

Spectrum of a large N gauge theory near the transition from confinement to screening

David J. Gross and Akikazu Hashimoto

Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030

Igor R. Klebanov

Department of Physics, Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544-0708

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We study the spectrum of $(1+1)$ -dimensional large N QCD coupled to an adjoint Majorana fermion of mass m . As $m \rightarrow 0$ this model makes a transition from confinement to screening. We argue that in this limit the spectrum of single-string states develops a continuous part. The continuum begins at exactly twice the mass of the lightest bound state. This critical mass is nothing but the threshold for a decay into two lightest states. We present numerical results based on DLCQ that appear to support our claim. [S0556-2821(98)03110-5]

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I. INTRODUCTION

Deriving the low-energy properties of QCD continues to be a largely unsolved problem. For this reason one often resorts to simpler low-dimensional models in order to gain intuition about the $(3+1)$ -dimensional case. A classic such model is the 't Hooft model [1], which is the $(1+1)$ -dimensional $SU(N)$ gauge theory coupled to Dirac fermions in the fundamental representation. Two key elements in the solution of the model are the large N limit and the light-cone quantization. The large N limit simplifies the dynamics by removing the interactions between confined states. In the 't Hooft model one thus finds a single Regge trajectory of non-interacting mesons. Quantization on the light cone, originally introduced in [2], is a useful tool as well, because all unphysical degrees of freedom become manifestly non-dynamical and can be eliminated using the constraints.

In light cone quantization, one treats one of the null coordinates, usually chosen to be x^+ , as the time. The other null coordinate is then treated as spatial, and we could imagine compactifying it into a circle. Then the spectrum of the longitudinal momentum p_- becomes discrete, hence the name discrete light cone quantization (DLCQ) [3,4]. This approach to theories on the light cone is often useful as a conceptual tool (as, for example, in the matrix formulation of M theory [5]). It is also a practical device for solving theories numerically. By now there is an enormous body of literature on QCD on the light cone. Readers are referred, for example, to [6] for a recent review and list of references.

While the 't Hooft model provides the simplest demonstration of confinement in a non-Abelian gauge theory, it does not capture the complexity of $(3+1)$ -dimensional gauge dynamics. This is because it contains no dynamical degrees of freedom in the adjoint representation of $SU(N)$. In order to model the physics of transverse gluons, one may consider $(1+1)$ -dimensional QCD coupled to matter in the adjoint representation [7]. Such degrees of freedom can be thought of as arising from dimensional reduction of QCD in higher dimensions. A particular model which has received some attention recently [7–9] is that of a single Majorana fermion in the adjoint representation coupled to two-dimensional QCD:

$$S = \int d^2x \operatorname{Tr} \left(i\bar{\Psi}^T \not{D} \Psi - m\bar{\Psi}^T \Psi - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} \right). \quad (1)$$

In many ways, this is the simplest model exhibiting some interesting physical features. This theory is manifestly finite (unlike the model coupled to an adjoint scalar which requires a mass renormalization) and its numerical investigation can be easily set up using the discrete light cone quantization [7,9–11]. The model contains one adjustable dimensionless parameter $x = \pi m^2 / g^2 N$. Several interesting features of this model have been noted in the literature. For example, for $x = 1$ the model becomes supersymmetric [8], while for $x = 0$ it is essentially equivalent to $SU(N)$ gauge theory coupled to N flavors of fundamental fermions [12].

Unlike the 't Hooft model, the theory (1) has an exponentially increasing density of bound states, $\rho(M) \sim e^{M/T_H}$ [8,9]. Thus, at temperature T_H a deconfinement transition occurs. Surprisingly, the temperature T_H exhibits a non-trivial dependence on mass; for example, $T_H \rightarrow 0$ as $m \rightarrow 0$. This is because in the massless limit the theory undergoes a phase-transition from the confining phase to the screening phase [13,14]. The string tension scales linearly with the fermion mass and vanishes at the point of the phase transition.

In order to improve our insight into the transition to screening, we need to understand, at least qualitatively, what happens to the spectrum of string-like bound states as $m \rightarrow 0$. On the one hand, for any $m > 0$ the theory is confining, hence the spectrum of single string states is strictly speaking discrete all the way to infinite mass. On the other hand, in the limit $m \rightarrow 0$, one should expect that the spectrum becomes dense, at least above a certain mass. A heuristic argument for this goes as follows [13]. In a screening theory there is finite range attraction between color non-singlets which may be strong enough to create a few bound states. However, since the attractive potential flattens at infinity, we expect the spectrum to be continuous above a certain mass. In this paper we present numerical evidence for the appearance of a continuous part of the single-string spectrum as $m \rightarrow 0$.

Our results are also consistent with [12,15]. There it was argued that for $m \rightarrow 0$ one can identify certain "basic" bound states (single particles). The spectrum of these par-

ticles is discrete. For small m most of the single-string states may be thought of as loosely bound multi-particle states. These multi-particle states form a continuum as $m \rightarrow 0$.

The principal new result of this paper is that the continuum begins at twice the mass of the lightest particle. We believe that the spectrum is discrete below this critical mass. While this structure of the $m \rightarrow 0$ limit of the spectrum could have been anticipated from the arguments reviewed above, we support it by careful analysis of the DLCQ numerical diagonalizations.

