results of the LSND experiment are confirmed by future experiments, the upper bound for the parameter  $|\langle m \rangle|$  is about  $3 \times 10^{-2}$  eV. Such small values of  $|\langle m \rangle|$  can be explored only by  $(\beta\beta)_{0\nu}$  decay experiments of future generations (see Ref. [17]).

# IV. CONSTRAINTS FROM ATMOSPHERIC NEUTRINO EXPERIMENTS

In the previous section we obtained constraints on the Majorana parameter  $|\langle m \rangle|$  from the results of reactor experiments and of the LSND experiment. In this section we present the allowed region for the parameter  $|\langle m \rangle|$  obtained from the data of atmospheric neutrino experiments in the scheme with mixing of three Majorana neutrinos and a neutrino mass hierarchy. The ratio of muon and electron atmospheric neutrino events has been found to be significantly smaller than the expected one in the Kamiokande [6], IMB [7], and Soudan [8] experiments. For the double ratio  $R = (\mu/e)_{data}/(\mu/e)_{MC}$  [ $(\mu/e)_{MC}$  is the Monte Carlo calculated ratio of muon and electron events under the assumption that neutrinos do not oscillate], in the regions of neutrino energies less than 1.3 GeV (sub-GeV) and more than 1.3 GeV (multi-GeV) the Kamiokande Collaboration found

$$R_{\text{Kamiokande}}^{\text{sub-GeV}} = 0.60^{+0.06}_{-0.05} \pm 0.05,$$

$$R_{\text{Kamiokande}}^{\text{multi-GeV}} = 0.57^{+0.08}_{-0.07} \pm 0.07.$$
(4.1)

The IMB [7] and Soudan [8] Collaborations found

$$R_{\rm IMB} = 0.54 \pm 0.05 \pm 0.12, \quad R_{\rm Soudan} = 0.75 \pm 0.16 \pm 0.10.$$
(4.2)

On the other hand, the values of the double ratio found in the Frejus [48] and NUSEX [49] experiments,

$$R_{\text{Frejus}} = 0.99 \pm 0.13 \pm 0.08, \quad R_{\text{NUSEX}} = 1.04 \pm 0.25,$$
(4.3)

are compatible with unity (but cannot exclude the atmospheric neutrino anomaly because of large errors).

The existence of the atmospheric neutrino anomaly was recently confirmed by the results of the high statistics Super-Kamiokande experiment [9]:

$$R_{\text{Super -}K}^{\text{sub-GeV}} = 0.635_{-0.033}^{+0.034} \pm 0.010 \pm 0.052,$$

$$R_{\text{Super -}K}^{\text{multi-GeV}} = 0.604_{-0.058}^{+0.065} \pm 0.018 \pm 0.065.$$
(4.4)

Here the three errors are, respectively, the statistical errors of the data, the statistical error of the Monte Carlo, and the systematic error. The results of atmospheric neutrino experiments can be explained by neutrino oscillations. The recent results of the CHOOZ experiment [13] exclude the possibility of  $\nu_{\mu} \stackrel{\leftarrow}{\rightarrow} \nu_{e}$  oscillations. In the framework of two flavor  $\nu_{\mu} \rightarrow \nu_{\tau}$  oscillations, the following 90% C.L. allowed ranges for the oscillation parameters were found by the analysis of the Kamio-kande data [6]:

$$5 \times 10^{-3} \text{ eV}^2 \leq \Delta m^2 \leq 3 \times 10^{-2} \text{ eV}^2, \quad 0.7 \leq \sin^2 2 \vartheta \leq 1.$$
  
(4.5)

The preliminary analysis of the Super-Kamiokande data [9] indicate the following 90% C.L. allowed ranges for the oscillation parameters:

$$3 \times 10^{-4} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 6 \times 10^{-3} \text{ eV}^2, \quad 0.8 \lesssim \sin^2 2 \vartheta \lesssim 1.$$
(4.6)

The values of  $\Delta m^2$  allowed by the Super-Kamiokande data are significantly smaller than those allowed by the Kamiokande data. However, the two allowed ranges of  $\Delta m^2$  overlap at  $\Delta m^2 \simeq 5 \times 10^{-3}$  eV<sup>2</sup>, indicating that the two experimental results are compatible.

