Perturbation evolution in cosmologies with a decaying cosmological constant

Pedro T. P. Viana and Andrew R. Liddle
Astronomy Centre, University of Sussex, Falmer, Brighton BN1 9QH, United Kingdom
(Received 27 August 1997; published 18 December 1997)

Structure formation models with a cosmological constant are successful in explaining large-scale structure data, but are threatened by the magnitude-redshift relation for type Ia supernovae. This has led to discussion of models where the cosmological ''constant'' decays with time, which might anyway be better motivated in a particle physics context. The simplest such models are based on scalar fields, and general covariance demands that a time-evolving scalar field also supports spatial perturbations. We consider the effect of such perturbations on the growth of adiabatic energy density perturbations in a cold dark matter component. We study two types of model, one based on an exponential potential for the scalar field and the other on a pseudo Nambu Goldstone boson. For each potential, we study two different scenarios, one where the scalar field presently behaves as a decaying cosmological constant and one where it behaves as dust. The initial scalar field perturbations are fixed by the adiabatic condition, as expected from the inflationary cosmology, though in fact we show that the choice of initial condition is of little importance. Calculations are carried out in both the zero-shear (conformal Newtonian) and uniform-curvature gauges. We find that both potentials allow models which can provide a successful alternative to cosmological constant models. [S0556-2821(98)01002-9]

PACS number(s): 98.65.Dx, 04.40.Nr

I. INTRODUCTION

Ever since the demise of the standard cold dark matter model, which proved unable to simultaneously match the Cosmic Background Explorer (COBE) observations of cosmic microwave background anisotropies, the abundance of rich galaxy clusters and the shape of the galaxy correlation function, it has been fashionable to study variations on this basic theme. A popular alternative is to lower the matter density, thus shifting the epoch of matter-radiation equality in a desirable direction. To keep the possibility that standard models of inflation, generating a spatially flat Universe, might be responsible for the seed perturbations, the most popular version of this model is to introduce a cosmological constant Λ to maintain the spatial flatness. This has proven a compelling framework within which it is possible to understand the formation and evolution of large-scale structure in the Universe [1,2].

The favored models, until recently, had the energy density Ω_Λ in the cosmological constant lying in the range 0.5 to 0.7. As well as giving a good fit to large-scale structure observations, these models have support from two further sources. If one assumes the baryon density in the Universe is that predicted by standard big bang nucleosynthesis [3], and that large galaxy clusters provide a representative sample of the universal ratio between baryons and the total amount of matter in the Universe [4], then the matter density must be significantly below one. Further, the need to have the age of the Universe, t_0, exceeding the age of the globular clusters in our galaxy suggests that Ω_Λ should be as large as possible. Until recently the required value for t_0 was usually around 14 Gyr [5], but a revised Cepheid distance scale due to new distance estimates by the satellite Hipparcos has brought this down to something more like 12 Gyr [6,7]. Given that most measurements (even taking into account Hipparcos's revision of the Cepheid distance scale) seem to suggest that the present value of the Hubble parameter h is at least 0.6 (in the usual units of 100 km s^{-1} Mpc^{-1}) [8] and perhaps even larger [9], in a flat Universe we would then need $\Omega_\Lambda > 0.3$ (0.55 if $t_0 > 14$ Gyr).

Unfortunately, these models have been dealt a serious blow by the preliminary results from the Supernova Cosmology Project [10], which attempts to determine the magnitude-redshift diagram of type Ia supernovae. They place a 95 percent confidence upper limit of $\Omega_\Lambda < 0.5$ for flat Universes. This is significantly stronger than earlier limits from the galaxy velocity distribution [11], galaxy outflows from voids [12] and the statistical analysis of the frequency of gravitational lensing of high-redshift quasars [13], all coming in around $\Omega_\Lambda < 0.7$. While the cold dark matter with a cosmological constant (ΛCDM) model remains viable with these smaller Ω_Λ values, this is seen as much less attractive because, as in the critical-density case, the required matter density is well above that given by direct observation.

A way out of this dilemma is to move to cosmological models where the cosmological constant is substituted by a dynamical quantity which decays with time [14–23]. Within these models it is possible to relax the constraints resulting both from the frequency of gravitational lensing of high-redshift quasars and from type Ia supernovae [24,18,25,21]. While in some ways this is clearly a regressive step, introducing more freedom into the model, it can also be argued that such a situation may be more natural on particle physics grounds. For example, there is the well-known difficulty within quantum field theory to understand the very small vacuum energy density, $\mu_{\rm vac} = (0.003\ {\rm eV})^4 \Omega_\Lambda$, required by a cosmological constant. If not strictly zero, due to some yet unknown cancellation mechanism, one would expect $\mu_{\rm vac}$ to be between 50 and 120 orders of magnitude larger [26]. A decaying cosmological constant term would be a simple way of reconciling a very large vacuum energy density early on in the Universe with an extremely small one at present.

Some authors have simply assumed more or less *ad hoc* decay laws for the cosmological constant term [14]. By com-

paring predictions of the models with observations it was then hoped that the correct decay law could be recovered, which would then shed some light on the possible physical process behind the decaying cosmological constant term. However, a credible mechanism for obtaining such a term already exists, which is to assume the existence of a scalar field presently relaxing towards the minimum of its potential (see e.g. [15–23]). Scalar fields are not only predicted to exist by some particle physics theories that go beyond the standard model, but are also the most plausible engine behind a possible inflationary period in the very early Universe [27]. The overall dynamics of the Universe in the presence of a relaxing scalar field, and its consequences for several classical cosmological tests, has been studied in detail by various authors [24,28].

Our aim in this paper is to study the effect of spatial perturbations in the cosmological ''constant'' term given by such a field, in particular on the growth of the matter perturbations. When one has the standard constant Λ term, there is no possibility of any perturbations in it. However, as soon as one permits any form of time variation, general covariance immediately implies that it must be able to support spatial perturbations. Despite this, presumably because of the development of this line of research from the original constant case, with few exceptions [19–23] most authors have not looked at the possible effect of spatial perturbations in the background value of the scalar field on the growth of perturbations in the matter distribution.

II. EQUATIONS AND INITIAL CONDITIONS

We will assume that the background space-time contains an ideal fluid and a scalar field. The equation of state of the ideal fluid, relating its background pressure, p_γ, to its background energy density, μ_γ, is $p_\gamma=(\gamma-1)\mu_\gamma$, where γ is a constant. The background energy density and pressure associated with a minimally coupled real scalar field with potential $V(\phi)$ are given by

$$\mu_\phi=\frac{1}{2}\,\dot\phi^2+V(\phi);\quad p_\phi=\frac{1}{2}\,\dot\phi^2-V(\phi). \tag{1}$$

The overdots represent derivatives with respect to coordinate time t.

The evolution of these background quantities is described by the Friedmann equation

$$H^2=\frac{8\pi G}{3}\left[\mu_\gamma+\frac{1}{2}\,\dot\phi^2+V(\phi)\right], \tag{2}$$

together with the two energy conservation equations

$$\dot\mu_\gamma=-3H\gamma\mu_\gamma, \tag{3}$$

$$\ddot\phi+3H\dot\phi+V_{,\phi}=0, \tag{4}$$

for the ideal fluid and the scalar field respectively. Here $H\equiv\dot a/a$ is the Hubble parameter, a the cosmic scale factor, and $,_\phi$ represents a derivative with respect to ϕ.

We shall carry out a fully relativistic treatment of the perturbations, using a formalism based on a series of papers by Hwang [29–33]. As always, the equations describing the evolution of the perturbations are very complicated and we have relegated their discussion to our two Appendices. A crucial question when studying perturbations is the choice of gauge. We shall consider two choices. Mainly we shall use the zero-shear gauge (ZSG) [also known as conformal Newtonian or longitudinal], but we shall also solve the equations in the uniform-curvature gauge (UCG) as a check on the accuracy of the numerical calculations.