II. DECOUPLING BETWEEN MASSLESS AND MASSIVE SECTORS

Let us begin by briefly reviewing the arguments of [12]. Consider a conformal field theory invariant under global symmetry group G . Such a model arises as a representation of affine Lie algebra \hat{G} . Consider, for example, a Lagrangian for a theory with right handed quarks $\psi^{(r)}$ and left handed quarks $\chi^{(r')}$ in representations r and r' of G respectively:

$$\mathcal{L}_{CFT} = \sum_r \psi^{\dagger(r)} \partial_+ \psi^{(r)} + \sum_{r'} \chi^{\dagger(r')} \partial_- \chi^{r'}.$$

There is a natural way to couple such a theory to a non-Abelian gauge field based on gauge group G :

$$\mathcal{L} = \mathcal{L}_{CFT} + A_+ J^+ + A_- J^- + \frac{1}{2g^2} (F_{\mu\nu}^a)^2$$

with

$$J^+ = \psi^{\dagger(r)} \lambda^{a(r)} \psi^{(r)},$$

$$J^- = \chi^{\dagger(r')} \lambda^{a(r')} \chi^{(r')}.$$

This theory is believed to be consistent if the levels k and \bar{k} of the left and right moving Kac-Moody (KM) currents coincide.

In light cone quantization, one treats the light like coordinates x^- and x^+ as space and time, respectively. The natural gauge in this coordinate system is the light cone gauge $A_- = 0$. In this gauge, A_+ and χ are non-dynamical. Taking into account the constraints imposed by these non-dynamical fields, the light cone Hamiltonian becomes

$$P^- = - \int dx^- \frac{1}{2} g^2 J^{a+} \frac{1}{\partial_-^2} J^{a+}. \quad (2)$$

The dependence on χ 's has disappeared from the Hamiltonian, other than the basic requirement that their chiral anomaly matches that of the ψ 's. This implies that χ could have been replaced with any other representation of \hat{G} as long as they have the same chiral anomaly.

Strictly speaking, it is incorrect to conclude that the physics of these models depends only on the KM level of the matter content, because by going to light cone coordinates, the dynamics of massless degrees of freedom propagating along the x^- axis is lost. This implies, however, that all data specifying the details of matter representation other than its KM level is encoded in the massless sector propagating

along x^- . The massless left-moving sector is therefore decoupled from the massive sector.

However, instead of treating x^- as space and x^+ as time, one could have considered taking x^+ as space and x^- as time. Then, the natural gauge choice would have been $A_+ = 0$, and using the same argument as the one given above, one concludes that the massless right-moving sector is decoupled from the massive sector.

$SU(N)$ gauge theory coupled to a massless adjoint fermion introduced in (1) is an example of such a gauged Wess-Zumino-Witten (WZW) model. The current $J^{ab} = 2\psi^{ac}\psi^{cb}$ generates a KM algebra of level N . For generic mass, the single particle color singlet states in this model are of the form

$$\text{Tr}(\psi\psi\dots\psi)|0\rangle \quad (3)$$

and these states become non-interacting in the $N \rightarrow \infty$ limit.

For $m=0$, however, the form of the light-cone Hamiltonian (2) suggests that the Hilbert space of single particle states can be block diagonalized into current blocks labeled by the KM primaries. The simplest states in the current blocks are of the form

$$\text{Tr}(JJJ\dots J)|0\rangle$$

or

$$\text{Tr}(JJJ\dots J\psi)|0\rangle.$$

General highest weight states are of the form

$$\left(\prod_{i=1}^n \psi^{a_i b_i} \right) |0\rangle \quad (4)$$

with symmetrization of indices a_i and b_i encoded in terms of Young tableaux with n boxes. A generic state in the current block with n fermions in the primary will be of the form

$$\text{Tr}(J^{l_1} \psi J^{l_2} \psi \dots J^{l_n} \psi) |0\rangle.$$

$SU(N)$ gauge theory coupled to N flavors of massless fundamental fermions is also an example of a gauged level N WZW model. The decoupling theorem implies that the physics in the massive sector should agree with that of the adjoint fermion model described earlier. The highest weight states of the KM algebra are of the form

$$\left(\prod_{i=1}^n \psi^{\dagger a_i \alpha_i} \psi^{\dagger b_i \beta_i} \right) |0\rangle$$

where a_i and b_i are symmetrized just as in the adjoint fermion case and are characterized by Young tableaux with n boxes. A generic state in such a current block sector will be of the form

$$[\psi^{\alpha_1 a_1} (J^{l_1})^{a_1 b_1} \psi^{\dagger b_1 \beta_1}]$$

$$\times [\psi^{\alpha_2 a_2} (J^{l_2})^{a_2 b_2} \psi^{\dagger b_2 \beta_2}] \dots [\psi^{\alpha_n a_n} (J^{l_n})^{a_n b_n} \psi^{\dagger b_n \beta_n}] |0\rangle.$$

For $n \geq 1$, these states appear to correspond to n mesons built out of fundamental quarks.

In general, the states in the massive sector are labeled by the current block and the currents. Based on this classification, a state

$$|\Phi\rangle = \text{Tr}(J^m \psi) |0\rangle$$

of the adjoint fermion model is associated with a state

$$|\Sigma\rangle = [\psi^{\alpha\alpha} (J^m)^{ab} \psi^{\dagger b\beta}] |0\rangle$$

of the N -flavored fundamental model. There are two problems with this identification. First, $|\Phi\rangle$ is a fermion whereas $|\Sigma\rangle$ is a boson. Secondly, $|\Phi\rangle$ is a unique state whereas $|\Sigma\rangle$ has N^2 -fold degeneracy due to choice of flavors. The resolution to this apparent discrepancy lies in the massless sector we have ignored up till now. The state $|\Sigma\rangle$ is actually $|\Phi\rangle \otimes |\Xi^{\alpha\beta}\rangle$ where $|\Xi^{\alpha\beta}\rangle$ is a fermionic state from the massless sector of the theory. These states carry flavor index, as is expected of the massless sector which contains all information about the matter representation beyond its KM level. Dynamically, these states simply act as spectators and are otherwise decoupled.

Taking the massless sector into account, the structure of the full Hilbert space is expected to be of the form

$$\mathcal{H} = \sum_{s,s'} \oplus (\mathcal{H}_s^c \otimes \mathcal{H}_{s,s'}^{GI} \otimes \mathcal{H}_{s'}^c), \quad (5)$$

where s and s' labels the representation of the KM algebra on the left and right movers, respectively. \mathcal{H}_s^c encodes the massless spectrum of the theory which is model dependent.

