Renormalization scheme dependence and the problem of the determination of α_s and the condensates from semileptonic τ decays

P. A. Rączka*
Centre for Particle Theory, University of Durham, South Road, Durham, DH1 3LE, United Kingdom
(Received 12 August 1997; published 21 April 1998)

The QCD corrections to the moments of the invariant mass distribution in the semileptonic τ decays are considered. The effect of the renormalization scheme dependence on the fitted values of $\alpha_s(m_\tau^2)$ and the condensates is discussed, using a simplified approach where the nonperturbative contributions are approximated by the dimension six condensates. The fits in the vector and axial-vector channels are investigated in the next-to-leading and the next-to-next-to-leading order. The next-to-next-to-leading order results are found to be relatively stable with respect to change of the renormalization scheme. A change from the $\overline{\text{MS}}$ scheme to the minimal sensitivity scheme results in the reduction of the extracted value of $\alpha_s(m_\tau^2)$ by 0.01. [S0556-2821(98)06809-X]

PACS number(s): 13.35.Dx, 11.10.Hi, 12.38.Cy

I. INTRODUCTION

Recently there has been considerable progress in the determination of the vector and axial-vector hadronic spectral functions from the semileptonic decays of the τ lepton [1–6]. Using the QCD predictions for the moments of these spectral functions one may obtain constraints on α_s and the parameters characterizing the nonperturbative QCD dynamics ([7–15]). The accuracy of the obtained value of α_s appears to be quite high. In order to have a proper understanding of the phenomenological relevance of this determination of α_s it is important to make a careful estimate of the theoretical uncertainties in the QCD predictions [16–27]. In this paper we investigate the uncertainties in the evaluation of the perturbative part of the QCD prediction for the spectral moments, for which the next-to-next-to-leading order (NNLO) approximation is available. We study in some detail the sensitivity of the perturbative QCD predictions to the choice of the renormalization scheme (RS) and the effect of the RS dependence on the fitted values of the QCD parameters.

II. THEORETICAL FRAMEWORK

Let us begin by summarizing the theoretical framework adopted in the analysis of the τ decay data [7–12]. In [11] it was suggested to use the $R^{kl}_{\tau,V/A}$ moments, defined by the relation

$$R^{kl}_{\tau,V/A}=\frac{1}{\Gamma_e}\int_0^{m_\tau^2} ds\left(1-\frac{s}{m_\tau^2}\right)^k\left(\frac{s}{m_\tau^2}\right)^l\frac{d\Gamma^{V/A}_{ud}}{ds}, \qquad (1)$$

where $d\Gamma^{V/A}_{ud}/ds$ denotes the invariant mass distribution for the Cabibbo allowed semileptonic τ decays in the vector (V) or axial-vector (A) channel and Γ_e denotes the electronic width of the τ lepton. The QCD predictions for $R^{kl}_{\tau,V/A}$ have the form

$$R^{kl}_{\tau,V/A}=\frac{3}{2}|V_{ud}|^2 S_{EW} R^{kl}_0(1+\delta^{kl}_{pt}+\delta^{kl}_{npt,V/A}), \qquad (2)$$

where V_{ud} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element ($|V_{ud}|=0.9752$) and $S_{EW}=1.0194$ represents the correction from electroweak interactions [28,29]. The R^{kl}_0 factor denotes the parton model prediction — for later use we shall need $R^{00}_0=1$ and $R^{12}_0=13/210$. The δ^{kl}_{pt} term denotes the perturbative contribution, evaluated for three massless quarks. (The u and d quark mass effects are negligible, the s quark mass effects are very small for nonstrange decays, and the c quark is considered to be decoupled [30,31].) The δ^{kl}_{pt} term is universal for the V and A channels. The $\delta^{kl}_{npt,V/A}$ term in Eq. (2) denotes the contribution from the nonperturbative QCD effects, which are estimated using the Shifman-Vainshtein-Zakharov (SVZ) approach [32]:

$$\delta^{kl}_{npt,V/A}=\sum_{D=4,6\ldots}\delta^{kl}_{(D)V/A}=\sum_{D=4,6\ldots}\frac{1}{m_\tau^D}\sum_j c^{kl}_{D,j}\langle O^{V/A}_{D,j}\rangle, \qquad (3)$$

where $\langle O^{V/A}_{D,j}\rangle$ are the vacuum expectation values of the gauge invariant operators of dimension D and $c^{kl}_{D,j}$ are coefficients specific for the considered spectral moment and the type of the operator. The $c^{kl}_{D,j}$ coefficients are in principle power series in the strong coupling constant.

The object of greatest interest is of course the total decay rate for the Cabibbo allowed semileptonic τ decays in the vector or axial-vector channels, $R^{00}_{\tau,V/A}$. The perturbative correction to this moment is sizable and it is highly sensitive to the value of the strong coupling, due to the low characteristic energy scale of $m_\tau=1.777$ GeV. The higher R^{kl}_τ moments are introduced to take advantage of the the full information contained in the hadronic spectral functions. By using the predictions for several R^{kl}_τ moments one may obtain a simultaneous fit of $\alpha_s(m_\tau^2)$ and of some of the condensates $\langle O^{V/A}_{D,j}\rangle$. In this way the whole analysis becomes self-consistent and in addition one obtains a check on both the perturbative and the nonperturbative QCD contributions.

*Present address: Institute of Theoretical Physics, Warsaw University, ul. Hoża 69, PL-00-681 Warsaw, Poland.

The perturbative QCD corrections δ_{pt}^{kl} are evaluated using a contour integral expression [33,34], which relates them to the QCD correction δ_{Π}^{pt} to the so-called Adler function [35], i.e., the logarithmic derivative of the transverse part of the vector/axial-vector current correlator $\Pi_{ud}^{(1)V/A}$:

$$-12\pi^2\sigma\frac{d}{d\sigma}\Pi_{ud,pt}^{(1)V/A}(\sigma)=3[1+\delta_{\Pi}^{pt}(-\sigma)]. \tag{4}$$

(In the approximation of massless quarks the perturbative contributions to the Adler functions for the vector and axial-vector current correlators are identical.) We have

$$\delta_{pt}^{kl}=\frac{i}{\pi}\int_C\frac{d\sigma}{\sigma}f^{kl}\left(\frac{\sigma}{m_\tau^2}\right)\delta_{\Pi}^{pt}(-\sigma), \tag{5}$$

where $f^{kl}(\sigma/m_\tau^2)$ is a weight function specific to the considered moment and C is a contour running clockwise from $\sigma=m_\tau^2-i\epsilon$ to $\sigma=m_\tau^2+i\epsilon$ away from the region of small $|\sigma|$. In the following we shall need the weight functions f^{00} and f^{12}:

$$f^{00}(x)=\frac{1}{2}-x+x^3-\frac{1}{2}x^4. \tag{6}$$

$$f^{12}(x)=\frac{1}{2}-\frac{70}{13}x^3+\frac{105}{26}x^4+\frac{126}{13}x^5-\frac{175}{13}x^6+\frac{60}{13}x^7. \tag{7}$$

The NNLO renormalization group improved perturbative expansion for δ_{Π}^{pt} may be written in the form

$$\delta_{\Pi}^{pt}(-\sigma)=a(-\sigma)[1+r_1a(-\sigma)+r_2a^2(-\sigma)], \tag{8}$$

where $a=\alpha_s/\pi=g^2/(4\pi^2)$ denotes the running coupling constant that satisfies the NNLO renormalization group (RG) equation:

$$\sigma\frac{da}{d\sigma}=-\frac{b}{2}a^2(1+c_1a+c_2a^2). \tag{9}$$

In the modified minimal subtraction ($\overline{\rm MS}$) scheme (i.e. using $\overline{\rm MS}$ subtraction procedure and choosing $\mu^2=-\sigma$) we have [36–40] for $n_f=3$ $r_1^{\rm MS}=1.63982$ and $r_2^{\rm MS}=6.37101$. The renormalization group coefficients for $n_f=3$ are $b=4.5$, $c_1=16/9$ and $c_2^{\rm MS}=3863/864\approx4.471$.