In Sec. III we obtained restrictions on the parameter  $|\langle m \rangle|$ from the exclusion plots of reactor experiments and from the allowed plot of the LSND experiment. Here we present the allowed region of the Majorana parameter  $|\langle m \rangle|$  obtained from the results of a  $\chi^2$  analysis of the Kamiokande atmospheric neutrino data in the model with mixing of three neutrinos and a neutrino mass hierarchy [50]. In this case, the oscillation probabilities of atmospheric neutrinos depend on three parameters:  $\Delta m^2$ ,  $|U_{e3}|^2$ , and  $|U_{\mu3}|^2$   $(|U_{\tau3}|^2=1)$  $-|U_{\mu3}|^2 - |U_{\mu3}|^2$ ). The matter effect for the atmospheric neutrinos reaching the Kamiokande detector from below has been taken into account. The presence of matter is important because it modifies the phases of neutrino oscillations [51] and its effect is to enlarge the allowed region towards low values of  $\Delta m^2$  (see Ref. [50]). The best fit of the Kamiokande data is obtained for  $\Delta m^2 = 2.5 \times 10^{-2} \text{ eV}^2$ ,  $|U_{e3}|^2$ =0.26 and  $|U_{\mu3}|^2$ =0.49, with  $\chi^2$ =6.9 for 9 degrees of freedom, corresponding to a C.L. of 65%.

The range allowed at 90% C.L. in the  $|\langle m \rangle| - \Delta m^2$  plane is shown in Fig. 4 as the vertically shadowed region. The solid and dashed lines in Fig. 4 represent the upper bounds obtained with Eq. (3.10) from the 90% C.L. exclusion plots of the Bugey [33] and CHOOZ [13] experiments, respectively. The thick solid line represents the unitarity upper bound  $|\langle m \rangle| \leq \sqrt{\Delta m^2}$ . From Fig. 4 it can be seen that the results of the Kamiokande experiment, together with the exclusion plots of the Bugey and CHOOZ experiments, imply that

$$|\langle m \rangle| \lesssim 8 \times 10^{-3} \text{ eV.} \tag{4.7}$$

The horizontally shadowed region in Fig. 4 indicates the range (4.6) of  $\Delta m^2$  allowed at 90% C.L. by the preliminary analysis of the data of the Super-Kamiokande experiment [9]. This range covers values of  $\Delta m^2$  smaller by an order of magnitude with respect to the range of  $\Delta m^2$  allowed by the Kamiokande data. However, the two allowed ranges have an overlap around  $\Delta m^2 \approx 5 \times 10^{-3} \text{ eV}^2$ . If this is the value of



FIG. 4. Upper bounds for the effective Majorana mass  $|\langle m \rangle|$  obtained from the 90% C.L. exclusion plots of the Bugey (solid line) and CHOOZ (dashed line) neutrino reactor experiments. The vertically shadowed region is allowed at 90% C.L. by the data of the Kamiokande atmospheric neutrino experiment. The horizontally shadowed region corresponds to the range of  $\Delta m^2$  allowed at 90% C.L. by the preliminary analysis of the data of the Super-Kamiokande experiment. The thick solid line represents the unitarity upper bound  $|\langle m \rangle| \leq \sqrt{\Delta m^2}$ .

 $\Delta m^2$ , the exclusion curve of the CHOOZ experiment puts a very strong constraint on  $|\langle m \rangle|$ :

$$|\langle m \rangle| \lesssim 3 \times 10^{-3} \text{ eV.} \tag{4.8}$$

Thus we can conclude that in all possible scenarios with mixing of three massive Majorana neutrinos and a neutrino mass hierarchy, the existing neutrino oscillation data imply that the effective Majorana mass, which characterizes the matrix element of  $(\beta\beta)_{0\nu}$  decays, is very small.

