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Two boosted black holes in asymptotically de Sitter space-time: Relation between mass and apparent horizon formation

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We study the apparent horizon for two boosted black holes in asymptotically de Sitter space-time by solving the initial data on a space with punctures. We show that the apparent horizon enclosing both black holes is not formed if the conserved mass of the system (Abbott-Deser mass) is larger than a critical mass. The black hole with too large an AD mass therefore cannot be formed in asymptotically de Sitter space-time even though each black hole has any inward momentum. We also discuss the dynamical meaning of the AD mass by examining the electric part of the Weyl tensor (the tidal force) for various initial data. [S0556-2821(98)04810-3]

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I. INTRODUCTION

The inflationary universe scenario is one of the most favorable models to explain the present isotropy and homogeneity of the Universe [1]. The basic idea of this scenario is that the potential energy of a scalar field, which behaves as an effective cosmological constant, dominates and causes the de Sitter–like rapid cosmic expansion in the early stage of the Universe. Then it seems likely that due to this rapid cosmic expansion the initial anisotropy and inhomogeneities might be stretched out and the universe becomes homogeneous and isotropic. Such a picture is based on the so-called cosmic no hair conjecture (CNHC) which states that ''all'' space-times with a cosmological constant Λ approach de Sitter space-time asymptotically [2]. Of course, the CNHC is not always true without any additional conditions since we know that some inhomogeneities can gravitationally collapse into black holes in space-time with Λ. Hence the dynamics of the inhomogeneities is an important issue to acquire physical insight into the present homogeneity and isotropy of the Universe and there is much research on this problem [3].

Recently, we have studied numerically apparent horizons in the initial data in asymptotically de Sitter space-time [4–6]. The results suggest that there is an upper limit on the gravitational mass, the Abbott-Deser (AD) mass [7] of a black hole. The AD mass is the corresponding notion to the Arnowitt-Deser-Misner (ADM) mass in asymptotically flat space-time. Hence large inhomogeneities may not collapse into a black hole and, furthermore, large black holes may not collide in asymptotically de Sitter space-time. The same result was obtained by the analysis of the Oppenheimer-Snyder model with Λ [8]. Further, dynamical simulations for the Brill waves in asymptotically de Sitter space-time were performed and revealed the same results as the spherically symmetric dust collapse [9]. It was also shown that under some conditions there is an upper limit on the area of the event horizon in asymptotically de Sitter space-time [10]. Since, as in the case of asymptotically flat space-time, the area of the black hole does not decrease in asymptotically de Sitter space-time, the black holes with too large a total area of the event horizons may not collide and not merge if the cosmic censorship hypothesis holds. We note, however, that the relation between the AD mass and event horizon formation is not yet clear in asymptotically de Sitter space-time.

Although the above analysis shows some clear relation between the AD mass and apparent horizon formation, there still remain some unanalyzed effects, one of which is the initial motion of a black hole in two-black-hole system. If two black holes have inward velocity, we may expect that those black holes might collide each other and will form a single black hole. Then, in this paper, we shall make a further study of the AD mass in a two-black-hole system. Previous work on the two-Einstein-Rosen bridge system was focused only on the cases with no relative velocity other than the uniform background cosmic expansion [5]. Hence, in order to see the effect of the relative velocity, we shall investigate the axisymmetric initial data of two nonspinning black holes with finite velocity in addition to the background cosmic expansion. We solve the initial data on a space with punctures following [11]. Then we search for the apparent horizon enclosing both black holes in order to get some insight into the dynamics of the inhomogeneities in the inflationary universe. We find that such an apparent horizon does not appear no matter how close together each black hole is and how fast it moves when the AD mass of this system is larger than a critical value which agrees with that of Schwarzschild–de Sitter space-time [12].

It is worth noting that as pointed out in Ref. [13], the AD mass can vanish and furthermore can be negative even if inhomogeneities satisfy energy conditions. In this article, we shall explicitly show this fact in the case of the initial data containing two black holes. So a question arises: ''Is a gravitational field produced when the AD mass vanishes?'' Then, in order to see the dynamical meaning of the AD mass, we investigate the asymptotic behavior of the electric part of the Weyl tensor which corresponds to the tidal force. In contrast with asymptotically flat space-time, the tidal force comes from the anisotropic velocity as well as the AD mass. In particular, the contribution from the anisotropic velocity becomes the leading term in the asymptotic region when the conserved momentum does not vanish. When the AD mass vanishes, the tidal force is produced solely by the anisotropic momentum. However, even if the dynamical effects of the anisotropic velocity on the large scale inhomogeneities might be important, the apparent horizon formation is essentially determined by the AD mass and furthermore the critical mass coincides with that of Schwarzschild–de Sitter space-time. This fact suggests that the final state of nonrotating gravitational collapse in asymptotically de Sitter space-time might be Schwarzschild–de Sitter space-time if naked singularities are not formed. Hence, it seems that the inhomogeneities with too large an AD mass cannot collapse into a single black hole.

