Friction domination with superconducting strings

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We investigate the evolution of a superconducting string network with arbitrary, constant string current in the friction dominated regime. In the absence of an external magnetic field the network always reaches a scaling solution. However, for string current stronger than a critical value, it is different than the usual horizon scaling of the nonsuperconducting string case. In this case the friction domination era never ends. Whilst the superconducting string network can be much denser than usually assumed, it can never dominate the universe energy density. It can, however, influence the cosmic microwave background radiation and the formation of large scale structure. When embedded in a primordial magnetic field of sufficient strength, the network never reaches scaling and, thus, eventually dominates the universe evolution. [S0556-2821(98)00402-0]

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I. INTRODUCTION

The microphysics of cosmic strings has received considerable attention. In particular, Witten [1] showed that cosmic strings become superconducting as a result of boson condensates or fermion zero modes in the string core. Such strings are capable of carrying a sizeable current, with the maximum current being about 10^{20} A for a grand unified scale string. Inevitably, such currents have cosmological and astrophysical [2] consequences. The consequences for emission of synchrotron radiation [3] and for high energy γ rays [4–6] have been explored.

Unlike non-conducting strings, loops of superconducting string can be stabilized from collapse by the angular momentum of the current carriers, forming vortons [7]. If vortons are sufficiently stable then consistency with standard cosmology puts severe constraints on theories giving rise to such strings. However, such constraints have been derived assuming that the evolution of a network of superconducting strings is similar to that of ordinary strings. Early studies using both analytic [8] and numerical techniques [9–11] showed that the string evolution was indeed similar to that of ordinary cosmic strings. However, these studies neglected the very early times when the string is interacting strongly with the surrounding plasma. During this friction dominated period the string correlation length grows until it catches up with the horizon and a scaling solution sets in.

Since the interaction of particles with a superconducting string [3,12] is very different from that of an ordinary string [13], there is every reason to expect that the friction dominated period could be vastly different. Thus, it is important to investigate this and ascertain whether or not the superconducting string network really does reach a scaling solution. Even if the network does reach a scaling solution it could still be very different from that of the non-conducting case.

In this paper we address this issue. In Sec. II we discuss the interaction of the plasma particles with the string. We consider the string damping time and the evolution of the curvature radius. We compare and contrast the situation for ordinary strings and superconducting strings assuming a constant string current. Section III addresses the form of the scaling solution. We show that, in the absence of a primordial magnetic field, there is a critical current about which the friction dominated period never ends. Instead, the string reaches a so-called plasma scaling solution, where the density of strings is considerably greater than the usual horizon scaling. In contrast, below this critical current the string network reaches the usual horizon scaling solution of the ordinary cosmic strings. The cosmological consequences of the plasma scaling solution are investigated in Sec. IV. Despite the more dense scaling solution we show that the strings never dominate the energy density of the universe. We also show that the overall large scale structure and microwave background anisotropy are of the same magnitude as produced by ordinary cosmic strings. However, the imprint may produce a richer, more filamentary structure. In Sec. V we embed the string network in a primordial magnetic field. For a magnetic field of magnitude greater than a critical value a string dominated universe results. Finally, the validity of the assumptions used is investigated in Sec. VI. We first study the magnetocylinder around the string and the extent to which this can be penetrated by plasma particles. We then analyze current conservation and show that the coherent current is indeed constant for a scaling solution. Finally, we investigate the effects on our results in case where the current is switched on at a later phase transition. In the final section we discuss our conclusions. Unless stated otherwise, we use natural units ($\hbar = c = 1$).

II. FRICTION ON SUPERCONDUCTING COSMIC STRINGS

A. Cosmic string friction

Friction on a cosmic string is caused by particle interaction with the string as it moves through the plasma. The friction force per unit length is

$$f \sim \rho \sigma v \overline{v}, \tag{1}$$

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where ρ is the energy density of the plasma, σ is the interaction cross-section, v is the velocity of the string segment and $\overline{v} \simeq \Delta p/m$ is given by

$$\overline{v} \equiv \max\{v, v_{th}\}, \tag{2}$$

where Δp is the particle's momentum change, $v_{th} \sim \sqrt{T/m}$ is the thermal velocity of the plasma particles, $m^2 \simeq m_0^2 + \alpha T^2$ is the mass of a plasma particle with rest mass m_0, $\alpha = g^2/4\pi$ with g the gauge coupling) and T is the plasma temperature. Also, $m_P = 1.22 \times 10^{19}$ GeV is the Planck mass.

Curves and wiggles on the strings tend to untangle due to string tension, which results in oscillations of the curved string segments on scales smaller than the causal horizon and larger than their curvature radii. Friction dissipates the energy of these oscillations and leads to their gradual damping. Thus, the strings become smooth on larger and larger scales with their curvature radius growing accordingly. The characteristic damping timescale is [14],

$$t_d \equiv \frac{\mu v^2}{\dot{\mathcal{E}}} \sim \frac{\mu v}{f} \sim \frac{\mu}{\rho \sigma \overline{v}}, \tag{3}$$

where μ is the string mass per unit length and $\dot{\mathcal{E}} \sim fv$, is the energy loss per unit time per unit length. The above time scale corresponds to a length-scale called the friction length [15,16]. This is the smoothing length of the strings, over which the strings are conformally stretched by the universe expansion.

Kibble [14] has shown that the growth of long wavelength perturbations (or the straightening of the kinks) on a string is described by

$$\ddot{\zeta} + \frac{2}{t_d}\dot{\zeta} - \frac{2}{R^2}\zeta = 0, \tag{4}$$

where ζ is the normal displacement, R is the string curvature radius and the dots signify derivation with respect to time. The timescale of the growth of R is roughly equal to that of the long wavelength perturbations t_{lw},[1] i.e. $R/\dot{R} \sim t_{lw} \sim \zeta/\dot{\zeta}$.

