Weak phase γ from the ratio of $B \to K\pi$ rates

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The ratio of partial decay rates for charged and neutral B mesons to $K\pi$ final states provides information on the weak phase $\gamma \equiv \mathrm{Arg}(V_{ub}^*)$ when augmented with information on the CP-violating asymmetry in the $K^{\pm}\pi^{\mp}$ mode. The requirements for a useful determination of γ are examined in the light of present information about the decays $B^0 \to K^+\pi^-$, $B^+ \to K^0\pi^+$, and the corresponding charge-conjugate modes. The effects of electroweak penguin diagrams and rescattering corrections are noted, and proposals are made for estimating and measuring their importance. [S0556-2821(98)05311-9]

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I. INTRODUCTION

The leading candidate to describe the violation of CP symmetry in decays of neutral kaons [1] is the existence of phases in the weak charge-changing couplings of quarks to W bosons. These couplings are parametrized by a unitary 3×3 matrix, the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2] whose elements V_{ij} connect the quarks $i=u,c,t$ of charge 2/3 with those $j=d,s,b$ of charge $-1/3$.

The phase $\gamma = \mathrm{Arg}(V_{ub}^*)$ (in a standard convention [3]) is poorly known in this description. The unlikely possibility that $\gamma=0$ would require CP violation in the neutral kaon system to originate elsewhere than via CKM phases, e.g., via a superweak interaction [4]. Thus, it is important to seek independent information on γ, which is provided by the study of B meson decays.

Some time ago we proposed a method [5] of measuring the weak phases γ and α from the decays $B^0 \to K^+\pi^-$, $B^+ \to K^0\pi^+$, $B^0 \to \pi^+\pi^-$ and from charge-conjugated processes. In the present article we explore in detail the part of the method which determines γ utilizing primarily the ratio of decays of neutral and charged B mesons to $K\pi$ final states [6–8]. By combining information on the charge-averaged ratio

$$R \equiv \frac{\Gamma(B^0 \to K^+\pi^-) + \Gamma(\overline{B^0} \to K^-\pi^+)}{\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^- \to \overline{K^0}\pi^-)} \qquad (1)$$

with the CP-violating rate asymmetry

$$A_0 \equiv \frac{\Gamma(B^0 \to K^+\pi^-) - \Gamma(\overline{B^0} \to K^-\pi^+)}{\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^- \to \overline{K^0}\pi^-)}, \qquad (2)$$

we find an expression for γ which depends only on these quantities and on the ratio of tree to penguin amplitudes for which we provide an estimate based on $B \to \pi\pi$ and $B \to \pi l \nu_l$ decays. This method has become of particular interest now that the CLEO Collaboration has observed both the $B^0 \to K^+\pi^-$ and the $B^+ \to K^0\pi^+$ processes (and their charge conjugates) [9]. A similar idea can be applied to $B_s \to K^+K^-$ and $B_s \to K^0\overline{K^0}$ decays [8,10].

We define amplitudes and discuss their phases and magnitudes in Sec. II. The extraction of γ from $B \to K\pi$ decays occupies Sec. III. Several potential sources of systematic errors, involving electroweak penguin amplitudes and rescattering effects, are studied in Sec. IV. A generalization of the method to $B \to K^*\pi$ and $B \to K\rho$ decays is discussed in Sec. V, and Sec. VI concludes.

II. AMPLITUDES AND THEIR MAGNITUDES

A. Definitions

We adopt a flavor-SU(3) decomposition of amplitudes which has been used in several previous descriptions of B decays to pairs of light pseudoscalar mesons [5,11–14]. For present purposes the important amplitudes are strangeness-preserving (unprimed) and strangeness-changing (primed) amplitudes corresponding to color-favored tree (T,T'), penguin (P,P'), and color-suppressed tree (C,C') processes. The contributions of electroweak penguin diagrams [15,16] may be included by replacing $T \to t \equiv T + P_{EW}^C$, $P \to p \equiv P - (1/3)P_{EW}^C$, and $C \to c \equiv C + P_{EW}$, where the superscript on the electroweak penguin amplitude P_{EW} denotes color suppression. We stress that, although this general description of many processes in terms of just a few SU(3) amplitudes assumes flavor SU(3), in certain cases, such as the one discussed in the subsequent subsection, only isospin symmetry is required.

The phases of amplitudes for $\Delta S=0$ transitions are $\mathrm{Arg}(V_{ub}^* V_{ud}) = \gamma$ for tree amplitudes and $\mathrm{Arg}(V_{tb}^* V_{td}) = -\beta$ for top-dominated penguin amplitudes. For $|\Delta S|=1$ the corresponding phases are $\mathrm{Arg}(V_{ub}^* V_{us}) = \gamma$ (tree amplitude) and $\mathrm{Arg}(V_{tb}^* V_{ts}) = \pi$ (top-dominated penguin amplitude). Nothing changes in the $|\Delta S|=1$ penguin transitions if these receive important contributions from $c\bar{c}$ intermediate

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states [17,18]. While the phase of the $\Delta S=0$ penguin amplitude may be affected under such circumstances [17], we shall not be concerned with this phase.

In the rest of this section we ignore amplitudes which involve the participation of the spectator quark. Without rescattering, these amplitudes are expected to be suppressed by a factor of f_B/m_B, where $f_B \sim 200$ MeV is the B meson decay constant. The neglect of these contributions was noted to be equivalent to an assumption that some rescattering effects are unimportant, thus leading to the vanishing of certain final state interaction phase differences [12,19–21]. Such amplitudes can also be generated by rescattering from intermediate states obtained by $T^{(\prime)}$, $P^{(\prime)}$, $C^{(\prime)}$ amplitudes. Using a Regge analysis to demonstrate rescattering effects [22,23], it was shown [24] that such amplitudes may be suppressed only by a factor of about 0.2 [25] rather than by $f_B/m_B \sim 0.04$, in which case explicit tests for such rescattering can be performed. The effect of these rescattering amplitudes, assumed to be as large as estimated in Ref. [24], will be studied in Sec. IV B. Similarly, we begin by ignoring electroweak penguin contributions, deferring their treatment to Secs. IV A.