The QCD predictions for the δ_{pt}^{kl} are usually calculated in the modified minimal subtraction ($\overline{\rm MS}$) renormalization scheme [41]. However, in the NNLO approximation with massless quarks there is a two-parameter freedom in choosing the RS. This is a consequence of the fact that in each order of perturbation expansion the finite parts of the renormalization constant for the coupling constant may be chosen arbitrarily. Different choices of the finite parts of the renormalization constant result in different definitions of the coupling constant, which are related by a finite renormalization. This results in a change of values of the coefficients r_1, r_2 and c_2. (We restrict our discussion to the class of mass and gauge independent schemes, for which the coefficients b and c_1 are universal.) The formulas describing how the redefinition of the coupling affects the coefficients r_i and c_2 are collected for example in [42]. The dimensional QCD parameter Λ also depends on the choice of the RS [43]. In the NNLO there exists however a RS invariant combination of the expansion coefficients [44–47]:

$$\rho_2=c_2+r_2-c_1r_1-r_1^2. \tag{10}$$

For the δ_{Π} we have $\rho_2=5.23783$.

The change in the expansion coefficients and the change in the coupling constant compensate each other, but of course in the finite order of perturbation expansion such compensation may only be approximate, which results in the numerical RS dependence of the perturbative predictions. This RS dependence is formally of higher order in the coupling constant, but in the case of the τ decay it may be significant numerically, since the coupling is not very small at the energy scale of m_τ. It is therefore very important to verify to what extent the RS dependence affects the predictions and the fits to the experimental data.

The authors of [1,2,4] used the R_τ^{kl} moments with $k=1$ and $l=0,1,2,3$, and fitted the $D=4,6,8$ condensates. Since the aim of this paper is primarily to study the theoretical uncertainties in the whole procedure we shall adopt a simplified approach, which still has considerable phenomenological relevance. This approach is based on the fact that the dominant nonperturbative contribution to $R_{\tau,V/A}^{00}$ in the SVZ expansion comes from the $D=6$ term [7,8], because the $D=4$ term is suppressed by additional power of α_s. We shall therefore neglect the $D=4$ contribution to the total decay rate and — for consistency — the higher order correction to the $D=6$ coefficient. (Such an approximation was in fact made already in [8].) In the following we shall also neglect contributions from the $D\geqslant8$ condensates. In order to be able to fit the $D=6$ condensate together with α_s we shall use the QCD prediction for the $R_{\tau,V/A}^{12}$ moment, which similarly to $R_{\tau,V/A}^{00}$ has a suppressed contribution from the $D=4$ condensate. Neglecting in $R_{\tau,V/A}^{12}$ the contributions from $D\geqslant8$ condensates we obtain a simple set of self-consistent formulas. In our approximation

$$\delta_{npt,V/A}^{00}=\delta_{(6)V/A}^{00}=-\frac{24\pi^2}{m_\tau^6}\sum_j C_{6,j}\langle O_{6,j}^{V/A}\rangle, \tag{11}$$

$$\delta_{npt,V/A}^{12}=\delta_{(6)V/A}^{12}=\frac{1680\pi^2}{13m_\tau^6}\sum_j C_{6,j}\langle O_{6,j}^{V/A}\rangle, \tag{12}$$

where $\Sigma_jC_{6,j}\langle O_{6,j}^{V/A}\rangle$ is the leading $D=6$ contribution to the transverse part of the hadronic vacuum polarization function:

$$(-\sigma)^3\Pi_{ud(D=6)}^{(1)V/A}(\sigma)=\sum_j C_{6,j}\langle O_{6,j}^{V/A}\rangle. \tag{13}$$

This contribution is dominated by the four-quark condensates [10]. In the phenomenological analysis it is usually expressed in a simplified form, motivated by the chiral symmetry and the vacuum saturation approximation [10]:

$$\sum_j C_{6,j}\langle O_{6,j}^{V/A}\rangle=h_{V/A}\frac{32\pi}{81}\alpha_s\rho\langle\bar{q}q\rangle^2, \tag{14}$$

where $\langle\bar{q}q\rangle$ is the quark condensate, $h_V=-7$, $h_A=11$ and ρ is an effective parameter, characterizing the deviation from the strict vacuum saturation.

In our study of the RS dependence effects we parametrize the freedom of choice of the RS by the parameters r_1 and c_2, following the conventions of our previous work [24–26]. To obtain the perturbative QCD corrections δ_{pt}^{kl} we evaluate the contour integral numerically, using under the integral a numerical solution of the RG equation (9) in the complex energy plane. In this way we take full advantage of the RG-invariance properties of the perturbative prediction and we resum[1] to all orders some of the large terms which would otherwise appear in the perturbation expansion [48,49]. We assume that the integration contour C is a circle $\sigma=-m_\tau^2\exp(-i\theta)$, $\theta\in[-\pi,\pi]$. To determine the numerical value of the running coupling constant on the contour C we solve the transcendental equation, which results from integration of the RG equation (9) with a suitable boundary condition and analytic continuation to the complex energy plane:

$$b\ln\left(\frac{m_\tau}{\Lambda_{\overline{\rm MS}}^{(3)}}\right)-i\frac{b\theta}{2}=r_1^{\overline{\rm MS}}-r_1+c_1\ln\left(\frac{b}{2c_1}\right)+F^{(n)}(a), \tag{15}$$

where in NLO

$$F^{(1)}(a)=\frac{1}{a}+c_1\ln\left(\frac{c_1 a}{1+c_1 a}\right), \tag{16}$$

and in NNLO for $4c_2-c_1^2>0$

$$F^{(2)}(a)=\frac{1}{a}+c_1\ln(c_1 a)-\frac{c_1}{2}\ln(1+c_1 a+c_2 a^2)+\frac{2c_2-c_1^2}{(4c_2-c_1^2)^{1/2}}\arctan\left(\frac{a(4c_2-c_1^2)^{1/2}}{2+c_1 a}\right). \tag{17}$$

The presence of $\Lambda_{\overline{\rm MS}}^{(3)}$ in the expression valid for arbitrary scheme follows from our taking into account explicitly the relation between Λ parameters in different schemes [43] and using $\Lambda_{\overline{\rm MS}}^{(3)}$ as a reference parameter.