The perturbation equations, described in the Appendices, relate the perturbed part of the metric variables, α (perturbed part of the lapse function), φ (perturbed part of the spatial curvature), χ (perturbed part of the shear) and κ (perturbed part of the expansion scalar), to the perturbed part of the matter variables, $\epsilon=\epsilon_\gamma+\epsilon_\phi$ (perturbed part of the total energy density), $\varpi=\varpi_\gamma+\varpi_\phi$ (perturbed part of the total pressure) and $\Psi=\Psi_\gamma+\Psi_\phi$ (perturbed part of the total energy density flux, or total fluid four-velocity, depending on the frame chosen). In the ZSG χ is chosen to vanish, while in the UCG φ vanishes. In all, we will need to numerically integrate a system of seven simultaneous first-order ordinary differential equations, formed by the background equations, Eqs. (B1) to (B3), where Eq. (B4) gives μ_γ, and the four perturbation equations resulting from either Eqs. (B18) and (B19) (in the ZSG), or Eqs. (B20) and (B21) (in the UCG).

We must now specify the initial conditions required in order to solve this system. We shall do that by providing the initial values for the quantities ϵ_γ, Ψ_γ, $\delta\phi$ and $\delta\phi'$. The first two give the initial values for φ and φ' (ZSG) (χ and χ' in the UCG) by means of Eqs. (B8) and (B9) [Eqs. (B14) and (B15) in the UCG].

We make two requirements on the initial conditions. The first is to choose the initial value of Ψ_γ so that only the growing mode of the solution for the evolution of an ideal fluid in an Einstein–de Sitter Universe is present (see the exact analytical solutions in Table 1 of Ref. [30]), as this is the situation we would expect in the later phase of the evolution of the Universe if the density perturbations in the ideal fluid were produced in the very early Universe.

Secondly, we will assume that the energy density perturbations in the ideal fluid and the scalar field are related by the adiabatic condition [34]. Standard models of inflation [35] always give this type of perturbation, as there is only one dynamical degree of freedom during inflation, and so it is by far the most natural choice to make. The entropy perturbation should therefore vanish,

$$S_{\gamma\phi}\equiv\frac{\epsilon_\gamma}{\mu_\gamma+p_\gamma}-\frac{\epsilon_\phi}{\mu_\phi+p_\phi}=0, \tag{5}$$

together with its first time derivative,

$$\dot S_{\gamma\phi}\equiv\frac{k^2}{a^2}\Psi_{\gamma\phi}-3He_{\gamma\phi}=0, \tag{6}$$

where

$$\Psi_{\gamma\phi}\equiv\frac{\Psi_\gamma}{\mu_\gamma+p_\gamma}-\frac{\Psi_\phi}{\mu_\phi+p_\phi}, \tag{7}$$

and

$$e_{\gamma\phi} \equiv \frac{e_\gamma}{\mu_\gamma + p_\gamma} - \frac{e_\phi}{\mu_\phi + p_\phi}. \tag{8}$$

The quantities

$$e_\gamma \equiv \varpi_\gamma - \frac{\dot{p}_\gamma}{\dot{\mu}_\gamma}\epsilon_\gamma; \quad e_\phi \equiv \varpi_\phi - \frac{\dot{p}_\phi}{\dot{\mu}_\phi}\epsilon_\phi, \tag{9}$$

represent the internal entropy of each component. In the case of the ideal fluid we have $e_\gamma = 0$. From the adiabatic conditions we obtain the initial values of $\delta\phi$ and $\delta\phi'$:

$$\delta\phi = \frac{-2a\phi' V_{,\phi}\epsilon_\gamma + k^2 H \phi'^2 \Psi_\gamma}{(3\gamma\mu_\gamma)[2V_{,\phi} - k^2\phi'/(3a)]}, \tag{10}$$

$$\delta\phi' = \phi' \frac{\epsilon_\gamma}{\gamma\mu_\gamma} - \frac{\delta\phi V_{,\phi}}{a^2 H^2 \phi'} + \phi'\alpha. \tag{11}$$

We also need to specify the initial values of the background variables H, ϕ and ϕ' which determine the cosmological model. The initial value of μ_γ is obtained by the requirement that the Universe has critical density. The other three degrees of freedom for the initial background conditions are fixed by requiring specific present values of the cosmological quantities h, t_0 and Ω_γ^0. Note that for some $V(\phi)$, and given the required values for h and Ω_γ^0, it may not be possible to obtain the desired value for t_0. In general, for fixed h and Ω_γ^0 there will be a maximum value t_0 that can be reached for a given $V(\phi)$.

III. THE CHOICE OF SCALAR FIELD POTENTIAL

Before we go on to perform the numerical integration of the perturbation equations in both the ZSG and the UCG, we need to specify what type of ideal fluid we will consider and the shape of the scalar field potential. We are mainly interested in the growth of matter density perturbations during the matter dominated era, and there are good reasons to believe that most matter in the Universe has negligible intrinsic velocity, i.e. it is *cold*. In this paper we will assume that the ideal fluid has no pressure, that is $\gamma = 1$.

With relation to the type of scalar field we should consider the issue is not as clear. Many different scalar fields have been proposed, most of them with the specific aim of producing an inflationary expansion phase in the very early Universe, though some also originate from attempts at extending the standard model of particle physics.

We will consider two different scalar field potentials, an exponential potential of the form

$$V(\phi) = V_0 \exp(-\beta\phi), \tag{12}$$

and the potential associated with a pseudo Nambu-Goldstone-boson (PNGB) field [36],

$$V(\phi) = M^4[\cos(\phi/f) + 1]. \tag{13}$$

Both these potentials have been extensively studied, within the context of power-law inflation [37] and natural inflation [38] respectively. The PNGB field has also been proposed explicitly as the most natural candidate for a presently-existing minimally-coupled scalar field [18,21].

In both cases we have two degrees of freedom, and in principle one should explore the full 2-dimensional parameter space defined by them. However, our main objective in this paper is to draw attention to the importance of taking into account the possibility of spatial perturbations in a cosmological scalar field when one is assumed to exist, due to their influence on the growth of matter density perturbations. Therefore, in this paper we will only consider two sets of values for the constants associated with each of the two potentials.

In two of the models thus obtained, one for each potential, the scalar field presently behaves like a slowly-decaying cosmological constant. In these models, we will choose the initial values of the background variables H, ϕ and ϕ', and the constants associated with the potentials, so that we end up with $h = 0.6$, $\Omega_{\rm m}^0 = 0.4$, and an age for the Universe of $t_0 = 14$ Gyr. In the other two models, again one for each potential, the scalar field starts behaving like nonrelativistic matter, scaling as a^{-3}, at a redshift close to 100. Therefore, the age of the Universe in these models is pretty much the same as if the Universe was always matter dominated. In order to obtain an acceptable age we lower h slightly to $h = 0.55$ to give $t_0 \simeq 12$ Gyr.

Our numerical integration of the perturbation equations will begin at a redshift of $z = 1100$, roughly at electron-photon decoupling, and end at the present time. One reason for this choice is that through the COBE satellite measurement of the amplitude of cosmic microwave background anisotropies we have good knowledge of the amplitude of energy density perturbations existing at the horizon scale at this redshift [39]. The main reason though is that for reasonable parameter values we are well into matter domination and the effects of radiation on the matter power spectrum of density perturbations have already run their course. At this redshift, the power spectrum is well described by the cold dark matter transfer function, for example as parametrized by Bardeen *et al.* [40,41]. We are thus able to analyze the effects of the scalar field on the matter power spectrum, without them being concealed within the full Boltzmann code machinery. We cannot however make predictions for the full microwave anisotropy power spectrum.

A. The exponential potential

For the exponential potential there is a particularly interesting situation, which we will call EXP1, where the relative energy densities of the scalar field and the ideal fluid remain constant with time, thus implying $p_\phi = (\gamma - 1)\mu_\phi$, after a transitional period. The energy density associated with the scalar field is then a fixed fraction, $24\pi G\gamma/\beta^2$, of the total energy density in the Universe [16,42,22]. This scaling solution of the cosmological background equations is one of two attractor points for the system, the other being the well-known power-law inflation solution. The former is the one which is attained if the potential is steep enough. As homogeneous perturbations around the scaling solution typically have complex eigenvalues, the system usually approaches the attractor point through oscillations in the relative energy densities of the scalar field and the ideal fluid. It has recently been studied by Ferreira and Joyce [22], though for different parameters than the model we will look at.

