

## Double-scattering contribution to $b_1(x, Q^2)$ in the deuteron

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We study the tensor structure function  $b_1(x, Q^2)$  in deep inelastic scattering of an electron from a polarized deuteron target. We model the electron-nucleon cross section at the starting point for  $Q^2$  evolution by vector-meson dominance. Shadowing due to the double scattering of vector mesons, along with the presence of a  $d$ -state admixture in a ground-state deuteron wave function gives rise to a nonvanishing contribution to  $b_1(x, Q^2)$ . We find a large enhancement at low  $x$  in qualitative agreement with other recent estimates of double-scattering contributions to  $b_1(x, Q^2)$ . If the model is valid, it should apply within the range of present fixed-target experiments. [S0556-2821(98)06111-6]

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### I. INTRODUCTION

Deep inelastic scattering from polarized targets continues to excite interest among both theorists and experimentalists. When an electron scatters off a spin-one target, such as a deuteron, new information not present in the case of a spin- $\frac{1}{2}$  target can be obtained [1]. A leading twist tensor-polarized structure function,  $b_1(x, Q^2)$ , can be determined by measuring the cross section from a target aligned along the beam, and subtracting the cross section for an unpolarized target. The function  $b_1(x, Q^2)$  vanishes if the spin-one target is made up of spin- $\frac{1}{2}$  nucleons at rest or in a relative  $s$  wave. In the parton model it measures the difference in the quark momentum distributions of a helicity 1 and 0 target,

$$b_1 = \frac{1}{2}(2q_{\uparrow}^0 - q_{\uparrow}^1 - q_{\downarrow}^1), \quad (1)$$

where  $q_{\uparrow}^m$  ( $q_{\downarrow}^m$ ) is the probability to find a quark with momentum fraction  $x$  and spin up (down) along the  $z$  axis, in a hadron (nucleus) with helicity  $m$ , moving with infinite momentum along the  $z$  axis.  $b_1(x, Q^2)$  has not yet been measured experimentally. Two recent papers have studied the effect of multiple scattering on  $b_1$  and found large contributions at small  $x$ . Our aim is to explore these issues in the context of vector-meson dominance (VMD), where some of the uncertainties evident in Refs. [2,3] are more explicit. Within the range of these uncertainties we find that multiple scattering does produce a large contribution to  $b_1(x, Q^2)$  at small  $x$ . Our estimates are smaller than those of Refs. [2,3] by factors of 1.5 – 2.5, differences which are not unexpected given the conceptual differences between their approaches and ours. Nikolaev and Schäfer [2] use the pomeron structure function in the proton to extract the diffractive shadowing

contribution. Their results have been presented for  $Q^2 = 10 \text{ GeV}^2$ . Edelmann *et al.* [3] estimate  $b_1$  by expressing it in terms of  $\delta F_2/F_2$ . Our analysis does not support such a simple scaling relation between  $b_1$  and the shadowing of  $F_2$ , although the two originate in the same double-scattering mechanism. Furthermore, the authors of Ref. [3] do not specify the scale at which their results should apply. Given these differences of approach, we view our results as qualitative confirmation of the work of Refs. [2,3] in a specific, rather well defined, model.

Deep inelastic scattering (DIS) from nuclear targets is usually discussed in the context of the ‘‘convolution model’’ [4], where it is assumed that the constituents of the nucleus scatter incoherently. An essential assumption of the convolution model is that a quark residing inside the nucleon absorbs the virtual photon (in a typical DIS process), while the fragments of nucleus and the constituents propagate into the final state without interaction or interference. In the convolution model  $b_1$  vanishes if the  $d$ -state admixture in the deuteron is ignored [1]. The contribution to  $b_1$  from the deuteron  $d$  state was studied in Ref. [5], along with the contribution from double scattering from the two nucleons, which amounts to a coherent contribution to the amplitude. In Ref. [5] the double-scattering process was studied at the parton level.