After evaluating the predictions for δ_{pt}^{00} and δ_{pt}^{12} we perform the fits to the experimental data for $R_{\tau,V/A}^{00}$ and $D_{\tau,V/A}^{12}=R_{\tau,V/A}^{12}/R_{\tau,V/A}^{00}$. We use the experimental values reported recently by ALEPH [4]: $R_{\tau,V}^{00}=1.782\pm0.018$, $D_{\tau,V}^{12}=0.0532\pm0.0007$, $R_{\tau,A}^{00}=1.711\pm0.019$, $D_{\tau,A}^{12}=0.0639\pm0.0005$. In the fits we assume for simplicity that the experimental errors for R_τ^{00} and D_τ^{12} are not correlated.

We express the results of our fits in terms of $\alpha_s(m_\tau^2)$ and $\delta_{(6)}^{00}$. [We actually fit the value of $\Lambda_{\overline{\rm MS}}^{(3)}$, which we then convert to $\alpha_s(m_\tau^2)$ using the RG equation in the $\overline{\rm MS}$ scheme.] For comparison with other determinations of the strong coupling constant we extrapolate α_s from m_τ^2 to m_Z^2. Our procedure for extrapolation relies on the matching formula relating $\alpha_s(\mu^2,n_f+1)$ to $\alpha_s(\mu^2,n_f)$:

$$\alpha_s(\mu^2,n_f+1)=\alpha_s(\mu^2,n_f)+\frac{L}{3}\frac{\alpha_s^2(\mu^2,n_f)}{\pi}+\frac{1}{9}\left(L^2+\frac{57}{4}L-\frac{11}{8}\right)\frac{\alpha_s^3(\mu^2,n_f)}{\pi^2}, \tag{18}$$

where $L=\ln(\mu/\tilde{m}_q)$ and $\tilde{m}_q$ is the running quark mass $m_q(\mu^2)$ of the heavy quark evaluated at the scale $\mu=m_q$. (The NNLO matching formula of that form was originally proposed in [50–52], see also the discussion in [53]. However, in [31] it was found that a numerical coefficient in the NNLO term is actually different, which was subsequently confirmed in [54]. We use the coefficient of [31].) In order to evolve α_s from scale μ_1 to the scale μ_2 we solve the equation:

$$F^{(k)}[\alpha_s(\mu_1^2)/\pi]-F^{(k)}[\alpha_s(\mu_2^2)/\pi]=b\ln(\mu_1/\mu_2). \tag{19}$$

To obtain $\alpha_s(m_Z^2)$ from the given value of $\alpha_s(m_\tau^2)$ we first evolve α_s from the scale m_τ to the scale of $2\tilde{m}_c$ using the $n_f=3$ RG equation, then we use the matching formula (18) to obtain $\alpha_s[(2\tilde{m}_c)^2,n_f=4]$, evolve this to the scale of $2\tilde{m}_b$ using the $n_f=4$ RG equation, use the matching formula to obtain $\alpha_s[(2\tilde{m}_b)^2,n_f=5]$, and finally evolve this to the scale of m_Z using the $n_f=5$ RG equation. We use $\tilde{m}_c=1.3$ GeV and $\tilde{m}_b=4.3$ GeV, which are the central values recommended by the Particle Data Group [55]. In Table I we give for reference some values of $\alpha_s(m_\tau^2)$ and $\alpha_s(m_Z^2)$ as a function of $\ln(m_\tau/\Lambda_{\overline{\rm MS}}^{(3)})$ in the NNLO approximation.

III. FITS IN THE VECTOR CHANNEL

Let us begin with the calculation in the $\overline{\rm MS}$ scheme, to see how our approximate treatment compares with a more complete analysis reported in [4]. In Table II we give the values for δ_{pt}^{12} in the $\overline{\rm MS}$ scheme, in NLO and NNLO, as a function of $\ln(m_\tau/\Lambda_{\overline{\rm MS}}^{(3)})$. For completeness, we also include precise values for δ_{pt}^{00}, which may be compared with those given previously in [10]. Fitting the experimental results for $R_{\tau,V}^{00}$ and $D_{\tau,V}^{12}$ we obtain in NNLO $\Lambda_{\overline{\rm MS}}^{(3)}=441\pm32$ MeV, which corresponds to $\alpha_s(m_\tau^2)=0.367\pm0.018$ and $\alpha_s(m_Z^2)=0.1238\pm0.0018$. This is very close to the value $\alpha_s(m_\tau^2)=0.360\pm0.022$ obtained by ALEPH in a fit involving more R_τ^{kl} mo-

[1]As was discussed in [49], and in more detail in [24], the resummation via the contour integral is very important in precise phenomenological applications since it greatly reduces the RS dependence of the finite order perturbative predictions. On the other hand, in [21] it was argued that this type of resummation leads in the case of higher order renormalon corrections to a less precise asymptotic expansion than the conventional perturbation expansion for δ_{pt}^{00}. A detailed discussion of this problem goes beyond the scope of the present paper. Let us observe, however, that the authors of [21] do not address the problem of RS dependence in higher orders of perturbation expansion. For example a recent investigation of a similar issue reported in [23] seems to indicate that in the effective charge scheme the ''contour improved'' expansion has better convergence properties than the conventional expansion.

TABLE I. Table of values of $\alpha_s(m_\tau^2)$ and $\alpha_s(m_Z^2)$ in NNLO related by the matching procedure described in the text.

$\ln(m_\tau/\Lambda^{(3)}_{\overline{\rm MS}})$	$\Lambda^{(3)}_{\overline{\rm MS}}$	$\alpha_s(m_\tau^2)$	$\alpha_s(m_Z^2)$
1.30	484	0.392	0.1261
1.35	461	0.378	0.1249
1.40	438	0.366	0.1236
1.45	417	0.354	0.1224
1.50	397	0.343	0.1212
1.55	377	0.333	0.1201
1.60	359	0.323	0.1190
1.65	341	0.314	0.1179
1.70	325	0.306	0.1168
1.75	309	0.298	0.1157
1.80	294	0.291	0.1147
1.85	279	0.284	0.1136
1.90	266	0.277	0.1126

ments and $D=4,6,8$ nonperturbative contributions [4]. We also obtain $\delta^{00}_{(6)V}=0.0147\pm0.0025$, which is in reasonable agreement with the value of the nonperturbative contribution obtained by ALEPH [4]. This confirms our expectation that the $D=6$ approximation adopted in this work provides a good approximation to the more complete analysis involving a larger set of parameters.

Let us now consider the same fit, but in a different renormalization scheme. As is well known, the theoretical and phenomenological motivation for the widely used $\overline{\rm MS}$ scheme is not very strong, and there has been extensive discussion on the problem of the optimal choice of the renormalization scheme [44–47,56–60]. One of the interesting approaches is based on the so-called principle of minimal sensitivity (PMS) [44]. The philosophy behind this approach is very simple — since the theoretical predictions of any theory should be in principle independent of the RS, then in the finite order of perturbation expansion one should look for the RS, which mimics this as closely as possible.