This paper is organized as follows. In Sec. II, we present initial data of two masses with linear momenta in asymptotically de Sitter space-time and search for the apparent horizon, numerically. In Sec. III, we investigate the electric part of the Weyl tensor of the conformally flat initial data and discuss the effect of the AD mass and the linear momentum on the electric part of the Weyl tensor. Then Sec. IV is devoted to a discussion. We adopt units of $c=G=1$. Our notations follow those of Misner, Thorne, and Wheeler (MTW) [14].

II. INITIAL DATA FOR TWO BOOSTED MASSES IN ASYMPTOTICALLY de SITTER SPACE-TIME

A. Setting up initial data

In order to obtain initial data, we have to solve the Hamiltonian and momentum constraints:

$$^{(3)}R-\hat{K}_i^j\hat{K}_j^i+\frac{2}{3}K^2=6H^2+16\pi\rho_H, \tag{2.1}$$

$$D_j\left(\hat{K}_i^j-\frac{2}{3}\delta_i^jK\right)=8\pi J_i, \tag{2.2}$$

with

$$H\equiv\sqrt{\frac{\Lambda}{3}}, \tag{2.3}$$

where $^{(3)}R$ is the Ricci scalar of three-space. $\hat{K}_i^j$ and K are the traceless part and the trace of the extrinsic curvature of three-space, respectively. D_j is the covariant derivative with respect to the intrinsic metric of three-space. ρ_H and J_i are, respectively, the energy density and the momentum density of the matter fluid measured by the normal line observer.

In order to solve Eq. (2.1), we follow the York's conformal prescription [15] and assume the conformally flat metric:

$$dl^2=\psi^4(R,z)f_{ij}dx^idx^j, \tag{2.4}$$

where f_{ij} is the flat Euclidean metric. We set the trace part of the extrinsic curvature K to

$$K=-3H. \tag{2.5}$$

Note that the condition (2.5) turns out to be the Friedmann equation $(\dot{a}/a)^2=H^2$ for the scale factor a in the homogeneous and isotropic universe. Hence, the condition (2.5) is regarded as the assumption of a uniformly expanding background universe.

Then Eqs. (2.1) and (2.2) become

$$\tilde{\Delta}\psi=-\frac{1}{8}\tilde{K}_i^j\tilde{K}_j^i\psi^{-7}-2\pi\tilde{\rho}_H\psi^{-3}, \tag{2.6}$$

$$\tilde{D}_j\tilde{K}_i^j=8\pi\tilde{J}_i, \tag{2.7}$$

where $\tilde{K}_{ij}=\hat{K}_{ij}\psi^2$, $\tilde{\rho}_H=\rho_H\psi^8$, $\tilde{J}_i=J_i\psi^6$, $\tilde{D}_i$ is the covariant derivative with respect to the flat metric f_{ij}, and $\tilde{\Delta}$ is the Laplacian operator of flat space. The traceless part of the extrinsic curvature $\tilde{K}_{ij}$ is decomposed into the transverse traceless part and the longitudinal traceless part

$$\tilde{K}_{ij}=\tilde{K}_{ij}^{TT}+(\tilde{L}\tilde{W})_{ij}, \tag{2.8}$$

with

$$(\tilde{L}\tilde{W})_{ij}\equiv\tilde{D}_i\tilde{W}_j+\tilde{D}_j\tilde{W}_i-\frac{2}{3}f_{ij}\tilde{D}_l\tilde{W}^l. \tag{2.9}$$

As for the transverse traceless part of the extrinsic curvature, we assume $\tilde{K}_{ij}^{TT}=0$. Restricting to the vacuum case, i.e., $\widetilde{\rho_H}=\tilde{J}_i=0$, Eq. (2.7) becomes

$$\tilde{\Delta}\tilde{W}_i+\frac{1}{3}\tilde{D}_i\tilde{D}^j\tilde{W}_j=0. \tag{2.10}$$

The analytic monopole solution for $\tilde{W}^i$ is given by [16]

$$\tilde{W}^i=-\frac{1}{3r}(7P^i+n^iP^kn_k), \tag{2.11}$$

where $n^i=x^i/r$. P^i is the linear momentum defined by

$$P^i=\frac{1}{8\pi}\oint\tilde{K}^{ij}d^2\tilde{S}_j, \tag{2.12}$$

where $d^2\tilde{S}_i$ is the area element of the sphere in the conformal flat space. Just for simplicity, we consider only the initial data of two black holes with equal mass and equal magnitude of momentum. Since Eq. (2.10) is linear, we can easily find the extrinsic curvature for the two-black-hole system with linear momenta by superposition as

$$\tilde{K}_{ij}=K_{ij}^{(+)}+K_{ij}^{(-)}, \tag{2.13}$$

where $K_{ij}^{(\pm)}$ comes from each hole located at $(0,0,\pm a)$. Then $K^{(\pm)ij}$ is given as

