

## Mass spectrum of light and heavy hadrons from improved lattice actions

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We use improved lattice actions for glue, light quarks, and heavy quarks for which we use lattice NRQCD to compute hadron masses. Our results are in good agreement with experiment, except for charmed hadrons. It seems that charmed quarks are not well approximated as heavy quarks nor as light quarks. [S0556-2821(98)03311-6]

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### I. INTRODUCTION

A notorious feature of the perturbative theory of lattice QCD is that the perturbation series converges slowly at a moderate lattice spacing. Predictions based only on the first two or three terms in the series are in most cases still not reliable. A recent study [1] shows that a partial reason for this is the large contribution from tadpole diagrams, which are absent if one uses dimensional regularization instead of lattice regularization. Based on this the method of tadpole improvement [1] for the lattice perturbation series has been suggested. This has many practical implications for studies of lattice QCD. Most importantly it makes the idea of improved actions practical in simulations of lattice QCD. The idea of improved actions was proposed a long time ago [2] to systematically improve lattice actions using perturbation theory to remove the effect introduced by the finite lattice spacing  $a$ . If such an improvement really works, one can simulate lattice QCD on a coarse lattice with a lower cost in CPU time, but still obtain reliable results for physics in the continuum limit. Several simulations [3–6], where the tadpole improvement is implemented already show that this is possible.

With an improved action for glue at the one-loop level and the tree-level improved action for nonrelativistic QCD (NRQCD) the mass spectrum of charmonium is obtained in [3] from lattices with a lattice spacing  $a$  between 0.5 fm and 0.2 fm, and it agrees with experiment. Similar results were also obtained in other simulations [4], where the mass spectra of bottomonium and the  $B_c$  meson, in addition to that of charmonium, were obtained. With an improved action for light quarks, the D234 action, the mass spectrum of light hadrons is calculated from simulations on coarse lattices [5], and good agreement with experiment is found. In general improved actions are not unique. In [6] another improved action for the light fermion, which was proposed a long time ago [7], was employed, and the mass spectrum of light hadrons was also successfully calculated [6], where the gluonic action improved at tree level was used. Unlike the widely used Sheikholeslami-Wohler (SW) action for light quarks [8], in which the effect of  $O(a)$  is removed at the tree level, the actions used in these simulations are improved up to  $O(a^2)$  at tree level.

In this work we will present our results for the mass spectrum of light hadrons, strange hadrons, and hadrons containing a  $b$  or,  $c$  quark. In calculating the mass spectrum we use

the improved action for glue at the one-loop level and the action of [7] for light quarks. For the heavy  $b$  and  $c$  quarks we employ the action of lattice NRQCD. The actions for quarks are all improved at tree level up to  $O(a^2)$ . We use the quenched approximation for light quarks. Two lattices are used with the size  $6^3 \times 12$  and  $8^3 \times 16$ . Their lattice spacings determined by the charmonium spectrum are 0.41 fm and 0.23 fm, respectively. The whole calculation is performed on two UNIX workstations, which have computational power equivalent to that of a high-performance personal computer (such as a Pentium Pro 200). Our paper is organized as follows. In Sec. II we introduce the actions used in this work. Our results for light hadrons and strange hadrons are given in Sec. III. The results of  $c$ -flavored and  $b$ -flavored hadrons are given in Sec. IV, where our results [4] for simulations of quarkonia with improved actions are also given briefly. Section V is the summary.

### II. THE IMPROVED ACTIONS

#### A. The gluonic action

We take the one-loop improved action for glue [9], where the action consists of plaquette, rectangle, and parallelogram terms and is accurate up to errors of  $O(\alpha_s^2 a^2, a^4)$ . Implementing tadpole improvement the action becomes [3]

$$S(U) = \beta \sum_{pl} \frac{1}{3} \text{Re Tr}(1 - U_{pl}) + \beta_{rt} \sum_{rt} \frac{1}{3} \text{Re Tr}(1 - U_{rt}) + \beta_{pg} \sum_{pg} \frac{1}{3} \text{Re Tr}(1 - U_{pg}), \quad (1)$$

$$\beta_{rt} = -\frac{\beta}{20u_0^2} (1 + 0.4805\alpha_s), \quad \beta_{pg} = -\frac{\beta}{u_0^2} 0.03325\alpha_s, \quad (2)$$

$$u_0 = \left( \frac{1}{3} \text{Re Tr}(U_{pl}) \right)^{1/4}, \quad \alpha_s = -\frac{\ln \left( \frac{1}{3} \text{Re Tr}(U_{pl}) \right)}{3.06839}. \quad (3)$$

We used this action to generate gluonic configurations at  $\beta=7.4$ , on a lattice whose size is  $8^3 \times 16$ , and also at  $\beta=6.8$  with the size  $6^3 \times 12$ . The parameter  $u_0$  is determined by self-consistency. It is 0.8631 and 0.8267 at  $\beta=7.4$  and

$\beta=6.8$  respectively. The pseudo heat bath method [10] was used to update the links, and the three  $SU(2)$  subgroups were updated three times in each overall update step. For each lattice we generated 100 configurations for our mass calculations.

### B. The improved action for light quarks

The improved action proposed in [7] can be written as

$$S_{\text{light}} = - \sum_x \left\{ m \bar{\psi}(x) \psi(x) + \sum_{\mu} \bar{\psi}(x) \gamma_{\mu} \Delta_{\mu} (1 - c_1 \Delta_{\mu}^{(2)}) \right. \\ \left. \times \psi(x) + r \sum_{\mu} \bar{\psi}(x) \Delta_{\mu}^{(2)} \Delta_{\mu}^{(2)} \psi(x) \right\}, \quad (4)$$

where  $\Delta_{\mu}$  and  $\Delta_{\mu}^{(2)}$  are lattice derivatives with the gauge link  $U_{\mu}(x)$ :

$$\Delta_{\mu} \psi(x) = \frac{1}{2} [U_{\mu}(x) \psi(x + \hat{\mu}) - U_{\mu}^{\dagger}(x - \hat{\mu}) \psi(x - \hat{\mu})], \quad (5)$$

$$\Delta_{\mu}^{(2)} \psi(x) = U_{\mu}(x) \psi(x + \hat{\mu}) + U_{\mu}^{\dagger}(x - \hat{\mu}) \psi(x - \hat{\mu}) - 2 \psi(x). \quad (6)$$