B. Decomposition for $B\to K\pi$ decays

The amplitude for $B^+\to K^0\pi^+$ is given by a QCD-penguin contribution:

$$A(B^+\to K^0\pi^+) = -|P'|, \tag{3}$$

where we have adopted a convention in which all strong phases are expressed relative to that in the $|\Delta S|=1$ penguin amplitude. As a consequence, one expects no CP-violating difference between the partial widths $\Gamma_{0+}\equiv\Gamma(B^+\to K^0\pi^+)$ and $\Gamma_{0-}\equiv\Gamma(B^-\to \overline{K^0}\pi^-)$. (We use a notation in which the subscripts denote the charges of the final kaon and pion.) For brevity we shall thus define $\Gamma_C\equiv\Gamma_{0+}=\Gamma_{0-}$ to be the partial width for a charged B to decay to a neutral kaon and a charged pion.

The amplitude for $B^0\to K^+\pi^-$ is expected to be dominated by the penguin contribution P'. One only uses isospin symmetry to relate the penguin amplitudes in neutral and charged B decays to $K\pi$ states. The tree contribution can be roughly estimated [12], $|T'/P'|\sim 0.2$. We shall refine this estimate presently. Thus

$$A(B^0\to K^+\pi^-) = |P'| - |T'|e^{i\delta}e^{i\gamma},$$

$$A(\overline{B^0}\to K^-\pi^+) = |P'| - |T'|e^{i\delta}e^{-i\gamma}, \tag{4}$$

where δ is the strong phase difference between the tree and penguin amplitudes. The corresponding rates may be defined as $\Gamma_{+-}\equiv\Gamma(B^0\to K^+\pi^-)$ and $\Gamma_{-+}\equiv\Gamma(\overline{B^0}\to K^-\pi^+)$. A CP-violating rate asymmetry $\Gamma_{+-}\neq\Gamma_{-+}$ may arise whenever both $\sin\delta$ and $\sin\gamma$ are nonvanishing. At the same time, even if $\sin\delta=0$ so that $\Gamma_{+-}=\Gamma_{-+}$, these two partial rates may differ from Γ_C as a result of the extra T' contribution they contain [6–8]. This difference can shed light on the weak phase γ. Our purpose in the present paper is to estimate the experimental demands on such a determination.

C. Magnitudes of amplitudes

The CLEO Collaboration [9] has observed both $B^0\to K^+\pi^-$ and $B^+\to K^0\pi^+$ (here we do not distinguish between a process and its charge conjugate). The observed branching ratios are

$$\mathcal{B}(B^0\to K^+\pi^-) = (15^{+5}_{-4}\pm 1\pm 1)\times 10^{-6}, \tag{5}$$

$$\mathcal{B}(B^+\to K^0\pi^+) = (23^{+11}_{-10}\pm 3\pm 2)\times 10^{-6}. \tag{6}$$

We shall express squares of amplitudes in units of B branching ratios times 10^6. In Ref. [14] we averaged the rates (5) and (6) to obtain $|P'|^2=16.3\pm 4.3$. However, here we shall leave open the possibility that a significant T' contribution is affecting the $B^0\to K^+\pi^-$ rate, and take $|P'|^2=23\pm 10.5$ from the $B^+\to K^0\pi^+$ rate.

We estimate $|T'|$ by relating it through flavor SU(3) to the corresponding strangeness-preserving amplitude $|T|$ governing such decays as $B^0\to\pi^+\pi^-$ and $B^+\to\pi^+\pi^0$. Using the phase conventions of Ref. [12], we find

$$A(B^0\to\pi^+\pi^-) = -(T+P),$$

$$A(B^+\to\pi^+\pi^0) = -(T+C)/\sqrt{2}. \tag{7}$$

Although neither process has been observed with a statistically significant signal, Ref. [9] quotes a 2.8σ signal of

$$\mathcal{B}(B^+\to\pi^+\pi^0) = (9^{+6}_{-5})\times 10^{-6} \tag{8}$$

and a 2.2σ signal of

$$\mathcal{B}(B^0\to\pi^+\pi^-) = (7\pm 4)\times 10^{-6}. \tag{9}$$

Taking Eq. (8) as an estimate of $|T|^2/2 = 9\pm 5.5$ (neglecting the color-suppressed amplitude C in $B^+\to\pi^+\pi^0$), and Eq. (9) as an estimate of $|T|^2=7\pm 4$ (neglecting the penguin amplitude P in $B^0\to\pi^+\pi^-$), we find [14] that $|T|^2=8.3\pm 3.8$. The observed rate for the semileptonic decay $B^0\to\pi^- l^+\nu_l$ [26],

$$\mathcal{B}(B^0\to\pi^- l^+\nu_l) = (1.8\pm 0.4\pm 0.3\pm 0.2)\times 10^{-4}, \tag{10}$$

is compatible with this estimate if one calculates the $B^0\to\pi^+\pi^-$ decay via factorization. An early estimate [27] based on a form factor dominated by the B^* pole,

$$\frac{\Gamma(B^0\to\pi^- l^+\nu_l)}{\Gamma(B^0\to\pi^+\pi^-)} = \frac{M_B^2}{12\pi^2 f_\pi^2} \simeq 13, \tag{11}$$

would imply $\mathcal{B}(B^0\to\pi^+\pi^-) = (1.4\pm 0.4)\times 10^{-5}$ and hence $|T|^2 = 14\pm 4$ on the basis of the observed semileptonic rate (10). More recent estimates [28] yield a similar range of values. An improvement of the data will allow one to focus on the q^2 value appropriate to pion (or kaon) production and thus to reduce the dependence on models drastically. The direct CLEO upper limit [9] $\mathcal{B}(B^0\to\pi^+\pi^-) < 1.5\times 10^{-5}$ (90% C.L.) gives a poorer upper limit on $|T|^2$ than our estimate.