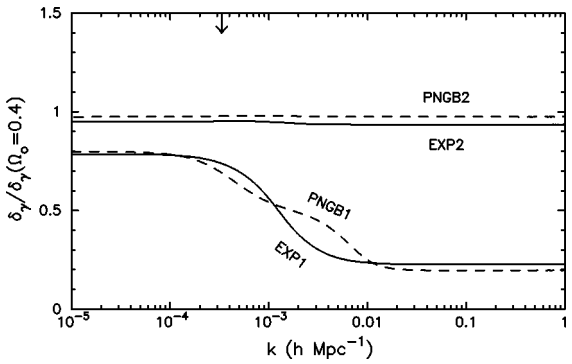

FIG. 1. This shows the present amplitude of fractional perturbations in the energy density of the ideal fluid in the ZSG for our four different models, relative to the one obtained in the case where no scalar field is present and the Universe is flat with $\Omega_0=0.4$. The full and dashed lines represent respectively the two exponential potential and the two PNGB models. The arrow indicates the inverse Hubble radius, $k=aH$, at present.

That the Universe eventually reaches the scaling solution with $\Omega_m=0.4$ demands that $\beta=\sqrt{40\pi G}$. We choose the scalar field energy density at the start of the simulation to be much smaller than that of the scaling solution; this is not necessary, though it does have to be true much earlier at nucleosynthesis [42,22]. This restricts the possible combinations for the values of $\dot{\phi}$ at $z=1100$ and V_0. The value ϕ takes at $z=1100$ is a matter of definition and we set it to zero. We will assume $\dot{\phi}$ to be extremely small at $z=1100$, thus implying that $V_0=(0.025\ \mathrm{eV})^4$. With this choice of parameters the scalar field begins to contribute significantly to the total energy density in the Universe by a redshift of about 70. In accordance with the discussion above, this model therefore gives the same age as the Einstein–de Sitter case, and to make it large enough we choose $h=0.55$.

The second situation we consider, denoted EXP2, is where the system is presently entering a period of power-law inflationary expansion, analogous to the exponential expansion which would arise if a cosmological constant is present in the Universe. To obtain our desired values (stated above) for Ω_m^0, h and t_0 implies roughly that $\beta=\sqrt{28\pi G}$. On the other hand, these constraints only very weakly define the initial splitting of the energy density associated with the scalar field at $z=1100$ into its potential and kinetic parts. We will again arbitrarily assume that $\dot{\phi}(z=1100)\simeq 0$, thus implying $V_0=(0.0025\ \mathrm{eV})^4$.

B. The PNGB potential

In the case of the PNGB potential we will also examine two distinct possible evolution histories for the Universe. In the model we will call PNGB1, where $M=0.04\ h^{1/2}$ eV and $f=1.25\times 10^{18}$ GeV, the scalar field starts to roll down its potential at a redshift of about 100, and is presently coherently oscillating at the bottom of the potential, behaving dynamically as cold matter. This model, like EXP1, gives the same age as a Universe containing only matter, so again we take h to be 0.55, giving an age for the Universe of 12 Gyr. The constraint $\Omega_m^0=0.4$ implies that $\phi(z=1100)=f$ and $\dot{\phi}(z=1100)\simeq 0$.

In model PNGB2, where $M=0.003\ h^{1/2}$ eV and $f=1/\sqrt{8\pi G}=2.4\times 10^{18}$ GeV, the scalar field begins moving down its potential at around $z=1$, and has not yet reached the bottom of the potential by the present time. The values of M and f in this model coincide with those chosen for a more detailed analysis by Frieman *et al.* [18] in their paper dealing with PNGB motivated dynamical cosmological constant models.[1] Imposing the boundary condition $\Omega_m^0=0.4$ leads to $\phi(z=1100)=1.75f$ and $\dot{\phi}(z=1100)\simeq 0$, while $h=0.6$ and $t_0=14$ Gyr are obtained by simply choosing the correct value for H at $z=1100$.

IV. RESULTS

We will work in the ZSG, and display the fluid density perturbation $\delta_\gamma^\chi\equiv\epsilon_\gamma^\chi/\mu_\gamma$. We arbitrarily take its initial value at redshift 1100 to be 10^{-5}; as the equations are linear, and our main results only show power spectra relative to one another, the initial magnitude of the density perturbation is irrelevant. Note that the density perturbations given in different gauges coincide on scales well within the horizon, but otherwise, though uniquely defined as long as the gauge is given, do not coincide. Care must therefore be taken in interpreting any long-wavelength behavior. Most of the literature uses the comoving gauge if the large-scale power spectrum is shown.

We integrated the background and perturbation equations both in the ZSG and the UCG using the NAG Fortran Library Routine D02CJF, which is based on a variable-order, variable-step Adams method, for values of k in the range of 10^{-5} to 1 h Mpc^{-1}. We did this not only for the four scalar field models chosen in the previous section, but also for two cases where no scalar field was assumed present, which will serve as comparison. In one the Universe has critical density, while in the other the Universe is flat with $\Omega_0=0.4$, thus implying the presence of a cosmological constant.

In Fig. 1 we plot the present amplitude obtained for $\delta_\gamma^\chi(k)$ (for simplicity henceforth omitting the superscript and always meaning this gauge) for our four models, relative to that found in the case of a flat Universe with $\Omega_0=0.4$ (with h in this comparison model adjusted to match that of each of the four scalar field models we consider). The main integration runs were done in the ZSG, but when the same runs were performed in the UCG, with $\epsilon_\gamma^\varphi(z=1100)$ calculated from $\epsilon_\gamma^\chi(z=1100)$ using Eq. (B28) and then $\epsilon_\gamma^\chi(z=0)$ from $\epsilon_\gamma^\varphi(z=0)$ using Eq. (B31), the difference in the present value

[1] In a more recent paper, Coble *et al.* [21] considered a slightly different model with $M=0.005$ eV and $f=1.885\times 10^{18}$ GeV, where the scalar field starts moving down the PNGB potential at $z\simeq 10$ and is presently already oscillating around its minimum. These values were chosen so that the model yields $\Omega_m^0=0.4$ and $h=0.7$. This implies an age for the Universe only slightly above 10 Gyr. We prefer the PNGB2 model over theirs. If one chose to obtain $h=0.55$, and thus an age close to 12 Gyr, one would end up with $\Omega_m^0=0.5$.

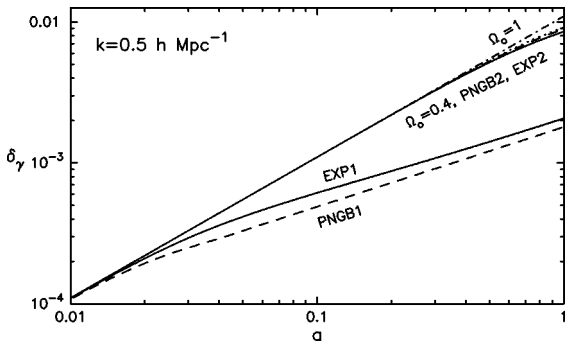

FIG. 2. The evolution in the ZSG of the perturbations with a comoving wave number of $k=0.5\ h$ Mpc^{-1}. The dash-dotted and dotted lines represent the cases where respectively the Universe has critical density and the Universe is flat with $\Omega_0=0.4$, with no scalar field present. Solid lines show the EXP models and dashed ones the PNGB models.

of ϵ_γ^χ was less than one percent over the whole range for k in the four models.