# V. NONHIERARCHICAL NEUTRINO MASS SPECTRA

In this section we consider the following two possibilities. (I) "Inverted" mass hierarchy of three neutrinos. In the previous sections we have assumed that there are three Majorana neutrinos with a hierarchy of masses and that  $\Delta m_{21}^2$  is relevant for the suppression of the flux of solar  $\nu_e$ 's. Another possibility to explain the solar neutrino data in the framework of three neutrino mixing is to assume that the neutrino mass spectrum has the form [37,38]

$$m_1 \ll m_2 \simeq m_3, \tag{5.1}$$

and  $\Delta m_{32}^2$  is relevant for the suppression of the flux of solar  $\nu_e$ 's. In this case, SBL neutrino oscillations are described by the expressions (3.2) and (3.3), with the change  $U_{\ell/3} \rightarrow U_{\ell/1}$ . From the exclusion plots of reactor experiments it follows that

$$|U_{e1}|^2 \le a_e^0, \tag{5.2}$$

with  $a_e^0$  given by Eq. (3.8). The value of  $a_e^0$  depends on  $\Delta m^2 \equiv m_3^2 - m_1^2 \simeq m_3^2$ . From Fig. 1 one can see that  $a_e^0$  is small for  $\Delta m^2$  in the range (3.4). In this case, the effective Majorana mass is given by

$$|\langle m \rangle| \simeq |U_{e2}^2 + U_{e3}^2| \sqrt{\Delta m^2}.$$
 (5.3)

If *CP* is conserved in the lepton sector and the relative *CP* parity of  $\nu_2$  and  $\nu_3$  is equal to unity,  $|\langle m \rangle|$  is (practically) equal to  $\sqrt{\Delta m^2}$  [31,38]. In general, we have

$$|\langle m \rangle| \leq \sqrt{\Delta m^2}. \tag{5.4}$$

Thus, in the case of the neutrino mass spectrum (5.1), the upper bound for the effective Majorana mass could be in the eV region (if the LSND result is confirmed). If the spectrum (5.1) is realized in nature, from the inequality (5.2) it follows also that neutrino mass  $m(^{3}\text{H})$  measured in <sup>3</sup>H-decay experiments is practically equal to the heaviest mass [38]:

$$m(^{3}\mathrm{H}) \simeq \sqrt{\Delta m^{2}}.$$
 (5.5)

Let us notice that in the case of a hierarchy of three neutrino masses the contribution of the term that depends on  $m_3 \approx \sqrt{\Delta m^2}$  to the  $\beta$  spectrum of <sup>3</sup>H is suppressed by the factor  $|U_{e3}|^2 \leq a_e^0$ . Therefore, the observation of the effect of a neutrino mass in the experiments measuring the high-energy part of the  $\beta$  spectrum of <sup>3</sup>H [52,53] would be an indication in favor of the neutrino spectrum (5.1) with an "inverted" mass hierarchy.

(II) Four massive neutrinos. All the existing indications in favor of neutrino mixing (solar neutrinos, atmospheric neutrinos, LSND) cannot be described by any scheme with three massive neutrinos [54–57]. If we take all data seriously, we need to consider schemes of mixing with (at least) four massive neutrinos [58], that include not only  $\nu_e$ ,  $\nu_{\mu}$ , and  $\nu_{\tau}$ , but also (at least) one sterile neutrino. In Refs. [54,55] it was shown that among all the possible mass spectra of four neutrinos only two can accommodate all the existing data:

(A) 
$$\underbrace{\underbrace{m_1 < m_2}_{\text{LSND}} \ll \underbrace{m_3 < m_4}_{\text{LSND}}}_{\text{SND}}$$
, (B)  $\underbrace{\underbrace{m_1 < m_2}_{\text{M}_1 < m_2} \ll \underbrace{m_3 < m_4}_{\text{LSND}}}_{\text{LSND}}$ .

In the case of scheme (A),  $\Delta m_{21}^2$  is relevant for the atmospheric neutrino anomaly and  $\Delta m_{43}^2$  for the suppression of solar  $\nu_e$ 's, whereas in scheme (B) the roles of  $\Delta m_{21}^2$  and  $\Delta m_{43}^2$  are reversed. In both schemes two groups of close masses are separated by the "LSND gap" of the order of 1 eV. In scheme (B), the upper bound for the effective Majorana mass is given by

$$|\langle m \rangle| \leq a_e^0 \sqrt{\Delta m^2},\tag{5.7}$$

with  $\Delta m^2 \equiv m_4^2 - m_1^2 \simeq m_4^2$ . Hence, in scheme (B) the effective Majorana mass  $|\langle m \rangle|$  must satisfy the constraints discussed in Sec. III and presented in Fig. 3. This means that in scheme (B) the contribution of Majorana neutrino masses to the amplitude of  $(\beta \beta)_{0\nu}$  decay is strongly suppressed. In scheme (A), the effective Majorana mass is bounded by

$$|\langle m \rangle| \leq \sum_{i=3,4} |U_{ei}|^2 \sqrt{\Delta m^2} \leq \sqrt{\Delta m^2}.$$
 (5.8)

Hence, no *a priori* suppression of the Majorana mass contribution to  $(\beta\beta)_{0\nu}$  decay is expected in scheme (A).