For the adjoint fermion there are no massless states (this is a consequence of the fact that the central charge vanishes for the infrared conformal field theory). Nevertheless, \mathcal{H}_s^c is non-trivial: it is built of certain discrete topological states which, roughly speaking, label the distinct vacua for each of the current block sectors. Thus, the spectrum of the adjoint fermion model in the limit $m \rightarrow 0$ should exhibit

- (1) A set of ‘‘basic’’ bound states (particles) with masses m_1, m_2, \dots . They give rise to n -body thresholds at $m^2 = (\sum_{j=1}^n m_j)^2$. The lowest such threshold is the most obvious because this is where the spectrum becomes continuous. It is expected to occur at $m^2 = 4m_1^2$.
- (2) There should be degeneracy arising from the tensor product structure (5) of the physical states, which includes the topological sector of the theory. One manifestation of it will be the match between the continuous parts of the spectra of bosons and fermions.

In the following section, we will present numerical evidence based on DLCQ for both of these features.

III. NUMERICAL RESULTS FROM DISCRETE LIGHT CONE QUANTIZATION

A. Review of adjoint fermion model in DLCQ

The application of DLCQ techniques to the gauged adjoint fermion model has been developed in [7,9–11]. We will briefly review the construction below.

We start with the action (1), where Ψ is an adjoint Majorana fermion whose spinor components are given by $\begin{pmatrix} \psi \\ \chi \end{pmatrix}$.

In lightcone coordinates and in the lightcone gauge, (1) becomes

$$S = \text{Tr} \int dx^+ \int dx^- \left(i\psi \partial_+ \psi + i\chi \partial_- \chi - i\sqrt{2}m\chi\psi + \frac{1}{2g^2} (\partial_- A_+)^2 + A_+ J^+ \right).$$

Here, $J_{ij}^+ = 2\psi_{ik}\psi_{kj}$. We see that χ and A_+ are non-dynamical and simply lead to constraints.

In DLCQ, one compactifies the x^- direction into a circle of period L and assign periodic boundary condition to the gauge fields. The original two dimensional model is recovered in the decompactification limit of this theory. Having assigned periodic boundary condition for the gauge fields, the equation of motion allows two possible boundary conditions for the fermions: periodic or anti-periodic. If periodic boundary condition are used, the mode expansion of ψ includes a zero momentum component. It is customary to ignore this mode when computing the DLCQ spectrum. This is justified for generic m because the constraint due to the zero-momentum component of χ will set the zero momentum component of ψ to zero. At $m=0$, however, this constraint disappears, and one is no longer justified in throwing away the zero momentum component of ψ . Unfortunately, the simplest DLCQ cannot be applied in the presence of such a zero-mode, as will be made clear shortly. One could simply discard the zero-mode (at least for $m>0$ this should not affect the spectrum in the $K \rightarrow \infty$ limit). The price we pay is that the rate of convergence to the decompactification limit becomes much slower. This problem does not arise if one chooses to quantize the fermions using anti-periodic boundary conditions, since the zero momentum modes are absent from the beginning. We have found empirically that a DLCQ computation using anti-periodic boundary conditions for the fermions indeed converges much faster compared to the periodic boundary conditions. Therefore, we will adopt the anti-periodic boundary conditions as the method of choice, as did [9]. It should be stressed, however, that the decompactification in DLCQ is impossible to achieve in practice. Thus, a careful extrapolation is needed to extract the features of the spectrum alluded to at the end of the previous section.

In light cone quantization, fermions are made to satisfy the canonical anti-commutation relations imposed at equal light cone time x^+ :

$$\{\psi_{ij}(x^-), \psi_{kl}(y^-)\} = \frac{1}{2} \delta(x^- - y^-) \left(\delta_{il} \delta_{jk} - \frac{1}{N} \delta_{ij} \delta_{kl} \right).$$

In terms of the modes

$$\psi_{ij}(x) = \frac{1}{\sqrt{2L}} \sum_{n \in \text{odd}} B_{ij}(n) e^{-\pi i n x / L},$$

the anticommutation relations become

$$\{B_{ij}(m), B_{kl}(n)\} = \delta(m+n) \left(\delta_{il} \delta_{jk} - \frac{1}{N} \delta_{ij} \delta_{kl} \right),$$

TABLE I. Number of states as a function of K in adjoint fermion model using anti-periodic boundary conditions.

K	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Dim	18	28	40	58	93	141	210	318	492	762	1169	1791	2786	4338	6712

where $B_{ij}(-n)$ for $n > 0$ refers to $B_{ji}^\dagger(n)$ in the notation of [10].

Taking the appropriate constraints into account, the light-cone momentum and energy become

$$P^+ = \sum_{n \in \text{odd}} \left(\frac{\pi n}{L} \right) B_{ij}(-n) B_{ji}(n),$$

$$P^- = \frac{m^2}{2} \sum_{n \in \text{odd}} \left(\frac{L}{\pi n} \right) B_{ij}(-n) B_{ji}(n)$$

$$+ \sum_{n \in \text{even}} \frac{g^2 L}{(\pi n)^2} J_{ij}(-n) J_{ji}(n).$$

Restricting to the sector where $P^+ = \pi K/L$, we find

$$M^2 = 2P^+ P^- = \frac{g^2 N}{\pi} K \left(\frac{m^2 \pi}{Ng^2} \sum_{n \in \text{odd}} \frac{1}{n} B_{ij}(-n) B_{ji}(n) \right.$$

$$\left. + \frac{1}{N} \sum_{n \in \text{even}} \frac{1}{n^2} J_{ij}(-n) J_{ji}(n) \right).$$

We are interested in the $m \rightarrow 0$ limit of the spectrum. For any $m > 0$ the use of DLCQ is completely justified, and we expect the spectrum to converge as $K \rightarrow \infty$. Thus, we will imagine taking m very small in the formula above. If m is small enough (say 10^{-10}), then our numerical calculations will not feel it at all. Hence, we will simply set $m = 0$ in the numerics, and think of the extrapolation $K \rightarrow \infty$ as a representation of the $m \rightarrow 0$ limit of the spectrum.