Although we believe that Eqs. (10) and (11) should be used to obtain the initial values for $\delta\phi$ and $\delta\phi'$, in accordance with the adiabatic relations Eqs. (5) and (6) one expects from inflationary generated perturbations, we also looked at what occurs when arbitrary initial values for these quantities are assumed. After performing several integration runs for a variety of initial values for $\delta\phi$ and $\delta\phi'$, we reached the conclusion that the evolution of the energy density perturbations in the ideal fluid is almost (to a few percent) independent of the initial values one considers for these quantities, as long as they are less than of order unity. In particular, this holds true (to better than 1 percent) when it is assumed that there are no initial scalar field perturbations either in the ZSG, i.e. $(\delta\phi')^\chi=\delta\phi^\chi=0$, or in the UCG, i.e. $(\delta\phi')^\varphi=\delta\phi^\varphi=0$. So the initial condition for the scalar field perturbations does not seem particularly important.

In Fig. 2 we plot the evolution of $\delta_\gamma(a)$ for perturbations with a comoving wave number of $k=0.5\ h$ Mpc^{-1} between $z=100$ and the present. All the four scalar field models are shown, together with the cases of a critical-density Universe and a flat Universe with $\Omega_0=0.4$. The integration runs were actually started at $z=1100$, but all models behave very much as the critical-density case up to $z=100$. We chose the wavenumber shown as it corresponds to a scale which was already well inside the Hubble radius at the beginning of the integrations, so we need not worry about specifying the gauge. In fact, comparing the present amplitude of δ_γ obtained in the ZSG and in the UCG for the various models under consideration, we find that the density perturbations in the two gauges coincide for values of k down to about $10^{-3}\ h$ Mpc^{-1}, and can therefore be regarded as gauge-independent. For smaller k values one needs to be careful to specify the gauge used.

As mentioned in Sec. III, Fig. 1 represents the distortion in the square root of the matter power spectrum suffered since redshift 1100, $P(k)\propto\delta_\gamma^2$, relative to a flat Universe with $\Omega_0=0.4$ and no scalar field. Since the full spectrum in that model, including all known effects of radiation and neutrinos, is well known, we can therefore use it to obtain the expected present shape of $P(k)$ for our four scalar field models.

We show the power spectra in Fig. 3, where we simply multiplied the expected present shape of $P(k)$ in a flat Universe with $\Omega_0=0.4$, $\Omega_B=0.016\ h^{-2}$ and either $h=0.55$ or $h=0.6$, with no scalar field present, by $\delta_\gamma^2/\delta_\gamma^2(\Omega_0=0.4)$. This procedure is correct since up to $z=1100$ the background and perturbation evolution of any of the scalar field models is equivalent to that of a cosmological constant model which has the same values for Ω_0 and h as each individual scalar field model. We also show in Fig. 3 the expected present shape of $P(k)$ in a critical-density Universe for the two values of h considered. We obtained the expected present shape of $P(k)$ for both the critical density and the cosmological constant cases by assuming a Harrison–Zel'dovich (scale-invariant) primordial power spectrum for the energy density perturbations and using the cold dark matter transfer function initially derived by Bardeen *et al.* [40], and later modified by Sugiyama [41] to include the contribution of baryons.

The critical-density and cosmological constant models are normalized to COBE [39]. The amplitude of the other models, which has been computed relative to the latter, is almost correct. It includes the two main effects—that the early time matter power spectrum in low-density models is higher by a factor $1/\Omega_0$ than in critical-density models is encoded in the initial conditions through the different initial value of H, and the growth suppression factor from the scale factor dynamics is computed in the subsequent evolution. As each model (ex-

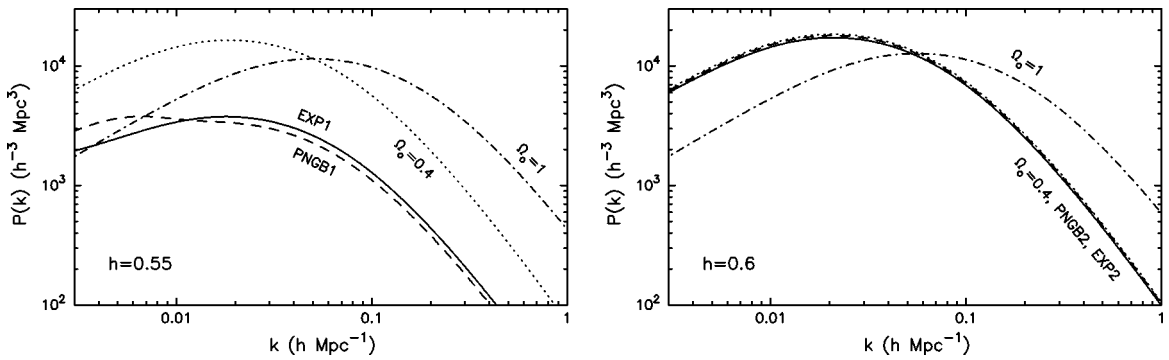

FIG. 3. The power spectrum of the energy density perturbations in the matter component for the models, all normalized to COBE as described in the text. The line styles represent $P(k)$ for the same situations as in the previous two figures. Note that the PNGB2 and EXP2 models give a very similar spectrum to the standard cosmological constant case.

cept the critical-density one) has the same redshift of matter-radiation equality, all that is omitted is the line-of-sight contribution to the Sachs-Wolfe effect, which requires a full Boltzmann code for accurate computation. The effect of this term on the COBE normalization is nonexistent for the critical-density case, and known to be negligible in the cosmological constant case (see e.g. Refs. [43,2]). It may be slightly more significant for the decaying cosmological constant case [21] and future observations would probably require an accurate computation, while present ones do not.

We only show the power spectra for values of k larger than $10^{-3}\ h\ \mathrm{Mpc}^{-1}$. On smaller scales the amplitude of the power spectrum is gauge-dependent, though of course well-defined in any particular gauge.

V. DISCUSSION

The results presented in this paper extend previous work on the effects of a presently-existing scalar field, with regard to the evolution of energy density perturbations in a pressureless fluid. In accordance with general covariance we allow for the presence of spatial perturbations in the scalar field. Contrary to previous authors, we explicitly relate the energy density perturbations in the pressureless fluid to those in the scalar field through the adiabatic condition, as expected if those perturbations arose from a period of inflation in the very early Universe.

The study of the perturbation evolution was performed both in the zero-shear and uniform-curvature gauges, in the latter case for the first time.

We considered two possible potentials for the scalar field, an exponential and that associated with a pseudo Nambu-Goldstone-boson field. Each of these potentials has two degrees of freedom, and we worked with two different sets of parameters for each potential. Those models, EXP2 and PNGB2, which were chosen so that the scalar field becomes dynamically important only very recently, at $z\sim 1$, and is presently behaving as a cosmological constant, yield a shape for the power spectrum of energy density perturbations in the matter component extremely close to that one obtains in a cosmological constant model where $\Omega_0=0.4$ and $h=0.6$, though the amplitude is a few percent smaller.

The situation is rather different for the other two models, EXP1 and PNGB1, chosen so that the scalar field becomes dynamically important at a much earlier time, around a redshift of 100. They presently behave as pressureless matter. In the case of model EXP1 the shape of the power spectrum of energy density perturbations in the matter component is again very close to that one obtains in a cosmological constant model, this time with $\Omega_0=0.4$ and $h=0.55$. Its amplitude is however only about 5 percent of that for the cosmological constant model. Model PNGB1 has the further interesting feature that the shape of $P(k)$ is altered for values of k below $0.01\ h\ \mathrm{Mpc}^{-1}$. Unfortunately this corresponds to scales above around $100\ h^{-1}$ Mpc, at the limit of what present cluster and galaxy surveys can probe. The feature in the shape of $P(k)$ for model PNGB1 is due to oscillations in the rate of decrease of the Hubble parameter when the scalar field becomes dynamically important at $z\sim 100$. On smaller scales the suppression of the growth of the energy density perturbations in the matter component is similar to that in model EXP1.

The results just described suggest that the presence of spatial perturbations in a presently-existing scalar field substantially affects the evolution of energy density perturbations in the matter component only if the scalar field has contributed significantly to the total energy density in the Universe for several Hubble times. If the scalar field is becoming dynamically important only now, the presence of spatial perturbations does not seem to have much effect.