In the case of the conventional perturbative QCD expansion the RS parameters of the PMS scheme are determined by a system of transcendental and algebraic equations [44]. Unfortunately, in the case of perturbative predictions obtained via numerical evaluation of the contour integral these equations do not apply, so the optimized parameters have to be determined by direct computation of δ^{kl} for different values of r_1 and c_2.

TABLE II. Table of values of the NLO and NNLO predictions for δ^{00}_{pt} and δ^{12}_{pt} in the $\overline{\rm MS}$ scheme.

$\ln(m_\tau/\Lambda^{(3)}_{\overline{\rm MS}})$	$\delta^{00}_{\overline{\rm MS},\rm NLO}$	$\delta^{00}_{\overline{\rm MS},\rm NNLO}$	$\delta^{12}_{\overline{\rm MS},\rm NLO}$	$\delta^{12}_{\overline{\rm MS},\rm NNLO}$
1.30	0.1967	0.2285	0.1267	0.1420
1.35	0.1891	0.2187	0.1235	0.1364
1.40	0.1820	0.2095	0.1206	0.1315
1.45	0.1753	0.2009	0.1178	0.1273
1.50	0.1690	0.1928	0.1151	0.1235
1.55	0.1631	0.1852	0.1126	0.1201
1.60	0.1576	0.1781	0.1102	0.1170
1.65	0.1523	0.1714	0.1078	0.1142
1.70	0.1474	0.1652	0.1056	0.1115
1.75	0.1428	0.1593	0.1034	0.1090
1.80	0.1384	0.1538	0.1013	0.1066
1.85	0.1342	0.1486	0.0993	0.1043
1.90	0.1303	0.1437	0.0973	0.1021

TABLE III. Table of values of the NLO and NNLO predictions for δ^{12}_{pt} obtained in a scheme preferred by the principle of minimal sensitivity ($r_1=-0.64$ in NLO and $r_1=0$, $c_2=1.5\rho_2=7.857$ in NNLO).

$\ln(m_\tau/\Lambda^{(3)}_{\overline{\rm MS}})$	$\delta^{12}_{\rm PMS,NLO}$	$\delta^{12}_{\rm PMS,NNLO}$
1.30	0.1382	0.1634
1.35	0.1329	0.1521
1.40	0.1284	0.1432
1.45	0.1245	0.1360
1.50	0.1209	0.1300
1.55	0.1177	0.1250
1.60	0.1148	0.1208
1.65	0.1120	0.1171
1.70	0.1095	0.1138
1.75	0.1070	0.1108
1.80	0.1047	0.1081
1.85	0.1025	0.1055
1.90	0.1004	0.1032

The dependence of δ^{00}_{pt} on the scheme parameters r_1 and c_2 was discussed in detail in [24] and the RS dependence of δ^{12}_{pt} was investigated in [25,26]. In both cases it was found that for moderate values of $\Lambda^{(3)}_{\overline{\rm MS}}$ the NNLO predictions have a saddle point type of behavior[2] as a function of r_1 and c_2 and that the position of the saddle point is well approximated by $r_1=0$ and $c_2=1.5\rho_2=7.857$. (Incidentally, these scheme parameters correspond to the approximate solution [62] of the algebraic PMS equations for $\delta_{\rm II}$ evaluated for spacelike momenta.) For very large values of $\Lambda^{(3)}_{\overline{\rm MS}}$ the RS-dependence pattern is more complicated than a simple saddle point, but even then the scheme parameters distinguished above belong to the region of extremely small RS dependence. We shall therefore accept these parameters as the PMS parameters in NNLO. The values of the NLO and NNLO PMS predictions for δ^{12}_{pt} are given in Table III. (The values for δ^{00}_{pt} have been already given in [24]. The contour plots provided there in principle allow one to obtain predictions for δ^{00}_{pt} in arbitrary scheme with reasonably large expansion coefficients.)

Using the PMS predictions we obtain from the NNLO fit in the vector channel $\delta^{00}_{(6)V}=0.0156\pm0.0023$ and $\Lambda^{(3)}_{\overline{\rm MS}}$

[2]The relative stability of the perturbative predictions for δ^{00}_{pt} and δ^{12}_{pt} with respect to change of the RS found in [24–26] is a direct consequence of the use of the contour resummation. Another important consequence of the ''contour improvement'' is that both quantities have quite similar patterns of RS dependence. This is rather natural, since both quantities originate from the same Adler function. However, if the conventional expansion is used, the perturbative expressions for both quantities look quite unrelated, and they have different patterns of RS dependence.

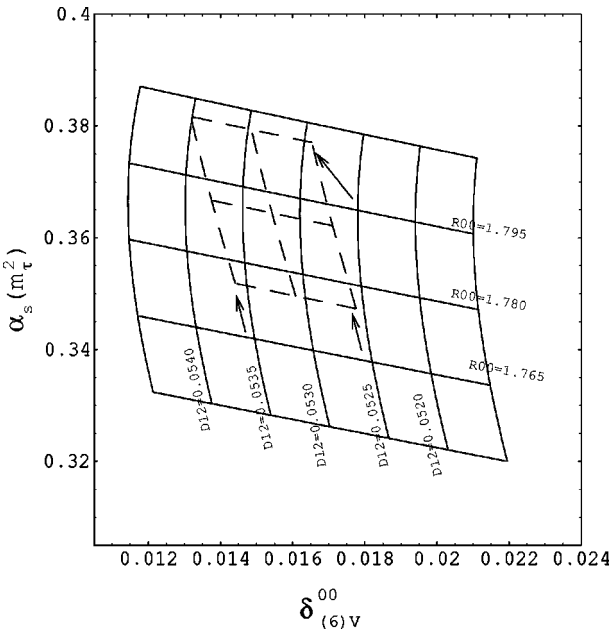

FIG. 1. Plot of the fitted values of $\alpha_s(m_\tau^2)$ and $\delta^{00}_{(6)}$ in the vector channel as a function of $R^{00}_{\tau,V}$ and $D^{12}_{\tau,V}$, obtained using the NNLO PMS predictions. The dashed lines indicate the change in the plot when the $\overline{\text{MS}}$ NNLO predictions are used instead.

$=421\pm30$ MeV, which corresponds to $\alpha_s(m_\tau^2)=0.356\pm0.017$ and $\alpha_s(m_Z^2)=0.1226\pm0.0018$. We see that in NNLO the change from the $\overline{\text{MS}}$ scheme to the PMS scheme results in the reduction of the fitted value of α_s by an amount significant compared for example to the presently available experimental precision.

In order to make our calculations more generally useful we show in Fig. 1 the results of the NNLO fit of $\alpha_s(m_\tau^2)$ and $\delta^{00}_{(6)V}$, obtained using the PMS predictions, as a function of the experimental values of $R^{00}_{\tau,V}$ and $D^{12}_{\tau,V}$. For comparison we indicate the results of the NNLO fit in the $\overline{\text{MS}}$ scheme (dashed lines).