Initially the friction length is smaller than the curvature radius. In this case Eq. (4) suggests that the growth of the string curvature radius is

$$\frac{dR}{dt} \sim \frac{t_d}{R}. \tag{5}$$

If the growth of the friction length is faster than that of R, then at some point it will catch up with the curvature radius and Eq. (5) will cease to be valid. This occurs when the curvature radius reaches the horizon [14] and the strings become smooth on horizon scales. From then on Eq. (4) gives, $R \sim t$, the friction domination era ends and the string network satisfies the well-known horizon-scaling solution. Thus, in order to see whether a network of superconducting cosmic strings ever reaches a scaling solution one has to study the evolution of the string curvature radius. For this, a closer look at the superconducting string system is required.

B. The superconducting string system

The current of a charged-current carrying superconducting string creates a magnetic field around the string core. This field is very strong near the string and does not allow the plasma particles to approach. As the charged particles encounter the magnetic field their motion is diverted in such a way that they create a surface current of opposite orientation than the string current [3], which screens the magnetic field. Thus, while the string moves in the plasma it is shielded by a magnetocylinder, which contains the magnetic field and does not allow the plasma to penetrate.

The flow of the plasma around the string creates a shock front, which is the border of the magnetocylinder [3], whose distance from the string is determined by the pressure balance between the magnetic field and the plasma. The string magnetic field is given by the Biot-Savart law[2]

$$B_s(r) \simeq \frac{2J}{r}, \tag{6}$$

where J is the string current and r is the distance from the string. All through this paper, unless stated otherwise, we will assume that the string current J is constant. The validity of this assumption will be discussed later but here we can just mention that it is primarily based on dynamical reasons. The magnitude of the current will be treated as a free parameter and can have any value up to $J_{max} \sim e\sqrt{\mu} \sim \eta$, where $e = \sqrt{4\pi\alpha_{em}}$ is the charge of the current carriers (α_{em} is the fine structure constant) and we have used that, $\mu \sim \eta^2$ with η being the scale of the symmetry breaking that produced the string network. We also assume, for simplicity, that the current switches on at the phase transition that creates the string network.

The pressure balance, $B_s^2(r_s) \sim \rho \overline{v^2}$ suggests that the dimensions of the magnetocylinder are of order

$$r_s \sim \frac{J}{\sqrt{\rho}\,\overline{v}}. \tag{7}$$

The magnetocylinder cross-section is not circular of course [18] but Eq. (7) is a good estimate of the minimum distance of the shock front from the string [3]. We can use, therefore, r_s as the superconducting, charged-current carrying string cross-section per unit length. From Eq. (7) it is evident that the linear cross-section increases with the string current, in agreement with [12].

The above linear cross-section r_s has to be compared with the usual linear cross section for a cosmic string:

$$\sigma_{cs} \sim k^{-1}, \tag{8}$$

[1]Note that for t_{lw} we have $R \sim v\ t_{lw}$, which implies $v \sim \dot{R}$.

[2]For temperatures higher than the electroweak energy scale the electroweak symmetry is unbroken and the ''magnetic'' field of the string is actually due to the hypercharge generator. However, as shown in [17], its magnitude still follows, $B \sim J/r$ as in Eq. (6).

where $k\sim m\overline{v}$ is the momentum of the incident particle in the frame of the string [13].

III. EVOLUTION OF THE NETWORK

A. The one-scale model

The one-scale model gives a reliable, order of magnitude description of the evolution of a string network according to the behavior of one parameter only, the curvature radius R [14]. As suggested by numerical simulations [10], the intercommuting process of superconducting strings is very similar to the non-superconducting string case, i.e. the probability of producing string loops is of order unity. This suggests [14] that the typical inter-string distance of the network is $\sim R$. Then, the number P of string intersections of a string segment during a time interval Δt is $P\sim v\Delta t/R$. Thus, the number density of loops produced is

$$\dot{n}_{pr}\sim\frac{P}{\Delta V\Delta t}\sim\frac{v}{R^4}\Rightarrow\left.\frac{\dot{n}}{n}\right|_{pr}\sim\frac{v}{R},\tag{9}$$

where $\Delta V\sim R^3$ is the volume element (in order not to double-count string segment intersections) and $n\sim R^{-3}$ is the loop number density.

The string energy density may be estimated as $\rho_s\sim\mu R/\Delta V\sim\mu/R^2$. Thus, for scales larger than the curvature radius, conformal stretching suggests that $\rho\propto a^{-2}$, where a is the scale factor of the expansion of the universe ($a\propto t^{1/2}$ in the radiation era). The initial size of the loops produced is also $\sim R$ and, thus, the mass of a loop is approximately $\sim\mu R$. Therefore, Eq. (9) suggests that the rate of energy density loss of the open-string network due to loop production is $\dot{\rho}_{pr}\sim\mu v/R^3$, where $v\sim\dot{R}$. Thus, overally, the network density evolves according to[3]

$$\frac{\dot{\rho}_s}{\rho_s}=-2\frac{\dot{a}}{a}-\frac{v}{R}.\tag{10}$$

From the above it can be inferred that the network would satisfy a scaling solution, $\rho_s/\rho=$const, provided $R\propto t$ for the radiation era. If R manages to grow up to horizon size then scaling is ensured since $R\sim t$. In this case the inter-string distance is the horizon size and the string velocity is $v\sim1$. We will refer to this scaling solution as horizon scaling.