The parameter  $c_1$  is determined to be  $1/6$  at tree level to remove the  $O(a^2)$  effect,  $m$  is the mass parameter for a quark. The last term in the action with the parameter  $r$  is introduced in analogy to the Wilson term in the Wilson action to solve the doubling problem of lattice fermions. However, the doubling problem is not totally solved. In the free case one can solve the equation of motion determined by the action to see whether the doublers are removed or not. An analysis in [8], and the analysis in [5] for the D234 action, show that in the low-energy regime the actions describe one particle in the sense that the propagator has only one pole. In the high-energy regime there are additional ‘‘unphysical’’ poles. As we are only interested in the low-energy regime in this work, we can expect the effect from other ‘‘unphysical poles’’ to be negligible. We will take  $r=1/6$ . With this choice the action is the same as employed in [6], in which it is shown that there is indeed no effect in the low-energy regime which can be related to the ‘‘unphysical poles.’’ In this work we used the stabilized biconjugate gradient algorithm [11] to calculate propagators for light quarks. On our lattices this algorithm is at least three times faster than the conventional conjugate gradient algorithm.

Currently there are two types of improved actions proposed for fermions. The action given in Eq. (4) is obtained firstly by improving the action for naive fermions, and then a dimension-7 operator is introduced as a Wilson term to solve the doubling problem. The D234 action of [5] is obtained by improving the Wilson action. In this action, one needs not only to reduce the effect at order of  $O(a^2)$ , but also the effect at order of  $O(a)$ , as the Wilson term there is built with a dimension-5 operator. For this reason the action in Eq. (4) may be preferred because there is no effect at order of  $O(a)$ . However, from the numerical results in [5,6] and from this work physical masses of light hadrons can be obtained reli-

ably with both actions on lattices with lattice spacing from 0.2 fm to 0.4 fm. Throughout this work we use the action in Eq. (4) for light quarks.

### C. The action of lattice NRQCD

Heavy quarks whose mass is larger than 1 in lattice units cannot be simulated directly as above with reliable results. To simulate them one uses the heavy quark effective theory, HQET. The formulation of HQET for hadrons with zero velocity is equivalent to that of NRQCD on the lattice, except that the expansion parameters are different. As we will only create hadronic states on the lattice with zero space momenta, we may use lattice NRQCD for heavy quarks such as the  $b$  and  $c$  quarks. On the lattice one needs to calculate the propagator of heavy quarks satisfying its appropriate evolution equation. We take the evolution equation proposed in [12,13]. The propagator  $G(t)$  [where  $G(t)=0$  for  $t \leq 0$ ] can be calculated on the lattice as

$$G(1) = \left( 1 - \frac{H_0}{2n} \right)^n U_4^{\dagger} \left( 1 - \frac{H_0}{2n} \right)^n \delta_{\mathbf{x},0}, \quad (7)$$

$$G(t+1) = \left( 1 - \frac{H_0}{2n} \right)^n U_4^{\dagger} \left( 1 - \frac{H_0}{2n} \right)^n (1 - \delta H) G(t), \quad (8)$$

where

$$H_0 = - \frac{\Delta^{(2)}}{2M_Q}, \quad (9)$$

$$\delta H = - \frac{g}{2M_Q} \boldsymbol{\sigma} \cdot \mathbf{B} + \frac{\Delta^{(4)}}{24M_Q} - \frac{(\Delta^{(2)})^2}{16nM_Q^2}. \quad (10)$$

In Eq. (10)  $\Delta^{(4)}$  is the lattice version of the continuum operator  $\Sigma_i D_i^4$ .  $M_Q$  is the mass parameter for the heavy quark  $Q$ . The last two terms in  $\delta H$  are the correction terms to remove the effect at order  $O(a^2)$ . The first term is responsible for spin splitting in the mass spectrum, where  $\mathbf{B}$  is the chromomagnetic field. We use the definition of  $\mathbf{B}$  in terms of gauge links given in [12], and it is also improved up to errors of order  $O(a^4, g^2 a^2)$ . The parameter  $n$  is introduced to avoid numerical instability when high-momentum modes occur. With propagators calculated with Eqs. (7),(8) we reach an accuracy of order  $1/M_Q$  in the mass spectrum. With lattice NRQCD we have calculated the mass spectrum of quarkonium at the above  $\beta$  values [4], and have determined that at  $\beta=7.4$  the mass parameters for  $b$  and  $c$  quarks are  $M_b=4.6$  and  $M_c=1.4$ , respectively. We will use these parameters for our calculations of mass spectra of  $b$ - and  $c$ -flavored hadrons. We take  $n=3$  for the  $c$  quark and  $n=1$  for the  $b$  quark.

In our calculations of quark propagators with the action in Eq. (4) or with Eqs. (7),(8) tadpole improvement is implemented.

### III. THE MASS OF LIGHT HADRONS

We create hadronic states on the lattice by using standard local operators  $O_H(x)$ :