Besides PMS another frequently discussed approach to the optimization is the effective charge (EC) method [56,47], which amounts to the absorption of all the higher order radiative corrections to the physical quantity — in this case δ_{Π} — into the definition of the coupling constant. In the EC scheme we have $r_1=0$ and $c_2=\rho_2$. The NLO and NNLO predictions for δ^{00}_{pt} and δ^{12}_{pt} in the EC scheme are given in Table IV. We see that in NNLO the difference between EC and PMS is very small. This is reflected by the results of the fit: in NNLO we obtain $\alpha_s(m_\tau^2)=0.356$ and $\delta^{00}_{(6)V}=0.0158$.

To have a broader picture of the renormalization scheme dependence we perform the fit in a more general class of schemes. It is clear that regardless of our choice of the concrete optimal scheme there is a continuum of schemes, which seem to be equally reasonable. Predictions in such schemes should also be somehow taken into account in the phenomenological analysis. A natural way to do this is to supplement the prediction in a preferred scheme with an estimate of the variation of the predictions over a whole set of *a priori* acceptable schemes. A concrete realization of this idea was presented in [61], based on the existence of the RS invariant

TABLE IV. Table of values of the NLO and NNLO predictions for δ^{00}_{pt} and δ^{12}_{pt} in the effective charge scheme.

$\ln(m_\tau/\Lambda^{(3)}_{\overline{\text{MS}}})$	$\delta^{00}_{\text{EC,NLO}}$	$\delta^{00}_{\text{EC,NNLO}}$	$\delta^{12}_{\text{EC,NLO}}$	$\delta^{12}_{\text{EC,NNLO}}$
1.30	0.2151	0.2357	0.1360	0.1624
1.35	0.2063	0.2261	0.1312	0.1524
1.40	0.1979	0.2168	0.1271	0.1439
1.45	0.1901	0.2080	0.1233	0.1368
1.50	0.1828	0.1996	0.1200	0.1308
1.55	0.1759	0.1917	0.1169	0.1257
1.60	0.1694	0.1842	0.1141	0.1213
1.65	0.1634	0.1771	0.1114	0.1175
1.70	0.1577	0.1704	0.1089	0.1141
1.75	0.1523	0.1642	0.1065	0.1110
1.80	0.1473	0.1583	0.1043	0.1082
1.85	0.1425	0.1528	0.1021	0.1057
1.90	0.1380	0.1476	0.1000	0.1033

ρ_2, which provides a natural[3] RS independent characterization of the magnitude of the NNLO corrections for the considered physical quantity. In [61] it was proposed to calculate variation of the predictions over the set of schemes for which the expansion coefficients satisfy the condition

$$\sigma_2(r_1,r_2,c_2)\leqslant l|\rho_2|, \tag{20}$$

where

$$\sigma_2(r_1,r_2,c_2)=|c_2|+|r_2|+c_1|r_1|+r_1^2. \tag{21}$$

A motivation for the condition (20) is that it eliminates schemes in which the expansions (8) and (9) involve unnaturally large expansion coefficients that introduce large cancellations in the expression for the RS invariant ρ_2. The constant l in the condition (20) controls the degree of cancellation that we want to allow in the expression for ρ_2. In our case we have for the PMS parameters $\sigma_2(\text{PMS})\approx2|\rho_2|$, so in order to take into account the schemes, which have the same — or smaller — degree of cancellation as the PMS scheme we take $l=2$. One may expect, that the estimate of the RS dependence obtained according to this prescription would be useful for a quantitative comparison of reliability of perturbative predictions for different physical quantities, evaluated at different energies. It is also clear that any large variation of the predictions over a set of schemes satisfying the constraint (20) with $l=2$ would be an unambiguous sign of a limited applicability of the NNLO expression.

In Fig. 2 we show how the value of $\alpha_s(m_\tau^2)$ resulting from the fit depends on the parameters r_1 and c_2 specifying the renormalization scheme in NNLO. In the region of scheme parameters satisfying condition (20) with $l=2$ the minimum

[3]The definition of the RS invariant ρ_2 is not unique, since by adding any RS independent constant we also obtain a RS invariant. However, as has been discussed in detail in [61], the form (10) is distinguished by the fact, that it appears as a NNLO expansion coefficient in a manifestly RS invariant evolution equation for a physical quantity [45–47].

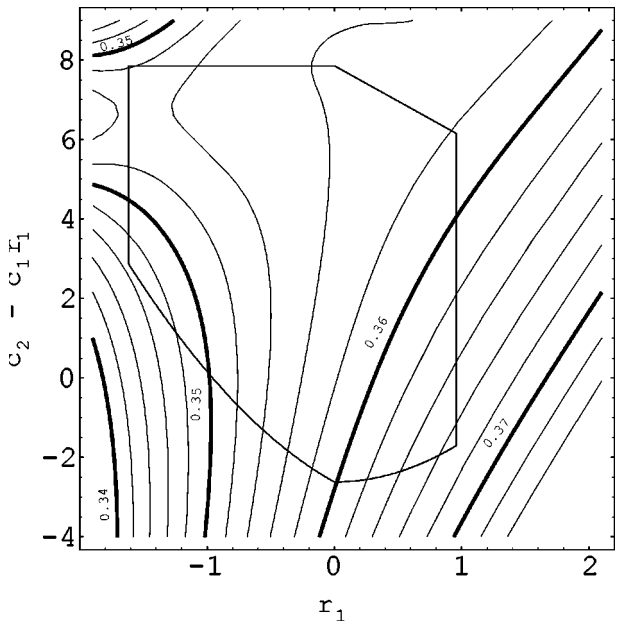

FIG. 2. The contour plot of the fitted value of $\alpha_s(m_\tau^2)$ in the vector channel as a function of the RS parameters r_1 and c_2. For technical reasons we use $c_2 - c_1 r_1$ as an independent variable on the vertical axis. The region of the scheme parameters satisfying the condition (20) with $l=2$ has been also indicated.

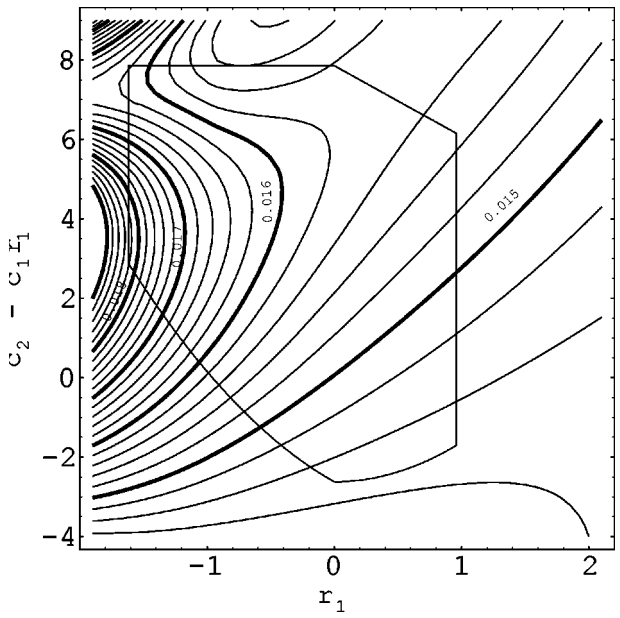

FIG. 3. The contour plot of the fitted value of $\delta^{00}_{(6)}$ in the vector channel as a function of the RS parameters r_1 and c_2. The region of the scheme parameters satisfying the condition (20) with $l=2$ has been also indicated.