In the present work, we investigate the behavior of  $b_1(x, Q^2)$  in a vector-meson dominance (VMD) [6] model. Of course deep inelastic scattering at large  $Q^2$  should be discussed in terms of quarks and gluons. If taken literally at large  $Q^2$ , VMD has the wrong  $Q^2$  dependence. VMD can be used, however, to provide ‘‘boundary value data’’ — i.e., starting values for parton distribution functions — at low  $Q^2$  where the assumptions of VMD are well founded. We choose VMD because it lends itself to the treatment of multiple-scattering effects that violate the convolution model and may give rise to a significant contribution to  $b_1$ . Also,

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because vector-meson production data are available, cross sections necessary for our analysis can be found in the literature. The cost of this increased certainty is the need to identify a scale  $Q_0^2$  to be assigned to the output of the model.

Double scattering (which we will refer to as ‘‘shadowing’’) and the  $d$ -state admixture in the deuteron play a crucial role in our VMD treatment as they do in Refs. [5,2,3]. According to VMD, the virtual photon can fluctuate between a bare photon state and a superposition of hadronic states with same quantum numbers as the photon ( $J^{PC} = 1^{--}$ ). In the simplest form of VMD this state is taken to be a superposition of  $\rho$ ,  $\omega$ , and  $\phi$  mesons,

$$\sqrt{\alpha}|h\rangle = \sum_V \frac{e}{f_V} \frac{m_V^2}{m_V^2 + Q^2} |V\rangle, \quad (2)$$

where  $em_V^2/f_V$  is the photon vector-meson coupling,  $\sqrt{\alpha}|h\rangle$  is the hadronic component of the photon, and  $Q^2$  is the virtuality of the spacelike virtual photon. As usual in VMD, we assume that the vector meson interacts diffractively with the nucleon and that the  $t$  dependence of the VMD amplitudes can be taken from vector-meson photoproduction.

The VMD contribution to  $b_1$  is constrained at both large and small  $x$  by simple physical effects. For multiple-scattering effects to be significant, the time (or distance) over which the virtual vector meson can propagate through the target nucleus (known as the ‘‘coherence length’’  $\equiv \lambda$ ) must be long enough for the meson to undergo more than one interaction with the target. We shall see that the coherence length is determined kinematically by the uncertainty principle. At large Bjorken  $x$  ( $x \geq 0.3$ )  $\lambda$  is smaller than a single nucleon, so double-scattering contributions to  $b_1$  are suppressed. At small  $x$ , double scattering can be important. In order to contribute to  $b_1$  it must distinguish between the helicity  $\pm 1$  and helicity 0 states of the deuteron. If the amplitude for  $\gamma^* p \rightarrow VX$  fell quickly with (transverse) momentum transfer, corresponding to long range in impact parameter space, then shadowing could not distinguish the orientation of the nucleons in the deuteron and  $b_1$  would be small at small  $x$ . At the opposite extreme, if  $\gamma^* p \rightarrow VX$  were flat in momentum transfer, corresponding to a  $\delta$  function in impact parameter, then shadowing would occur only when one nucleon was directly ‘‘in front of’’ the other. The quadrupole admixture in the deuteron wave function produces just such a deformation of the wave function in one helicity state relative to the other. The deuteron is a relatively large bound state, and the range of vector-meson electroproduction is limited, so the actual situation most closely resembles the second scenario and leads to a significant enhancement in  $b_1$  at small  $x$ .

The paper has been organized as follows. In Sec. II, we present the theoretical formulation of the model, reviewing VMD and the double-scattering analysis. Section III contains calculations and results. Throughout the paper, we have attempted to keep the analysis simple and self-explanatory.

## II. FORMULATION OF THE MODEL

$b_1$  measures a tensor (spin-two) correlation of the momentum distribution of quarks with the spin of the target in

DIS. Such a correlation must vanish in a spin- $\frac{1}{2}$  target on account of the Wigner-Eckart theorem. In principle, any spin-one target can have a nonvanishing  $b_1$ . Two nucleons bound in an  $s$ -wave cannot give  $b_1 \neq 0$ . What is not so obvious, perhaps, is that a  $d$ -state admixture in a  $J=1$  bound state of two nucleons will generically give  $b_1 \neq 0$  [1,5]. It is well known that the ground state of deuteron is not spherically symmetric: it is (primarily) an admixture of the states  ${}^3S_1$  ( $\ell=0, S=1$ ) and  ${}^3D_1$  ( $\ell=2, S=1$ ). This admixture produces (and was first detected through) the observation of an electromagnetic quadrupole moment of the deuteron. The observation of a nonzero  $b_1$  through the asymmetry  $b_1/F_1$  probes the same aspects of the nucleon-nucleon force as does the deuteron’s quadrupole moment.