B. Friction and network scaling

Whether the string network reaches a scaling solution or over-closes the universe instead is determined by the evolution of the string curvature radius. From Eqs. (1) and (8) we find that the friction force for non-superconducting strings is

$$f_{ns}\sim\frac{\rho v}{m}.\tag{11}$$

Similarly, from Eqs. (1) and (7) we obtain, for the plasma friction force,

$$f_{pl}\sim Jv\sqrt{\rho}.\tag{12}$$

Note that the friction forces are not affected by the transition from the radiation to the matter era, since $\rho\sim(m_P/t)^2$ at all times.

Inserting the above into Eq. (3) one finds

$$t_d^{ns}\sim\frac{\mu m}{\rho}\tag{13}$$

and

$$t_d^{pl}\sim\frac{\mu}{J\sqrt{\rho}}.\tag{14}$$

From the above it can be inferred that the temperature dependence of the damping time is changed only due to the variation of the particle mass. This has an effect only when the plasma friction force is subdominant. In this case, using Eq. (5) we obtain

$$R_{ns}\sim\frac{\eta}{m_P^{3/4}}t^{5/4}\tag{15}$$

and

$$R'_{ns}\sim\left[\frac{\eta}{\sqrt{m_0m_P}}\right]t_m^{-1/2}t^{3/2},\tag{16}$$

where Eqs. (15) and (16) correspond to temperatures higher and lower than $T(t_m)\sim m_0$, respectively[4] ($m_0\sim1$ GeV).

Similarly, in the case of plasma friction domination, for all temperatures, we have

$$R_{pl}\sim\frac{\eta}{\sqrt{J\ m_P}}t.\tag{17}$$

The above suggest that, if the plasma friction is subdominant, the curvature radius grows more rapidly than the horizon and will reach the horizon size at

$$t_*\sim\frac{m_P^3}{\eta^4}\tag{18}$$

or at

$$t'_*\sim\frac{m_P^2}{\eta^2m_0}\sim\left[\frac{\eta}{\sqrt{m_0m_P}}\right]^2t_*\tag{19}$$

for high and low temperatures, respectively. Comparing with t_m we find

$$\eta\gtrless\sqrt{m_0m_P}\Leftrightarrow t_*\lessgtr t'_*\lessgtr t_m\tag{20}$$

Thus, for $\eta>\sqrt{m_0m_P}\sim10^9$ GeV the network would reach horizon scaling before t_m.

[3]A similar result has been derived independently in [16].

[4]The prime when used denotes the low temperature case.

If, on the other hand, plasma friction dominates, then R follows Eq. (17) and the curvature radius will always remain in constant proportion to the horizon size. Also, the strings of the network assume a constant, terminal velocity of order

$$v_T \sim \dot{R} \sim \frac{\eta}{\sqrt{J\, m_P}}. \tag{21}$$

Both the friction forces Eqs. (11) and (12) are decreasing with time. However, the plasma friction force decreases less rapidly, for all temperatures. Thus, there is always a time when plasma friction will come to dominate. By comparing the forces, for a given string velocity v and for high and low temperatures, respectively, we find

$$t_c \sim \frac{m_P}{J^2} \tag{22}$$

and

$$t'_c \sim \frac{m_P}{m_0 J} \sim \left(\frac{J}{m_0}\right) t_c. \tag{23}$$

However, the evolution of the network will be affected by this only if $t_c \leqslant t_*$ (or $t'_c \leqslant t'_*$). In the opposite case f_{ns} remains dominant until the curvature radius increases up to horizon size and the horizon-scaling solution begins. Comparing the two critical times gives the critical string current

$$J_c \sim \frac{\eta^2}{m_P} \sim J_{max}\sqrt{G\mu} \tag{24}$$

for all temperatures, where G is Newton's gravity constant ($G = m_P^{-2}$). Note that, for high temperatures, $T_c \equiv T(t_c) \sim J$ and $J_c \sim T_* \equiv T(t_*)$.

If the string current is smaller than J_c then the evolution of the curvature radius follows the usual pattern described in the literature [19]. If, however, $J > J_c$, the curvature radius grows more rapidly than the horizon until t_c (or t'_c). After that, it follows Eq. (17). Since, $R_{pl} \propto t$, the network evolves again in a self-similar way but with typical inter-string distance smaller than the usual horizon scaling by a factor corresponding to the terminal velocity. We will call this scaling solution *plasma scaling.*

Using Eq. (24) we can rewrite Eq. (21) as

$$v_T \sim \sqrt{\frac{J_c}{J}}. \tag{25}$$

Also, from the above we find

$$J \gtrless m_0 \Leftrightarrow t_c \lessgtr t'_c \lessgtr t_m. \tag{26}$$

Thus, if $J > J_c$ and $J > m_0$ the network reaches plasma scaling before t_m.

Summing up, the above suggest that for $J < J_c$, *the network always reaches horizon scaling* at $t = \min\{t_*, t'_*\}$, and for $J \geqslant J_c$, *the network always reaches plasma scaling* at $t = \min\{t_c, t'_c\}$.

For currents weaker than $J \sim m_0 \sim 1$ GeV, it will be shown later that the magnetocylinder system is not impenetratable to plasma particles and, thus, for $J_c < J < m_0$, the above analysis is not entirely reliable.[5]

IV. THE PLASMA-SCALING SOLUTION

A. Open-string energy density

Let us now estimate the string network's energy density during plasma scaling. In the case of horizon scaling ($J \leqslant J_c$) the energy density of the network of open strings can be easily found to be

$$\frac{\rho_s}{\rho} \sim \left(\frac{\eta}{m_P}\right)^2 \sim G\mu. \tag{27}$$

The above result is the usual estimate for a non-superconducting string network energy density [19].