The similarity between the scalar field models EXP2 and PNGB2, and a flat model with $\Omega_0=0.4$, $h=0.6$ and no scalar field, makes the first two models attractive substitutes of the latter. Further, given that the above mentioned flat model is able to reproduce all the most reliable observational data presently available on large-scale structure and cosmic microwave background temperature anisotropies [2], we expect models EXP2 and PNGB2 to do as well. As they are quite similar to the standard cosmological constant case, a detailed evaluation of the supernova constraint would be interesting; Frieman *et al.* [18] suggest that the constraint will weaken so they should be viable.

With regard to models EXP1 and PNGB1, their much smaller growth for the matter perturbations seems very problematic. We expect the dispersion of the density contrast smoothed on spheres of $8\ h^{-1}$ Mpc, usually represented by σ_8, required for both models so that they can reproduce the present abundance of high mass x-ray emitting galaxy clusters to be the same as for the critical-density case, due to the two models being dynamically equivalent to it since well before redshift 10. This conservatively requires σ_8 in the range 0.45 to 0.8 for both models [44]. On the other hand, as the growth suppression factor of the matter perturbations is about 4.5 times larger in the EXP1 and PNGB1 models than in a flat model with $\Omega_0=0.4$ and $h=0.55$, to a first approximation this means that the value of σ_8 implied by COBE for the two scalar field models would be about 4.5 times smaller than for the flat model [21]. Inclusion of the integrated Sachs-Wolfe effect may lead to a slight further decrease in σ_8 [21]. If the primordial power spectrum of energy density perturbations is assumed scale-invariant, then this would mean that $\sigma_8<0.18$ for both EXP1 and PNGB1 [2]. One would need a very ''blue'' primordial power spectrum, with at least $n>1.45$, for σ_8 to reach the minimum requirement of 0.45. Ferreira and Joyce [22] advocate a much larger value of Ω_0, which resolves the amplitude problem while giving a more modest change in the shape of the spectrum.

As in both EXP1 and PNGB1 the scalar field goes through a period at a redshift of about 100 when it behaves dynamically like a cosmological constant, we expect the full angular anisotropy spectrum for the cosmic microwave background radiation in these models to display a distinctive signature. This clearly merits detailed investigation.

Note. As we were completing this paper, a report by Caldwell *et al.* [23] appeared which covers similar issues. They use an enhanced Boltzmann code to generate microwave anisotropy power spectra. Their matter power spectra appear in good agreement with ours. They did not use adiabatic initial conditions for the scalar field perturbations, but we have shown that this is unlikely to have significant effects. In another paper, Frieman and Waga [45] use the recent observa-

tions of high-redshift type Ia supernovae [10] to constrain models of the type we consider. All our models are found to be viable.

ACKNOWLEDGMENTS

P.T.P.V. was supported by PPARC and A.R.L. by the Royal Society. We thank Scott Dodelson, Pedro Ferreira and Martin White for discussions, and Wo-Lung Lee for pointing out an error in an earlier version of this paper. We acknowledge use of the Starlink computer system at the University of Sussex.

APPENDIX A: THE EVOLUTION EQUATIONS

1. Gauge considerations

A cosmological perturbation is defined by means of a correspondence between an arbitrary background spacetime and the real physical Universe. A *gauge transformation* is a change in this correspondence, keeping the background spacetime fixed. Therefore, in general the value of the perturbed part of some physical quantity will not be invariant under a gauge transformation. Further, the degrees of freedom due to gauge transformations give rise to spurious unphysical modes in the solutions to the evolution of the perturbed part of gauge-dependent quantities, which can always be removed by a convenient gauge transformation. In order to avoid these spurious modes one either evolves gauge-invariant quantities related to the gauge-dependent quantities one is actually interested in, obtaining the latter from the former at any one time, or one has to provide a gauge-fixing condition which completely specifies the way through which spacetime is to be split into background and perturbed components.

The linear analysis of cosmological perturbations was initiated by Lifshitz, with a seminal paper published in 1946 [46], who used the so-called synchronous gauge-fixing condition. Unfortunately this condition, though considerably simplifying the perturbation equations, still leaves a residual gauge degree of freedom. The spurious modes thus arising are difficult to distinguish from the real physical ones, and their identification was a source of controversy for some time. The use of gauge-invariant quantities in the calculation of the evolution of cosmological perturbations only really took off with the paper by Bardeen in 1980 [47]. Though it avoids the problem of obtaining unphysical modes in the solutions to the perturbation equations, it really does not offer any extra advantage over the gauge-specific methods which remove any gauge degree of freedom by completely fixing the background/perturbed splitting.

In this paper we will use gauge-specific methods to solve the perturbation equations. We will adopt the notation and use the equations laid down in a series of papers by Hwang [29–33]. We will be solely interested in the evolution of density (scalar) perturbations. The system is composed of an ideal fluid plus a single minimally coupled real scalar field, ϕ, evolving in a background Einstein–de Sitter universe. Given that the spatial part of the background spacetime is thus homogeneous and isotropic, the perturbations in any physical quantities will necessarily be gauge-invariant under purely spatial gauge transformations. We will therefore only worry about the temporal gauge transformation.

2. Notation and general equations

We will now introduce the gauge nonspecific perturbation equations obtained by Hwang. They relate the perturbed part of the metric variables, α (perturbed part of the lapse function), φ (perturbed part of the spatial curvature), χ (perturbed part of the shear) and κ (perturbed part of the expansion scalar), to the perturbed part of the matter variables, $\epsilon=\epsilon_\gamma+\epsilon_\phi$ (perturbed part of the total energy density), $\varpi=\varpi_\gamma+\varpi_\phi$ (perturbed part of the total pressure)[2] and $\Psi=\Psi_\gamma+\Psi_\phi$ (perturbed part of the total energy density flux, or total fluid four-velocity, depending on the frame chosen). We have

$$\varpi_\gamma=(\gamma-1)\epsilon_\gamma, \tag{A1}$$

and one can derive that [32]

$$\epsilon_\phi=\dot\phi\dot{\delta\phi}-\dot\phi^2\alpha+V_{,\phi}\delta\phi, \tag{A2}$$

$$\varpi_\phi=\dot\phi\dot{\delta\phi}-\dot\phi^2\alpha-V_{,\phi}\delta\phi, \tag{A3}$$

$$\Psi_\phi=-\dot\phi\delta\phi, \tag{A4}$$

where $\delta\phi$ is the perturbed part of the scalar field. We will be particularly interested in the evolution of the fractional perturbation in the energy density of the ideal fluid, $\delta_\gamma\equiv\epsilon_\gamma/\mu_\gamma$.

We will express the perturbed parts of both the metric and matter variables by means of Fourier expansions. For example,

$$\delta\phi(\mathbf{x},t)=\sum_k \delta\phi_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{A5}$$

where

$$\delta\phi_{\mathbf{k}}(t)=\frac{1}{V}\int\delta\phi(\mathbf{x},t)e^{-i\mathbf{k}\cdot\mathbf{x}}d\mathbf{x}, \tag{A6}$$

being $k\equiv|\mathbf{k}|$ a fixed comoving wave number. The Fourier expansions are made in a large enough box that the induced periodicity is irrelevant.

As in this paper we are only interested in the linear evolution of cosmological perturbations, we will assume that the different Fourier modes for each variable behave independently of each other. We will drop the suffices $\mathbf{k}$ identifying each Fourier mode in order to lighten the notation.