One then uses factorization which introduces SU(3) breaking through a factor f_K/f_π [12] to predict

$$|T'/T| = (f_K/f_\pi)|V_{us}/V_{ud}| = 0.27 \tag{12}$$

with an error estimated to be about 20% [the typical breaking of flavor SU(3) symmetry] [29]. Combining the estimates for amplitudes and their ratio, we then find

$$r\equiv|T'/P'|=0.16\pm0.06. \tag{13}$$

The estimate of $|T'|$ is likely to improve in the future once the spectrum for the semileptonic decay $B^0\rightarrow\pi^- l^+\nu_l$ is measured at $q^2=m_K^2$. One uses factorization directly [30,31] to predict

$$\Gamma(B^0\rightarrow K^+\pi^-)|_{\rm tree} = 6\pi^2 f_K^2|V_{us}|^2 a_1^2 \left.\frac{d\Gamma(B^0\rightarrow\pi^- l^+\nu_l)}{dq^2}\right|_{q^2=m_K^2}, \tag{14}$$

with $a_1=1.08\pm0.04$ [32]. This value was obtained from a fit to $b\rightarrow c\bar{u}d$ subprocesses in $B\rightarrow D^{(*)}+\pi(\rho)$ decays. The value appropriate to the subprocesses $b\rightarrow u\bar{u}d$ and $b\rightarrow u\bar{u}s$ which contribute to the tree amplitudes in $B\rightarrow\pi\pi$ and $B\rightarrow K\pi$ may be slightly different. It may be difficult to determine a_1 to an accuracy of better than 10% in these processes as a result of penguin (P) amplitudes accompanying the factorizable color-allowed ($T\sim a_1$) and non-factorizable color-suppressed ($C\sim a_2$) $\Delta S=0$ amplitudes. (In contrast, $B\rightarrow D\pi$ has no penguin contributions.)

Since the present branching ratio (10) is known to about 30%, a factor of 100 increase in the data sample (envisioned in future high-intensity studies) would permit this branching ratio to be known to about 3%. More crucial is the error on the differential rate on the right-hand side of Eq. (14). As we shall see, a 10% determination of $|T'|$ (hence a 20% accuracy in the differential rate at $q^2=m_K^2$) is the accuracy that will be required in order for the present method to be reasonably useful.

III. EXTRACTION OF γ FROM $B\rightarrow K\pi$ RATES

A. Fleischer-Mannel bound

Recalling the definitions of Sec. II, we may form the ratio

$$R\equiv\frac{\Gamma(B^0\rightarrow K^+\pi^-)+\Gamma(\overline{B^0}\rightarrow K^-\pi^+)}{\Gamma(B^+\rightarrow K^0\pi^+)+\Gamma(B^-\rightarrow \overline{K}^0\pi^-)}=\frac{\Gamma_{+-}+\Gamma_{-+}}{2\Gamma_C} \tag{15}$$

which has the simple form [6–8]

$$R=1-2r\cos\gamma\cos\delta+r^2. \tag{16}$$

Fleischer and Mannel [7] have pointed out that if $R<1$ a useful bound on γ can be obtained regardless of the value of r or δ:

$$\sin^2\gamma\leqslant R. \tag{17}$$

The present value of R is 0.65 ± 0.40, and so a reduction of errors by a factor of 3 with no change in central value would begin to provide a useful limit excluding some region around $\gamma=\pi/2$.

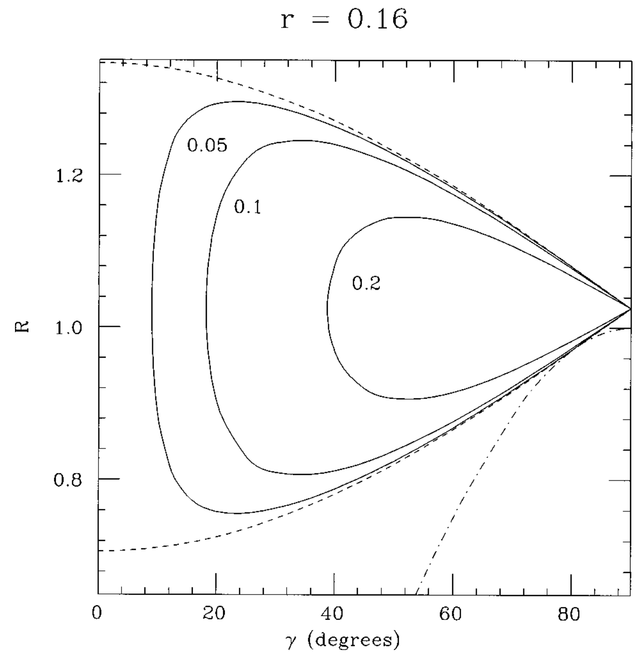

FIG. 1. Value of R (ratio of neutral to charged $B\rightarrow K\pi$ partial widths) as a function of $\gamma=\mathrm{Arg}(V_{ub}^*)$ for $r=0.16$. Solid lines are labeled by values of the pseudo-asymmetry parameter $|A_0|$. Dotted boundary lines correspond to $A_0=0$. The case $\gamma=0$ for arbitrary R between the bounds of the dashed lines also corresponds to $A_0=0$. Also shown (dot-dashed lines) is the Fleischer-Mannel bound $\sin^2\gamma\leqslant R$.

B. Use of information on r

In the presence of information on r (see also Ref. [8]) one can provide a more precise estimate of γ by measuring the difference in $B^0\rightarrow K^+\pi^-$ and $\overline{B^0}\rightarrow K^-\pi^+$ decay rates. One forms the pseudo-asymmetry

$$A_0\equiv\frac{\Gamma(B^0\rightarrow K^+\pi^-)-\Gamma(\overline{B^0}\rightarrow K^-\pi^+)}{\Gamma(B^+\rightarrow K^0\pi^+)+\Gamma(B^-\rightarrow \overline{K}^0\pi^-)}=\frac{\Gamma_{+-}-\Gamma_{-+}}{2\Gamma_C}. \tag{18}$$

Note that the denominator is taken to be $2\Gamma_C$ in order to divide by $|P'|^2$ without any complication from the T' amplitude. Since

$$A_0=2r\sin\delta\sin\gamma, \tag{19}$$

one can combine Eqs. (16) and (19) to eliminate δ. The result is

$$R=1+r^2\pm\sqrt{4r^2\cos^2\gamma-A_0^2\cot^2\gamma}. \tag{20}$$

This quantity is plotted in Fig. 1 for $r=0.16$ (the central value of our estimate) and various values of A_0. (Note that the results are insensitive to the sign of A_0.) We vary r between its limits and plot R for $r=0.10$ in Fig. 2 and for $r=0.22$ in Fig. 3.

Let us assume for the moment that r is known. Figures 1–3 have several interesting features.

(1) The maximum value of $|A_0|=2r$ occurs only for $\gamma=\delta=90°$ [as one sees from Eq. (19)].