$$K^{(\pm)ij}=\frac{3}{2r_{\pm}^2}[P^{(\pm)i}n^{(\pm)j}+P^{(\pm)j}n^{(\pm)i}-(f^{ij}-n^{(\pm)i}n^{(\pm)j})P^{(\pm)k}n_k^{(\pm)}], \qquad (2.14)$$

where $n^{(\pm)i}$ and $P^{(\pm)i}$ are defined by

$$n^{(\pm)i}=\frac{1}{r_{\pm}}(x,y,z\mp a), \qquad (2.15)$$

$$P^{(\pm)i}=(0,0,\pm P), \qquad (2.16)$$

with $r_{\pm}=\sqrt{x^2+y^2+(z\mp a)^2}$. Substituting $\widetilde{K}_{ij}$ obtained above into Eq. (2.6), we get the equation for the conformal factor ψ. We rewrite the conformal factor following [11],

$$\psi=\frac{M_0}{2r_+}+\frac{M_0}{2r_-}+u\equiv\frac{1}{\alpha}+u, \qquad (2.17)$$

where M_0 is a constant. Then the Hamiltonian constraint becomes

$$\widetilde{\Delta}u=-\frac{1}{8}\alpha^7\widetilde{K}_i^j\widetilde{K}_j^i(1+\alpha u)^{-7}. \qquad (2.18)$$

It should be noted that the coefficient in Eq. (2.18) is no longer singular as opposed to Eq. (2.6) because the monopole terms are subtracted by Eq. (2.17). The conformal factor ψ (or u) is obtained by solving iteratively Eq. (2.18) by a finite difference method typically with 200×200 grid numbers in cylindrical (R,z) coordinates. The boundary conditions are given by

$$\psi\to1+\frac{M}{2r} \quad \text{for} \quad r\equiv\sqrt{x^2+y^2+z^2}\to+\infty, \qquad (2.19)$$

$$\left.\frac{\partial u}{\partial R}\right|_{R=0}=\left.\frac{\partial u}{\partial z}\right|_{z=0}=0. \qquad (2.20)$$

Note that $2M_0$ agrees with M in the limit $P^i\to0$. By virtue of the time slicing condition (2.5), the boundary condition (2.19) guarantees that the space-time is an asymptotically de Sitter one. In practice, however, numerical solutions cover only a finite region. We therefore use the Robin boundary condition at the outer boundary:

$$\frac{\partial u}{\partial r}=\frac{1-u}{r}. \qquad (2.21)$$

B. Finding apparent horizons

Now we search for apparent horizons in the initial data prescribed above. A marginally trapped surface is a closed two-surface S where the expansion Θ of future-directed outgoing null vectors l^μ normal to it vanishes [17]. The apparent horizon is defined as the outer boundary of a connected component of the trapped region. The apparent horizon is believed to agree with the marginally outer trapped surface by the proof in Ref. [17]. However, the proof is correct only if the apparent horizon is smooth and hence, exactly speaking, the apparent horizon might not agree with the marginally outer trapped surface [18]. In the situation considered here, however, we find a smooth apparent horizon and hence we may regard the outer marginally trapped surface as the apparent horizon.

Let s^μ be the outward-pointing spacelike unit normal to S and n^μ be the unit normal to a time slice; then l^μ can be written as $l^\mu=n^\mu+s^\mu$ and thus

$$\Theta=D_is^i+K_{ij}s^is^j-K=0 \qquad (2.22)$$

on S. For conformally flat space (2.4), Eq. (2.22) is rewritten as

$$\Theta=-\frac{r}{\psi^2(r^2+r_\theta^2)^{3/2}}\left[r_{\theta\theta}+\left(\frac{r_\theta^3}{r^2}+r_\theta\right)\left(4\frac{\psi_\theta}{\psi}+\cot\theta\right)-r_\theta^2\left(\frac{3}{r}+4\frac{\psi_r}{\psi}\right)-2r-4r^2\frac{\psi_r}{\psi}\right]+K_{ij}s^is^j-K=0, \qquad (2.23)$$

where suffices r,θ denote those partial derivatives. To find an apparent horizon, we solve Eq. (2.23) as a two-point boundary value problem with boundary conditions $r_\theta=0$ at $\theta=0,\pi/2$ following Sasaki *et al.* [19]. This method was also used recently by Cook *et al.* [20].

Like asymptotically flat space-time, when an apparent horizon forms, there always exists an event horizon enclosing it in asymptotically de Sitter space-time if there is no naked singularity and the null convergence condition is satisfied [21]. Hence, by investigating the existence of the apparent horizon in various initial data, we obtain some insight into the evolution of inhomogeneities.

As in the case of asymptotically flat space-time, when the separation a between each black hole is short enough, an apparent horizon encompassing both black holes appears (see Fig. 1). The existence of such an apparent horizon means the merging of two black holes and we shall search for such an apparent horizon in an asymptotically de Sitter space.