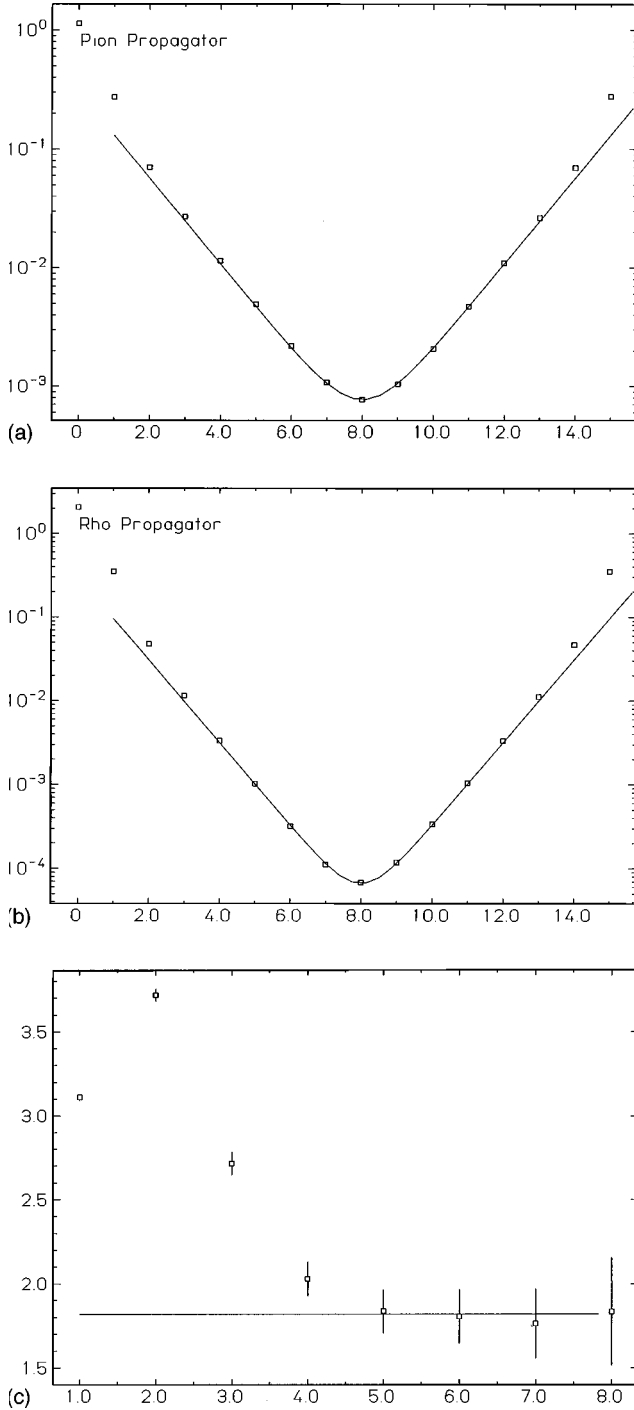


FIG. 1. (a) The propagator  $C_\pi(t)$  with  $m = -0.65$ . The points with error are measured, the line is from the fit. The  $x$  axis is for  $t$ . (b) The propagator  $c_\rho(t)$  with  $m = -0.65$ . (c) The effective mass plot for the proton propagator with  $m = -0.65$ , the effective mass is defined as  $m_{\text{eff}} = \ln[C_\rho(t-1)/C_\rho(t)]$ . The  $x$  axis is for  $t$ .

$$O_\pi(x) = \bar{u}(x) \gamma_5 d(x), \quad (11)$$

$$O_\rho^\mu(x) = \bar{u}(x) \gamma^\mu d(x), \quad (12)$$

$$O_P(x) = \varepsilon_{abc} u_a(x) [u_b^T(x) C \gamma_5 d_c(x)] \quad (13)$$

for  $\pi$ ,  $\rho$ , and proton, respectively. In Eqs. (11)–(13)  $u(x)$  and  $d(x)$  stand for  $u$  and  $d$  quarks, the indices  $a, b$ , and  $c$  are

TABLE I. The hadron masses in lattice unit at  $\beta=7.4$ .

$\beta=7.4$	$m = -0.5$	$m = -0.6$	$m = -0.65$	$m = -0.7$	$m = -0.75$
$m_\pi$	1.142(9)	0.941(11)	0.832(14)	0.713(18)	0.577(27)
$m_\rho$	1.362(20)	1.211(24)	1.138(32)	1.068(46)	1.010(95)
$m_P$	2.23(9)	1.96(7)	1.82(6)	1.69(7)	1.56(16)

color indices. With these operators one can measure the corresponding hadron correlations  $C_H(t)$ , in which the hadron has zero space momentum and its spin is averaged if it has spin. We only use local sources to calculate  $C_H(t)$ , and  $u$  and  $d$  quarks are taken to be degenerate in mass. The hadron correlation  $C_H(t)$  is fitted in a certain time interval as

$$a_H(e^{-m_H t} + e^{-m_H(T-t)}) \quad \text{for } H = \pi, \rho, \quad (14)$$

$$a_H e^{-m_H t} \quad \text{with } t < \frac{T}{2} \quad \text{for } H = \text{proton}, \quad (15)$$

where  $T$  is the lattice size in the time direction.

Using the lattice configurations on an  $8^3 \times 16$  lattice for  $\beta=7.4$  we calculated these hadron correlations varying the mass parameter for the light quark from  $-0.5$  to  $-0.75$ , and fit them according to Eqs. (14),(15). The negative values of the mass parameters simply reflect the difference in sign between the mass parameter and the ‘‘Wilson’’ parameter  $r$ . The range of light quark mass parameters investigated here is the same order as the mass of the  $s$  quark. The fitting window for  $C_\pi(t)$  and  $C_\rho(t)$  is chosen from  $t=6$  to  $t=10$ , for the proton we can see a plateau in the region of  $t=4$  to  $t=8$ , so we take the window to be from  $t=4$  to  $t=7$ . As examples which show our fit, we plot the measured  $C_H(t)$  for  $\pi$  and  $\rho$  with the fitted results in Fig. 1(a) and Fig. 1(b), respectively. Figure 1(c) is the plot for the effective mass of the proton. In all these figures the results are at  $m = -0.65$ . Our results for the fitted masses are given in Table I.

We also constructed hadron correlations for the pion with a minimal lattice momentum  $|\mathbf{p}| = 2\pi/L$ , where  $L$  is the lattice size in the space direction. Measuring these correlations one can investigate the dispersion relation  $E^2(\mathbf{p}) - m^2$

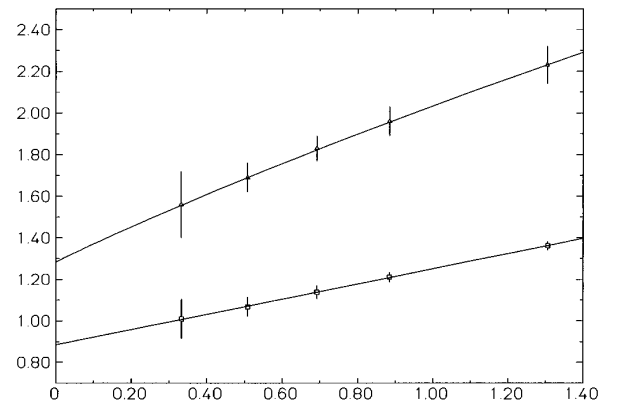


FIG. 2. The chiral extrapolation for  $\rho$  and proton. The upper line with data points is for proton. The lower line with data points is for  $\rho$ . The  $x$  axis is read to be  $m_\pi^2$ .