To expose the physical significance of various structure functions, it is useful to describe Compton scattering in terms of helicity amplitudes. The lepton scattering cross section from a hadron target involves the hadron tensor

$$W_{\mu\nu}(p, q, H_1, H_2) = \frac{1}{4\pi} \int d^4x e^{iq \cdot x} \langle p, H_2 | [J_\mu(x), J_\nu(0)] | p, H_1 \rangle, \quad (3)$$

which is the imaginary part of the forward current-hadron scattering amplitude. Here  $H_1$  and  $H_2$  are components of the target spin along a quantization axis and  $J_\mu$  is the electromagnetic current.  $W_{\mu\nu}$  can be decomposed into a set of four linearly independent structure functions for a spin-half target using parity and time-reversal invariance, while for a spin-one target, the number of linearly independent structure functions is eight [1]. Thus,  $b_1(x, Q^2)$  can be related to the helicity amplitudes  $A_{h_1 H_1, h_2 H_2}$  for the process  $h_1 + H_1 \rightarrow h_2 + H_2$ , where  $h_j$  ( $H_j$ ) labels the helicity of the photon (target), and

$$A_{h_1 H_1, h_2 H_2} = \epsilon_{h_2}^{*\mu} \epsilon_{h_1}^\nu W_{\mu\nu}, \quad (4)$$

$\epsilon_h^\mu$  is the polarization vector of photon of helicity  $h$ . It can be shown that

$$b_1(x, Q^2) = \frac{1}{2} (2A_{+0,+0} - A_{++,++} - A_{+-,+-}). \quad (5)$$

The structure functions  $W_1$  and  $W_2$  in unpolarized DIS of an electron from a proton target can be described in terms of the photoabsorption cross section  $\sigma_T$  and  $\sigma_L$  for transverse (helicity  $\pm 1$ ) and longitudinal (helicity 0) photons, respectively, as

$$W_1 = \frac{K}{4\pi^2 \alpha} \sigma_T, \quad (6)$$

$$W_2 = \frac{K}{4\pi^2 \alpha} (\sigma_T + \sigma_L) \frac{Q^2}{Q^2 + \nu^2}. \quad (7)$$

$K$  is the incident virtual photon flux,  $\nu$  is its energy in the laboratory frame.

Taking suitable combinations of helicities we can separate out  $b_1$ . We separate the contributions to  $b_1$  into single- and

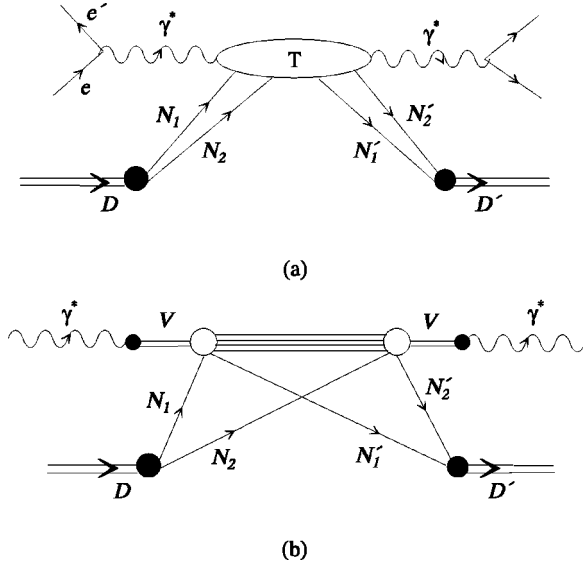


FIG. 1. Double-scattering diagrams in deuteron, with coherent contributions to amplitude from proton and neutron.

double-scattering terms. The single-scattering terms are given by the convolution formalism and reflect the  $d$ -state admixture in the deuteron ground state. [1,5]. We put these aside and focus on the double-scattering contributions, which are given by

$$b_1^{(2)}(x, Q^2) = \frac{Q^2}{8\pi^2 x \alpha} \{ \delta\sigma_{\gamma^*D}^{(2)}|_{m=0} - \delta\sigma_{\gamma^*D}^{(2)}|_{m=1} \}, \quad (8)$$

where  $\delta\sigma_{\gamma^*D}^{(2)}$  signifies the double-scattering shadowing correction contribution to the deuteron cross section:

$$\sigma_{\gamma^*D} = \sigma_{\gamma^*p}^{(1)} + \sigma_{\gamma^*n}^{(1)} + \delta\sigma_{\gamma^*D}^{(2)}. \quad (9)$$

In all cases  $\sigma$  refers to the cross section for transverse photons — the subscript  $T$  has been dropped for simplicity.