In order to control the strength of the inhomogeneities, we vary a, $P_i^{\pm}$, and M_0. However, these parameters are not conserved quantities of this system. In asymptotically de Sitter space-time, there are ten conserved quantities associated with ten Killing vectors of background de Sitter space-time. In the initial data considered here, only the AD mass associated with the timelike Killing vector of background de Sitter space-time is the nontrivial conserved Killing quantity. (Conserved total linear and angular momenta vanish here.) Hence, we utilize the AD mass as a measure characterizing the strength of inhomogeneities in addition to a and P.

In general, in conformally flat initial data with the time slicing condition Eq. (2.5), the AD mass M_{AD} is expressed as [7,13]

$$M_{AD}=M+\Delta M, \qquad (2.24)$$

with

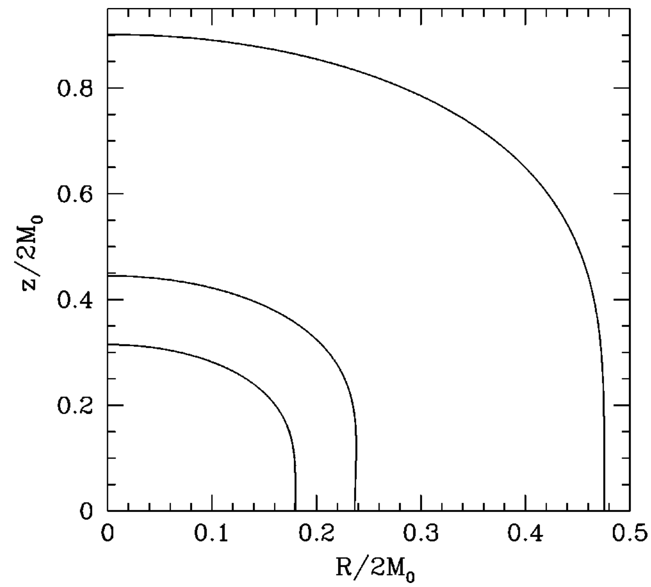

FIG. 1. The shapes of apparent horizons near critical separations for $P/M_0=-5$ and $H/2M_0=0,0.1,0.15$ from outside to inside.

$$M \equiv -\frac{1}{2\pi}\oint d^2\tilde{S}_i f^{ij}\partial_j\psi, \tag{2.25}$$

$$\Delta M \equiv \frac{H}{8\pi}\oint d^2\tilde{S}_i\tilde{K}^i_j\tilde{\xi}^j. \tag{2.26}$$

Here the surface integral is taken over an infinitely large sphere and $\tilde{\xi}^i=(x,y,z)$ is the conformal Killing vector orthogonal to the sphere. Although M agrees with the definition of the ADM mass and is non-negative, ΔM can be negative. For example, for the initial data that we consider, we obtain, from Eq. (2.14) using the Gauss's theorem in the region $\mathbf{R}^3-\{\text{two points}\}$,

$$\Delta M = \frac{H}{8\pi}\int_{\mathbf{R}^3-\{\text{two points}\}} d\tilde{v}\tilde{D}_i(\tilde{K}^i_j\tilde{\xi}^j)+2HaP=2HaP, \tag{2.27}$$

where $d\tilde{v}$ is the volume element in the conformal space and we have made use of Eq. (2.7) and the conformal Killing equation $(\tilde{L}\tilde{\xi})_{ij}=0$. Therefore, M_{AD} can be negative for $P<0$ (inward velocity) if

$$M<2Ha|P|. \tag{2.28}$$

We shall numerically investigate the critical separation a_c such that the apparent horizon surrounding black holes disappears for $a>a_c$. In Fig. 2, we display a_c/H^{-1} with respect to M_{AD} normalized by M_c which is defined by

$$M_c \equiv \frac{1}{\sqrt{27}H}. \tag{2.29}$$

In fact, M_c is the critical mass of Schwarzschild–de Sitter space-time. Schwarzschild–de Sitter space-time is characterized by M_{AD}, and when $M_{AD}<M_c$, it contains a black hole.