TABLE II. The masses of  $K$  and  $K^*$  in lattice unit at  $\beta=7.4$ .

$\beta=7.4$	$m = -0.5$	$m = -0.6$	$m = -0.65$	$m = -0.7$	$m = -0.75$
$m_K$	0.979(12)	0.870(13)	0.814(14)	0.755(16)	0.694(20)
$m_{K^*}$	1.239(25)	1.163(29)	1.127(33)	1.092(40)	1.061(50)

$=c^2|\mathbf{p}|^2$ . We find that  $c$  is close to 1. For example, we obtained  $c=1.02(7)$  at  $m=-0.5$  and  $c=1.00(18)$  at  $m=-0.75$ . This fact indicates that rotational invariance is restored on the lattice. To find the masses in the chiral limit, we fit the hadron masses with

$$m_\rho = v_0 + v_1 m_\pi^2, \quad m_P = u_0 + u_1 m_\pi^2 + u_2 m_\pi^3. \quad (16)$$

We find that the masses given in Table I are well described by these relations as shown in Fig. 2. For the mass of the proton we clearly see the effect of the cubic term. With these fits one can extrapolate the masses to the chiral limit.

If we assume that the quark mass is obtained through multiplicative and additive renormalizations of the mass parameter, then the square of  $m_\pi$  should be linear in the mass parameter  $m$ . With the data in Table I we find good agreement with this relation. Hence we are able to determine the critical mass parameter  $m_0$  at which  $m_\pi=0$ . We also determine the mass parameter  $m_s$  of the strange quark from the experimental value  $m_K^2/m_\rho^2=0.412$ . These parameters at  $\beta=7.4$  are

$$m_0 = -0.830(40), \quad m_s = -0.6658. \quad (17)$$

With the  $m_s$  above we calculated the propagator of the  $s$  quark and then the hadron correlations for  $K$ ,  $K^*$ , and  $\phi$ , where  $C_K(t)$  and  $C_{K^*}(t)$  are calculated with these mass parameters in Table I for light quark. The masses obtained for  $m_K$ ,  $m_{K^*}$  with nonzero mass for the  $u$  and  $d$  quark are given in Table II.

We fit these masses with

$$m_K^2 = b_0 + b_1(m - m_0), \quad m_{K^*} = c_0 + c_1(m - m_0) \quad (18)$$

to extrapolate to the limit of zero-mass  $u$  and  $d$  quarks. The data and the fitted line are drawn in Fig. 3. The data points

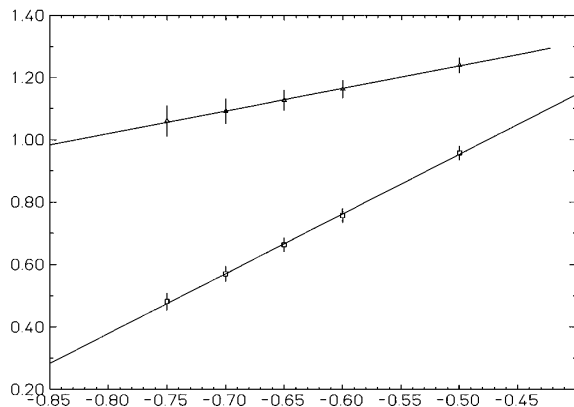


FIG. 3. The chiral extrapolation for the strange meson.

TABLE III. The hadron masses in lattice unit at  $\beta=6.8$ .

$\beta=6.8$	$m = -0.8$	$m = -0.9$	$m = -1.0$	$m = -1.05$	$m = -1.1$
$m_\pi$	1.353(9)	1.179(10)	0.986(11)	0.877(13)	0.754(14)
$m_\rho$	1.703(21)	1.587(29)	1.470(48)	1.409(62)	1.340(81)
$m_N$	2.770(77)	2.523(77)	2.262(79)	2.113(86)	1.941(106)
$m_K$	1.218(10)	1.125(10)	1.028(11)	0.977(12)	0.924(12)
$m_{K^*}$	1.614(27)	1.554(33)	1.495(43)	1.467(49)	1.439(56)

are well described by the linear fit giving confidence in the extrapolation.

Doing the same calculations of  $C_H(t)$  with our configurations at  $\beta=6.8$ , for which we choose the region of the light quark mass parameter  $m$  from  $-0.8$  to  $-1.1$ , we obtain hadron masses which are given in Table III.

At this  $\beta$ -value we have determined

$$m_0 = -1.233(56), \quad m_s = -0.9513. \quad (19)$$

With our results we obtain hadron masses in the chiral limit and the mass  $m_\phi$  which depends only on  $m_s$ . They are given in Table IV as a ratio to  $m_\rho$ , which is itself given in lattice units.

Having chosen the parameter  $m_s$  to fit the mass of the  $K$ , our predictions for the meson masses are in good agreement with experiment, although the quenched approximation was used. The mass of the proton obtained at  $\beta=7.4$  is 20% larger than the experimental value, while that from  $\beta=6.8$  agrees with the experimental value within the statistical error. A possible reason for the deviation at  $\beta=7.4$ , apart from the quenched approximation, could be the effect of the finite volume. Another possible reason is the use of a local source in our work, a result of which is that the signal of the ground state in  $C_P(t)$  is rather weak, and the fitting quality of  $C_P(t)$  is worse than that of meson propagators. This situation may be improved if smeared sources are used.

From the calculated  $\rho$  mass in lattice units we can determine the physical lattice spacing. We obtain

$$a_\rho^{-1} = 0.64(4) \text{ GeV, at } \beta=6.8, \quad (20)$$

$$a_\rho^{-1} = 0.87(4) \text{ GeV, at } \beta=7.4. \quad (21)$$

It is instructive to compare the lattice spacings determined from the quarkonium system. In our previous work [4] we obtained

TABLE IV. The hadron masses in the chiral limit.