The Hilbert space on which the M^2 operator acts can be constructed by acting on the vacuum with the mode operators $B(-n)$,

$$\text{Tr}[B(-n_1)B(-n_2)\dots B(-n_l)]|0\rangle,$$

subject to the condition that $\sum n_i = K$. These states are generated by a set of ordered partitions of K into odd integers, up to graded cyclic permutations. There are only finitely many such states. The M^2 matrix can be evaluated explicitly by commuting the oscillators. This is what makes DLCQ a powerful tool: the Hamiltonian is a finite dimensional matrix which can be diagonalized numerically (this feature breaks down in the presence of zero-momentum modes).

The decompactification limit of this theory is obtained by sending L to infinity, keeping P^+ constant. It is then necessary to scale K with L . This is exactly the sense in which the decompactification limit is a challenging limit in DLCQ. In general, the number of partitions of a positive integer into other positive integers grows exponentially. Solving models with adjoint matter in DLCQ therefore requires working with exponential algorithms.

In practice, the set of states and the elements of the Hamiltonian matrix can be generated with the aid of a com-

puter program. The number of states in each sector labeled by K is summarized in Table I.

The Hamiltonian preserves the number of oscillators modulo 2. The states with odd oscillator number correspond to fermions and arise for odd K . For even K we necessarily have an even number of oscillators; therefore, these states describe the bosonic part of the spectrum. The Hamiltonian also preserves a Z_2 symmetry corresponding to reversing the order in which the modes act on the vacuum:

$$\text{Tr}[B(-n_1)B(-n_2)\dots B(-n_l)]|0\rangle$$

$$\leftrightarrow \text{Tr}[B(-n_l)B(-n_{l-1})\dots B(-1)]|0\rangle,$$

so that the Hilbert space decomposes into sectors odd and even under the action of this Z_2 group.

B. Numerical results and extrapolations

We will now present the results of our numerical analysis. We evaluated the M^2 matrix explicitly and computed the first several eigenvalues in each of the Z_2 sectors for both bosons and fermions. We are interested in tracking the mass squared of a given state as we vary K . For this purpose we found it useful to plot the probability that a given state has n bits, which is encoded in its wave function, and to track these probabilities as we increase K . As an example, we illustrate in Fig. 1 the probabilities of various bit numbers for each of the low-lying eigenstates. One of the features visible in Fig. 1 is the existence of states which are sharply peaked in bit number distributions (e.g. states 1 and 5 in Fig. 1). These states can be readily distinguished from the rest of the states which are superpositions of various bit number sectors. In Fig. 1 we indicate by arrows the patterns with which states are tracked as we vary K . These choices are based on continuity in the shape of the distribution and of the eigenvalues. Although there is some element of guesswork in making such assignments, the steady pattern we observe in the shape of the distribution provides us with confidence that we are tracking the states correctly. As K increases, we also observe evidence for new tracks of states appearing in the spectrum. This is to be expected since the dimension of the Hilbert space increases rapidly with K . We indicate the likely ‘‘trail heads’’ of these tracks with ‘‘*’’ in Fig. 1.

We can now follow the arrows in Fig. 1 and plot M^2 as a function of K . We summarize this data in Fig. 2 where we plot M^2 against $1/K$ for each of the Z_2 sectors for both bosons and fermions.

Several comments are in order regarding the data contained in Fig. 2. First of all, in all but the bosonic Z_2 odd sector, the lowest eigenvalue appears to be well separated from the rest of the spectrum, and depends relatively smoothly on K . We studied convergence to the large K limit by performing a best fit to a curve of the form