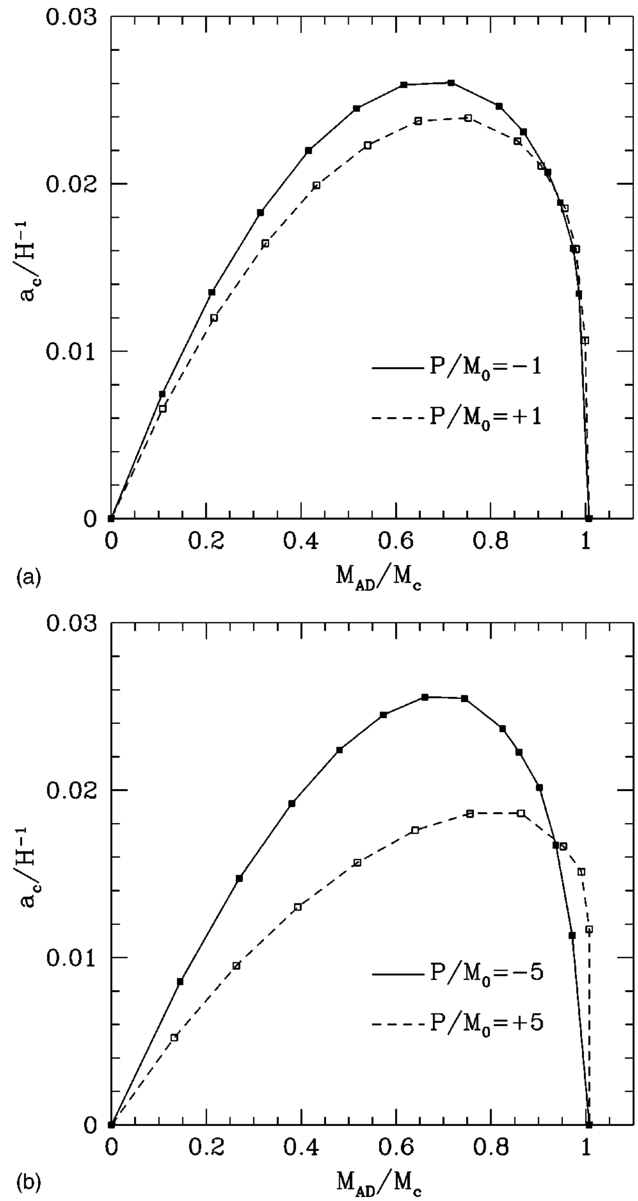

FIG. 2. The relation between the critical separation a_c/H^{-1} and AD mass M_{AD}. (a) is the case with $|P|/M_0=1$. The white square is for the case with $P>0$ and the black square corresponds to that of $P<0$. (b) is the same as (a) but with $|P|/M_0=5$. The critical value looks slightly shifted upward (by 0.7%) due to a numerical error.

However, in the case of $M_{AD}\geqslant M_c$, Schwarzschild–de Sitter space-time contains no black hole. As a test of our numerical code, we searched for an apparent horizon with $a=0$ and $P=0$, which corresponds to Schwarzschild–de Sitter space with $M_{AD}=2M_0$. For $M_{AD}/M_c>1.007$, an apparent horizon is not found. Therefore, our numerical code has an accuracy of less than 0.7%.

In Fig. 2(a), we set $|P|/M_0$ equal to 1. The white square shows that of $P>0$ while the black square corresponds to that of $P<0$. Figure 2(b) is the same but for $|P|/M_0=5$. In the asymptotically flat case, the critical separation a_c with fixed P/M_{AD} is proportional to M_{AD} and increases monotonically with increasing M_{AD} which coincides with the ADM mass, since there is no physical scale except for M_{AD}. This behavior means that larger M_{AD} produces larger gravity

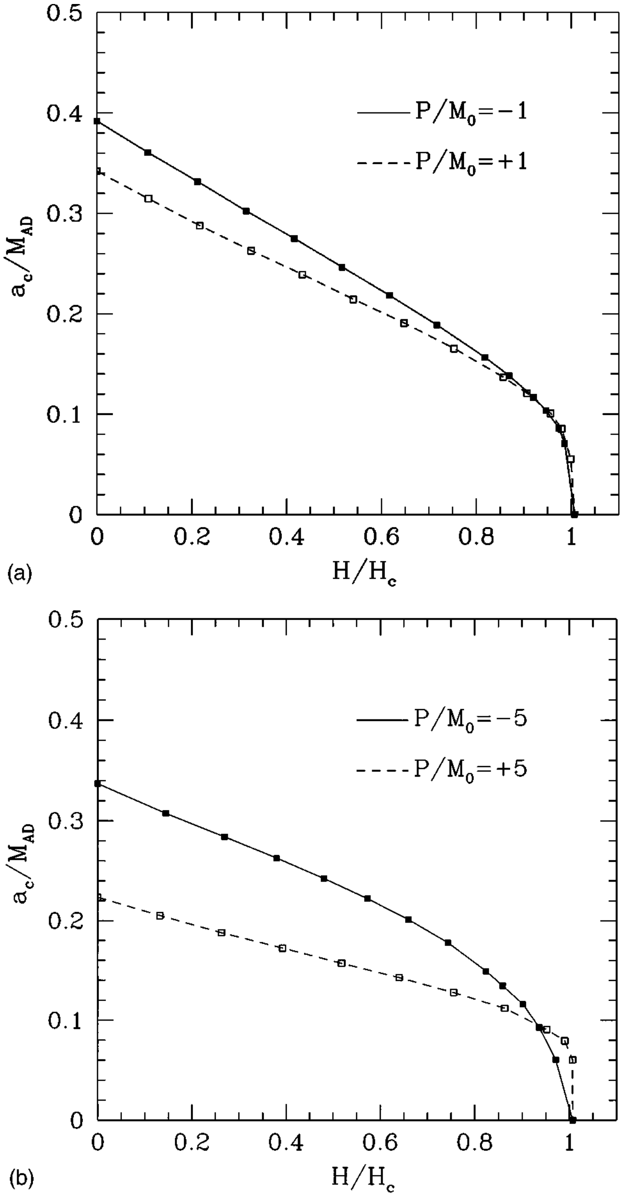

FIG. 3. The relation between the critical separation a_c/M_{AD} and cosmological constant $H/H_c = \sqrt{\Lambda/\Lambda_c}$.