	$\beta=6.8$	$\beta=7.4$	Expt.
$am_\rho$	1.198(67)	0.885(45)	
$m_P/m_\rho$	1.16(10)	1.45(11)	1.22
$m_K/m_\rho$	0.63(4)	0.64(3)	0.64
$m_{K^*}/m_\rho$	1.14(7)	1.13(6)	1.16
$m_\phi/m_\rho$	1.27(8)	1.26(9)	1.32

$$a_{J/\Psi}^{-1} = 0.477(5) \text{ GeV at } \beta = 6.8, \quad (22)$$

$$a_{J/\Psi}^{-1} = 0.749(4) \text{ GeV},$$

$$a_Y^{-1} = 0.861(5) \text{ GeV at } \beta = 7.4. \quad (23)$$

The lattice spacing at  $\beta = 7.4$  determined from  $\rho$  is close to that obtained from  $Y$ , while there is a deviation of 14% in comparison with  $a_{J/\Psi}^{-1}$ . At  $\beta = 6.8$  the deviation is 30%. Similar lattice spacings were also obtained with D234 action in [5]. Possible reasons for these deviations include: the effects neglected in the quenched approximation being different in different systems, or the effect from higher orders in lattice NRQCD being large for the charmonium system because the charm mass is not large enough. In general one should not be surprised that the lattice spacings determined from different systems are different when approximations are used in the calculations.

From the dependence of  $m_\rho$  on  $m_\pi$  we can obtain the quantity  $J = m_\rho dm_\rho^2 / dm_\pi^2$  at the experimental value  $m_\rho / m_\pi = 1.8$ . This quantity is introduced in [14] to judge the quality of the quenched approximation. For the ‘‘real world’’ it is  $J = 0.48(2)$ , while the ‘‘world averaged’’ result from the quenched approximation is  $J = 0.37$ . From our results we obtain

$$J = 0.375(87) \text{ at } \beta = 6.8,$$

$$J = 0.363(63) \text{ at } \beta = 7.4, \quad (24)$$

which are consistent with  $J = 0.37$ .

In [6] masses of light hadrons are also calculated with the same action for fermions given in Eq. (4), but with the gluonic action improved at the classical level. Two lattices with lattice spacings similar to ours were used. The masses of light hadrons are calculated with improved actions roughly in the range of  $\pi$  mass from 0.7 to 1.3 in lattice units and then are extrapolated to the chiral limit. A detailed comparison with results from large lattices was made. Our results are in agreement with their results. From this fact one cannot see any significant difference introduced at different level improved actions for gluon. However, there may be a difference in the determination of the mass parameter for the  $s$  quark. The mass parameter  $m$  in Eq. (4) is related to the hopping parameter  $\kappa$  in [6] via

$$m = \frac{1 - 2\kappa}{2\kappa}. \quad (25)$$

With this relation  $\kappa_s$  is 0.1640 and 0.1500 for lattices with the lattice spacing  $a_\rho^{-1} = 0.64$  GeV and  $a_\rho^{-1} = 0.87$  GeV respectively, while  $\kappa_s$  is determined with tree-level improved action for the gluon to be 0.166 and 0.1558 for lattices with the lattice spacing  $a_\rho^{-1} = 0.648$  GeV and  $a_\rho^{-1} = 0.855$  GeV, respectively [6]. It should be noted that this difference can also be due to other reasons, e.g.,  $\kappa_s$  in [6] it is determined by experimental data of  $m_{K^*}/m_K$ , while ours is determined by  $m_K/m_\rho$ .

TABLE V. The results for quarkonia at  $\beta = 7.4$ .

	$aE_{1S}$	$a\Delta E_{2S-1S}$	$aE_{1P}$	$aE_{1S}( \mathbf{p} =p_{\min})$
$aM_b^0 = 4.6$	0.4925(5)	0.712(3)	1.002(2)	0.5203(5)
$aM_c^0 = 1.4$	0.821(2)	0.89(2)	1.431(1)	0.898(2)

#### IV. THE MASS SPECTRUM OF $c$ - AND $b$ - FLAVORED HADRONS

Before going on to our results for  $c$ - and  $b$ - flavored hadrons, we briefly give our results for simulations of quarkonia in [4], and we discuss the relationship between our analysis and the slightly different one of [16]. We used the same action in Eq. (1) for the gluon and the NRQCD action, which is the one given in Eqs. (9),(10), but the term with  $\sigma \cdot \mathbf{B}$  is neglected. With this action the mass spectrum of quarkonia can be calculated at the accuracy of  $O(v^2)$ , where  $v$  is the velocity of the heavy quark in a quarkonium in its rest frame. Under this approximation only the spin averaged mass spectrum can be obtained. In [4] we calculated the mass of  $S$ - and  $P$ -wave charmonium and bottomonium, and matched the results with the experimental data to find the mass parameter in Eqs. (9),(10) corresponding to the  $c$  and  $b$  quark. We have found that  $aM_c = 1.4$  and  $aM_b = 4.6$  for  $\beta = 7.4$ . In our simulation we measure an energy  $E_H$  which is related to the mass  $M_H$  by

$$M_H = 2\Delta_Q + E_H, \quad (26)$$

where  $H$  stands for  $S$ - or  $P$ - wave quarkonium,  $\Delta_Q$  is the difference between the renormalized mass of the heavy quark and the zero point energy of the heavy quark on lattice, and it does not depend on the type of hadron. The exact mass can be obtained by a dispersion relation and by calculating  $E_H$  for which the quarkonium has nonzero spatial momentum. Our results for  $\beta = 7.4$  are given in Table V.