Glauber multiple-scattering theory [7] is usually used to describe the interaction of high-energy particles with nuclei. The basic assumption of the Glauber treatment is that the amplitude for a high-energy hadron to interact with a nucleus can be built from the scattering amplitude off individual nucleons. Here we shall employ another analysis [8] that uses a Feynman diagram technique, which reduces to the optical model results of Glauber theory in the limit of large  $A$  with a general one-particle nuclear density. The double-scattering contribution to photon-nucleus scattering can be represented diagrammatically as shown in Fig. 1(a). The following assumptions are made in this analysis: spin and any other internal degrees of freedom are neglected (except where necessary to isolate  $b_1$ ), all the nucleons are assumed to be equivalent, the momentum transfer from the incident hadron (here the vector meson) to target nucleus is small, the nucleus and nucleons are nonrelativistic. The vector mesons act as the intermediate states during the double scattering. Therefore the double-scattering picture looks as in Fig. 1(b), and the singularities of the amplitude  $T$  as a function of momenta of the intermediate vector mesons  $V$  are isolated. They correctly correspond to the propagation of the vector

meson between nucleons. We fix the momentum of the virtual photon to be, in the  $z$  direction,

$$q^\mu = (\nu, 0, 0, \sqrt{Q^2 + \nu^2}), \quad (10)$$

and define  $\vec{x} = (\vec{b}, z)$ . The momentum  $V^\mu$  of the vector meson can be seen to be

$$V^\mu = q^\mu + P^\mu - P'^\mu = (\nu, k_x, k_y, q_z + k_z), \quad (11)$$

where  $P^\mu, P'^\mu$  are the four-momenta of initial and final nucleon states,  $N$  and  $N'$ , respectively, and  $t = -(P - P')^2 = k^2$  is the momentum transfer squared. Finally, the expression for the double scattering takes the form

$$A_{\gamma^*D}^{(2)} = \frac{-1}{(2\pi)^3 2M} \int d^2b dz |\psi(\vec{b}, z)|^2 \sum_V \int d^3k T'_{\gamma^*N \rightarrow NV} \times \frac{e^{i(k_z z + \vec{k}_\perp \cdot \vec{b})}}{\nu^2 - M_V^2 - \vec{k}_\perp^2 - (q_z + k_z)^2 + i\epsilon} T'_{NV \rightarrow \gamma^*N}, \quad (12)$$

here  $N, V$  represent the nucleon and intermediate vector meson, respectively,  $T'_{\gamma^*N \rightarrow NV}$  is the production amplitude for vector mesons ( $\rho^0, \omega, \phi$ ),  $M$  is nucleon's mass, and  $M_V$  is the mass of vector meson. The amplitude  $T'$  depends on the momentum transfer in the  $t$  channel — the subprocess  $\gamma^*N \rightarrow NV$  is not limited to the *forward* direction. For small  $t$ ,  $t \approx -\vec{k}_\perp^2$ , so the  $t$  dependence determines the range of shadowing in impact parameter space. Even if the nucleons are misaligned in  $\vec{b}$  space by a distance of the order of the range of  $\gamma^*p \rightarrow VX$ , the vector meson can still undergo a second interaction with the other nucleon. If the range of the production amplitude in  $\vec{b}$  were smaller than the deuteron wave function, then we could approximate  $T'$  by its value at  $t=0$ . Since this is not the case we shall have to integrate over  $t$ . The vector meson is not on-shell, hence the  $i\epsilon$ . The energy of the meson state is given by  $E_V = \sqrt{M_V^2 + q_z^2}$  (for  $\nu^2 \gg |k|^2$ ),  $q_z^2 = Q^2 + \nu^2$ . The energy difference that defines the *virtuality* of the vector meson is therefore given by  $\Delta E = \sqrt{M_V^2 + \nu^2 + Q^2} - \nu$ , which for large values of photon energy can be written as  $\Delta E = (Q^2 + M_V^2)/2\nu$ . Therefore the vector mesons can exist for a time  $\Delta t \sim 1/\Delta E$ , and can propagate a distance  $\lambda$ , called its coherence length,  $\lambda = \Delta t = 1/\Delta E$ :