value of $\alpha_s(m_\tau^2)=0.347$ [$\Lambda^{(3)}_{\overline{\rm MS}}=403$ MeV, $\alpha_s(m_Z^2)=0.1217$] is attained for $r_1=-1.62$ and $c_2=0$, and the maximum value of $\alpha_s(m_\tau^2)=0.367$ [$\Lambda^{(3)}_{\overline{\rm MS}}=440$ MeV, $\alpha_s(m_Z^2)=0.1238$] is attained for $r_1=0.96$ and $c_2=0$. It should be noted that the $\overline{\rm MS}$ scheme parameters lie outside the $l=2$ region, but the fitted value of $\alpha_s(m_\tau^2)$ in this scheme coincides with the maximal value quoted above. It may also be interesting to note that one of the other schemes of potential interest, for which the NNLO expressions recently became available [64] — the so-called V scheme [63] — corresponds to the RS parameters $r_1=-0.109$ and $c_2=26.200$, which lie very far outside the $l=2$ region. The extracted value of $\alpha_s(m_\tau)$ appears to be quite stable with respect to change of the RS. This is a direct consequence of the relative stability of the predictions for δ^{00}_{pt} and δ^{12}_{pt} and the fact that the patterns of RS dependence of these quantities are similar. This in turn results from our use of the ''contour improved'' expansion.

It is of some interest to perform the same fits using instead the NLO predictions. Using the NLO predictions in the $\overline{\rm MS}$ scheme, we obtain $\delta^{00}_{(6)V}=0.0148$ and $\Lambda^{(3)}_{\overline{\rm MS}}=527$ MeV, which translates via NLO RG equation into $\alpha_s(m_\tau^2)=0.388$, which in turn corresponds via NLO extrapolation to $\alpha_s(m_Z^2)=0.1261$. We see that the value of $\delta^{00}_{(6)V}$ is practically identical to that obtained in the NNLO fit. However, the value of the strong coupling constant is surprisingly high and it is significantly different from the NNLO value. Using the NLO predictions in the PMS scheme we obtain $\delta^{00}_{(6)V}=0.0150$ and $\Lambda^{(3)}_{\overline{\rm MS}}=465$ MeV, which corresponds to $\alpha_s(m_\tau^2)=0.358$ and $\alpha_s(m_Z^2)=0.1230$. We see that although the difference in the NLO and NNLO values of $\Lambda^{(3)}_{\overline{\rm MS}}$ obtained in the PMS fits is still considerable, it is nevertheless much smaller than in the case of the $\overline{\rm MS}$ scheme. [If we look at $\alpha_s(m_\tau)$ instead of $\Lambda^{(3)}_{\overline{\rm MS}}$ this difference is even smaller, although this may be partially a result of a fortuitous compensation between different factors influencing the evaluation of α_s in various orders.] For completeness we give the NLO results in the EC scheme: $\delta^{00}_{(6)V}=0.0149$ and $\Lambda^{(3)}_{\overline{\rm MS}}=472$ MeV [$\alpha_s(m_\tau^2)=0.361$]. These numbers provide a nice illustration of the fact that the difference between the NLO and NNLO predictions is strongly scheme dependent. Therefore, if such a difference is to be used in any way to estimate the precision of the QCD prediction, some way of making a preferred choice of the renormalization scheme must be employed.

So far our discussion has concentrated primarily on the value of $\alpha_s(m_\tau^2)$ coming from the fit. In Fig. 3 we show the RS dependence of the fitted value of $\delta^{00}_{(6)V}$. We find that in the set of schemes satisfying the constraint (20) with $l=2$ the fitted value of $\delta^{00}_{(6)V}$ changes in the range 0.0145–0.0182. Using Eq. (14) we may translate this into the range $(2.22-2.78)\times10^{-4}$ GeV^6 for the commonly used parameter $\alpha_s\rho\langle\overline{q}q\rangle^2$. This seems to be in reasonable agreement with the values obtained previously by other authors, for example 1.8×10^{-4} GeV^6 obtained in the original work of SVZ [32] or $(3.8\pm2.0)\times10^{-4}$ GeV^6 obtained in a more recent analysis [65].

IV. FITS IN THE AXIAL-VECTOR CHANNEL

Similarly as in the case of the vector channel we start with the fit in the $\overline{\rm MS}$ scheme. We obtain $\delta^{00}_{(6)A}=-0.0168\pm0.0021$ and $\Lambda^{(3)}_{\overline{\rm MS}}=398\pm37$ MeV, which corresponds to $\alpha_s(m_\tau^2)=0.344\pm0.019$ and $\alpha_s(m_Z^2)=0.1213\pm0.0021$. The result of this fit should be compared with $\alpha_s(m_\tau^2)=0.365\pm0.025$, obtained by ALEPH in a more complete fit [4]. We find that the difference between our results and the ALEPH

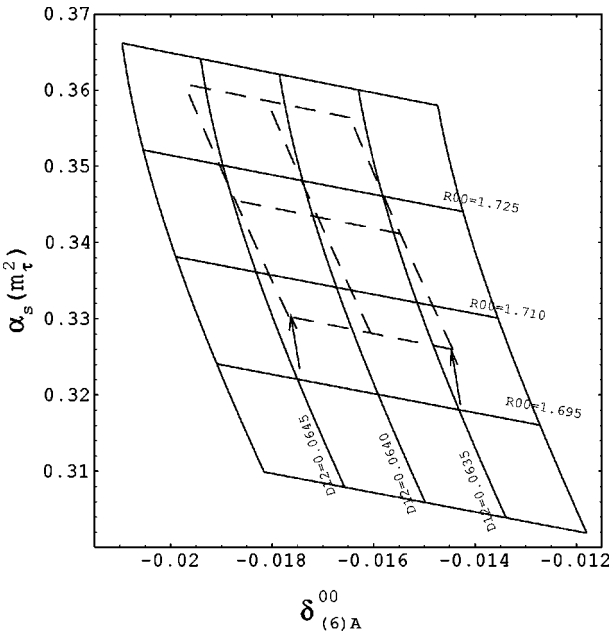

FIG. 4. Plot of the fitted values of $\alpha_s(m_\tau^2)$ and $\delta^{00}_{(6)}$ in the axial-vector channel as a function of $R^{00}_{\tau,A}$ and $D^{12}_{\tau,A}$, obtained using the NNLO PMS predictions. The dashed lines indicate the change in the plot when the $\overline{\text{MS}}$ NNLO predictions are used instead.