Using the mass splitting between  $S$ - and  $P$ - wave quarkonia we determined the lattice spacing to be

$$a_c^{-1} = 0.749(4) \text{ GeV from charmonium}, \quad (27)$$

$$a_b^{-1} = 0.861(5) \text{ GeV from bottomonium}. \quad (28)$$

and obtained the mass for  $S$ -wave charmonium and bottomonium:

$$M_{1S} = 3.00(2) \text{ GeV},$$

$$M_{2S} = 3.67(5) \text{ GeV for charmonium}, \quad (29)$$

$$M_{1S} = 9.5(3) \text{ GeV},$$

$$M_{2S} = 10.1(3) \text{ GeV for bottomonium}. \quad (30)$$

These results agree well with experiment. The above results are obtained from simulations on a lattice with the size  $8^3 \times 10$  at  $\beta = 7.4$ . For the simulations 200 configurations were used. We also simulated quarkonium systems at  $\beta = 6.8$ . But bottomonium is problematic at this  $\beta$  value. In the region around the mass of the  $b$  quark, the energy, e.g.,  $E_{1S}$  is around 0.1 to 0.2 in lattice units, or, the correlation length of

the system is 5 to 10 in lattice units. Hence a large effect due to finite volume can be expected, because our lattice volume is  $6^3$  at  $\beta=6.8$ . This prevents us from determining the mass parameter  $aM_b$  as precisely as that for  $\beta=7.4$ . At  $\beta=6.8$  the inverse of the lattice spacing is  $a^{-1}=0.477(1)$  GeV determined with charmonium systems. With this lattice spacing it is expected that  $E_{1S}$  for bottomonium is of order of  $10^{-1}$  in lattice unit.

Because of this we will present only our results for  $c$ - and  $b$ - flavored hadrons at  $\beta=7.4$ . On the lattice with  $\beta=6.8$  we calculated only masses of  $c$ -flavored hadrons and we will mention these results briefly. With the configurations at  $\beta=7.4$  used in the previous section we will calculate masses of the hadrons  $B^0, B^*, B_s, B_s^*$ , and  $\Lambda_b$  and masses of the  $D^0, D^*, D_s, D_s^*$ , and  $\Lambda_c$ . We take the values of the mass parameters in Eqs. (9),(10), determined from quarkonia, as the input for the following calculation. It should be noted that the determination is done without the term of  $\sigma \cdot \mathbf{B}$  in Eqs. (9),(10). It can be expected that the values of the mass parameter can be slightly different, if we determine them by adjusting them to match simulation results with Eqs. (9), (10) with experimental results for heavy flavor hadrons. The effect from this difference may be expected to be small.

For the hadrons we use operators analogous to the operators listed in Eqs. (11)–(13) to create these hadronic states on lattice. The difference is that a light quark field in Eqs. (11)–(13) is replaced by a heavy quark field  $Q(x)$  and the field  $Q(x)$  only has two nonzero components. The propagation of the field is determined by Eqs. (7),(8). For light quarks we use the propagators of the last section. For calculating propagators of heavy quarks we use a source smeared by a Gaussian:

$$F(\mathbf{x}, \mathbf{y}, t) = \left[ 1 + \epsilon \sum_i \Delta_i^{(2)} \right]^{n_s} (\mathbf{x}, \mathbf{y}, t). \quad (31)$$

The smearing parameter  $n_s$  is fixed at  $n_s=10$ , while the parameter  $\epsilon$  is adjusted so that the smearing radius is about the half that of the simulated system. Using a smeared source is essential for our simulations. With a local source for heavy quark propagators the signal of hadron correlations is overwhelmed by the statistical noise after three or four time slices from the source. We calculate hadron correlations only with smeared sources, while the sinks remain unsmeared. The measured hadron propagators are fitted to the form

$$C_H(t) \sim a_H e^{-E_H t}, \quad (32)$$

where the index  $H$  refers to the hadrons mentioned above. With the fitted energy  $E_H$  one can obtain the hadron mass  $M_H$

$$M_H = \Delta_Q + E_H, \quad (33)$$

where  $\Delta_Q$  is the same as in Eq. (26). Therefore one can predict the mass difference between two types of hadrons if the  $E_H$ 's are known.

Although we used a smeared source for the heavy quark propagator, the signal is buried in statistical noise after ten or eleven time slices from the source for mesons and after eight or nine time slices for baryons. For the meson we already see a plateau in the effective mass. We will take the fit window

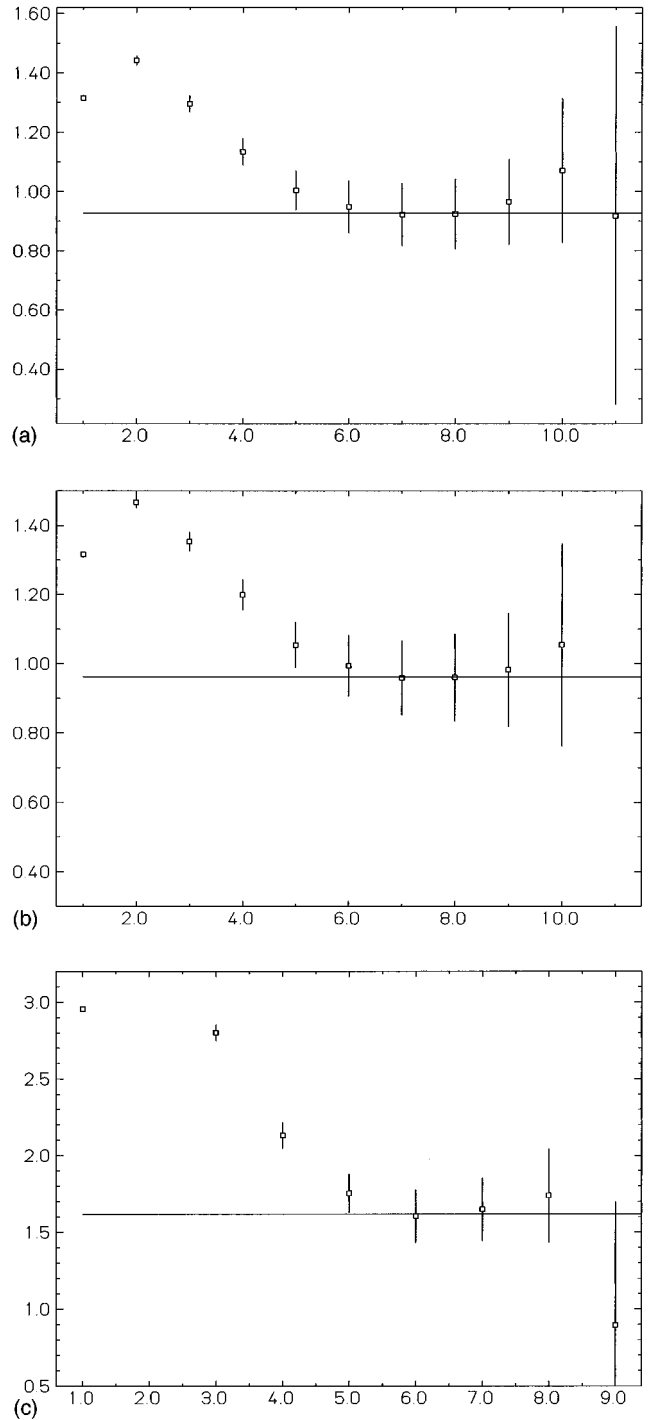


FIG. 4. (a) The effective mass plot for  $B$  with  $m = -0.65$ . (b) The effective mass plot for  $B^*$  with  $m = -0.65$ . (c) The effective mass plot for  $\Lambda_b$  with  $m = -0.65$ .