In the case of our system, formed by an ideal fluid plus a minimally coupled real scalar field, the perturbation equations take the form [29,30,32]

$$3\dot\varphi=3H\alpha-\kappa+\frac{k^2}{a^2}\chi, \tag{A7}$$

[2]Here we changed the notation from π to ϖ to avoid confusion with the number π.

$$-\frac{k^2}{a^2}\varphi+H\kappa=-4\pi G(\epsilon_\gamma+\dot\phi\dot\delta\phi-\dot\phi^2\alpha+V,_\phi\delta\phi), \tag{A8}$$

$$\kappa-\frac{k^2}{a^2}\chi=-12\pi G(\Psi_\gamma-\dot\phi\delta\phi), \tag{A9}$$

$$\dot\chi+H\chi=\alpha+\varphi, \tag{A10}$$

$$\dot\kappa+2H\kappa=\left(\frac{k^2}{a^2}-3\dot H\right)\alpha+4\pi G[(3\gamma-2)\epsilon_\gamma+4\dot\phi\dot\delta\phi-4\dot\phi^2\alpha-2V,_\phi\delta\phi], \tag{A11}$$

$$\dot\epsilon_\gamma+3H\gamma\epsilon_\gamma=\gamma\mu_\gamma(\kappa-3H\alpha)+\frac{k^2}{a^2}\Psi_\gamma, \tag{A12}$$

$$\dot\Psi_\gamma+3H\Psi_\gamma=-\gamma\mu_\gamma\alpha-(\gamma-1)\epsilon_\gamma, \tag{A13}$$

$$\ddot\delta\phi+3H\dot\delta\phi+\left(\frac{k^2}{a^2}+V,_{\phi\phi}\right)\delta\phi=\dot\phi(\kappa+\dot\alpha)-(3H\dot\phi+2V,_\phi)\alpha. \tag{A14}$$

It should again be stressed that these equations were obtained without reference to any gauge-fixing condition. No anisotropic pressure term appears as in both the case of an ideal fluid and a minimally coupled real scalar field the anisotropic pressure is zero. The last three equations are, in order, the energy and momentum conservation equations for the perturbations in the ideal fluid, and the energy conservation equation for the perturbations in the scalar field. The momentum conservation equation for the perturbations in the scalar field is identically satisfied.

The most obvious and fundamental gauge-fixing conditions follow from requiring that the perturbed part of one of the metric or matter variables is zero. We thus have: the synchronous gauge, $\alpha\equiv0$; the uniform-curvature gauge, $\varphi\equiv0$; the zero-shear gauge, $\chi\equiv0$; the uniform-expansion gauge, $\kappa\equiv0$; the uniform-density gauge, $\epsilon\equiv0$; the uniform-pressure gauge, $\varpi\equiv0$; and the comoving gauge, $\Psi\equiv0$. Except for the synchronous gauge, all the other gauge-fixing conditions completely remove the gauge modes from the solutions to the perturbation equations. We will use two of these gauge-fixing conditions to derive two different sets of perturbation equations from the system given above. With the aid of expressions relating quantities in the two gauges we will thus be able to estimate the numerical errors arising from the numerical integration of both sets of perturbation equations. We will consider the zero-shear gauge (ZSG, also known as the longitudinal or conformal Newtonian gauge [48]), and the uniform-curvature gauge (UCG). These choices are the ones which lead to the two simplest sets of perturbation equations, thus decreasing the probability of numerical errors creeping into the solutions. The two sets can be obtained by simply getting rid of χ and $\dot\chi$ in the case of the ZSG, and φ and $\dot\varphi$ for the UCG, in Eqs. (A7) to (A14).

APPENDIX B: NUMERICAL SOLUTIONS

1. Background and perturbation equations

The evolution of the background variables H and ϕ is obtained by numerically solving the system of first-order differential equations formed by Eq. (2) and the two first-order differential equations that can be obtained from Eq. (4),

$$\frac{d\phi}{da}=f, \tag{B1}$$

$$\frac{df}{da}=-4\frac{f}{a}+4\pi G\left(\frac{\gamma\mu_\gamma f}{aH^2}+af^3\right)-\frac{V,_\phi}{a^2H^2}, \tag{B2}$$

$$\frac{dH}{da}=-4\pi G\left(aHf^2+\frac{\gamma}{aH}\mu_\gamma\right), \tag{B3}$$

where μ_γ is given by analytically solving Eq. (3),

$$\mu_\gamma=\mu_\gamma^0\left(\frac{a}{a_0}\right)^{-3\gamma}. \tag{B4}$$

The suffix ''0'' indicates present-day values as usual. Note that the independent variable has been changed from coordinate time, t, to the scale factor, a. They are related by the first-order differential equation

$$\frac{dt}{da}=a^{-1}H^{-1}, \tag{B5}$$

the integration of which gives the time elapsed in the Universe between two given values of the scale factor. Derivatives with respect to the scale factor will be represented by a prime. We choose the independent variable to be the scale factor as it is easier to work with numerically and is more meaningful from the point of view of structure formation.

We have 8 perturbation equations for 11 dependent perturbation variables in both the ZSG and the UCG: ϵ_γ, ϵ_γ', Ψ_γ, Ψ_γ', $\delta\phi$, $\delta\phi'$, $\delta\phi''$, α and κ in either, along with φ and φ' in the ZSG, or χ and χ' in the UCG. Eqs. (A7) to (A10), (A12) and (A13) will be used to describe the evolution of α, κ and the quantities associated with the ideal fluid, ϵ_γ, ϵ_γ', Ψ_γ and Ψ_γ', in terms of $\delta\phi$, $\delta\phi'$, φ and φ' (ZSG) (the last two variables are replaced by χ and χ' in the UCG), and the background variables. We thus have in the ZSG,

$$\alpha=-\varphi, \tag{B6}$$

$$\kappa=-3H\varphi-3aH\varphi', \tag{B7}$$

$$\epsilon_\gamma=\frac{\varphi}{4\pi G}\frac{k^2}{a^2}+\frac{3H^2\varphi+3aH^2\varphi'}{4\pi G}-a^2H^2\phi'\delta\phi'-a^2H^2\phi'^2\varphi-V,_\phi\delta\phi, \tag{B8}$$

$$\Psi_\gamma=\frac{H\varphi+aH\varphi'}{4\pi G}+aH\phi'\delta\phi, \tag{B9}$$

$$\epsilon_\gamma'=-3\gamma\frac{\epsilon_\gamma}{a}-3\gamma\mu_\gamma\varphi'+\frac{k^2}{a^2}\frac{\Psi_\gamma}{aH}, \tag{B10}$$

$$\Psi'_\gamma = -3\frac{\Psi_\gamma}{a} + \gamma\mu_\gamma\frac{\varphi}{aH} - (\gamma-1)\frac{\epsilon_\gamma}{aH}, \tag{B11}$$

and for the UCG,

$$\alpha = H\chi + aH\chi', \tag{B12}$$

$$\kappa = \left(3H^2 + \frac{k^2}{a^2}\right)\chi + 3aH^2\chi', \tag{B13}$$

$$\epsilon_\gamma = -\left(3H^2 + \frac{k^2}{a^2}\right)\frac{H\chi}{4\pi G} - \frac{3aH^3\chi'}{4\pi G} - a^2H^2\phi'\,\delta\phi' + a^2H^2\phi'^2(aH\chi' + H\chi) - V_{,\phi}\delta\phi, \tag{B14}$$

$$\Psi_\gamma = -\frac{H^2\chi + aH^2\chi'}{4\pi G} + aH\phi'\,\delta\phi, \tag{B15}$$

$$\epsilon'_\gamma = -3\gamma\frac{\epsilon_\gamma}{a} + \gamma\mu_\gamma\frac{k^2}{a^2}\frac{\chi}{aH} + \frac{k^2}{a^2}\frac{\Psi_\gamma}{aH}, \tag{B16}$$

$$\Psi'_\gamma = -3\frac{\Psi_\gamma}{a} - \gamma\mu_\gamma\frac{\chi + a\chi'}{a} - (\gamma-1)\frac{\epsilon_\gamma}{aH}. \tag{B17}$$