Now, if friction continues to dominate ($J > J_c$) then the network reaches plasma scaling and the energy density of the open string network is larger. Indeed, using $\rho_s \sim \mu / R_{pl}^2$ we find

$$\frac{\rho_s}{\rho} \sim \frac{J}{m_P} \sim \frac{J}{J_c}(G\mu) \leqslant 1. \tag{28}$$

The above shows that *the current carrying string network can never dominate the universe energy density*. For overcritical currents the network density and the string terminal velocity are

$$J_c \leqslant J \leqslant J_{max},$$
$$1 \geqslant v_T \geqslant (G\mu)^{1/4}, \tag{29}$$
$$G\mu \leqslant \rho_s/\rho \leqslant \sqrt{G\mu}.$$

From the above it can be inferred that the larger the current the slower the strings move. Thus, the inter-string distance and the curvature radius are smaller (since $R \sim vt$). This is because low-velocity strings untangle slowly and have a small intercommuting range. As a result, although the number of intercommutations per unit time per unit volume $\sim v/R$ [20] is not reduced, as also suggested by Eq. (9), the loops produced by the network are smaller and, therefore, less string length is lost. This results in a larger open string energy density. However, even the maximum ρ_s cannot dominate the overall energy density of the universe.

B. Cosmological consequences

Even so, the existence of an over-dense string network could have other observable consequences. Indeed, were such a network to seed the large scale structure, it would produce structure of a smaller correlation. However, we show that it would *not* create density inhomogeneities that may be incompatible with the galaxy formation scenaria neither will it generate excessive cosmic microwave background anisotropies.

[5] From Eq. (24) it follows that this regime can only be realized when, $\eta \leqslant \sqrt{m_0 m_P}$.

1. Density inhomogeneities

The number M of open strings inside a horizon volume may be estimated as follows. The number of correlated domains inside the horizon is $\sim(H^{-1}/R)^3$, with $H^{-1}\sim t$ being the Hubble radius. Strings are expected to lie in the boundaries of the correlated domains. However, the open strings are one-dimensional objects and extend throughout the entire Hubble volume. Thus, the same string lies between the boundaries of $\sim(H^{-1}/R)$ domains while crossing the Hubble volume. Thus, the number of open strings inside the horizon is

$$M\sim\left(\frac{H^{-1}}{R}\right)^2. \tag{30}$$

The density contrast, due to strings on the horizon scale is, $\delta\rho\sim M(\mu t/t^3)\sim\rho_s$, where $\rho_s\sim\mu/R^2$. Therefore, the fractional density fluctuation of the string network at horizon crossing is (see also [19], p. 318),

$$\left(\frac{\delta\rho}{\rho}\right)_S\sim\frac{G\mu}{v_T^2}\geqslant G\mu. \tag{31}$$

However, the over-densities generated by the strings are smaller. Indeed, while moving through the plasma, the open strings generate wakes of over-dense matter due to the conical string metric [21]. The over-density inside a newly formed wake is of the order of $\delta\rho_W/\rho\simeq1$, its length is $l_W\sim v_T t$ and its thickness is $d_W\simeq v_T t\tan(\Delta/2)\sim(G\mu)v_T t$, where $\Delta\simeq8\pi G\mu$ is the deficit angle of the string metric. The linear mass over-density of a wake is $\delta\mu_W=\delta\rho_W d_W l_W\sim\rho(G\mu)(v_T t)^2$. Thus, since $R_{pl}\sim v_T t$, the total over-density due to open string wakes is

$$\left(\frac{\delta\rho}{\rho}\right)_W\simeq\frac{1}{\rho}\,\frac{\delta\mu_W}{R_{pl}^2}\sim G\mu. \tag{32}$$

The above shows that the over-densities generated by the string network are independent of the string velocity. This is so, because, although a denser network will create more wakes per horizon volume the length of such wakes will be shorter corresponding to filaments of small linear mass density. Thus, superconducting strings in grand unified theories (GUT's) could still account for the large scale structure observed even if they carry substantial currents.

2. Temperature anisotropies

Similar results are obtained for the temperature anisotropies in the microwave sky. The latter are produced due to the boost of radiation from the string deficit angle. The anisotropy generated by a single string is given by

$$\left(\frac{\Delta T}{T}\right)_S\sim(G\mu)\gamma v_T\leqslant G\mu, \tag{33}$$

where $\gamma\sim1$ is the Lorentz factor.[6]

However, the overall anisotropy includes contributions of all the strings that have crossed the line of sight until the present time. An analytical model to calculate the rms anisotropy from a string network is described in [22], where it is shown that, $(\Delta T/T)_{rms}\simeq(G\mu)\gamma v\sqrt{M}$. Thus, using Eq. (30) the rms temperature anisotropy in the case of a plasma scaling string network is

$$\left(\frac{\Delta T}{T}\right)_{rms}\simeq(G\mu)\gamma v_T\frac{H^{-1}}{R_{pl}}\sim G\mu. \tag{34}$$

The above shows that the rms anisotropy is again independent of the string velocity. Thus, GUT superconducting strings could generate the observed anisotropy regardless of their current. The only effect that a denser network would have on the pattern of temperature fluctuations would be to hide its non-Gaussian profile in smaller angular scales than the usual estimates of 1° [23], since the inter-string distance would be smaller than the horizon size at decoupling.

Superconducting strings could add to the temperature anisotropies by emitting radiation themselves, mostly due to the decay of loops or small scale structure. However, as shown in [3], the radiational contribution of the open string network is of minor importance. The latter could have an effect only on ultra high energy radiation [4] and be a γ-ray source.