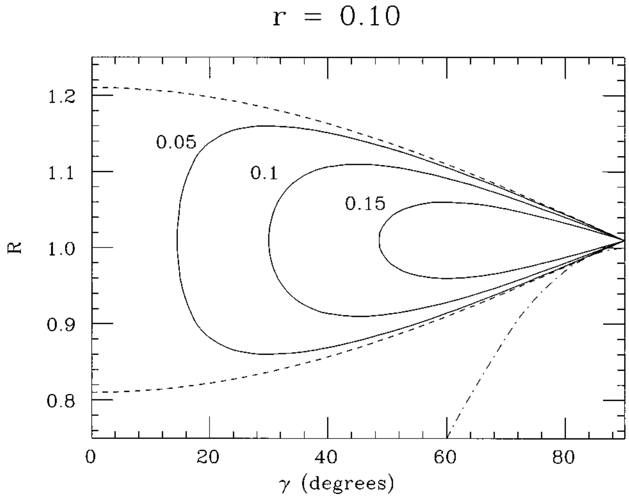

FIG. 2. Same as Fig. 1 but for $r=0.10$.

(2) The result is symmetric with respect to $\gamma \to \pi-\gamma$, since A_0 is only sensitive to sin γ and the cos δ term in R involves a sign arbitrariness.

(3) The sensitivity to γ in the range $\gamma > 45°$ is greatest for $A_0=0$.

(4) As long as $A_0 \neq 0$ there will be two solutions for γ in the range $0 \leqslant \gamma \leqslant \pi/2$ (and two in the range $\pi/2 \leqslant \gamma \leqslant \pi$) for any given R. Observation of a non-zero A_0 would rule out the possibility of $\gamma=0$ mentioned in the Introduction and hence would disprove a superweak model of CP violation.

The sign of cos δ, which would resolve the ambiguity between γ and $\pi-\gamma$, can be studied theoretically. One model-dependent calculation [33] finds $\delta < \pi/2$, implying that R is smaller (larger) than $1+r^2$ for γ smaller (larger) than $\pi/2$. This model calculation ignores possible phases due to soft final state interactions [22,34].

Equation (20) can be inverted to obtain a quadratic equation for $\sin^2 \gamma$ in terms of r, A_0, and R. We then find

$$4r \sin \gamma = \pm\{[(1+r)^2-(R+A_0)][(R-A_0)-(1-r)^2]\}^{1/2} \pm\{[(1+r)^2-(R-A_0)][(R+A_0)-(1-r)^2]\}^{1/2}. \tag{21}$$

This relation also follows directly from the geometry of a triangle formed by the amplitudes $A(B^+ \to K^0\pi^+)$, T', $A(B^0 \to K^+\pi^-)$ and the charge-conjugated triangle. The triangle construction is similar to one employed in Ref. [35] for obtaining γ from $B \to DK$ decays. One measures the sides of two triangles (for three processes and their charge conjugates), sharing a common base (P' in the present case), where another pair of sides in both triangles is equal in length (T' in the present case), and forms an angle 2γ. Just as in the $B \to DK$ case, one of the sides of each triangle (T') is much smaller than the others. This leads to large experimental uncertainties in determining γ, following from relatively small experimental errors in the side measurements (square root of rates). Furthermore, the magnitude of the small T' suffers the largest theoretical uncertainty. So, if one draws these triangles in roughly correct proportions, one can see why the uncertainty in γ is likely to be large.

C. Required precision

The precision in R, r, and A_0 required to measure γ to a given level depends on their values. We consider two extreme cases of final state phases which bracket others. In the first, with sin $\delta \simeq 0$, corresponding to parameters near the dashed boundary curves in Figs. 1–3, the error in γ is dominated by the errors in R and r. In the second, corresponding to parameters midway between the dashed boundary curves in Figs. 1–3, with cos $\delta \simeq 0$ and $R \simeq 1+r^2$, the error in γ is due to the errors in r and A_0.

(1) When sin $\delta=0$, we have the simple result

$$\cos \gamma = \pm \frac{R-1-r^2}{2r}, \tag{22}$$

so that

$$\left.\frac{\partial \gamma}{\partial R}\right|_r = \mp \frac{1}{2r \sin \gamma}, \quad \left.\frac{\partial \gamma}{\partial r}\right|_R = \frac{\cos \gamma \pm r}{r \sin \gamma}. \tag{23}$$

Let us take as an example the case $r=0.16$, $A_0=0$, and $\gamma=90°$. Then $\Delta\gamma=5°$ corresponds to $|\Delta R|=0.028$ for fixed r and $|\Delta r|=0.087$ for fixed R. The requirement on R is more stringent. In order to satisfy it, we need about 2500 events in both charged and neutral modes of $B \to K\pi$, or roughly 200 times the present sample. This is thought to be within the capabilities of an upgraded version of the Cornell Electron Storage Ring (CESR) [36], as well as dedicated hadronic B production experiments at the Fermilab Tevatron and the CERN Large Hadron Collider (LHC) [37].

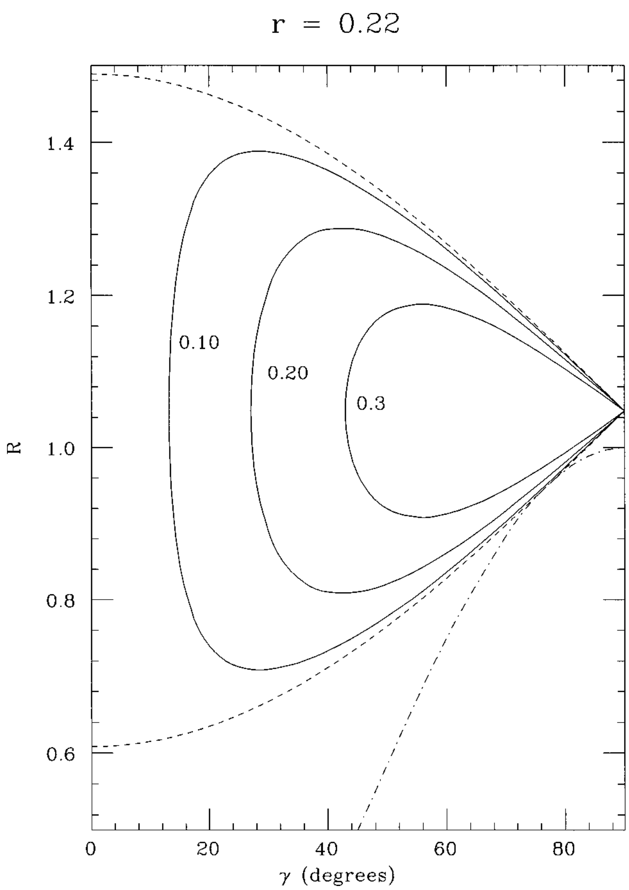

FIG. 3. Same as Fig. 1 but for $r=0.22$.