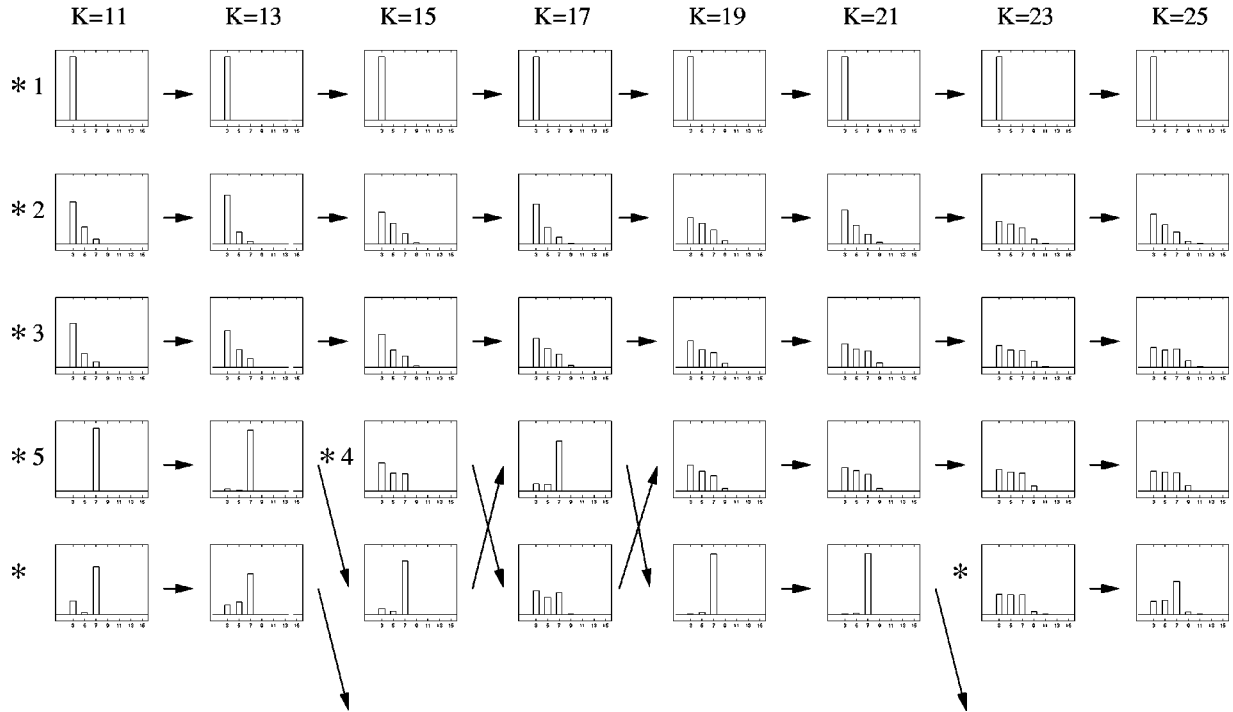


FIG. 1. Probability distributions in bit number space of the low-lying mass eigenstates of the light-cone Hamiltonian in the fermionic Z_2 sector. The arrows indicate the likely tracking pattern for these states as we vary K . The symbol “*” indicates a “trail head” where a new state appears in the spectrum.

$$a_0 + a_1 \left(\frac{1}{K} \right) + a_2 \left(\frac{1}{K} \right)^2. \quad (6)$$

We included the quadratic term in the fit to account for the curvature in the data. Thus, a_0 gives the extrapolated value of the mass squared. Our extrapolation indicates that the lightest state is a fermion with $M_{F1}^2 = 5.7$, followed by a boson with $M_{B1}^2 = 10.8$, followed by a fermion with $M_{F2}^2 = 17.3$ (in units of $g^2 N / \pi$). These states are approximate eigenstates of the number of bits with eigenvalues 3, 2 and 5 respectively. These results are in good agreement with the extrapolations performed in [9,10]. The best fit curves also indicate that the lightest states have converged fairly well, although for a linear extrapolation to be justifiable, K must be of order 10^2 .

The states which are not approximately pure in bit number (e.g. states 2, 3, and 4 in Fig. 1) behave somewhat differently. These states oscillate strongly in K with a period of 4. We will have more to say about these oscillations in the following subsection but, for the time being, simply note that they get smaller with increasing K . Figure 2 gives an indication that these levels will ultimately converge at large K . It is not sensible to fit a curve of the form (6) to such a wildly oscillating data. As an alternative, we adopt the procedure where we fit (6) to the valleys and the peaks of the data separately. Unfortunately, this cuts in half the amount of data used in each extrapolation. Since (6) is a 3 parameter fit, we performed the extrapolation only in cases where at least 4 data points are available.

As we illustrate in Fig. 2, these states show an indication that they are converging toward roughly the same mass in the large K limit. What makes this particularly interesting is the fact that the extrapolated value based on the fit (6) is

approximately $M_G^2 = 22.9$. The states becoming degenerate at a particular value of M_G^2 is suggestive of the onset of continuous spectrum. Notice that $M_G^2 = 22.9$ equals $4M_{F1}^2$, which is where one expects the first band of continuum corresponding to the free two body spectrum of particles of mass M_{F1} .

One could in principle perform a 3 parameter fit to 3 data points. These fits also give extrapolations close to 22.9. The extent to which an extrapolation misses this mark increases as we go up in energy levels. However, since higher levels start at trail heads with higher values of K , it is necessary to go to higher values of K to achieve the same degree of convergence. We expect these higher levels to converge to a mass-squared of 22.9 when the calculation is pushed to sufficiently high K to allow for a reliable extrapolation. The data illustrated in Fig. 2 is quite suggestive of such degeneracies at large K .

A picture that appears to be emerging from these observations is the following. M_{F1} , M_{B1} , and M_{F2} are the masses of the lightest particles to which the single trace states (3) dissociate in the deconfinement limit. The states piling up at M_G suggest a continuous two-body spectrum of $|F1\rangle$.

There is evidence for other “single-particle” states buried in the continuum. A clear example is the bosonic Z_2 odd state (state 2 in Fig. 2) which, to a good approximation, consists of 4 bits and has the extrapolated mass-squared equal to 25.6. Furthermore, we find evidence for a very pure 7-bit state (state 5 in Fig. 2) of extrapolated mass-squared 35.3 in the fermionic Z_2 even sector. The masses of these “pure” states appear to vary smoothly with K similarly to those of the “single-particle” states $|F1\rangle$, $|B1\rangle$, and $|F2\rangle$. Thus, we speculate that there is an infinite sequence of “single-particle” states. At least the first few such states are