The plasma-scaling solution for the open string network is a direct consequence of assuming that intercommuting produces loops with efficiency of order unity. This scaling solution, however, is much different from the usual horizon scaling of non-superconducting strings, since the network is denser with slower moving strings. Note that, in this case, the friction force never becomes negligible, i.e. *the friction domination era never ends* (see also Sec. VI B 2).

In the above calculations we have implicitly assumed that the overall energy density is not dominated by the loops produced by the network. This is not a trivial assumption since current carrying string loops can avoid total collapse by forming stable vortons which could have lethal consequences to the universe evolution [7]. However, vorton production is beyond the scope of this paper.

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[6]The inequality in Eq. (33) could be reversed for ultra-relativistic velocities. However, in order for this to occur one requires $\gamma v\geqslant1$ which gives $v\geqslant1/\sqrt{2}\sim0.7$. Simulations of cosmic strings have shown that the typical string velocity is $v\simeq0.2$ for ordinary non-superconducting strings [19]. This is due to the fact that such strings develop substantial microstructure (wiggly strings). In our treatment we do not consider such microstructure because, for current-carrying strings, it is suppressed due to electromagnetic radiation. Thus, although, in fact, wiggly strings are the limit of current-carrying strings for $J\rightarrow0$, we estimate the non-superconducting string velocity to be of order unity. Even so, it is realistic to assume that *the average coherent open-string velocity is never ultra-relativistic*, especially in the case of $J>J_c$ in which we are interested, when v_T can be several orders of magnitude smaller than unity.

V. IN A PRIMORDIAL MAGNETIC FIELD

It would be interesting to embed the whole network in a primordial magnetic field and observe how its evolution is going to be (if at all) affected. It is obvious that, in principle, the current carrying strings do interact with an external magnetic field, since, at a distance, they appear not too different from current carrying wires. Thus, the magnetic force per unit length on the strings is

$$f_B \sim JB\sin\theta, \tag{35}$$

where B is the magnitude of the external field and θ is the angle between the magnetic field lines and the string segment. Depending on θ the above force can accelerate or decelerate the string.

A. Magnetic force domination

The magnetic force can be compared with f_{ns} and f_{pl} in order to determine under which conditions it will be the dominating one. For simplicity, we will consider the high temperature case only.

If f_{ns} is the dominant friction force then by comparing Eqs. (11) and (35) (with $\sin\theta=1$) it can be found that the magnetic force will dominate at

$$t_c^B \sim \frac{\eta^{12}}{(B_0 J)^4 m_P}, \tag{36}$$

where B_0 is the magnitude of the magnetic field at network formation.

The magnetic force will never dominate provided the network reaches horizon scaling before t_c^B. The condition for this is

$$B_0 < \left(\frac{J_c}{J}\right)\eta^2. \tag{37}$$

If f_{pl} is the dominant friction force, then by comparing Eq. (12) with Eq. (35) we find that the magnetic force is subdominant if

$$B_0 < \sqrt{\frac{J_c}{J}}\eta^2. \tag{38}$$

In order for the magnetic field energy density $\rho_B \sim B^2/8\pi$ not to dominate the overall energy density of the universe, we require, $B_0 \leq \eta^2$. In view of Eqs. (37) and (38) this constraint implies that $J < J_c$ ensures that the magnetic field will never dominate the forces acting on the strings. Thus, *a weak current $J \leq J_c$ suggests that the string network will follow the standard non-superconducting string network evolution regardless of the existence of a primordial magnetic field*. On the other hand, in the case of a strong current the magnetic field can influence the network evolution provided it is stronger than the constraint (38).

B. String domination

If the magnetic force was dominant then the network would evolve according to its action on the strings. In this case however, the curvature radius would not grow larger than the coherence scale of the magnetic field R_B. Indeed, since the string tension is dominated by the magnetic force, there is no driving force to ''straighten'' the strings over larger scales. The incoherence of the magnetic field would twist the strings and curve them over scales of order R_B. Also, any loops with dimensions larger than R_B, will not contract. So, over these scales, no string length is lost and, effectively, there is no loop production. Thus, $R \propto a$ and we have string domination [24]. The above have been calculated in the high temperature case only. We expect similar results in low temperatures.

The ratio of energy densities is

$$\frac{\rho_s}{\rho} \sim \left(\frac{\eta t}{R_B m_P}\right)^2. \tag{39}$$

Thus, string domination could be avoided only if

$$R_B > \frac{\eta}{m_P} R_H, \tag{40}$$

where $R_H \sim t$ is the horizon size. For current values $R_B \sim 1$ kpc and $R_H \sim 10^4$ Mpc we find that we can avoid string domination if $\eta < 10^{11}$ GeV.

The above is dependent on the assumption that the string current J remains constant. However, if the external magnetic field is strong enough, it would affect the string current.[7] Still, even by taking into account the back-reaction of the external field on the string current, the qualitative picture is not significantly changed.

VI. THE VALIDITY OF THE ASSUMPTIONS

Our results are based on two assumptions. We have assumed that the magnetocylinder is impenetratable and free of plasma. Also, we assumed that the string current is constant and generated at the formation of the string network. These assumptions hold firmly in some regimes but could be unreliable in other. In what follows we try to explore their validity.

A. The magnetocylinder

In our description of the magnetocylinder in Sec. II B we have assumed that it is essentially free of plasma. Here we investigate whether substantial plasma can be trapped or generated inside the magnetocylinder and whether it can enter the latter by penetrating the magnetic shock front. We consider the case $J \geq J_c$ only since, in the opposite case, the magnetocylinder is irrelevant to the string kinetics.