and therefore an apparent horizon is formed even if the separation a is large. On the other hand, as shown in Fig. 2, in the case of asymptotically de Sitter space-time, the critical separation a_c is not proportional to M_{AD}. There is a maximum value near $M_{AD} \simeq 0.7 M_c$ and then the critical separation a_c decreases with increasing M_{AD}. Further, we find that when an apparent horizon forms the inequality (2.28) is *not* satisfied; that is, M_{AD} is strictly positive. Therefore, we may conclude as far as we have examined that if $M_{AD} < 0$ no apparent horizon encompassing both black holes appears.

In Fig. 3, a_c/M_{AD} as a function of H/H_c is shown, where $H_c \equiv 1/\sqrt{27}M$. We find that a_c/M_{AD} drops sharply near $H/H_c \simeq 1$. Further, there does not appear the apparent horizon enclosing both black holes for all a when $M_{AD} > M_c$ (or $H_{AD} > H_c$) within our numerical accuracy. This behavior essentially coincides with the two-Einstein-Rosen-bridge system investigated in Ref. [5]. When there is a positive cosmological constant, it is impossible for two black holes with large M_{AD} to coalesce and to form a single black hole.

III. AD MASS AND LINEAR MOMENTUM AS A SOURCE OF GRAVITY

In the previous section, we have adopted the AD mass as a measure of inhomogeneities and shown its relation to apparent horizon formation. However, the AD mass can vanish and could be negative even if the inhomogeneities due to ordinary matter fields exist. Then, we cannot conclude, only from the condition that the AD mass vanish, that such a space-time is de Sitter.

In order to show that M_{AD} can be negative, let us focus on conformally flat space. Then, we will first rewrite ΔM as

$$\Delta M = \frac{H}{8\pi}\int \left[\tilde{\xi}^i \tilde{D}_j \tilde{K}^j_i + \frac{1}{2}\tilde{K}^{ij}(\tilde{L}\tilde{\xi})_{ij}\right] d\tilde{v} = H\int \tilde{\xi}^i \tilde{J}_i d\tilde{v}, \tag{3.1}$$

where the first equality comes from the Gauss's theorem and in the second one use has been made of the momentum constraint (2.2) and the conformal Killing equation $(\tilde{L}\tilde{\xi})_{ij} = 0$. From Eq. (3.1), we can see that when $\xi^i J_i$ is negative, ΔM is also negative. Hence, in contrast with the ADM mass in asymptotically flat space-time, M_{AD} is reduced by the imploding motion of matter fields. Furthermore, in this case, M_{AD} can be negative since it is possible to consider arbitrary large H or, in other words, the support of the integrand in Eq. (3.1) can be arbitrary large.

Here we shall estimate the condition to make M_{AD} negative. Assuming the dominant energy condition $\rho_H \geqslant (J_i J^i)^{1/2}$, we obtain

$$|\Delta M| \simeq HL \times |\tilde{J}_r| V \lesssim HL \times \tilde{\rho}_H V, \tag{3.2}$$

where L and V are, respectively, the length scale and volume scale of the region in which inhomogeneities exist. On the other hand, from Eqs. (2.6) and (2.25), we obtain

$$M = \int \left(\tilde{\rho}_H \psi^{-3} + \frac{1}{16\pi}\tilde{K}^j_i \tilde{K}^i_j \psi^{-7}\right) d\tilde{v} \gtrsim \tilde{\rho}_H V. \tag{3.3}$$

From the above equation, we find the condition of $M < |\Delta M|$ to be

$$L \gtrsim H^{-1}. \tag{3.4}$$

Hence, in order that M_{AD} be negative, the size of the inhomogeneities should be larger than the cosmological horizon scale.

In the case of asymptotically flat space-time, the ADM mass is the conserved energy. Furthermore, it can be a source of gravity (tidal force) and therefore it is called the *gravitational mass*. As mentioned, the AD mass is also a conserved quantity and has a meaning of energy in asymptotically de Sitter space-time. However, a question arises here: Does the AD mass produce the gravitational field in the same manner as the ADM mass in asymptotically flat space-time? In order to answer this question, by analyzing the conformally flat initial data, we shall investigate the asymptotic behavior of the electric part of the Weyl tensor which corresponds to the tidal force for a timelike geodesic normal to the spacelike hypersurface considered here. Here we shall consider three cases, i.e., initial data in a spherical symmetric space (A), with a spherical source with a linear momentum (B), and with two boosted masses of which the total momentum vanishes (C). To obtain the ''physical'' components, we introduce an orthonormal triad of basis vectors $e^i_{(\alpha)}$ as