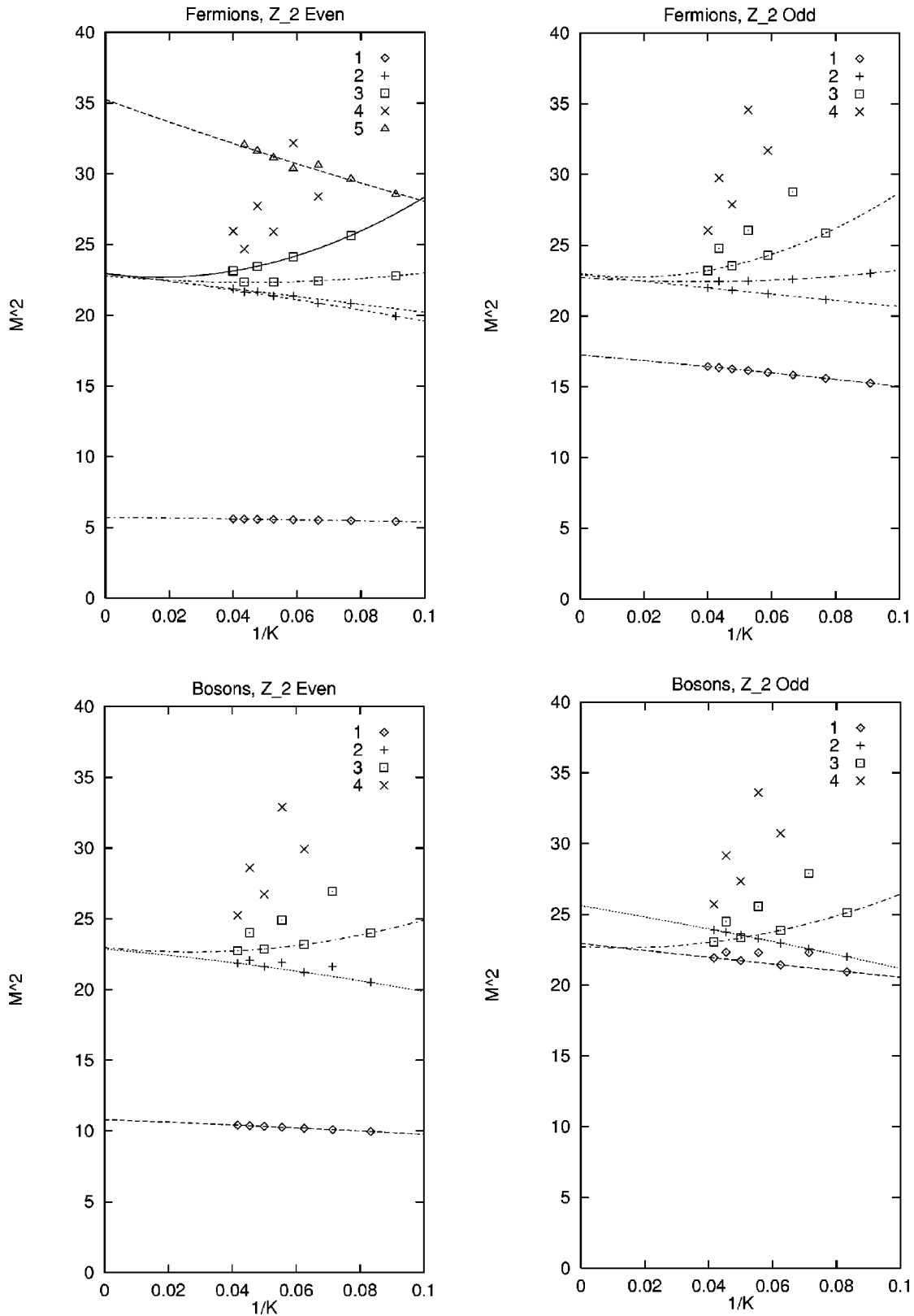


FIG. 2. M^2 eigenvalues for $m=0$ as a function of $1/K$. In each of the 4 sectors (Z_2 even and Z_2 odd for bosons and fermions) we exhibit extrapolations of the lightest states that are pure in bit number, and also graph some of the lightest states that appear to converge to the continuum.

distinguished by their purity in the number of bits (the ‘‘multi-particle’’ states tend to be far from being eigenstates of the number of bits).

It follows that there should also be other two-body thresh-

olds at mass-squared $(M_{F1} + M_{B1})^2 = 32.2$, $(M_{F1} + M_{F2})^2 = 42.8$, $(M_{B1} + M_{F2})^2 = 55.4$, etc. Perhaps these could be detected by a sufficiently detailed examination of the spectrum.

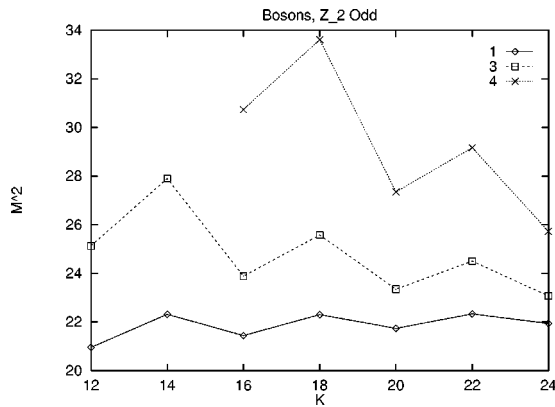


FIG. 3. The first three states in the bosonic Z_2 odd sector of the adjoint fermion model that appear to be converging to the continuum.

One final point we wish to emphasize is the fact that the continuum at $M_G^2 = 22.9$ appears to exist both in the bosonic and the fermionic sectors. The interpretation of these states as the two-body continuum coming from the $|F1\rangle$ particle suggests that these states should be bosonic, $|F1\rangle \otimes |F1\rangle$. How then should we interpret the continuum at $M_G = 22.9$ in the fermionic sector? Recalling a similar issue of statistics in the case of adjoint vs N -flavor fundamental correspondence suggests the following explanation. The states in the fermionic continuum must correspond to a state of the form

$$|F1\rangle \otimes |F1\rangle \otimes |\Xi\rangle$$

where $|\Xi\rangle$ is a companion fermionic state arising from the topological sector of the theory, which is otherwise decoupled from the dynamics.

C. Oscillations

So far, our evidence for the appearance of a continuum of states at $M^2 = 4M_{F1}^2$ has been based on numerical extrapolations. In this subsection, we will present stronger evidence by studying the pattern of oscillations exhibited by these states as they converge toward the large K limit.

Oscillations similar to the ones we illustrated in Fig. 2 arise in the DLCQ spectrum of a pair of free particles of mass m . For a finite K , the spectrum is given by

$$M^2 = m^2 K \left(\frac{1}{n} + \frac{1}{K-n} \right)$$

where n and $K-n$ are the numbers of units of momentum carried by the individual particles, and $1 \leq n \leq K-1$. This spectrum oscillates in K . We will show that the oscillations seen in Fig. 2 are due to a similar mechanism.