1. Trapped plasma in the magnetocylinder

One question that needs to be addressed is whether any possible ''blocked'' plasma inside the magnetocylinder

[7]One could argue that, since the current grows as $\dot{J} \sim vB$ [1], it will eventually reach its maximum value, $J_{max} \sim \eta$ and from then on remain constant.

would have any effect on the dynamics of the string. Such blocked plasma could conceivably arise at the initiation of the string current. If this is the case, the addition to the string's mass per unit length would be

$$\mu_{pl}\sim\rho(T\sim\eta)\ \ r_s^2(T\sim\eta)\sim J^2. \tag{41}$$

Thus, since $J\leqslant\sqrt{\mu}$, μ_{pl} is, in general, negligible compared to the string linear density. Also, Eq. (41) suggests that the density of the magnetocylinder plasma would be, $\rho_{pl}(t)\sim\mu_{pl}/r_s(t)^2\sim\rho(t)$. Thus, the plasma density inside the magnetocylinder does not disturb the pressure balance since the magnetic pressure inside the magnetocylinder is dominant. Therefore, *the existence of initially trapped plasma inside the magnetocylinder does not seriously affect the string system.*

The situation is not as clear in the case of particle production from the string. Indeed, for fermionic charge carriers [1], in the presence of an external magnetic field, after the string current assumes its maximum value, the string produces particles at the rate

$$\frac{d^2N}{dt\ dl}\sim e^2\overline{v}B. \tag{42}$$

Thus, if m_0 is the particle mass, the plasma mass per unit length inside the magnetocylinder increases as

$$\dot{\mu}_{pl}\sim e^2 m_0\overline{v}B. \tag{43}$$

Taking $B\propto a^{-2}\propto t^{-1}$ and using Eq. (41) we obtain

$$\mu_{pl}\sim J^2+B_0\frac{m_0 m_P}{\eta^2}. \tag{44}$$

Thus, the linear mass density of the string system would be influenced only if

$$B_0>\eta^2\left[\frac{\eta}{\sqrt{m_0 m_P}}\right]^2. \tag{45}$$

For $\eta>\sqrt{m_0 m_P}$, the above suggests that we can never have μ_{pl} domination over μ.

Additional particle production inside the magnetocylinder may be generated by an ac component of the string current [11,32], created by the string small scale structure. ac currents, though, are likely to be suppressed due to radiational back-reaction [5,9,11]. Moreover, the small scale structure of the strings is expected to decay due to electromagnetic radiation emission [8].

2. Penetration of plasma in the magnetocylinder

Usually, it is assumed that the plasma, being a perfect conductor, is frozen into the magnetic field lines and, thus, could never penetrate the shock front of the magnetocylinder.[8] However, even with high conductivity, the dimensions of the magnetocylinder could be small enough for diffusion to be important.

The diffusion length is, $l_d\sim\sqrt{t/\sigma_c}$ [25], where $\sigma_c\sim T/e^2$ is the plasma conductivity at high temperatures. By comparing l_d with r_s, it can be easily shown that the time t_{diff} when diffusion becomes negligible is

$$t_{diff}\sim\left(\frac{e}{J}\right)^4 m_P^3. \tag{46}$$

By comparing the above with t_* and t_c we see that *diffusion becomes negligible only after a scaling solution is achieved.* This could be a very serious problem to our considerations. Fortunately, it can be shown that, even if diffusion allowed the plasma particles to penetrate, in most cases their Larmor rotation would eject them outwards before they could significantly penetrate the magnetic shield.

Indeed, as shown in [26], for a head-on collision of a particle with the magnetocylinder's magnetic field we have

$$\overline{v^2}-\dot{r}^2=\frac{1}{m^2r^2}+\left(\frac{J}{m}\right)^2\ln^2\left(\frac{r}{r_s}\right)\pm\frac{J}{m^2r}, \tag{47}$$

where $\overline{v}$ is the initial velocity of the in-falling particle and $r\geqslant r_s$.

In the above the coupling of the particle spin to the magnetic field has been taken into account. All the angular momentum and spin components have been taken to be of the order of $\hbar=1$.

If $r_s>1/J$, then the dominant term in Eq. (47) is the logarithmic one (apart from very near r_s where there is a potential well[9]). By solving Eq. (47) we obtain

$$r_{min}\sim r_s\exp\left(-\frac{m\overline{v}}{J}\right). \tag{48}$$

Given that $J\geqslant J_c$, when $\eta>\sqrt{m_0 m_P}$ we have $J>m\geqslant m\overline{v}$ during the plasma scaling solution. Thus, the penetration in the magnetocylinder is always negligible. On the other hand, if $\eta\leqslant\sqrt{m\ m_P}$ then penetration is negligible only if $J>m\overline{v}$.

If, $r_s<1/J$, then, near r_s, the dominant term in Eq. (47) is the inverse square one. By solving Eq. (47) we find

$$r_{min}\sim(m\overline{v})^{-1}, \tag{49}$$

[8]One possible caveat to this argument is that, at early times, i.e. before the electroweak phase transition, the existence of non-Abelian terms in the field equations may destroy the freezing condition even for a high conductivity. In [25] it has been shown that the criterion for the freezing condition to hold is roughly, $T^2\geqslant B_s$. It is easy to see that the above is indeed satisfied outside the magnetocylinder. Thus, provided the conductivity is high, the in-falling, non-Abelian plasma is frozen into the magnetic field.

[9]This potential well around r_s corresponds to quasi-bound states of plasma particles temporarily trapped by the string [27].

which is, in fact the non-superconducting string linear cross-section (8).

