Instability of cosmological event horizons of nonstatic global cosmic strings. II. Perturbations of gravitational waves and massless scalar fields

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The stability of the cosmological event horizons (CEHs) of a class of nonstatic global cosmic strings is studied against perturbations of gravitational waves and massless scalar fields. It is found that the perturbations of gravitational waves always turn the CEHs into nonscalar weak spacetime curvature singularities, while the ones of massless scalar fields turn the CEHs either into nonscalar weak singularities or into scalar ones depending on the particular cases considered. The perturbations of test massless scalar fields are also studied, and it is found that they do not always give the correct prediction. [S0556-2821(98)07208-7]

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I. INTRODUCTION

Cosmic strings which may have been formed in the early Universe have been studied extensively [1], since the pioneering work of Kibble [2]. Recently, Banerjee *et al.* [4] and Gregory [5] studied nonstatic global strings, and some interesting results were found. In particular, Gregory showed that the spacetime singularities usually appearing in the static case [3] can be replaced by cosmological event horizons (CEH's). This result is very important, as it may make the structure formation scenario of cosmic strings more likely, and may open a new avenue to the study of global strings.

However, our recent studies [6] showed that these CEHs in general were not stable to the perturbations of null dust fluid, and always turned into spacetime singularities. The singularities are strong in the sense that the distortion of the test particles diverges logarithmically.

In this paper, we shall study the stability of the CEHs against perturbations of massless scalar fields and gravitational waves. Specifically, the paper is organized as follows: in Sec. II we consider the perturbations of a test massless scalar field, while in Secs. III and VI, we consider the ''physical'' perturbations of gravitational waves and massless scalar fields, respectively. The word ''physical'' here means that the back reaction of the perturbations is taken into account. The paper is closed by Sec. V, where our main conclusions are derived.

The main purpose of studying perturbations of test massless scalar fields is to generalize the Helliwell-Konkowski (HK) conjecture about the stability of quasiregular singularities [7] to the stability of CEHs. As a matter of fact, in [6] which will be referred to as paper I, it was shown that the conjecture works well and gives the correct predictions about the stability of the CEHs, as far as the perturbations of null dust fluid are concerned.

The notations used in this paper will closely follow the ones used in paper I, and to avoid of repeating, some results given there will be directly used without any further explanations.

II. THE PERTURBATIONS OF TEST MASSLESS SCALAR FIELDS

By requiring that the string have fixed proper width and that the spacetime have boost symmetry in the (t,z) plane, Gregory managed to show that the spacetime for a U(1) global string (vortex) is given by the metric [5]

$$ds^2=e^{2A(r)}dt^2-dr^2-e^{2[A(r)+b(t)]}dz^2-C^2(r)d\theta^2. \tag{2.1}$$

For the cases where $b(t)=\ln[\cosh(\beta t)],+\beta t,-\beta t$, with β being a positive constant, the metric coefficients inside the core of a string have the asymptotic behavior

$$e^{A(r)}\sim\beta(r_0-r),\quad C(r)\sim C_0+O(r_0-r)^2, \tag{2.2}$$

as $r\to r_0^-$, where C_0 is a constant (cf. Eq. (3.14) in Ref. [5]). It was shown that in all the three cases the hypersurface $r=r_0$ represent a conelike CEH [5,6].

To study the stability of these CEHs against perturbations of massless scalar fields and gravitational waves, it is found convenient to introduce two null coordinates, u and v, via the relations

$$u=\frac{t+R}{\sqrt{2}},\quad v=\frac{t-R}{\sqrt{2}}, \tag{2.3}$$

where

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$$R \equiv \int e^{-A(r)} dr = -\frac{1}{\beta}\ln[\beta(r_0 - r)]. \tag{2.4}$$

In terms of u and v, the Gregory solutions can be cast in the form

$$ds^2 = 2e^{-M_{(0)}}du dv - e^{-U_{(0)}}[e^{V_{(0)}}dz^2 + e^{-V_{(0)}}d\theta^2], \tag{2.5}$$

where $M_{(0)} = \sqrt{2}\beta(u-v)$, and

$$U_{(0)} = \begin{cases} -\ln\left[\cosh\left(\dfrac{\beta}{\sqrt{2}}(u+v)\right)\right] & \\ +\dfrac{\beta}{\sqrt{2}}(u-v) - \ln C_0, & b(t) = \ln[\cosh(\beta t)], \\ -\sqrt{2}\beta v - \ln C_0, & b(t) = \beta t, \\ \sqrt{2}\beta u - \ln C_0, & b(t) = -\beta t. \end{cases} \tag{2.6}$$