Let us focus our attention on the oscillating states in the bosonic Z_2 odd sector from Fig. 2. For the sake of illustration, we plot these states again in Fig. 3. Our claim is that these states arise as the free two-body spectrum of $|F1\rangle$ particles. Since each of these particles is a composite state, whose mass is determined with a finite resolution, the correct formula for a finite K is

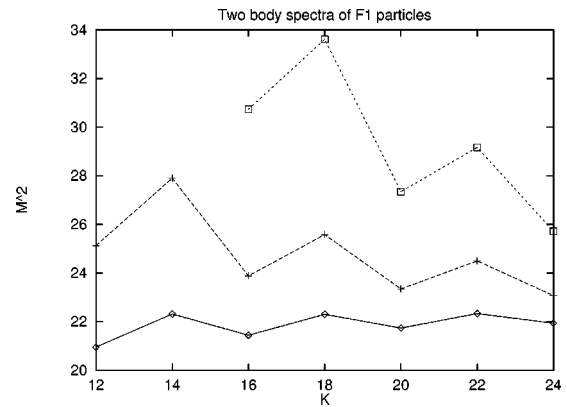


FIG. 4. The spectrum of a pair of non-interacting $F1$ particles with total momentum K .

$$M^2 = K \left(\frac{M_{F1}^2(n)}{n} + \frac{M_{F1}^2(K-n)}{K-n} \right), \tag{7}$$

where n is an odd integer $1 \leq n \leq K-1$, and $M_{F1}(n)$ is the mass of the $|F1\rangle$ in the $K=n$ sector. We illustrate the spectrum determined using (7) in Fig. 4. (Since $|F1\rangle$ is a fermion, we only keep the states which are antisymmetric with respect to their exchange.)

Figures 3 and 4 are generated using completely independent methods. It is therefore quite remarkable that the resulting plots are *identical*. This can be easily verified by laying one on top of the other. Thus, the identification of bosonic Z_2 odd states as non-interacting two-body states of $|F1\rangle$ appears to be exact even for finite K .

Having found such a remarkable structure in the Z_2 odd sector of the bosonic spectrum, it is natural to expect a similar situation to hold in the Z_2 even sector. However, the correspondence here is not exact. In Fig. 5, we illustrate both the bosonic Z_2 spectrum from previous section and the expectation based on (7). What we seem to be finding here is that (7) captures the qualitative features of the oscillations of the Z_2 even sector states, but the DLCQ data contains additional “noise.” We will speculate on the source of this noise at the end of this section. Empirically, we find that the noise decays as $1/K$ in the large K limit. Therefore, despite the fact

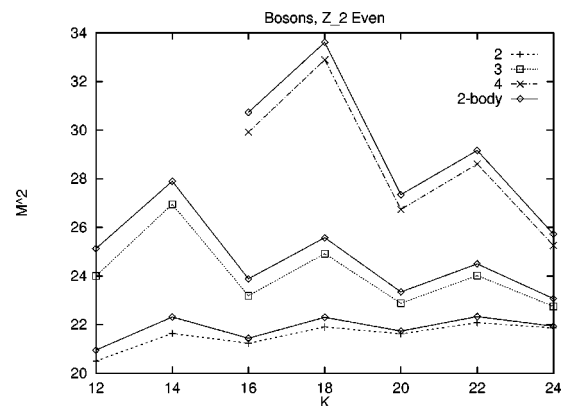


FIG. 5. Dashed lines label the first three states in the bosonic Z_2 even sector of the adjoint fermion model that appear to be converging to the continuum. The solid line is the spectrum of a non-interacting pair of $F1$ particles.

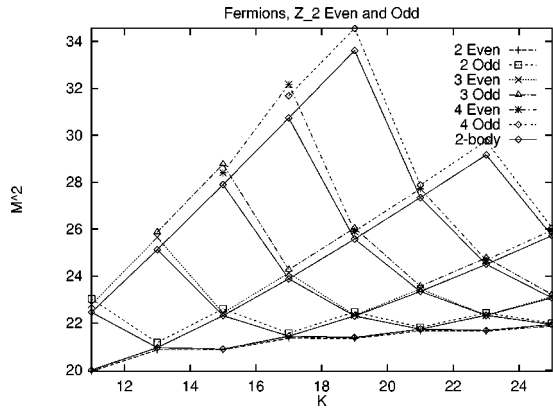


FIG. 6. Dashed lines label the first three states for both the fermionic Z_2 even and the fermionic Z_2 odd sectors of the adjoint fermion model that appear to be converging to the continuum. The solid line is the spectrum of a non-interacting pair of $F1$ particles.

that the correspondence is not exact at finite K , we believe that the pattern of oscillation seen in the bosonic Z_2 even sector is also consistent with the picture that a continuum is formed in the large K limit.

There are additional subtleties in attempting to extend this picture to the states in the fermionic sector. As was described at the end of the previous subsection, the only way one can make a fermionic state out of a pair of $|F1\rangle$ particles is to introduce states from the topological sector of the theory carrying fermionic statistics. In DLCQ with anti-periodic boundary conditions, such a state cannot carry zero light-cone momentum, as these states are required to carry momentum in half-integer units. The closest we can get to the situation we hope to describe is to distribute K units of momentum according to $(|F\rangle, |F\rangle, |\Xi\rangle) = (n, K-n-1, 1)$. It should be stressed, however, that $|\Xi\rangle$ is not really a free particle, and it is not *a priori* clear how to generalize (7) taking the ‘‘topological sector’’ into account. A rough guess is to take

$$M^2 = (K-1) \left(\frac{M_{F1}^2(n)}{n} + \frac{M_{F1}^2(K-n-1)}{K-n-1} \right). \quad (8)$$

The data from the fermionic sector of the adjoint fermion model suggests the presence of two-particle states which are both symmetric and antisymmetric under their exchange. This is puzzling in light of the fact that $|F1\rangle$'s constitute a pair of identical fermions. Perhaps $|\Xi\rangle$ is binding with one of the $|F1\rangle$'s so that the resulting pair of constituents are no longer identical. Here we have included the symmetric wavefunctions for the sake of comparison with the DLCQ data.