$$\lambda = \frac{2\nu}{M_V^2 + Q^2} = \frac{Q^2}{Mx(M_V^2 + Q^2)}. \quad (13)$$

For significant shadowing or double scattering to occur, the coherence length of the intermediate vector meson should be of the order of the typical internucleon separation in the nucleus,  $\sim 1.7$  fm. Thus, these effects increase as  $x$  decreases. Multiple scattering is most prominent at small  $x$  and for low-mass vector mesons. One can thus justify the use of only lowest mass vector mesons in the present case. Our model resembles partonic approaches to low- $Q^2$  shadowing (see, for example, [9]), where the virtual photon converts to a  $q\bar{q}$  pair at a distance before the target proportional to  $1/x$  in the laboratory frame. Shadowing is then explained in terms

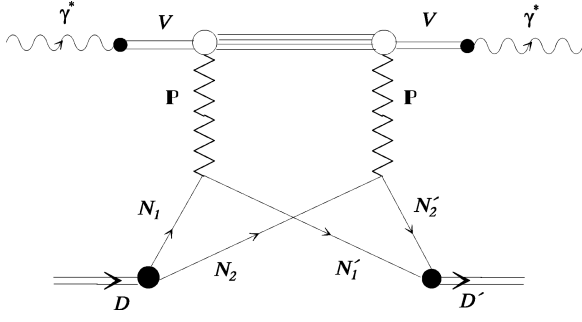


FIG. 2. Double-scattering diagram, showing production of intermediate vector mesons  $V$  via pomeron exchange.

of  $\bar{q}$ -nucleon scattering amplitude. The symmetric  $q\bar{q}$  pairs at not too large  $Q^2$ , with a transverse separation  $\sim 1/\sqrt{Q^2}$  can be viewed as a meson, the strong color interaction between quark and antiquark increasing with increasing separation.

Now we return to Eq. (12). The optical theorem relates the total cross section to the imaginary part of the forward-scattering amplitude as  $\delta\sigma_{\gamma^*D} = (1/W_D^2) \text{Im} A_{\gamma^*D}^{(2)}|_{t=0}$ , with  $W_D^2$  the total center-of-mass energy of the  $\gamma^*D$  system,  $W_D^2 = 2W_N^2$ ,  $W_N^2 = (p+q)^2 \equiv 2M\nu - Q^2$ . To simplify Eq. (12), we carry out the  $k_z$  integration. Given the sign of the exponential, only the singularity in the upper half of the  $k_z$  plane contributes. Since the vector meson interacts diffractively with the nucleon, the double-scattering diagram looks as shown in Fig. 2. The optical theorem relates the resulting on-shell amplitude to the differential cross section for vector-meson photoproduction,

$$\frac{d\sigma}{dt} \Big|_{t=k^2} = \frac{1}{16\pi} \sum_V \frac{|T'_{\gamma^*N \rightarrow NV}|_{t=k^2}^2}{W_N^4}, \quad (14)$$

where

$$\frac{d\sigma}{dt} \Big|_{t=k^2 \approx -\tilde{k}_\perp^2} = \frac{d\sigma}{dt} \Big|_{t=0} e^{-a\tilde{k}_\perp^2}.$$

We estimate the  $t$  dependence from photoproduction data where  $a \approx 10.4, 10.0$ , and  $7.3 \text{ GeV}^{-2}$  for  $\rho, \omega$ , and  $\phi$  vector mesons, respectively.