$$e^i_{(r)}=\frac{1}{\psi^2 r}(x,y,z), \tag{3.5}$$

$$e^i_{(\theta)}=\frac{1}{\psi^2 r\sqrt{x^2+y^2}}(xz,yz,-x^2-y^2), \tag{3.6}$$

$$e^i_{(\varphi)}=\frac{1}{\psi^2\sqrt{x^2+y^2}}(-y,x,0). \tag{3.7}$$

Using Eq. (2.5), the triad components of the electric part of the Weyl tensor in the vacuum region are written as

$$E_{(\alpha)(\beta)}\equiv C_{(\alpha)\mu(\beta)\nu}t^\mu t^\nu={}^{(3)}R_{(\alpha)(\beta)}-3H\hat{K}_{(\alpha)(\beta)}-\hat{K}^i_{(\alpha)}\hat{K}_{(\beta)i}, \tag{3.8}$$

where t^μ is the unit vector normal to the hypersurface.

Since we consider the conformally flat initial data, the asymptotic behavior of the metric is given by Eq. (2.19). Hence the triad components of the Ricci tensor ${}^{(3)}R_{(\alpha)(\beta)}$ of three-space asymptotically behave as

$${}^{(3)}R_{(r)(r)}\to-\frac{2M}{r^3}, \tag{3.9}$$

$${}^{(3)}R_{(\theta)(\theta)}\to\frac{M}{r^3}, \tag{3.10}$$

$${}^{(3)}R_{(\varphi)(\varphi)}\to\frac{M}{r^3}, \tag{3.11}$$

and the other components are of order $O(r^{-4})$. Thus the traceless part of the extrinsic curvature $\hat{K}_{ij}$ gives rise to the difference among the various conformally flat initial data.

A. Spherically symmetric initial data

In a spherically symmetric space, Eq. (2.7) is easily integrated as

$$\tilde{K}_{(r)(r)}=-2\tilde{K}_{(\theta)(\theta)}=-2\tilde{K}_{(\varphi)(\varphi)}\to\frac{S_J}{r^3},\quad\text{for } r\to+\infty, \tag{3.12}$$

where

$$S_J\equiv 8\pi\int_0^{+\infty} r'^3\tilde{J}_r dr', \tag{3.13}$$

and other components of $\tilde{K}_{(\alpha)(\beta)}$ vanish. Then, it is easily seen that for $r\to+\infty$, $E_{(\alpha)(\beta)}$ behaves as

$$E_{(r)(r)}=-2E_{(\theta)(\theta)}=-2E_{(\varphi)(\varphi)}\to-\frac{2M}{r^3}-H\tilde{K}_{(r)(r)}$$
$$=-\frac{2}{r^3}\left(M+\frac{H}{8\pi}\oint\tilde{K}^j_i\tilde{\xi}^i d\tilde{S}_j\right)=-\frac{2M_{AD}}{r^3}, \tag{3.14}$$

and other components are $O(r^{-4})$. Equation (3.14) shows that M_{AD} produces the tidal force by the same way as the ADM mass in asymptotically flat space-time.

B. A Spherical source with linear momentum

The solution of the momentum constraint (2.7) for a single black hole located at the origin is obtained by keeping only $K^{(+)ij}$ nonzero with $a=0$ in Eq. (2.13). Then P is a conserved linear momentum associated with the translational Killing vector of background de Sitter space-time. Then the asymptotic behavior of the extrinsic curvature is given by

$$\tilde{K}_{(r)(r)}\to\frac{3}{r^2}P\cos\theta, \tag{3.15}$$

$$\tilde{K}_{(\theta)(\theta)}=\tilde{K}_{(\varphi)(\varphi)}\to-\frac{3}{2r^2}P\cos\theta, \tag{3.16}$$

and the other components vanish. We can easily verify that in this case M_{AD} coincides with M considering Eqs. (2.24)–(2.26). However, the asymptotic behavior of $E_{(\alpha)(\beta)}$ is different from the asymptotically flat case,

$$E_{(r)(r)}\to-\frac{2M_{AD}}{r^3}-\frac{3H}{r^2}P\cos\theta, \tag{3.17}$$

$$E_{(\theta)(\theta)}=E_{(\varphi)(\varphi)}\to-\frac{M_{AD}}{r^3}-\frac{3H}{2r^2}P\cos\theta, \tag{3.18}$$

and other components vanish. M_{AD} produces the tidal force by the same manner as asymptotically flat space-time. However, in the limit of $r\to\infty$, the leading term of $E_{(\alpha)(\beta)}$ comes from the linear momentum P and depends on the polar angle θ.[1] The relative strength between the tidal force due to the momentum P and that due to M_{AD} is given by $Hr\times|P|/M_{AD}$. Hence, in order for the tidal force due to the linear momentum to dominate, the cosmological constant

[1]This effect can be regarded as a kinematical effect due to taking the comoving coordinate system. In fact, the linear momentum can be eliminated by a coordinate transformation. We note, however, that the dipole term in the tidal force for a test particle cannot be eliminated by the coordinate transformation [22].

should satisfy $|P|/M_{AD} \gtrsim 1$ if the inhomogeneity distributes within a region smaller than the cosmological horizon scale. On the other hand, if the inhomogeneity distributes over the cosmological horizon scale and there is a coherent imploding motion in addition to the translational motion, the M_{AD} can vanish. In such a case, the effect of the linear momentum on the tidal force also dominates. It should be noted that the tidal force is produced by the linear momentum of a matter field even if M_{AD} vanishes.