Although (8) is admittedly *ad hoc*, it appears to capture the general structure of the oscillations seen in the spectrum computed for the adjoint fermion model, as we illustrate in Fig. 6. Again, we find qualitative agreement accompanied by some ‘‘noise’’ which decays as $1/K$. This time, however, there is a natural suspect for the culprit responsible for the noise. The topological sector plays an important role in assigning appropriate statistics for the states in this sector. By using anti-periodic boundary conditions, however, we were forced to mutilate the structure of the topological sector by forcing it to carry small but finite momenta. This effect is not

properly accounted for in (8). The amount of error in momentum we introduced is of order $1/K$, and this is indeed the magnitude of the noise we see in the spectrum.

This also suggests the probable cause of the noise in the bosonic Z_2 even sector. The structure of these states might very well be of the form

$$|F1\rangle \otimes |F1\rangle \otimes |\Omega\rangle$$

where $|\Omega\rangle$ is a topological state carrying even fermion numbers. Even in theories with only fermionic matter, such a state can arise easily from bilinears. The noise in this sector may be due to the fact that (7) does not account for the presence of the topological sector of this type. In the large K limit, however, we would expect all dynamics in the topological sector to decouple. It is therefore satisfying to find that the noise decays as $1/K$ in the large K limit.

Although in general the correspondence between the spectra derived from (7) and (8) with the DLCQ spectrum is not as precise as what we found in the bosonic Z_2 odd sector, the qualitative agreement, and the fact that the discrepancy shrinks with increasing K , provides a strong indication that the oscillations seen in the DLCQ spectrum are a signature of states forming a continuum. Furthermore, we feel that the small discrepancy is actually probing the topological sector of the theory. It would be extremely interesting to understand this structure from first principles.

IV. CONCLUSIONS

By performing explicit DLCQ analysis of QCD coupled to adjoint fermions, we found isolated ‘‘single-particle’’ states $|F1\rangle$, $|B1\rangle$ and $|F2\rangle$ at mass-squared equal to 5.7, 10.8, and 17.3 respectively (in units of $g^2 N/\pi$). These states are approximate eigenstates of the number of bits with eigenvalues 3, 2 and 5 respectively. In addition, we found an indication that a continuum of states is appearing at $M_G^2 = 22.9$ in both the bosonic and the fermionic sector. The fact that $M_G^2 = 4M_{F1}^2$ suggests that these states are a two-particle continuum built out of the $|F1\rangle$'s. To account for the statistics, the states are interpreted to be of the form $|F1\rangle \otimes |F1\rangle$ or $|F1\rangle \otimes |F1\rangle \otimes |\Xi\rangle$, where $|\Xi\rangle$ describes a fermionic state in the topological sector of the theory, which is otherwise decoupled from the dynamics. The existence of fermionic states converging to the continuum at $M_G^2 = 22.9$ thus provides some numerical evidence for the ‘‘direct sum of tensor products’’ structure of the Hilbert space,

$$\mathcal{H} = \sum_{s,s'} \oplus (\mathcal{H}_s^c \otimes \mathcal{H}_{s,s'}^{GI} \otimes \mathcal{H}_{s'}^c)$$

suggested in [12]. The oscillatory behavior of M^2 for the states converging to the continuum can be understood exactly (at least for the bosonic Z_2 odd states) in terms of the spectrum of a non-interacting 2-body system (7). This provides strong support for our claim about the continuity of the spectrum for $M^2 > 4M_{F1}^2$.

In addition, we find evidence for other ‘‘single-particle’’ states whose mass is higher than M_G . At least the first few such states are distinguished from the continuum states by their purity in bit number. For example, in the bosonic Z_2

odd sector there is a state of mass-squared 25.6 which is, to a high accuracy, a 4-bit state. We are thus led to speculate that there is an infinite sequence of “single-particle” states. Perhaps these states can be grouped into one Regge trajectory of the fermions and one Regge trajectory of the bosons.

Our analysis indicates clearly that the adjoint fermion model contains string-like states made out of adjoint bits which dissociate in the $m \rightarrow 0$ limit into the stable constituent “particles.” For small m these states can be thought of as loosely bound state of such “particles.” In the $m \rightarrow 0$ limit, these “particles” are free, as can be inferred from the threshold of the continuum.

While we have seen that the DLCQ gives convincing evidence for the existence of constituent “particles” and their 2-body continua, some puzzles about the structure of the spectrum remain. A paradox having to do with the state counting of this model was noted in [15]. Since the tension of the QCD string vanishes in the $m \rightarrow 0$ limit, one expects to find a spectrum with Hagedorn temperature $T_H \rightarrow 0$. On the other hand, one expects the spectrum of a screening theory to have $T_H = \infty$. Since for $m = 0$ the spectrum decomposes into the more basic building blocks (single particles), we should

only count these particles as fundamental. It is natural to expect that these particles form a single Regge trajectory, hence they do not have an exponentially growing density of states. The problem is that the particles from a single Regge trajectory, and the multiparticle states built out them, do not have enough degeneracy to form an exponentially rising density of states when m is turned on [15]. Thus, it seems necessary to take into account additional large degeneracy due to the presence of certain topological states [12,15]. It would be interesting to see how the resolution to this apparent paradox manifests itself in the DLCQ numerical analysis.

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