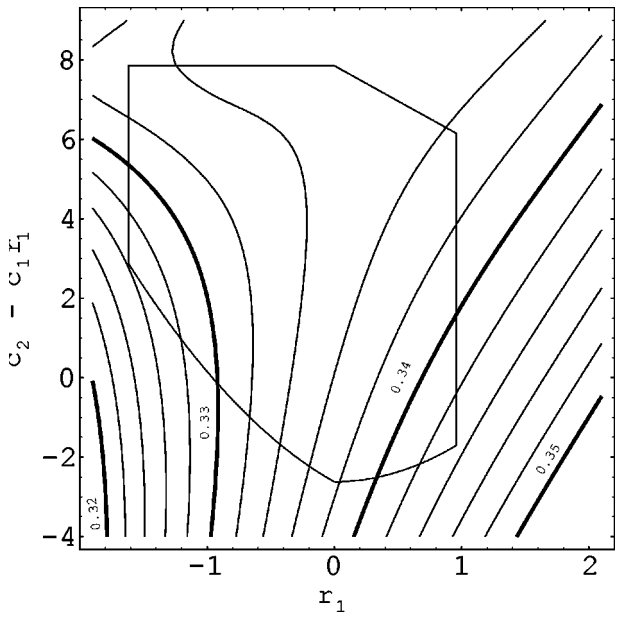

FIG. 5. The contour plot of the fitted value of $\alpha_s(m_\tau^2)$ in the axial-vector channel as a function of the RS parameters r_1 and c_2. The region of the scheme parameters satisfying the condition (20) with $l=2$ has been also indicated.

result is slightly bigger than in the case of vector channel. It should be noted however that the ALEPH fit in the axial-vector channel has a surprisingly large $D=4$ contribution, which would be difficult to justify theoretically.

Performing the NNLO fit in the PMS scheme we obtain $\delta^{00}_{(6)A}=-0.0165\pm0.0018$ and $\Lambda^{(3)}_{\overline{\text{MS}}}=380\pm34$ MeV, which corresponds to $\alpha_s(m_\tau^2)=0.335\pm0.018$ and $\alpha_s(m_Z^2)=0.1203\pm0.0021$. Similarly as in the case of the vector channel we find appreciable reduction in the extracted value of $\alpha_s(m_\tau^2)$ as compared to the values obtained in the $\overline{\text{MS}}$ scheme.

To make our results useful in the case of future improvements of the experimental analysis we show in Fig. 4 the results of the NNLO fit of $\alpha_s(m_\tau^2)$ and $\delta^{00}_{(6)A}$, obtained with the PMS predictions, as a function of the experimental values of $R^{00}_{\tau,A}$ and $D^{12}_{\tau,A}$. For comparison we indicate the results of the NNLO fit in the $\overline{\text{MS}}$ scheme (dashed lines).

In Fig. 5 we show how the fitted value of $\alpha_s(m_\tau^2)$ depends on the RS parameters r_1 and c_2. In the region of the scheme parameters satisfying the condition (20) with $l=2$ we have $0.326<\alpha_s(m_\tau^2)<0.343$ [$364\ \text{MeV}<\Lambda^{(3)}_{\overline{\text{MS}}}<397$ MeV, $0.1193<\alpha_s(m_Z^2)<0.1212$]. A similar figure may be obtained for the RS dependence of $\delta^{00}_{(6)A}$: it is found that $0.015<-\delta^{00}_{(6)A}<0.017$.

Having obtained the results for $\delta^{00}_{(6)}$ in the vector and axial-vector channels it is of some interest to verify the simple relation between them, implied by the generalized vacuum saturation approximation: $\delta^{00}_{(6)A}/\delta^{00}_{(6)V}=-11/7\approx-1.57$. Using the numbers from the NNLO PMS fits in both channels we obtain $\delta^{00}_{(6)A}/\delta^{00}_{(6)V}=-1.06\pm0.28$.

In order to have a full picture of the perturbative uncertainties in the axial-vector channel we also perform the NLO fits. Using the $\overline{\text{MS}}$ scheme, we obtain $\delta^{00}_{(6)A}=-0.0166$ and $\Lambda^{(3)}_{\overline{\text{MS}}}=473$ MeV, which translates via NLO RG equation into $\alpha_s(m_\tau^2)=0.362$, which in turn corresponds via NLO extrapolation to $\alpha_s(m_Z^2)=0.1235$. In the PMS scheme we obtain $\delta^{00}_{(6)A}=-0.0167$ and $\Lambda^{(3)}_{\overline{\text{MS}}}=419$ MeV, which implies $\alpha_s(m_\tau^2)=0.336$ and $\alpha_s(m_Z^2)=0.1207$. Similarly as in the vector channel case we note a rather large difference between NLO and NNLO in the case of the $\overline{\text{MS}}$ scheme, which is significantly reduced if the preferred scheme is PMS. In NLO in the EC scheme we obtain $\delta^{00}_{(6)A}=-0.0167$ and $\Lambda^{(3)}_{\overline{\text{MS}}}=425$ MeV [$\alpha_s(m_\tau^2)=0.339$].

V. DISCUSSION AND CONCLUSIONS

Our discussion of the RS dependence of the QCD predictions and the fits may be summarized as follows.

(1) Changing the scheme from the $\overline{\text{MS}}$ scheme to the PMS scheme we obtain a reduction in the extracted value of $\alpha_s(m_\tau^2)$ by approximately 0.01 [$\alpha_s(m_Z^2)$ is reduced by 0.001]. Also, the difference between the NLO and NNLO results is much smaller in the PMS scheme than in the $\overline{\text{MS}}$ scheme.

(2) Varying the scheme parameters r_1 and c_2 in the region satisfying the condition (20) with $l=2$ we obtain an uncertainty in the extracted value of $\alpha_s(m_\tau^2)$ of approximately 0.02 [uncertainty in $\alpha_s(m_Z^2)$ is 0.002]. As was argued in the text, this set of scheme parameters seems to be a minimal set that one should take into account in the discussion of the RS dependence.

Our general conclusion is that the perturbative predictions for the QCD effects in the inclusive decay rates for the semileptonic τ decays appear to be relatively precise, despite the

rather low energy scale. It should be emphasized however, that in the discussion of the final precision of α_s extracted from the τ decays one should also take into account the approximate character of the SVZ expansion itself [32,66,67].

ACKNOWLEDGMENTS

Discussions with R. Alemany, M. Beneke, M. Davier, A. Höcker, C. Maxwell, and M. Neubert are gratefully acknowledged.