A second example with sin $\delta=0$ exhibits greater sensitivity to r. When $\gamma=45°$ or 135° (which is about as far from $\gamma=90°$ as allowed by present fits [38]) we have

$$\left.\frac{\partial\gamma}{\partial R}\right|_r=\mp\frac{1}{\sqrt{2}r},\quad \left.\frac{\partial\gamma}{\partial r}\right|_R=\pm\sqrt{2}+\frac{1}{r}. \tag{24}$$

Choosing $r=0.16$ and the positive sign in the second of the above two equations to exhibit the more stringent requirement, we find that an error of $|\Delta R|=0.028$, which as noted in the previous example is thought to be within reach in future experiments, corresponds to $\Delta\gamma=7°$. An error in γ of this same magnitude (for $r=0.16$) is associated with $\Delta r=0.016$, i.e., a 10% error in r or a 20% error on the quantity $d\Gamma(B^0\to\pi^- l^+\nu_l)/dq^2$ at $q^2=m_K^2$, as noted at the end of Sec. II C. Thus, one can envision determining γ to an overall error of $[(7°)^2+(7°)^2]^{1/2}\simeq 10°$ if $\gamma=45°$ or 135°, and if the required precision on r can be achieved.

(2) When cos $\delta=0$, the magnitude of the asymmetry $|A_0|$ is just $2r$ sin γ, so

$$\left.\frac{\partial\gamma}{\partial|A_0|}\right|_r=\frac{1}{2r\cos\gamma},\quad \left.\frac{\partial\gamma}{\partial r}\right|_{|A_0|}=-\frac{\tan\gamma}{r}. \tag{25}$$

Taking $\gamma=45°$ and $r=0.16$, we find $\Delta\gamma=7°$ corresponds to $\Delta|A_0|=0.028$ for fixed r and $\Delta r=0.02$ for fixed $|A_0|$. Measurement of $|A_0|$ to ±0.028 requires 1250 events each of $B^0\to K^+\pi^-$ and $\overline{B^0}\to K^-\pi^+$. Thus, a measurement of γ with an overall error of less than 10° appears feasible in this case as well.

The reduction of $\Delta r/r$ by about a factor of 4 to $\simeq$10% appears to be at the limits of understanding of form factors which would permit determination of T in $B\to\pi^+\pi^-$ from factorization and $B\to\pi l\nu_l$. (The $B\to\pi l\nu$ decay is claimed to be capable of yielding an accuracy of 10% in determining $|V_{ub}|$ [39].) As noted at the end of Sec. II C, if factorization is found to be reliable in comparing $B^0\to\pi^+\pi^-$ and $B\to\pi l\nu$ decays, one may be able to pass directly from $B\to\pi l\nu$ decays to an estimate of the tree (T') contribution to $B^0\to K^+\pi^-$, since all that is required is for the weak current to produce a kaon, a process which we can estimate reliably.

IV. SYSTEMATIC ERRORS

Aside from the statistical errors analyzed in Sec. III, we have noted that a systematic theoretical error in determining r of $\simeq$10% seems to be unavoidable. An error of this magnitude is encountered whether we determine r from $B^0\to\pi^+\pi^-$ and $B^+\to\pi^+\pi^0$ decays, thereby omitting nonfactorizable P and C terms, respectively, or from $B\to\pi l\nu_l$ decays. In addition, two smaller contributions to the amplitudes should be noted: (A) the effects of the color-suppressed electroweak penguin terms $P_{EW}'^C$ in $B^0\to K^+\pi^-$ and $B^+\to K^0\pi^+$, and (B) the effect of the annihilation amplitude (called A' in Ref. [12]) (or rescattering effects) on $B^+\to K^0\pi^+$.

Fleischer and Mannel find a very small electroweak penguin term [7], $|P_{EW}'^C/P'|<0.01$. The small value is obtained as a result of a delicate cancellation among larger contributions from electroweak penguin operators which have a different color structure. The calculation is based on factorization of hadronic matrix elements of QCD and electroweak penguin operators. With all uncertainties involved, a conservative estimate should allow values of $|P_{EW}'^C/P'|$ at a level of 5% [40], given roughly by a product of the ratio of corresponding Wilson coefficients and a color factor [12]. The effect of electroweak penguin contributions on our analysis will be studied in Sec. IV A. We will also suggest ways of measuring electroweak penguin contributions in related processes.

A small A' term should also be allowed in $B^+\to K^0\pi^+$. A naive estimate of A' neglecting rescattering, $|A'/T'|\sim f_B/m_B$, yields $|A'/P'|\sim0.01$. (A similar estimate applies to the u-quark contribution to P'.) Rescattering effects are hard to calculate. Regge-model estimates [23,24], in which rescattering from intermediate states such as $K^+\pi^0$ is described by a ρ-trajectory exchange, suggest $|A'/T'|\sim0.2$. The consequences of such rescattering effects will be studied in Sec. IV B. A few methods for direct measurements of rescattering effects in SU(3)-related processes will also be described.

A. Modification due to electroweak penguins

In the presence of electroweak penguin contributions, which carry the same weak phase as P', Eqs. (3) and (4) take the form [5]

$$A(B^+\to K^0\pi^+)=A(B^-\to\overline{K^0}\pi^-)=-|p'|,$$

$$p'\equiv P'-(1/3)P_{EW}'^C, \tag{26}$$

$$A(B^0\to K^+\pi^-)=|p'+P_{EW}'^C|-|T'|e^{i\delta}e^{i\gamma},$$

$$A(\overline{B^0}\to K^-\pi^+)=|p'+P_{EW}'^C|-|T'|e^{i\delta}e^{-i\gamma}. \tag{27}$$

A common unmeasurable strong phase in the B^0 and $\overline{B^0}$ decay amplitudes has been omitted; δ is the corrected strong phase difference. Consequently, the expressions for R and A_0 are modified as follows:

$$R/a^2=1-2r'\cos\gamma\cos\delta+r'^2,$$

$$A_0/a^2=2r'\sin\delta\sin\gamma, \tag{28}$$

where $a=|1+(P_{EW}'^C/p')|$ and $r'=(1/a)|T'/p'|=|T'/(p'+P_{EW}'^C)|$. The Fleischer-Mannel bound becomes $\sin^2\gamma\leq R/a^2$.