Using these expressions we can now convert Eqs. (A11) and (A14) into the following second-order differential equations

$$\delta\phi'' = -\left(\frac{4}{a} + \frac{H'}{H}\right)\delta\phi' - \left(\frac{k^2}{a^2} + V_{,\phi\phi}\right)\frac{\delta\phi}{a^2H^2} - 4\phi'\varphi' + 2\frac{\varphi V_{,\phi}}{a^2H^2}, \tag{B18}$$

$$\varphi'' = -\left[\frac{(3\gamma+2)}{a} + \frac{H'}{H}\right]\varphi' - \left[\frac{3\gamma}{a^2} + \frac{2H'}{aH} + (\gamma-1)\frac{k^2}{a^4H^2} - 4\pi G(\gamma-2)\phi'^2\right]\varphi + 4\pi G\left[(\gamma-2)\phi'\,\delta\phi' + \gamma\frac{\delta\phi V_{,\phi}}{a^2H^2}\right], \tag{B19}$$

in the ZSG and

$$\delta\phi'' = -\left[\frac{4}{a} + \frac{H'}{H} + 4\pi G(\gamma-2)a\phi'^2\right]\delta\phi' - \left(\frac{k^2}{a^2} + V_{,\phi\phi} + 4\pi G\gamma a\phi' V_{,\phi}\right)\frac{\delta\phi}{a^2H^2} - \left[3\gamma H\phi' + 2aH'\phi' + \frac{2V_{,\phi}}{aH} - 4\pi G(\gamma-2)a^2H\phi'^3\right]\chi' - \left[2H'\phi' + \frac{3\gamma H\phi'}{a} + \frac{2V_{,\phi}}{a^2H} + (\gamma-2)\frac{k^2}{a^2}\frac{\phi'}{aH} - 4\pi G(\gamma-2)aH\phi'^3\right]\chi, \tag{B20}$$

$$\chi'' = -\left[\frac{3\gamma+2}{a} + 3\frac{H'}{H} - 4\pi G(\gamma-2)a\phi'^2\right]\chi' - \left[\frac{3H'}{aH} + \frac{3\gamma}{a^2} + (\gamma-1)\frac{k^2}{a^4H^2} - 4\pi G(\gamma-2)\phi'^2\right]\chi - 4\pi G\left[(\gamma-2)\frac{\phi'\,\delta\phi'}{H} + \gamma\frac{\delta\phi V_{,\phi}}{a^2H^3}\right], \tag{B21}$$

in the UCG. Each of these equations can be split into two first-order differential equations, in the same way as we did for Eq. (4), which we will then numerically integrate in order to determine the evolution of $\delta\phi$, $\delta\phi'$, φ and φ' (ZSG) (χ and χ' in the UCG).

In all we will need to simultaneously numerically integrate a system of seven first-order ordinary differential equations, formed by the background Eqs. (B1) to (B3), where expression (B4) gives μ_γ, and the four perturbation equations resulting from either (B18) and (B19) (in the ZSG), or (B20) and (B21) (in the UCG). The question of initial conditions for this procedure is addressed in the main text of this paper.

2. Relations between quantities in different gauges

Once we have calculated the evolution of the perturbation variables in some particular gauge, we can use gauge-invariant variables and the gauge nonspecific set of perturbation Eqs. (A7) to (A14) to obtain the evolution of such variables in any other gauge.

We will use this possibility to control the errors arising from the numerical integration of the perturbation equations. We will express both our initial conditions and the final results for the perturbation variables in the ZSG, and use the UCG simply as an estimator of the numerical integration errors. As an example, we will derive the relations between the perturbations in the energy density of the ideal fluid in the ZSG and the UCG. The change from the ZSG to the UCG, and vice-versa, for the other perturbation variables can be obtained in a similar way.

The quantities

$$\epsilon_\gamma^\chi \equiv \epsilon_\gamma + 3H(\mu_\gamma + p_\gamma)\chi, \tag{B22}$$

$$\Psi_\gamma^\chi \equiv \Psi_\gamma + (\mu_\gamma + p_\gamma)\chi, \tag{B23}$$

and

$$\epsilon_\gamma^\varphi \equiv \epsilon_\gamma + 3(\mu_\gamma + p_\gamma)\varphi, \tag{B24}$$

$$\Psi_\gamma^\varphi \equiv \Psi_\gamma + (\mu_\gamma + p_\gamma)\frac{\varphi}{H}, \tag{B25}$$

are invariant under temporal gauge transformations, as can be seen by using the relations provided by Hwang [30]. The first two variables simply take the values of ϵ_γ and Ψ_γ, respectively, when these quantities are calculated in the ZSG, while the same occurs for the last two variables with relation to the UCG. We thus want to express in the UCG ϵ_γ^φ as a function of ϵ_γ^χ and Ψ_γ^χ, in order to obtain the initial value of ϵ_γ to be used in the UCG calculations from that originally given in the ZSG. Also, we want to know how to

obtain ϵ_γ^χ from ϵ_γ^φ and Ψ_γ^φ, so that we can compare the final value of ϵ_γ obtained in the two gauges.

In the UCG we have

$$\epsilon_\gamma^\varphi = \epsilon_\gamma^\chi - 3H(\mu_\gamma + p_\gamma)\chi, \tag{B26}$$

and, by using Eqs. (A8) and (A9),

$$\frac{k^2}{a^2}H\chi - 12\pi GH(\Psi_\gamma + \Psi_\phi) = -4\pi G(\epsilon_\gamma + \epsilon_\phi). \tag{B27}$$

Through some algebraic manipulation of the above relations we then obtain

$$\epsilon_\gamma^\varphi = \epsilon_\gamma^\chi - 12\pi G(\mu_\gamma + p_\gamma)\frac{a^2}{k^2}[3H(\Psi_\gamma^\chi + \Psi_\phi^\chi) - (\epsilon_\gamma^\chi + \epsilon_\phi^\chi)], \tag{B28}$$

where ϵ_ϕ^χ and Ψ_ϕ^χ are defined in an analogous fashion to the ideal fluid gauge-invariant variables.

On the other hand, in the ZSG we have

$$\epsilon_\gamma^\chi = \epsilon_\gamma^\varphi - 3(\mu_\gamma + p_\gamma)\varphi, \tag{B29}$$

and, by using the same equations as previously,

$$-\frac{k^2}{a^2}\varphi - 12\pi GH(\Psi_\gamma + \Psi_\phi) = -4\pi G(\epsilon_\gamma + \epsilon_\phi). \tag{B30}$$

Hence, we get

$$\epsilon_\gamma^\chi = \epsilon_\gamma^\varphi - 12\pi G(\mu_\gamma + p_\gamma)\frac{a^2}{k^2}[(\epsilon_\gamma^\varphi + \epsilon_\phi^\varphi) - 3H(\Psi_\gamma^\varphi + \Psi_\phi^\varphi)], \tag{B31}$$

where again ϵ_ϕ^φ and Ψ_ϕ^φ are defined in the same way as the ideal fluid gauge-invariant variables.

[1] L. M. Krauss and M. S. Turner, Gen. Relativ. Gravit. **27**, 1137 (1995); J. P. Ostriker and P. J. Steinhardt, Nature (London) **377**, 600 (1995); R. Stompor, K. M. Górski, and A. J. Banday, Mon. Not. R. Astron. Soc. **277**, 1225 (1995).

[2] A. R. Liddle, D. H. Lyth, P. T. P. Viana, and M. White, Mon. Not. R. Astron. Soc. **282**, 281 (1996).

[3] C. J. Copi, D. N. Schramm, and M. S. Turner, Science **267**, 192 (1995); Phys. Rev. Lett. **75**, 3981 (1995); M. S. Turner, J. W. Truran, D. N. Schramm, and C. J. Copi, Astrophys. J. **466**, L59 (1996).

[4] S. D. M. White, J. F. Navarro, A. E. Evrard, and C. S. Frenk, Nature (London) **366**, 429 (1993); D. A. White and A. C. Fabian, Mon. Not. R. Astron. Soc. **273**, 72 (1995); D. A. Buote and C. R. Canizares, Astrophys. J. **457**, 565 (1996); A. E. Evrard, C. Metzler, and J. F. Navarro, *ibid.* **469**, 464 (1996).