Note that in paper I, the perturbations were considered in both regions $r \leqslant r_0$ and $r \geqslant r_0$. However, as shown there, the conclusions obtained in these two regions are the same. Thus, without loss of generality, in the rest of the paper, we shall restrict ourselves only to the region $r \leqslant r_0$. Then, the Klein-Gordon equation $\phi,_{\mu}\phi,_{\nu}g^{\mu\nu} = 0$ takes the form

$$2\phi_{,uv} - U_{(0),v}\phi_{,u} - U_{(0),u}\phi_{,v} = 0, \tag{2.7}$$

where $()_{,x} \equiv \partial()/\partial x$.

To study the above equation, let us first consider the case $b(t) = +\beta t$. In this case, it can be shown that Eq. (2.7) has the general solution

$$\phi(u,v) = F(u)e^{(\alpha u - \beta v)/\sqrt{2}} + G(v), \quad [b(t) = +\beta t], \tag{2.8}$$

where $F(u)$ and $G(u)$ are arbitrary functions of their indicated arguments. To have the perturbation be finite initially ($t = -\infty$), we require that the two arbitrary functions be finite as $t \to -\infty$ and $\alpha \geqslant \beta$. Then, the trace of the energy-momentum tensor (EMT) $T_{\mu\nu}$ for the test massless scalar field is given by

$$\begin{aligned} T = T^{\lambda}_{\lambda} &= -\phi_{,\mu}\phi_{,\nu}g^{\mu\nu} \\ &= -2\left[F'(u) + \frac{\alpha F(u)}{\sqrt{2}}\right]e^{[\alpha u + \beta(2u - 3v)]/\sqrt{2}} \\ &\quad \times \left[G'(v) - \frac{\beta F(u)}{\sqrt{2}}e^{(\alpha u - \beta v)/\sqrt{2}}\right], \end{aligned} \tag{2.9}$$

which diverges as $r \to r_0^-$, where a prime denotes the ordinary derivative with respect to their indicated arguments. Therefore, when the back reaction of the perturbations is taken into account, we would expect that the CEH will be turned into scalar curvature singularity, provided that the HK conjecture continuously holds for CEHs [7].

Similarly, it can be shown that the same conclusion is also true for the case $b(t) = -\beta t$.

When $b(t) = \ln[\cosh(\beta t)]$, Eq. (2.7) has the general solutions

$$\phi(u,v) = \sum_n \frac{b_n e^{\beta(u-v)/\sqrt{2}}}{[(a_n e^{\sqrt{2}\beta u} + 2)(a_n e^{-\sqrt{2}\beta v} - 2)]^{1/2}}, \tag{2.10}$$

where $\{b_n\}$ and $\{a_n\}$ are integration constants. Projecting the corresponding EMT onto the parallel propagated orthogonal (PPON) frame defined by Eqs. (A3) and (A4) in paper I, we find that the nonvanishing components are given by

$$\begin{aligned} T_{(0)(0)} &= T_{(1)(1)} = C_+^2\phi_{,u}^2 + C_-^2\phi_{,v}^2, \\ T_{(0)(1)} &= C_+^2\phi_{,u}^2 - C_-^2\phi_{,v}^2, \\ T_{(2)(2)} &= T_{(3)(3)} = e^{2\beta R}\phi_{,u}\phi_{,v}, \end{aligned} \tag{2.11}$$

where

$$\begin{aligned} C_{\pm} &\equiv \frac{1}{\sqrt{2}\beta^2(r_0 - r)^2}\{E \pm \epsilon[E^2 - \beta^2(r_0 - r)^2]^{1/2}\}, \\ \phi_{,u} &= \sqrt{2}\beta\sum_{n=1}^{\infty}\frac{b_n e^{\beta R}[a_n e^{\beta(R-t)} - 2]}{[a_n^2 e^{2\beta R} - 4a_n \sinh\beta t e^{\beta R} - 4]^{3/2}}, \\ \phi_{,v} &= \sqrt{2}\beta\sum_{n=1}^{\infty}\frac{b_n e^{\beta R}[a_n e^{\beta(R+t)} - 2]}{[a_n^2 e^{2\beta R} - 4a_n \sinh\beta t e^{\beta R} - 4]^{3/2}}, \end{aligned} \tag{2.12}$$