Using $|P_{EW}'^C/p'|<0.05$, and assuming an arbitrary relative strong phase difference between p' and P_{EW}^C, one has $a^2=1.0\pm0.1$. This factor normalizes both the ratio of rates R and the asymmetry A_0. The electroweak penguin terms introduce an additional 5% uncertainty in the ratio of tree-to-penguin amplitudes r'.

An important question is whether a is larger or smaller than 1. Model-dependent perturbative calculations [7,33] of QCD and electroweak penguin amplitudes suggest that the strong phase difference between p' and $P_{EW}'^C$ is smaller than $\pi/2$; hence $a>1$. This would imply that both R and A_0 can only be increased by electroweak penguin contributions and that the Fleischer-Mannel bound is maintained. Since such calculations disregard possible phases due to soft final state interactions [22,34], one cannot exclude, however, the possibility that $a<1$.

One way to obtain a clue to the sign of $a-1$ is to compare the rates for $B^+\to K^0\pi^+$ and $B^0\to K^0\pi^0$ [12]:

$$\frac{2\Gamma(B^0\to K^0\pi^0)}{\Gamma(B^+\to K^0\pi^+)}=\left|1-\frac{P'_{EW}}{p'}\right|^2, \tag{29}$$

where a smaller color-suppressed tree (C') term was neglected. A measurement of this ratio would determine whether the relative strong phase between P'_{EW} and p' is larger or smaller than $\pi/2$. Although, in principle, P'_{EW} and P'^{C}_{EW} can carry different strong phases, it seems likely that this information would be sufficient to determine whether a is larger or smaller than 1. The deviation of the ratio (29) from 1 would also provide some information on the magnitude of the color-allowed electroweak amplitude P'_{EW}, which could provide a useful measure for the smaller P'^{C}_{EW} term. Other ways of measuring the importance of electroweak penguins have been noted in Refs. [15,16,41].

The inclusion of electroweak penguins in Figs. 1–3 is straightforward. The figures are to be interpreted as plots of R/a^2 versus γ for different values of A_0/a^2 and for a fixed value of r'. The $1/a^2$ factor involves a 10% uncertainty. Such an uncertainty in R/a^2 is seen to lead to a rather large theoretical error in determining γ, typically of a few tens of degrees. The error decreases with increasing r'.

B. Modification due to rescattering

In the presence of final state rescattering, the general decomposition of the decay amplitudes of charged and neutral B mesons is given in terms of amplitudes carrying specific weak phases [12]:

$$A(B^+\to K^0\pi^+)=-|p'|+|A'|e^{i\Delta}e^{i\gamma}, \tag{30}$$

$$A(B^0\to K^+\pi^-)=|p'+P'^{C}_{EW}|-|T'|e^{i\delta}e^{i\gamma}. \tag{31}$$

We will assume that the magnitude of A' (acquiring an unknown strong phase Δ), dominated by rescattering from intermediate states such as $K^+\pi^0$ and multibody states, is given by $|A'/T'|\sim 0.2$. Estimates of the other ratios of amplitudes were given in previous sections, $r'\approx|T'/p'|\sim 0.2$ and $|P'^{C}_{EW}/p'|\sim 0.05$.

The general expressions for R and A_0 are

$$\left(\frac{f}{a^2}\right)R=1-2r'\cos\gamma\cos\delta+r'^2,$$

$$\left(\frac{f}{a^2}\right)A_0=2r'\sin\delta\sin\gamma, \tag{32}$$

where $f\equiv 1-2|A'/p'|\cos\Delta\cos\gamma+|A'/p'|^2$. The interference between p' and A' in $B^+\to K^0\pi^+$ leads to an asymmetry

$$A_+\equiv\frac{\Gamma(B^+\to K^0\pi^+)-\Gamma(B^-\to\overline{K^0}\pi^-)}{\Gamma(B^+\to K^0\pi^+)+\Gamma(B^-\to\overline{K^0}\pi^-)}, \tag{33}$$

which is given by $fA_+=2|A'/p'|\sin\Delta\sin\gamma$. Since $|A'/p'|\sim 0.05$, this asymmetry is not expected to exceed the level of 10%.

The factor f/a^2 in R and A_0 involves roughly equal hadronic uncertainties due to rescattering (f) and electroweak penguin (a^2) contributions. Although it is unlikely that this factor differs from unity by more than 20% ($a^2/f=1\pm 0.2$), it introduces sizable uncertainties in the determination of γ as described in Sec. III. The Fleischer-Mannel bound becomes $\sin^2\gamma\leqslant(f/a^2)R$. The usefulness of this method in determining (or at least constraining) γ depends crucially on future experimental limits on rescattering effects. Let us mention a few such possible measurements in SU(3)-related processes.

The most direct measurements of rescattering effects can be made in B decay processes which in the framework of a diagramatic SU(3) description [12] proceed only through annihilation of the b and spectator quarks. Such decays may also proceed via rescattering from other less-suppressed amplitudes. In this case, Regge-model estimates [23,24] suggest that ratios of amplitudes such as $|A'/T'|$ and $|E/T|$ are enhanced from $f_B/m_B\sim 0.04$ (without rescattering) to ~ 0.2 (with rescattering). A list of all such processes, in which a B meson decays to two pseudoscalars, is given in Ref. [24].

Consider, for instance, $B^0\to K^+K^-$ which is given by the amplitude E. In the absence of rescattering one expects $\mathcal{B}(B^0\to K^+K^-)/\mathcal{B}(B^0\to\pi^+\pi^-)\approx|E/T|^2\sim(f_B/m_B)^2\sim 0.002$. [SU(3) breaking and the penguin contribution to $B^0\to\pi^+\pi^-$ are neglected.] For $\mathcal{B}(B^0\to\pi^+\pi^-)=10^{-5}$, this would imply $\mathcal{B}(B^0\to K^+K^-)\sim 2\times 10^{-8}$, or 200 times smaller than the present CLEO upper limit [9]. On the other hand, a description of rescattering into K^+K^- from $\pi^+\pi^-$ and from other intermediate states, in terms of a K^* Regge-trajectory exchange, suggests that $|E/T|\sim 0.2$, thus implying $\mathcal{B}(B^0\to K^+K^-)\sim 4\times 10^{-7}$, only an order of magnitude below the present limit. A future stringent bound on $\mathcal{B}(B^0\to K^+K^-)$, at a level of 10^{-7} or lower, would provide a useful limit on rescattering effects.