[1] ALEPH Collaboration, D. Buskulic, Phys. Lett. B **307**, 209 (1993).
[2] CLEO Collaboration, T. Coan *et al.*, Phys. Lett. B **356**, 580 (1995).
[3] R. Alemany, in *TAU 96*, Proceedings of the Fourth Workshop on Tau Lepton Physics, Estes Park, Colorado, 1996, edited by J.G. Smith and W. Toki [Nucl. Phys. B (Proc. Suppl.) **55C**, 341 (1997)].
[4] A. Höcker, in *TAU 96* [3], p. 379.
[5] M. Davier, in *TAU 96* [3], p. 395.
[6] ALEPH Collaboration, R. Barate *et al.*, Z. Phys. C **76**, 151 (1997).
[7] E. Braaten, Phys. Rev. Lett. **60**, 1606 (1988); **63**, 577 (1989).
[8] S. Narison and A. Pich, Phys. Lett. B **211**, 183 (1988).
[9] E. Braaten, Phys. Rev. D **39**, 1458 (1989).
[10] E. Braaten, S. Narison, and A. Pich, Nucl. Phys. **B373**, 581 (1992).
[11] F. Le Diberder and A. Pich, Phys. Lett. B **289**, 165 (1992).
[12] S. Narison and A. Pich, Phys. Lett. B **304**, 359 (1993).
[13] C. A. Dominguez and J. Solà, Z. Phys. C **40**, 63 (1988).
[14] V. Giménez, J. A. Peñarrocha and J. Bordes, Phys. Lett. B **223**, 245 (1989).
[15] V. G. Kartvelishvili and M. V. Margvelashvili, Z. Phys. C **55**, 83 (1992).
[16] A. Pich, in *QCD 94*, Proceedings of the QCD Workshop, Montpellier, France, 1994, edited by S. Narison [Nucl. Phys. B (Proc. Suppl.) **39BC**, 326 (1995)].
[17] S. Narison, in *TAU 94*, Proceedings of the Third Workshop on Tau Lepton Physics, Montreux, Switzerland, 1994, edited by L. Rolandi [Nucl. Phys. B (Proc. Suppl.) **40**, 47 (1995)].
[18] G. Altarelli, in *TAU 94* [17], p. 59.
[19] G. Altarelli, P. Nason and G. Ridolfi, Z. Phys. C **68**, 257 (1994).
[20] C. A. Dominguez, Phys. Lett. B **345**, 291 (1995).
[21] P. Ball, M. Beneke, and V. M. Braun, Nucl. Phys. **B452**, 563 (1995).
[22] M. Neubert, Nucl. Phys. **B463**, 511 (1996).
[23] C. J. Maxwell and D. G. Tonge, Nucl. Phys. **B481**, 681 (1996); institution report DTP-97-30, hep-ph/9705314.
[24] P.A. Rączka and A. Szymacha, Z. Phys. C **70**, 125 (1996).
[25] P. A. Rączka, in *ICHEP 96*, Proceedings of 28th International Conference on High Energy Physics, Warsaw, Poland, 1996, edited by Z. Ajduk and A. K. Wróblewski (World Scientific, Singapore, 1997), p. 805.
[26] P. A. Pączka, in *TAU 96* [3], p. 403.
[27] K. A. Milton, I. L. Solovtsov, and O. P. Solovtsova, Phys. Lett. B **415**, 104 (1997).
[28] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **61**, 1815 (1988).
[29] E. Braaten and Chong Sheng Li, Phys. Rev. D **42**, 3888 (1990).
[30] K. G. Chetyrkin, Phys. Lett. B **307**, 169 (1993).
[31] S. A. Larin, T. van Ritbergen, and J. A. M. Vermaseren, Nucl. Phys. **B438**, 278 (1995).
[32] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979): **B147**, 519 (1979).
[33] C. S. Lam and T. M. Yan, Phys. Rev. D **16**, 703 (1977).
[34] K. Schilcher and M. D. Tran, Phys. Rev. D **29**, 570 (1984).
[35] S. L. Adler, Phys. Rev. D **10**, 3714 (1974).
[36] K. G. Chetyrkin, A. L. Kataev and F. V. Tkachov, Phys. Lett. **85B**, 277 (1979); M. Dine and J. Sapirstein, Phys. Rev. Lett. **43**, 668 (1979); W. Celmaster and R. J. Gonsalves, *ibid.* **44**, 560 (1979); Phys. Rev. D **21**, 3112 (1980).
[37] S. G. Gorishny, A. L. Kataev, and S. A. Larin, Phys. Lett. B **212**, 238 (1988).
[38] L. R. Surguladze and M. A. Samuel, Phys. Rev. Lett. **66**, 560 (1991); **66**, 2416(E) (1991).
[39] S. G. Gorishny, A. L. Kataev, and S. A. Larin, Phys. Lett. B **259**, 144 (1991).
[40] K. G. Chetyrkin, Phys. Lett. B **390**, 309 (1997).
[41] W. A. Bardeen, A. J. Buras, D. W. Duke, and T. Muta, Phys. Rev. D **18**, 3998 (1978).
[42] P. A. Rączka, Phys. Rev. D **46**, R3699 (1992).
[43] W. Celmaster and R. J. Gonsalves, Phys. Rev. D **20**, 1420 (1979).
[44] P. M. Stevenson, Phys. Lett. **100B**, 61 (1981); Phys. Rev. D **23**, 2916 (1981).
[45] A. Dhar, Phys. Lett. **128B**, 407 (1983).
[46] A. Dhar and V. Gupta, Pramana **21**, 207 (1983); Phys. Rev. D **29**, 2822 (1984).
[47] G. Grunberg, Phys. Rev. D **29**, 2315 (1984).
[48] A. A. Pivovarov, Z. Phys. C **53**, 461 (1992).
[49] F. Le Diberder and A. Pich, Phys. Lett. B **286**, 147 (1992).
[50] W. Wetzel, Nucl. Phys. **B196**, 259 (1982).
[51] W. Bernreuther and W. Wetzel, Nucl. Phys. **B197**, 228 (1982).
[52] W. Bernreuther, Ann. Phys. (N.Y.) **151**, 127 (1983); Z. Phys. C **20**, 331 (1983).
[53] G. Rodrigo and A. Santamaria, Phys. Lett. B **313**, 441 (1993).
[54] K. G. Chetyrkin, B. A. Kniehl and M. Steinhauser, Phys. Rev. Lett. **79**, 2184 (1997).
[55] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).
[56] G. Grunberg, Phys. Lett. **95B**, 70 (1980).
[57] W. Celmaster and D. Sivers, Phys. Rev. D **23**, 227 (1981).
[58] A. Dhar and V. Gupta, Phys. Lett. **101B**, 432 (1981).
[59] S. J. Brodsky, G. P. Lepage, and P. Mackenzie, Phys. Rev. D **28**, 228 (1983).
[60] For a summary of early contributions see D. W. Duke and R. G. Roberts, Phys. Rep. **120**, 275 (1985).
[61] P.A. Rączka, Z. Phys. C **65**, 481 (1995).
[62] M. R. Pennington, Phys. Rev. D **26**, 2048 (1982); J. C. Wrig-

ley, *ibid.* **27**, 1965 (1983); P. M. Stevenson, *ibid.* **27**, 1968 (1983); J. A. Mignaco and I. Roditi, Phys. Lett. **126B**, 481 (1983).

[63] W. Fischler, Nucl. Phys. **B129**, 157 (1977); T. Appelquist, M. Dine, and I. J. Muzinich, Phys. Lett. **69B**, 231 (1977); Phys. Rev. D **17**, 2074 (1978): A. Billoire, Phys. Lett. **92B**, 343 (1980).

[64] M. Peter, Phys. Rev. Lett. **78**, 602 (1997); Nucl. Phys. **B501**, 471 (1997).

[65] V. Giménez, J. A. Peñarrocha, and J. Bordes, Nucl. Phys. **B357**, 3 (1991).

[66] S. Narison, Phys. Lett. B **361**, 121 (1995).

[67] B. Chibisov, R. D. Dikeman, M. Shifman, and N. Uraltsev, Int. J. Mod. Phys. A **12**, 2075 (1997).