Next, we consider the deuteron form factor terms in Eq. (12). We can write the deuteron wave function as mixture of  $s$  and  $d$  states ( $m=1$ )

$$\begin{aligned} \psi_{m=1} = & \frac{u_0(r)}{r} Y_0^0(\Omega) \chi_1^1 \\ & + \frac{u_2(r)}{r} \left\{ \sqrt{\frac{3}{5}} Y_2^2(\Omega) \chi_1^{-1} - \sqrt{\frac{3}{10}} Y_2^1(\Omega) \chi_1^0 \right. \\ & \left. + \sqrt{\frac{1}{10}} Y_2^0(\Omega) \chi_1^1 \right\}, \end{aligned} \quad (15)$$

where the  $Y$ 's are the spherical harmonics and the  $\chi$ 's are the spin-wave functions. Using the orthogonality of the  $\chi$  functions, we get

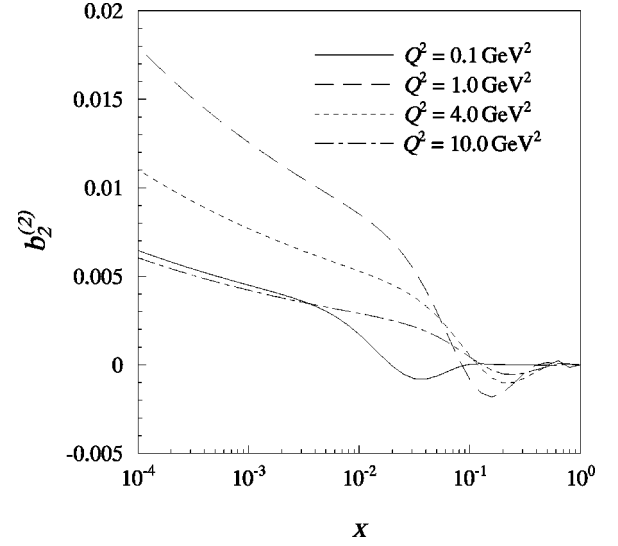


FIG. 3. Behavior of  $b_2^{(2)}(x, Q^2)$  with  $x$  using Eq. (20) at  $Q^2 = 0.1, 1.0, 4.0$ , and  $10.0 \text{ GeV}^2$ , with Bonn potential for deuteron.

$$\begin{aligned} |\psi_{m=1}|^2 = & \frac{u_0^2}{r^2} Y_0^0 * Y_0^0 + \sqrt{\frac{1}{10}} \frac{u_0 u_2}{r^2} Y_0^0 * Y_2^0 \\ & + \frac{u_2^2}{r^2} \frac{3}{5} Y_2^2 * Y_2^2 + \frac{3}{10} \frac{u_2^2}{r^2} Y_2^1 * Y_2^1 \\ & + \sqrt{\frac{1}{10}} \frac{u_0 u_2}{r^2} Y_2^0 * Y_0^0 + \frac{1}{10} \frac{u_2^2}{r^2} Y_2^0 * Y_2^0, \end{aligned} \quad (16)$$

with the wave functions normalized by

$$\int_0^\infty dr [u_0^2(r) + u_2^2(r)] = 1. \quad (17)$$

Similarly for  $m=0$ ,

$$\begin{aligned} |\psi_{m=0}|^2 = & \frac{u_0^2}{r^2} Y_0^0 * Y_0^0 - \sqrt{\frac{2}{5}} \frac{u_0 u_2}{r^2} Y_0^0 * Y_2^0 \\ & + \frac{3}{10} \frac{u_2^2}{r^2} Y_2^{-1} * Y_2^{-1} + \frac{2}{5} \frac{u_2^2}{r^2} Y_2^0 * Y_2^0 \\ & + \frac{3}{10} \frac{u_2^2}{r^2} Y_2^1 * Y_2^1 - \sqrt{\frac{2}{5}} \frac{u_0 u_2}{r^2} Y_2^0 * Y_0^0. \end{aligned} \quad (18)$$

Subtracting Eq. (16) from Eq. (18) gives

$$|\psi|_{m=0}^2 - |\psi|_{m=1}^2 = \frac{-3}{4\sqrt{2}\pi} \frac{u_0(r)u_2(r)}{r^2} (3\cos^2\theta - 1). \quad (19)$$