[5] B. Chaboyer, P. Demarque, P. J. Kernan, and L. M. Krauss, Science **271**, 957 (1996); B. Chaboyer, P. Demarque, P. J. Kernan, L. M. Krauss, and A. Sarajedini, Mon. Not. R. Astron. Soc. **283**, 683 (1996); R. Jimenez and P. Padoan, Astrophys. J. **463**, L17 (1996); R. Jimenez, P. Thejll, U. G. Jorgensen, J. MacDonald, and B. Pagel, Mon. Not. R. Astron. Soc. **282**, 926 (1996); M. Salaris, S. Degl'Innocenti, and A. Weiss, Astrophys. J. **479**, 665 (1997).

[6] M. W. Feast and R. M. Catchpole, Mon. Not. R. Astron. Soc. **286**, L1 (1997); M. W. Feast and P. A. Whitelock, to appear in Proceedings of the Hipparcos Venice 97 Symposium, Report No. ESA SP-402, astro-ph/9706097 (1997).

[7] R. G. Gratton, F. Fusi Pecci, E. Carretta, G. Clementini, C. E. Corsi, and M. Lattanzi, Astrophys. J. (to be published), astro-ph/9704150; I. N. Reid, Astron. J. **114**, 1 (1997).

[8] D. Branch, A. Fisher, E. Baron, and P. Nugent, Astrophys. J. **470**, L7 (1996); D. Branch, W. Romanishin, and E. Baron, *ibid.* **465**, 73 (1996); M. Hamuy, M. M. Phillips, R. A. Schommer, N. B. Suntzeff, J. Maza, and R. Aviiles, Astron. J. **112**, 2398 (1996); B. E. Schaefer, *ibid.* **111**, 1668 (1996).

[9] W. L. Freedman *et al.*, Nature (London) **371**, 757 (1994); B. P. Schmidt *et al.*, Astrophys. J. **432**, 42 (1994); J. Mould *et al.*, *ibid.* **449**, 413 (1995); N. R. Tanvir, T. Shanks, H. C. Ferguson, and D. R. T. Robinson, Nature (London) **377**, 27 (1995); A. G. Riess, W. H. Press, and R. P. Kirshner, Astrophys. J. **473**, 88 (1996); P. Ruiz-Lapuente, *ibid.* **465**, L83 (1996).

[10] S. Perlmutter *et al.*, Astrophys. J. **483**, 565 (1997).

[11] A. Nusser and A. Dekel, Astrophys. J. **405**, 437 (1993); F. Bernardeau, R. Juskiewicz, A. Dekel, and F. R. Bouchet, Mon. Not. R. Astron. Soc. **274**, 20 (1995).

[12] A. Dekel and M. J. Rees, Astrophys. J. **422**, L1 (1994).

[13] C. S. Kochanek, Astrophys. J. **466**, 638 (1996).

[14] See, e.g., M. Özer and M. O. Taha, Nucl. Phys. **B287**, 776 (1987); M. Reuter and C. Wetterich, Phys. Lett. B **188**, 38 (1987); W. Chen and Y. Wu, Phys. Rev. D **41**, 695 (1990); J. Carvalho, J. Lima, and I. Waga, *ibid.* **46**, 2404 (1992).

[15] K. Freese, F. C. Adams, J. A. Frieman, and E. Mottola, Nucl. Phys. **B287**, 797 (1987).

[16] C. Wetterich, Nucl. Phys. **B302**, 668 (1988).

[17] C. Wetterich, Astron. Astrophys. **301**, 321 (1995).

[18] J. A. Frieman, C. T. Hill, A. Stebbins, and I. Waga, Phys. Rev. Lett. **75**, 2077 (1995).

[19] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988).

[20] N. Weiss, Phys. Rev. D **39**, 1517 (1989).

[21] K. Coble, S. Dodelson, and J. Frieman, Phys. Rev. D **55**, 1851 (1997).

[22] P. G. Ferreira and M. Joyce, Phys. Rev. Lett. (to be published), astro-ph/9707286.

[23] R. R. Caldwell, R. Dave, and P. J. Steinhardt, Report No. astro-ph/9708069 (1997).

[24] B. Ratra and A. Quillen, Mon. Not. R. Astron. Soc. **259**, 738 (1992).

[25] P. J. Steinhardt, Nature (London) **382**, 768 (1996).

[26] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989); S. M. Carroll, W. H. Press, and E. L. Turner, Annu. Rev. Astron. Astrophys. **30**, 499 (1992); L. M. Krauss and M. S. Turner, Gen. Relativ. Gravit. **27**, 1137 (1995).

[27] A. H. Guth, Phys. Rev. D **23**, 347 (1981); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982); A. D. Linde, Phys. Lett. **108B**, 389 (1982).
[28] P. J. E. Peebles and B. Ratra, Astrophys. J. **325**, L17 (1988); N. Sugiyama and K. Sato, *ibid.* **387**, 439 (1992); Y. Yoshii and K. Sato, *ibid.* **387**, L7 (1992); L. F. Bloomfield-Torres and I. Waga, Mon. Not. R. Astron. Soc. **279**, 712 (1996); M. Birkel and S. Sarkar, Astropart. Phys. **6**, 197 (1997).
[29] J. Hwang, Astrophys. J. **375**, 443 (1991).
[30] J. Hwang, Astrophys. J. **415**, 486 (1993).
[31] J. Hwang, Astrophys. J. **427**, 533 (1994).
[32] J. Hwang, Astrophys. J. **427**, 542 (1994).
[33] See also J. Hwang, Class. Quantum Grav. **7**, 1613 (1990); J. Hwang and E. T. Vishniac, Astrophys. J. **353**, 1 (1990); J. Hwang, *ibid.* **380**, 307 (1991); J. Hwang and J. J. Hyun, *ibid.* **420**, 512 (1994); J. Hwang and H. Noh, Report No. astro-ph/9701137 (1997).
[34] H. Kodama and M. Sasaki, Prog. Theor. Phys. Suppl. **78**, 1 (1984); Int. J. Mod. Phys. A **1**, 265 (1986); **2**, 491 (1986).
[35] A. R. Liddle and D. H. Lyth, Phys. Rep. **231**, 1 (1993).
[36] C. Hill and G. Ross, Nucl. Phys. **B311**, 253 (1988); Phys. Lett. B **203**, 125 (1988).
[37] L. F. Abbott and M. B. Wise, Nucl. Phys. **B244**, 541 (1984); F. Lucchin and S. Matarrese, Phys. Rev. D **32**, 1316 (1985); J. D. Barrow, Phys. Lett. B **187**, 12 (1988); A. R. Liddle, *ibid.* **220**, 502 (1989).
[38] K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990); F. C. Adams *et al.*, Phys. Rev. D **47**, 426 (1993).
[39] See, e.g., G. F. Smoot *et al.*, Astrophys. J. **396**, L1 (1992); E. L. Wright *et al.*, *ibid.* **420**, 450 (1994); C. L. Bennett *et al.*, *ibid.* **464**, L1 (1996); E. F. Bunn, A. R. Liddle, and M. White, Phys. Rev. D **54**, R5917 (1996); E. F. Bunn and M. White, Astrophys. J. **480**, 6 (1997).
[40] J. M. Bardeen, J. R. Bond, N. Kaiser, and A. S. Szalay, Astrophys. J. **304**, 15 (1986).
[41] N. Sugiyama, Astrophys. J., Suppl. Ser. **100**, 281 (1995).
[42] D. Wands, E. J. Copeland, and A. R. Liddle, Ann. (N.Y.) Acad. Sci. **688**, 647 (1993).
[43] A. R. Liddle and D. H. Lyth, Mon. Not. R. Astron. Soc. **273**, 1177 (1995).
[44] P. T. P. Viana and A. R. Liddle, Mon. Not. R. Astron. Soc. **281**, 323 (1996).
[45] J. A. Frieman and I. Waga, astro-ph/9709063.
[46] E. M. Lifshitz, J. Phys. (Moscow) **10**, 116 (1946).
[47] J. M. Bardeen, Phys. Rev. D **22**, 1882 (1980).
[48] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).