for meson propagators from 6 to 9. For baryons we take the fit window from 5 to 7. In Figs. 4(a), 4(b), and 4(c) we give our effective mass plots for  $B$ ,  $B^*$ , and  $\Lambda_b$ , respectively. These plots are for propagators at the mass parameter  $m = -0.65$ . Our results for  $E_H$  of various hadrons with non-zero mass of light quarks are given in Table VI.

For  $c$ - and  $b$ -flavored hadrons with  $s$  quarks we have

$$E_{B_s} = 0.920(11), \quad E_{B_s^*} = 0.952(9), \quad (34)$$

TABLE VI. The results for  $c$ - and  $b$ -flavored hadrons at  $\beta=7.4$ .

	$m = -0.5$	$m = -0.6$	$m = -0.65$	$m = -0.7$	$m = -0.75$
$E_{D^0}$	1.069(10)	0.990(10)	0.950(9)	0.909(9)	0.863(9)
$E_{D^{*0}}$	1.143(8)	1.067(9)	1.030(9)	0.991(10)	0.948(10)
$E_{\Lambda_c}$	1.976(16)	1.810(17)	1.727(17)	1.643(15)	1.561(17)
$E_{B^0}$	1.043(14)	0.967(12)	0.928(12)	0.894(12)	0.856(13)
$E_{B^{*0}}$	1.066(11)	0.999(9)	0.962(9)	0.927(9)	0.891(10)
$E_{\Lambda_b}$	1.892(12)	1.713(12)	1.617(14)	1.504(16)	1.381(22)

$$E_{D_s} = 0.937(9), \quad E_{D_s^*} = 1.018(9). \quad (35)$$

To extrapolate to the zero light quark mass we assume the dependence of  $E_H$  on the mass parameter  $m$  to be

$$E_H = E_H^{(0)} + E_H^{(1)}(m - m_0), \quad (36)$$

and use this relation to fit our data. In Figs. 5(a) and 5(b) we show the fits to the data of Table VI. Except for  $\Lambda_b$ , our results in Table VI can be well fitted with this relation. In

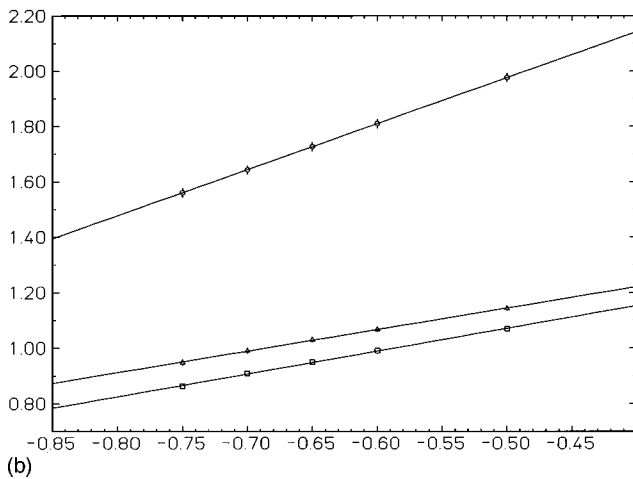
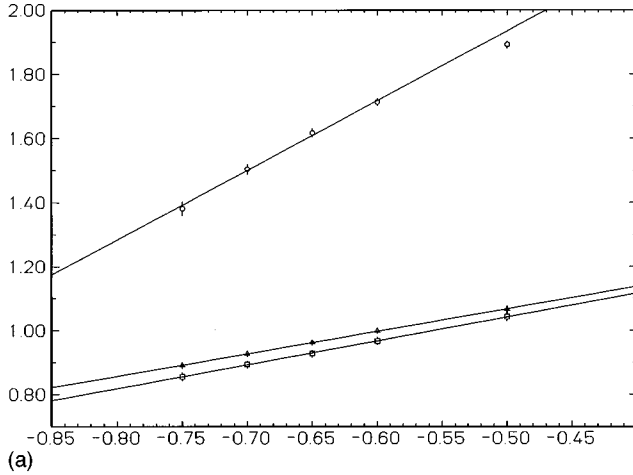


FIG. 5. (a) The chiral extrapolation for  $b$ -flavored hadrons. The upper line with data points is for  $\Lambda_b$ , the middle one with data points is for  $B^*$ , the lower one with data points is for  $B$ . The  $x$  axis is read as  $m$ . (b) The chiral extrapolation for  $c$ -flavored hadrons. The upper line with data points is for  $\Lambda_c$ , the middle one with data points is for  $D^*$ , the lower one with data points is for  $D$ . The  $x$  axis is read as  $m$ .