In view of Eq. (7), $r_s < 1/J$ implies that $\sqrt{\rho \overline{v}}/J^2 > 1$. However, from Eqs. (11) and (12) we have, $f_{pl} > f_{ns} \Rightarrow \sqrt{\rho \overline{v}}/J^2 < m\overline{v}/J$. Combining these two we see that $J \geqslant J_c$ and $r_s < 1/J$ can both be true only if $J < m\overline{v}$.

Thus, we have shown that, for $J \geqslant J_c$, penetration of plasma particles in the magnetocylinder is possible only if $J < m\overline{v}$ and $\eta < \sqrt{m_0 m_P}$. When this happens, the particles ''see'' the string core as implied by Eq. (49). In all cases $J > m_0$ is sufficient to ensure that the particles cannot penetrate the string magnetocylinder during plasma friction and, therefore, in this regime, our assumption is valid. The case $J < m_0$ could arise in non-realistic systems, but not in the usual particle theories.

B. The string current

In the above we have assumed that the current switches on at the time of formation of the string network and remains constant during the subsequent network evolution. In this paragraph we take a closer look at these assumptions.

1. Current conservation

Witten has suggested that the superconducting strings will most probably carry a strong dc current, which would persist due to topological index theorems [1]. Indeed, the current in this case is [1,19,28]

$$J_a = \frac{1}{eW} \partial_a \psi, \tag{50}$$

where $W \approx 2\ln(\Lambda R) \sim 100$ is the self-inductance of the string (with Λ^{-1} being the string width and R a suitable cut-off radius identified with the inter-string distance) and ψ is the phase of the scalar field that breaks electromagnetism inside the string. Since the latter cannot unwind for topological reasons the gradient is kept constant and the current is conserved.

In general, current conservation is a direct outcome of the field equations for a straight superconducting string. This can be immediately deduced from the string action, which due to the current includes the term [9,29–32]

$$\Delta S = -c \int d^2\xi \sqrt{-\gamma} J^2, \tag{51}$$

where ξ are the string world-sheet coordinates, γ is the determinant of the world-sheet metric, and c is a constant related to the charge of the current carriers. Varying the above action immediately gives the current conservation equation

$$\partial_a J^a = 0. \tag{52}$$

The above correspond to a straight, infinite string. However, realistic strings are much more complicated, since they are irregularly tangled and carry random-walk currents.

The string current on such strings has its own correlation length l. The orientation of the current on larger scales follows a one-dimensional random walk pattern. Thus, between two points with string length distance $L > l$, the average string current is $J_{rms} \sim J/\sqrt{(L/l)}$, where J is the local, coherent current inside a correlated string segment. Current conservation, suggests that the overall current J_{rms} remains constant if the string length is unaltered. Therefore, the local current has to diminish with time as l grows.[10] It is important to note that in our calculations we are dealing with the local current J, since it is this current that generates the magnetic field responsible for the existence of the string magnetocylinder and, thus, determines the magnitude of the plasma friction force.

Another important point is the fact that Brownian contraction gradually decreases the string length L between two distant fixed points on the string. Thus, $J_{rms} L = \text{const}$ and J_{rms} is expected to increase. Since, $L \sim d^2/R$ [19] where d is the distance between the points, this process suggests that, $\dot{J}_{rms}/J_{rms} \sim \dot{R}/R$. Putting together the two effects described above we see that the increase of J_{rms} due to Brownian contraction may be enough to hold the local current J more or less constant, although its coherence length would grow.

The balance between the two effects depends delicately on the growth rate of the current coherence length, which is as yet unknown. A reasonable guess would be that the correlation length l grows with the speed of light, since, inside the string, the charge carriers are massless. Now, from Eq. (5) we have $\dot{R} \leqslant 1$. Thus, if l grows with light-speed, *the current will always be coherent over smooth string segments*, i.e. segments of length $L \leqslant R$. This justifies the fact that we use the local, coherent current for our calculations since the rms current is relevant only for scales larger than R, i.e. much larger than the dimensions of the magnetocylinder.

Balancing the above effects gives, $J_{rms} L \sim \sqrt{LtJ}$. Thus, the time dependence of the current is

$$-2\frac{\dot{J}}{J} = \frac{1}{t} - \frac{\dot{R}}{R}. \tag{53}$$

The above suggests that the current is indeed constant at a scaling solution, when $R \propto t$. Thus, although the current will be time dependent during f_{ns} domination our results are unaffected since, in this period, the existence of a current does not influence the evolution of the network.

By means of Eq. (53), the initial critical current for the network can be found with the use of Eqs. (18) and (24) as

$$J_c^0 \sim J_c (G\mu)^{1/8} \tag{54}$$

for both low and high temperatures.

The evolution of the current magnitude on the open strings is still unclear. If the current was not kept constant, then its variation could destabilise the delicate balance of the plasma-scaling solution. In the case that the string current decreased with time, f_{pl} would become less effective, the curvature radius would grow faster than in Eq. (17) and, thus, it would eventually catch up with the horizon resulting in horizon scaling. If the opposite was true and the current increased with time, then plasma friction would become larger and the curvature radius would not be able to follow

[10]The growth of the current coherence length is due to the algebraic addition of the current at the interface between two initially uncorrelated domains, as have been shown numerically in [10].

the growth of the horizon at a constant ratio. Instead the strings would become slower and the network denser. However, the growth of J would have to end when it reached J_{max}. Then, the network would assume the plasma-scaling solution at its maximum density case. In all cases, though, we still manage to avoid domination of the universe energy density from the open string network.

2. Initiation of the current

Although we have assumed that the current switches on at network formation, in general, this could occur at a later stage. However, even if the network evolves initially without the presence of a current, the final picture is not severely altered.