Combining the results summarized in Eqs. (8), (12), (14), and (19) the final expression for the function  $b_2^{(2)}(x, Q^2)$  ( $= 2xb_1^{(2)}$ ) emerges as

$$b_2^{(2)}(x, Q^2) = \frac{-3}{\pi^4} \frac{Q^2}{16\sqrt{2}\alpha} \text{Im} i \int d^2b \int dz u_0(r) u_2(r) \frac{2z^2 - b^2}{(z^2 + b^2)^2} \sum_V \int d^2k_\perp e^{iz/\lambda} e^{ik_\perp \cdot \vec{b} - a\vec{k}_\perp^2} \frac{M_V^4}{(M_V^2 + Q^2)^2} \frac{d\sigma}{dt} \Big|_{\gamma N \rightarrow VN, t=0} \quad (20)$$

Note that the crucial quadrupole factor  $(3\cos^2\theta - 1)$  translates into  $(2z^2 - b^2)$  in Eq. (20). With similar arguments, the shadowing contribution to the unpolarized structure function can be shown to be

$$\begin{aligned} \delta F_1^{(2)} = & \frac{-Q^2}{16\pi^4\alpha} \text{Im} i \int d^2b \int dz \frac{1}{z^2 + b^2} \left\{ u_0^2(r) + \frac{3}{4} u_2^2(r) \frac{b^4}{(z^2 + b^2)^2} \right\} \\ & \times \sum_V \int d^2k_\perp e^{iz/\lambda} e^{ik_\perp \cdot \vec{b} - a\vec{k}_\perp^2} \frac{M_V^4}{(M_V^2 + Q^2)^2} \frac{d\sigma}{dt} \Big|_{\gamma N \rightarrow VN, t=0} \end{aligned} \quad (21)$$

Since the diffractive photoproduction of vector mesons takes place via pomeron exchange, the differential cross section for forward scattering is of the form  $d\sigma/dt|_{\gamma^*N \rightarrow VN, t=0} \sim W^{4[\alpha_P(0)-1]}$ , where  $\alpha_P(t=0) = 1 + \delta$  is the soft pomeron intercept. Thus it can be seen that the scaling violations in  $b_1^{(2)}(x, Q^2)$  are of the order of  $1/Q^{2(1-2\delta)}$ , and the contribution vanishes at large  $Q^2$ . In these models, the structure function vanishes at large  $Q^2$  and scaling can be restored within the context of the model only if one takes into account the continuum of heavier mesons (GVMD). Rather, we take the point of view that VMD should not describe the  $Q^2$  dependence because it is intrinsically a low- $Q^2$  effective theory. VMD provides an estimate of certain (in this case multiple scattering) contributions to the structure function at a low scale, which are then mapped into the large- $Q^2$  domain by standard QCD evolution.

### III. CALCULATIONS AND RESULTS

The resulting behavior of  $b_1^{(2)}(x, Q^2)$  using Eq. (20) is shown in Figs. 3 and 4. We have used the Bonn potential [10] for deuteron wave function in the calculations. The dif-

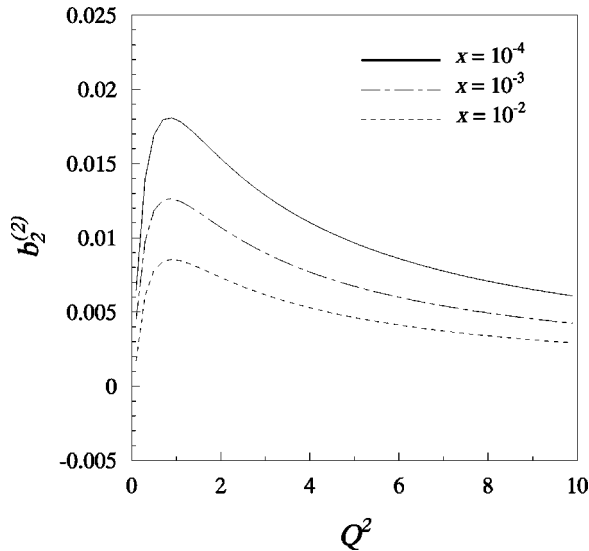


FIG. 4. Behavior of  $b_2^{(2)}(x, Q^2)$  with  $Q^2$  at  $x = 10^{-4}$ ,  $10^{-3}$ , and  $10^{-2}$ .