Another way to measure these effects is to compare $B^0\to K^0\overline{K^0}$ (given by P) with $B^+\to K^+\overline{K^0}$ (given by $P+A$) [12], both of which are anticipated to have branching ratios near 10^{-6}. If rescattering can be neglected, then $|A/T|\sim f_B/m_B\sim 0.04$, while $|P/T|\sim 0.2$ follows from recent CLEO measurements [14]. Therefore, the two branching ratios are expected to be equal within a factor of less than $\simeq 1.5$. (Electroweak penguin contributions do not affect this relation.) On the other hand, Regge-model rescattering implies $|A/T|\sim 0.2$, in which case the two branching ratios may differ substantially, by up to a factor of 4 or so. Also, the interference of P and A in $B^+\to K^+\overline{K^0}$ would lead to a sizable CP asymmetry between the rate of this process and its charge conjugate. These measurements could provide useful limits on final state rescattering.

V. GENERALIZATION TO $B\to(K^*\pi,\rho K)$ DECAYS

In addition to $B\to K\pi$ decays, one may also obtain information about γ from the analogous decays to a vector meson and a pseudoscalar, $B\to K\rho$ and $B\to K^*\pi$. Each of these two systems of neutral and charged B decays involves SU(3) amplitudes of a specific kind [42], and can be studied separately in a way very similar to $B\to K\pi$. The amplitudes of $B^+\to K^{*0}\pi^+$ and $B^0\to K^{*+}\pi^-$ are given in terms of P'_P and T'_P, whereas those of $B^+\to K^0\rho^+$ and $B^0\to K^+\rho^-$ are described by P'_V and T'_V. Here the subscript on each amplitude denotes whether the spectator quark is included in a pseudoscalar (P) or a vector (V) meson.

The values of the hadronic parameters, $|T'/P'|$, δ and the electroweak penguin corrections obtain different values in $B\to K\pi$, $B\to K^*\pi$ and $B\to K\rho$. As noted in Sec. III, the sensitivity of measuring γ increases with $|T'/P'|$ since the asymmetry A_0 and the deviation of R from 1 are both proportional to this ratio. In Ref. [42] we concluded from recent CLEO data [43] on $B^+\to K^+\omega$ and $B^+\to K^+\phi$ that $|P'_P| < |P'_V|$. Model-dependent calculations [18,44] predict $|T_P| > |T_V|$. If this turns out to be the case, namely if $\mathcal{B}(B^+\to\rho^+\pi^0) > \mathcal{B}(B^+\to\rho^0\pi^+)$ and $\mathcal{B}(B^0\to\rho^+\pi^-) > \mathcal{B}(B^0\to\rho^-\pi^+)$, then one would also conclude that $|T'_P| > |T'_V|$. Hence, $|T'_P/P'_P| > |T'_V/P'_V|$, implying that in this respect decays to $K^*\pi$ are more sensitive to a measurement of γ than decays to $K\rho$.

VI. CONCLUSIONS

By measuring the rates for $B^0\to K^+\pi^-$, $B^+\to K^0\pi^+$, and their charge conjugates, it is possible to extract information on the weak phase γ. A useful level of precision ($\Delta\gamma \simeq \pm 10°$) requires about 200 times the present data sample of about a dozen events in each channel. This appears to be within the reach of the highest luminosities attainable at e^+e^- colliders operating at the $\Upsilon(4S)$, and may also be feasible in hadronic production of B mesons.

A key source of uncertainty in the method appears to be the determination of the magnitude of the tree amplitude (T') interfering with the dominant penguin amplitude (P'). This requires one to measure the $B^0\to\pi^+\pi^-$ or $B^+\to\pi^+\pi^0$ rate to better than 20% and to make corresponding improvements in the understanding of how well factorization applies to the comparison of $B^0\to\pi^- l^+\nu_l$ with the tree amplitudes of $B^0\to\pi^+\pi^-$, $B^+\to\pi^+\pi^0$ and $B^0\to K^+\pi^-$ decays.

Another theoretical error follows from hadronic uncertainties in calculating electroweak penguin contributions to the $B\to K\pi$ decay amplitudes. A 5% contribution would lead to an uncertainty in γ of the order of tens of degrees. Finally, we argued that an uncertainty at a similar level follows from possible rescattering effects in $B^+\to K^0\pi^+$. The importance of such effects may be found by future measurements of the rates for $B^0\to K^+K^-$, $B^0\to K^0\overline{K^0}$ and $B^+\to K^+\overline{K^0}$.

Note added. After the present work was submitted for publication there appeared a related study [45] involving a slightly broader range of $r=0.20\pm0.07$. Very recently R. Fleischer [46] has discussed a set of measurements which permit constraints to be placed on rescattering and electroweak penguin effects.