Fig. 5(a) the fitted line for  $\Lambda_b$  is determined only by the first four data points. The fit has  $\chi^2 = 0.76$ . If we include the last point into the fit, the  $\chi^2$  jumps to 3.39, and  $E_{\Lambda_b}^{(0)}$  changes from 1.218(7) to 1.250(9). A reason for this may be the effect of higher orders of  $(m - m_0)$  omitted in Eq. (36). In our final results we will use the fit with the four data points for  $\Lambda_b$ . We obtain the energies at zero mass of light quarks:

$$E_{B^0}^{(0)} = 0.796(4), \quad E_{B^{*0}}^{(0)} = 0.836(3), \quad E_{\Lambda_b}^{(0)} = 1.218(7), \quad (37)$$

$$E_{D^0}^{(0)} = 0.800(3), \quad E_{D^{*0}}^{(0)} = 0.889(3), \quad E_{\Lambda_c}^{(0)} = 1.428(5). \quad (38)$$

It is interesting to note that the energies given in Eqs. (34),(37) for  $b$ -flavored hadrons are close to those of  $c$ -flavored hadrons. These energies  $E_H$ 's can be expanded in the inverse of  $M_Q$  with a leading order  $M_Q^0$  and are calculated with an accuracy of  $M_Q^{-1}$  in this work. The above fact indicates that the effect from the next-to-leading order and from higher orders is small in these spin averaged masses. However, as we will see, this is not true for spin-splittings. With the results in Eqs. (34),(35) and in Eqs. (37),(38) we are able to predict the mass differences. For doing this we take the lattice spacing determined by the bottomonium system for  $b$ -flavored hadrons,

$$a_b^{-1} = 0.86 \text{ GeV}. \quad (39)$$

This spacing is the same within errors as the lattice spacing determined from  $m_\rho$  in the last section. We obtain

$$M_{B^{*0}} - M_{B^0} = 34(6) \text{ MeV}, \quad M_{B_s} - M_{B^0} = 107(13) \text{ MeV}, \quad (40)$$

$$M_{B_s^*} - M_{B_s} = 27(17) \text{ MeV}, \quad M_{\Lambda_b} - M_{B^0} = 363(9) \text{ MeV}. \quad (41)$$

These results should be compared with the experimental results

$$M_{B^{*0}} - M_{B^0} = 46 \text{ MeV}, \quad M_{B_s} - M_{B^0} = 91 \text{ MeV}, \quad (42)$$

$$M_{B_s^*} - M_{B_s} = 47 \text{ MeV}, \quad M_{\Lambda_b} - M_{B^0} = 363 \text{ MeV}. \quad (43)$$

We find that our results for  $M_{\Lambda_b} - M_{B^0}$  and  $M_{B_s} - M_{B^0}$  agree well with experimental results, while the spin splitting  $M_{B^{*0}} - M_{B^0}$  and  $M_{B_s^*} - M_{B_s}$  are not in such good agreement, but differ by less than two standard deviations from the observed values. The value of  $M_{B^{*0}} - M_{B^0}$  is 28% lower than experimental value, but it agrees with the result from a large and fine lattice [15].

For  $c$ -flavored hadrons we take the lattice spacing determined by the charmonium system:

$$a_c^{-1} = 0.75 \text{ GeV} \quad (44)$$

and obtain

$$M_{D^*} - M_{D^0} = 67(5) \text{ MeV}, \quad M_{D_s^*} - M_{D^0} = 103(9) \text{ MeV}, \quad (45)$$

$$M_{D_s^*} - M_{D_s} = 66(14) \text{ MeV}, \quad M_{\Lambda_c} - M_{D^0} = 471(6) \text{ MeV}. \quad (46)$$

The experimental results are

$$M_{D^*} - M_{D^0} = 143 \text{ MeV}, \quad M_{D_s} - M_{D^0} = 104 \text{ MeV}, \quad (47)$$

$$M_{D_s^*} - M_{D_s} = 144 \text{ MeV}, \quad M_{\Lambda_c} - M_{D^0} = 420 \text{ MeV}. \quad (48)$$

Our value for  $M_{D_s} - M_{D^0}$  agrees well with the experimental value, but the predicted  $M_{\Lambda_c} - M_{D^0}$  is 12% larger the experimental value. Worse are the spin splittings, which are much lower than experimental values. Many reasons can be found for such discrepancies, e.g., the effects of quenching. However, there are two likely reasons, which can be mainly responsible: We used lattice NRQCD up to order of  $1/M_Q$ . For  $c$  quarks the effect from higher orders is very significant because the charm mass is not very large. A recent study of quarkonium systems [16] also shows that the effect of higher order terms in  $1/M_Q$  is large. We also have calculated masses of  $c$ -flavored hadrons at  $\beta=6.8$  with similar results. Another reason is the perturbative coefficients used in the NRQCD action in Eqs. (9),(10), where only the coefficients at tree-level are used. For the energy scale at the mass of the  $c$  quark, radiative corrections to these coefficients can be large, but one may take these corrections into account if they are available.

To obtain the absolute mass of a  $b$ - or  $c$ -flavored hadron we need to know  $\Delta_Q$  in Eq. (26) or in Eq. (33). This quantity can be calculated perturbatively. It can also be extracted by studying the mass spectrum of the quarkonium system. We extracted this number from our previous study at  $\beta=7.4$ , i.e., by using Eq. (26) and the results in Table V.

$$\Delta_c = 1.59(11), \quad \Delta_b = 5.25(18). \quad (49)$$

It should be noted that effects from the spin-dependent interaction are neglected in the extraction. Adding the spin-dependent interaction has little effect on these numbers, which have a quite large uncertainty. Using Eq. (49) we obtain

$$M_{D^0} = 1.79(8) \text{ GeV}, \quad M_{B^0} = 5.20(15) \text{ GeV} \quad (50)$$

where the lattice spacings in Eq. (44) and in Eq. (39) are used, respectively. These predictions are in good agreement with the experimental values:

$$M_{D^0} = 1.864 \text{ GeV}, \quad M_{B^0} = 5.278 \text{ GeV}. \quad (51)$$

However, the accuracy of our predictions is not good because of the large statistical error of  $\Delta_Q$ .

## V. SUMMARY

In this work we used improved actions for glue and quarks to calculate masses of light, strange,  $c$ - and  $b$ -flavored hadrons, where for  $c$  and  $b$  quarks we employed lattice NRQCD. The actions for quarks are improved at tree level to remove the effect at order of  $O(a^2)$ , while the gluonic action is improved at one-loop level. Tadpole improvement is implemented. The results obtained in this work for light and strange mesons are in agreement with experimental results. We obtain the mass spectrum of  $b$ -flavored hadrons with improved actions on a coarse lattice with the lattice spacing as 0.23 fm, and it is in agreement with experiment. For  $c$ -flavored hadrons the large effect from higher orders neglected in lattice NRQCD prevents us from obtaining spin splitting in mass comparable with experiment. In this work we have shown that improved actions with tadpole improvement can be used not only in light hadron sectors and in quarkonium systems as shown already in previous studies, but also in the case of hadrons containing one heavy quark.

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