ferential cross section for production of vector mesons  $\rho^0, \omega, \phi$  has been taken from Ref. [11] and earlier data from the references therein, and have values for forward scattering  $\sim 139.0, 10.4,$  and  $7.2 \mu\text{b}/\text{GeV}^2$  for  $\rho(W=70 \text{ GeV})$ ,  $\omega$  ( $W=80 \text{ GeV}$ ), and  $\phi(W=70 \text{ GeV})$ , respectively. Here  $W$  corresponds to the mean photon-proton center-of-mass energy. In Fig. 3 we have presented the variation of  $b_2^{(2)} = 2xb_1^{(2)}$  with  $x$ , for  $10^{-4} \leq x \leq 1.0$  at  $Q^2 = 0.1, 1.0, 4.0,$  and  $10.0 \text{ GeV}^2$ . We observe that  $b_2^{(2)}$  is significant toward small  $x$  values, behaving as  $\sim (1-x)^{2\delta}/x^{2\delta}$ , and is in general agreement with [2,3].

In Fig. 4 we have given the  $Q^2$  behavior of  $b_2^{(2)}(x, Q^2)$ , as predicted by the VMD model, at different values of  $x$ . That  $b_1$  vanishes at  $Q^2=0$  is clear from Eq. (20). It vanishes at large  $Q^2$  because the vector-meson propagators and the vector-meson electroproduction cross section both fall with  $Q^2$ . This can be explained by the reduction in the coherence length of the vector mesons as  $Q^2$  increases, at a fixed photon energy. Figure 5 shows the double-scattering contribution to  $F_2$  in deuteron using Eq. (21).

A few comments are in order here. Our results are more specific than those of Refs. [2,3] because we have made more specific assumptions about the nature of the intermedi-

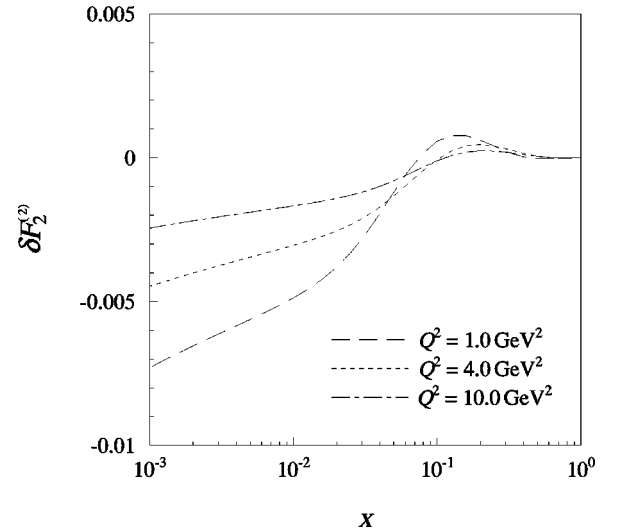


FIG. 5. Double-scattering contribution to  $F_2$  as given by Eq. (21).

ate hadronic state. Of course we could add further excited vector mesons to our calculation, however, their contribution would be suppressed at the low  $Q^2$  at which we work. We must still confront the question: At which scale should we graft our VMD results onto standard QCD evolution? Since we are interested in qualitative rather than quantitative behavior, some uncertainty can be tolerated.  $Q^2 = 0.1 \text{ GeV}^2$  is clearly too small — QCD evolution is not justified at such small  $Q^2$ .  $Q^2 = 10 \text{ GeV}^2$  is clearly too large — simple vector dominance is not justified at such large  $Q^2$ . A choice in the range of the  $\rho$  mass seems appropriate where both VMD and QCD have claims to applicability.

To summarize, we have presented a model for the double-scattering contribution to the tensor structure function  $b_1(x, Q^2)$  of the deuteron. The analysis is based on double scattering of vector mesons in electron-deuteron scattering.

We have found that the double-scattering contribution to  $b_1(x, Q^2)$  is significant for  $x \leq 0.1$  and behaves as  $\sim (1-x)^{2\delta}/x^{1+2\delta}$ . At large Bjorken  $x$  ( $x \geq 0.3$ ) the vector mesons can propagate only over distance scales of the order of the size of a single nucleon, and multiple-scattering contributions are not significant. At very small  $x$  ( $x \leq 10^{-2}$ ), the coherence length of the meson increases and hence the contribution increases. Our results agree qualitatively with those obtained in Refs. [2,3], and confirm the fact that a significant enhancement in  $b_1$  can be expected at small  $x$  due to the quadrupole deformation of the deuteron.

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