Also the values of the effective neutrino mass  $m({}^{3}\text{H})$  measured in experiments that investigate the high-energy part of the  $\beta$  spectrum of  ${}^{3}\text{H}[52,53]$  are different in schemes (A) and (B). In scheme (A) we have  $m({}^{3}\text{H}) \approx m_{4} \approx \sqrt{\Delta m^{2}}$ , whereas in scheme (B) the contribution of the term that depends on the heavy masses  $m_{3} \approx m_{4}$  to the  $\beta$  spectrum of  ${}^{3}\text{H}$  is suppressed by the factor  $\sum_{i=3,4} |U_{ei}|^{2} \leq a_{e}^{0}$ .

# **VI. CONCLUSIONS**

We have obtained various constraints on the parameter  $|\langle m \rangle|$  (which characterizes the contribution of Majorana neutrino masses to the matrix element of neutrinoless double- $\beta$  decay) from the results of neutrino oscillation experiments. We have shown that in the scheme with mixing of three Majorana neutrinos and a mass hierarchy (which corresponds to the seesaw mechanism for the generation of neutrino masses) the results of neutrino oscillation experiments put rather severe restrictions on the value of  $|\langle m \rangle|$ . The numerical value of the upper bound for  $|\langle m \rangle|$  depends rather strongly on the value of the parameter  $\Delta m^2 \equiv m_3^2 - m_1^2$ . If we take into account only the results of SBL reactor experiments

and the results of solar neutrino experiments, we can conclude that  $|\langle m \rangle| \leq 10^{-1}$  eV for  $\Delta m^2 \leq 10$  eV<sup>2</sup>. From the new results of the first LBL experiment CHOOZ and from the exclusion curve of the Bugey experiment it follows that for  $\Delta m^2 \leq 2$  eV<sup>2</sup> the parameter  $|\langle m \rangle|$  is less than  $3 \times 10^{-2}$  eV. If we take into account the results of the LSND experiment, we come to the conclusion that  $|\langle m \rangle| \leq 3 \times 10^{-2}$  eV.

We have also calculated the region of the parameter  $|\langle m \rangle|$ allowed by the data of the Kamiokande atmospheric neutrino experiment, using the recent three-neutrino  $\chi^2$  analysis presented in Ref. [50]. Taking into account this allowed range of  $|\langle m \rangle|$  and the constraints obtained from the results of the Bugey and CHOOZ experiments, we conclude that very small values of  $|\langle m \rangle|$  are allowed:  $|\langle m \rangle| \leq 8 \times 10^{-3}$  eV. Taking into account also the results of the preliminary analysis of Super-Kamiokande data, an even stronger constraint can be placed:  $|\langle m \rangle| \leq 3 \times 10^{-3}$  eV.

The constraints on the value of the effective Majorana mass  $|\langle m \rangle|$  that follow from the results of neutrino oscillation experiments must be taken into account in the interpretation of the data of  $(\beta\beta)_{0\nu}$  decay experiments. The observation of neutrinoless double- $\beta$  decay with a probability that corresponds to  $|\langle m \rangle| \gtrsim 10^{-1}$  eV (which is the sensitivity of future  $(\beta\beta)_{0\nu}$  decay experiments) would imply that the spectrum of three neutrinos does not follow a hierarchical pattern and the neutrino masses are not of seesaw origin, or that there are more than three massive neutrinos. This observation could also imply that nonstandard mechanisms for the violation of lepton number, such as right-handed currents (see Refs. [21,24,25]), supersymmetry with violation of R parity [26,24,27,28], and others [29,30], are responsible for neutrinoless double- $\beta$  decay. Thus, the observation of  $(\beta\beta)_{0\nu}$  decay could allow us to obtain information not only about the nature of massive neutrinos (Dirac or Majorana?), but also about the pattern of the mass spectrum of neutrinos and/or about nonstandard mechanisms of violation of the lepton number.

#### ACKNOWLEDGMENTS

S.M.B. would like to acknowledge support from the Dyson Visiting Professor Funds at the Institute for Advanced Study.

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