Indeed, were this the case, the network would initially evolve according to the standard cosmic string evolution scenario. Thus, the curvature radius would grow as in Eq. (15) until it reached horizon scaling. If $t_* < t_c$, or equivalently $J < J_c$, the network would reach horizon scaling regardless of when the current switches on since the latter would have no effect of the network dynamics. If, however, $t_* \geqslant t_c$, or equivalently $J \geqslant J_c$, the plasma friction force will always eventually dominate, as mentioned in Sec. IV. Indeed, in this case, if the time t_s, when current switched on, is before t_c then the evolution is identical to the one described in Sec. III for $J \geqslant J_c$. If, however, $t_s > t_c$, then the string network remains comovingly frozen due to excessive plasma friction, until the length-scale given by Eq. (17) reaches the size of the network curvature radius. From then on the evolution continues again as described earlier. This is expected regardless of whether the network has reached horizon scaling before t_s or not. Thus, for $J > J_c$, *in case* $t_s > t_*$ *the horizon-scaling solution is terminated and the network remains comovingly frozen until the plasma-scaling solution begins*.

Switching on the current in later times could have an effect on the magnitude of J, since the latter cannot be larger than the energy scale at the time. Indeed, suppose that the current switched on at the temperature, $T_s \sim \zeta\eta$, where $\zeta \leqslant 1$. Then, over a string segment of dimensions of the order of the curvature radius $R(t_s)$, the maximum average current at the current initiation time t_s would be, $(J_{rms})_{max} \sim \zeta\eta/\sqrt{n}$, where $n \sim R(t_s)/(\zeta\eta)^{-1}$. Since R, until t_s, would evolve according to Eq. (15), the current would be

$$(J_{rms})_{max} \sim \zeta^{7/4}\eta\left(\frac{\eta}{m_P}\right)^{1/4}. \tag{55}$$

Assuming that the coherence length of the current grows with light-speed, after some time the current in the segment considered will become coherent with magnitude $J \simeq J_{rms}$. Thus, Eq. (55) will be the maximum possible string current. Comparing with J_c of Eq. (24) we find that, in order for the current to affect the string network evolution ζ has to be greater than

$$\zeta_c \equiv \left(\frac{\eta}{m_P}\right)^{3/7}. \tag{56}$$

For GUT strings $\zeta_c \sim 10^{-2}$. Thus, the evolution of GUT strings that become current carrying at the electroweak transition [33] ($\zeta \sim 10^{-14}$) cannot be affected by their current.

VII. DISCUSSION AND CONCLUSIONS

In conclusion, we have investigated the evolution of a charged-current carrying, open string network. We have shown that, in the absence of a primordial magnetic field, the network always reaches a scaling solution. This ensures that *the network does not dominate the energy density of the universe*. If the network is embedded in a strong enough magnetic field, then it is possible that it will never reach a scaling solution and will dominate the energy density of the universe.

We have also demonstrated that, in all cases considered, the existence of a current on the strings will have an effect only if the current is larger than a critical value J_c, given in Eq. (24). We have found a similar critical value for the possible influence of a primordial magnetic field. It is interesting that J_c is also the critical current with respect to radiation emission from the string, over which electromagnetic radiation dominates gravitational radiation [34].

For string current stronger than J_c we have shown that friction never ends and the scaling solution is very different than the standard cosmic string horizon scaling, regardless of the time the string current initiates. The curvature radius and the inter-string distance follow the horizon growth in constant proportion, but they could be much smaller than the horizon size. As a consequence, the string network would be a lot denser. Inside a horizon volume the strings would be more curved and twisted and would move much slower. Thus, the loops produced by the network are smaller, although the intercommuting rates are unaffected. Therefore, even though the network is denser, it loses less mass by loop production and this is why so much of the string length is kept in the open strings. We called this scaling solution plasma scaling.

If the network reached plasma scaling then there are a number of consequences that may have observational importance. First of all, production of smaller loops could relax the vorton constraints [7]. Also, a denser network would generate large scale structure with much different features than the one produced by a horizon scaling network. Indeed, the slow moving strings would create filaments instead of thin wakes, whose separating distances could be much smaller than the horizon. Also the imprint of the strings on the microwave sky would be Gaussian in smaller angular scales than the horizon scale at decoupling. However, neither the magnitude of the overall density perturbations or the rms temperature fluctuations will be affected. Therefore, GUT superconducting strings can still satisfy observations even if they carry a substantial current.

The plasma scaling solution could also have important astrophysical effects. Indeed, a denser superconducting string network would result in substantial generation of high energy radiation [2,4–6]. Agreement with observations by adjusting accordingly the parameters of the model could provide information on the underlying theory.

The evolution of the open string network described in our paper is expected to be modified in the matter era due to streaming velocities developed by the plasma during the gravitational collapse of the protogalaxies. Such streaming velocities will tangle the strings since, if friction is dominant, the latter are more or less ‘‘glued’’ to the plasma [35]. The situation resembles the case of a dominant primordial magnetic field. The network curvature radius and inter-string distance would follow the scale of the plasma streaming and this could lead to string domination.

In our treatment we have made a number of assumptions. Since, in order to explore the evolution of the curvature radius, we were primarily interested in larger scales, we chose to ignore the small scale structure of the strings and its consequences (ac currents, string linear energy density and tension renormalisation) since it is likely to be substantially suppressed by radiation back-reaction and particle production. We have also assumed that the string magnetocylinder is impenetratable and free of plasma. We showed that this assumption is valid for $J \geqslant m_0 \sim 1$ GeV.

One fundamental assumption concerns current conservation. We have argued that the local string dc current would remain more or less constant. However, more work is required here, particularly on the evolution of the current's coherence length and the current's ac component. However, although a variable current may influence our results, it would never lead to string domination.

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