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[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Lett. **13**, 138 (1964).
[2] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[3] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).
[4] L. Wolfenstein, Phys. Rev. Lett. **13**, 562 (1964).
[5] M. Gronau and J. L. Rosner, Phys. Rev. Lett. **76**, 1200 (1996); A. S. Dighe, M. Gronau, and J. L. Rosner, Phys. Rev. D **54**, 3309 (1996); A. S. Dighe and J. L. Rosner, *ibid.* **54**, 4677 (1996).
[6] R. Fleischer, Phys. Lett. B **365**, 399 (1996).
[7] R. Fleischer and T. Mannel, Phys. Rev. D **57**, 2752 (1998).
[8] R. Fleischer, CERN Report No. CERN-TH/97-281, hep-ph/9710331 (unpublished).
[9] CLEO Collaboration, R. Godang *et al.*, Phys. Rev. Lett. **80**, 3456 (1998).
[10] R. Fleischer and I. Dunietz, Phys. Rev. D **55**, 259 (1997).
[11] D. Zeppenfeld, Z. Phys. C **8**, 77 (1981); M. Savage and M. Wise, Phys. Rev. D **39**, 3346 (1989); **40**, 3127(E) (1989); L. L. Chau *et al.*, *ibid.* **43**, 2176 (1991).
[12] M. Gronau, O. Hernández, D. London, and J. L. Rosner, Phys. Rev. D **50**, 4529 (1994); **52**, 6356 (1995); **52**, 6374 (1995).
[13] M. Gronau and J. L. Rosner, Phys. Rev. D **53**, 2516 (1996); A. S. Dighe, M. Gronau, and J. L. Rosner, Phys. Lett. B **367**, 357 (1996); **377**, 325(E) (1996).
[14] A. S. Dighe, M. Gronau, and J. L. Rosner, Phys. Rev. Lett. **79**, 4333 (1997).
[15] R. Fleischer, Z. Phys. C **62**, 81 (1994); Phys. Lett. B **321**, 259 (1994); **332**, 419 (1994).
[16] N. G. Deshpande and X.-G. He, Phys. Lett. B **336**, 471 (1994); Phys. Rev. Lett. **74**, 26 (1995); N. G. Deshpande, X.-G. He, and J. Trampetic, Phys. Lett. B **345**, 547 (1995).
[17] A. J. Buras and R. Fleischer, Phys. Lett. B **341**, 379 (1995).
[18] M. Ciuchini, E. Franco, G. Martinelli, and L. Silvestrini, Nucl. Phys. **B501**, 271 (1997); M. Ciuchini, R. Contino, E. Franco, G. Martinelli, and L. Silvestrini, *ibid.* **B512**, 3 (1998). This last work contains an extensive list of further references.
[19] L. Wolfenstein, Phys. Rev. D **52**, 537 (1995).
[20] M. Gronau, J. L. Rosner, and D. London, Report No. CERN-TH.7407/94, EFI-94-41, 1994 (unpublished); see also Sec. VI E in the second paper of Ref. [12].
[21] J.-M. Gérard and J. Weyers, Univ. Catholique de Louvain Report No. UCL-IPT-97-18, hep-ph/9711469, 1997 (unpublished); D. Delephine, J.-M. Gérard, J. Pesiau, and J. Weyers, Univ. Catholique de Louvain Report No. UCL-IPT-98-01,

hep-ph/9802361, 1998 (unpublished); J.-M. Gérard, J. Pestiau, and J. Weyers, Univ. Catholique de Louvain Report No. UCL-IPT-98-04, hep-ph/9803328, 1998 (unpublished).
[22] J. F. Donoghue, E. Golowich, A. A. Petrov, and J. M. Soares, Phys. Rev. Lett. **77**, 2178 (1996).
[23] B. Blok and I. Halperin, Phys. Lett. B **385**, 324 (1996).
[24] B. Blok, M. Gronau, and J. L. Rosner, Phys. Rev. Lett. **78**, 3999 (1997).
[25] Similar estimates of rescattering effects were obtained very recently by A. F. Falk, A. L. Kagan, Y. Nir, and A. A. Petrov, Phys. Rev. D **57**, 4290 (1998); see also M. Neubert, CERN Report No. CERN-TH/97-342, hep-ph/9712224, 1997 (unpublished); D. Atwood and A. Soni, hep-ph/9712287, 1997 (unpublished).
[26] CLEO Collaboration, J. P. Alexander *et al.*, Phys. Rev. Lett. **77**, 5000 (1996).
[27] M. B. Voloshin, Yad. Fiz. **50** , 166 (1989) [Sov. J. Nucl. Phys. **50**, 105 (1989)].
[28] L. Gibbons (private communication).
[29] We thank L. Gibbons for noting that flavor SU(3) seems to be obeyed to within 15% in the comparison of $D\to K^* l \nu_l$ and $D\to \rho l \nu_l$ decays when differences between CKM elements are taken into account.
[30] J. D. Bjorken, Nucl. Phys. B (Proc. Suppl.) **11**, 325 (1989).
[31] D. Bortoletto and S. Stone, Phys. Rev. Lett. **65**, 2951 (1990).
[32] M. Neubert, CERN Report No. CERN-TH/97-169, hep-ph/9707368, in Proceedings of the High-Energy Euroconference on Quantum Chromodynamics, Montpellier, France, 1997.
[33] J. M. Gérard and W. S. Hou, Phys. Lett. B **253**, 478 (1991); Phys. Rev. D **43**, 2902 (1991).
[34] L. Wolfenstein, Phys. Rev. D **43**, 151 (1991).
[35] M. Gronau and D. Wyler, Phys. Lett. B **265**, 172 (1991).
[36] See, e.g., A. Weinstein, presented at Beauty '97 Conference, University of California at Los Angeles, 1997, Nucl. Instrum. Methods (to be published).
[37] S. Stone (private communication). See, e.g., P. Kasper and N. Harnew, talks presented at Beauty '97 Conference [36].
[38] J. L. Rosner in *Masses of Fundamental Particles (Cargèse 1996)*, lectures given at Cargèse Summer Institute on Particle Physics, edited by M. Lévy *et al.* (Plenum, New York, 1997), p. 43.
[39] P. Ball, hep-ph/9709407, in Proceedings of the 7th International Symposium on Heavy Flavor Physics, Santa Barbara, CA, 1997.
[40] Enhancement of electroweak penguin contributions due to nonperturbative effects in hadronic matrix elements or due to new physics effects was considered by R. Fleischer and T. Mannel, University of Karlsruhe Report No. TTP97-22, hep-ph/9706261, 1997 (unpublished).
[41] A. J. Buras and R. Fleischer, Technical University of Munich Report No. TUM-HEP-275/97, hep-ph/9704376 (unpublished); M. Gronau, O. Hernández, D. London, and J. L. Rosner, Phys. Rev. D **52**, 6374 (1995).
[42] M. Gronau and J. L. Rosner, Phys. Rev. D **57**, 1783 (1998).
[43] CLEO Collaboration, M. S. Alam *et al.*, CLEO CONF-23, presented at Lepton-Photon Symposium, Hamburg, 1997; J. G. Smith, in Proceedings of the Seventh International Symposium on Heavy Flavor Physics, University of California, Santa Barbara, CA, 1997; CLEO Collaboration, T. Bergfeld *et al.*, Cornell University Report No. CLNS 97-1537 hep-ex/9803018 (unpublished).
[44] Chau *et al.* [11].
[45] F. Würthwein and P. Gaidarev, hep-ph/9712531, 1997 (unpublished).
[46] R. Fleischer, CERN Report No. CERN-TH/98-60, hep-ph/9802433 (unpublished).