C. Initial data for two black holes with linear momenta

In contrast with the previous case, the total linear momentum vanishes in the case of the initial data obtained in Sec. II. From Eqs. (2.9) and (2.11), the asymptotic behavior of $\widetilde{K}_{(\alpha)(\beta)}$ is

$$\widetilde{K}_{(r)(r)} \to \frac{6}{r^3} aP(\cos 2\theta + 1), \tag{3.19}$$

$$\widetilde{K}_{(\theta)(\theta)} \to -\frac{3}{2r^3} aP(\cos 2\theta + 3), \tag{3.20}$$

$$\widetilde{K}_{(\varphi)(\varphi)} \to -\frac{3}{2r^3} aP(3\cos 2\theta + 1), \tag{3.21}$$

and the other components are $O(r^{-4})$. Hence M_{AD} is given by

$$M_{AD} = M + 2HaP, \tag{3.22}$$

and for $r \to +\infty$, $E_{(\alpha)(\beta)}$ is given as

$$E_{(r)(r)} \to -\frac{2M_{AD}}{r^3} - \frac{6}{r^3} HaP(3\cos 2\theta + 1), \tag{3.23}$$

$$E_{(\theta)(\theta)} \to \frac{M_{AD}}{r^3} + \frac{1}{2r^3} HaP(3\cos 2\theta + 5), \tag{3.24}$$

$$E_{(\varphi)(\varphi)} \to \frac{M_{AD}}{r^3} + \frac{1}{2r^3} HaP(9\cos 2\theta - 1), \tag{3.25}$$

and the other components are of order $O(r^{-4})$. Equations (3.23)–(3.25) mean that when the motion of the inhomogeneities is not isotropic the tidal force depends on their directions even if the total momentum vanishes. It is worthy to notice that even if M_{AD} vanishes, the tidal force is indeed produced by the anisotropic motion of a matter field as in Sec. III B.

IV. DISCUSSION

We have numerically obtained initial data of two boosted black holes in asymptotically de Sitter space-time. We have searched for an apparent horizon enclosing both black holes and have shown the relation between apparent horizon formation and the AD mass M_{AD}. Interestingly, when M_{AD} is larger than the critical mass M_c of Schwarzschild–de Sitter space-time, the apparent horizon enclosing both black holes does not appear for any separation a between each black hole and for any momentum. This result is the same as that in the analysis of the Einstein-Rosen bridge system [5].

Furthermore, in order to understand the dynamical meaning of M_{AD}, we have examined the asymptotic behavior of the electric part of the Weyl tensor $E_{(\alpha)(\beta)}$, which corresponds to the tidal force, in three cases of initial data in asymptotically de Sitter space-time. In the spherically symmetric case, M_{AD} produces the tidal force by the same way as asymptotically flat space-time, i.e., M_{AD}/r^3. However, in the case of the initial data of a spherical source with linear momentum, the behavior of the tidal force is very different from the asymptotically flat case. In this case, although M_{AD} produces a part of the tidal force in the same way as asymptotically flat space-time, the leading term of $E_{(\alpha)(\beta)}$ is proportional to r^{-2}, which comes from the linear momentum, and depends on the direction. In the case of the initial data obtained in Sec. II, i.e., two boosted black holes, M_{AD} also produces the tidal force in the same way as the above examples but the contribution of the relative velocity produces the anisotropic dependence of the tidal force on the polar angle θ. However, in contrast with a spherical source with linear momentum, both contributions from M_{AD} and linear momenta on the tidal force have the same asymptotic behavior r^{-3} since the total momentum vanishes.

From these results, it seems that the anisotropic velocity is important for the dynamics of inhomogeneities in a scale comparable to the cosmological horizon scale. On the other hand, in the analysis of initial data, we have seen that apparent horizon formation crucially depends on whether the AD mass is larger than M_c or not. This fact does not depend on the anisotropic velocity and suggests that the final state of nonrotating gravitational collapse in asymptotically de Sitter space-time is Schwarzschild–de Sitter space-time if cosmic censorship holds. Inhomogeneities with too large an M_{AD} cannot collapse into a black hole again if the cosmic censorship holds.

If there exists a positive cosmological constant, large scale nonspherical gravitational collapse may be very different from that of asymptotically flat space-time due to the different behavior of $E_{(\alpha)(\beta)}$. In order to investigate such an effect on the dynamics of the inhomogeneities, we need to solve the Einstein equation numerically and follow the time evolution.

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