where E is a constant. From the above expressions we can see that, as $t \to -\infty$, these tetrad components vanish, and as $R \to +\infty (r \to r_0)$, the components $T_{(0)(0)}$, $T_{(1)(1)}$, and $T_{(0)(1)}$ become unbounded, while $T_{(2)(2)}$ and $T_{(3)(3)}$ remain finite. Thus, after the back reaction of the perturbations of the massless scalar field is taken into account, we would expect that the CEH is turned into a spacetime curvature singularity. However, unlike the last two cases, the nature of the singularity should be a nonscalar one, since now all the scalars built from $T_{\mu\nu}$ are finite, for example,

$$\begin{aligned} T = T^{\lambda}_{\lambda} &= -\frac{1}{2}e^{2\beta R}\phi_{,u}\phi_{,v} \sim \text{const}, \\ T^{\lambda\delta}T_{\lambda\delta} &= \frac{1}{4}e^{4\beta R}\phi_{,u}^2\phi_{,v}^2 \sim \text{const}, \end{aligned} \tag{2.13}$$

as $R \to +\infty$. To verify whether or not the above analysis gives the correct prediction for the stability of the CEHs, let us turn to consider real perturbations, that is, taking the back reaction of the perturbations into account.

III. PERTURBATIONS OF GRAVITATIONAL WAVES

In paper I, it was noted that, although the study of test null dust fluid and the one of real null dust fluid all gave the same results on the instability of the CEHs, the cause of the instability was different. For the real perturbations, it was caused by the nonlinear interaction of gravitational waves, rather than what the study of the test particles indicated that they should be caused by the back reaction of perturbations of null dust fluids. Thus, to study the role that gravitational waves can play, we devote this section to perturbations of pure gravitational waves. These perturbations are always expected to exist, since at the time when the strings were formed, the temperature of the Universe was very high, and the spacetime was filled with gravitational and particle radiation [1].

To study the general perturbation of gravitational waves, it is found difficult. In the following we shall study some particular cases. This does not lose any generality, since if the CEHs are stable, they should be stable against any kind of perturbations. Otherwise, they are not stable. Then, from [8] we can easily construct the following solutions to the Einstein vacuum field equations:

$$ds^2 = 2e^{-M} du dv - e^{-U}(e^{V} dz^2 + e^{-V} d\theta^2), \tag{3.1}$$

where the metric coefficients are given by

$$\begin{aligned} M &= -\ln[a'(u)b'(v)] - \delta[a(u)-b(v)] \\ &\quad - \frac{\delta^2}{4}[a(u)+b(v)]^2 + M_c, \\ V &= \ln[a(u)+b(v)] + \delta[a(u)-b(v)] - 2\ln C_0, \\ U &= -\ln[a(u)+b(v)], \end{aligned} \tag{3.2}$$

where $a(u)$ and $b(v)$ are arbitrary functions, and δ, C_0, and M_c are constants. The corresponding Kretschmann scalar is given by

$$\begin{aligned} \mathcal{R} &\equiv R_{\alpha\beta\gamma\sigma} R^{\alpha\beta\gamma\sigma} \\ &= \delta^4 [12 - \delta^2 [a(u)+b(v)]^2] \\ &\quad \times e^{-2[\delta(a-b)+\delta^2(a+b)^2/4 - M_c]}. \end{aligned} \tag{3.3}$$

Choosing the null tetrad

$$\begin{aligned} l_\mu &= e^{M/2}\delta^u_\mu, \quad n_\mu = e^{M/2}\delta^v_\mu, \\ m_\mu &= e^{-U/2}[e^{V/2}\delta^z_\mu + i e^{-V/2}\delta^\varphi_\mu], \\ \overline{m}_\mu &= e^{-U/2}[e^{V/2}\delta^z_\mu - i e^{-V/2}\delta^\varphi_\mu], \end{aligned} \tag{3.4}$$

we find that the nonvanishing components of the Weyl tensor $C_{\mu\nu\lambda\sigma}$ are given by

$$\begin{aligned} \Psi_0 &= -C_{\mu\nu\lambda\delta} l^\mu m^\nu l^\lambda m^\delta \\ &= \frac{\delta^2 b'(v)^2 e^M}{4}\{3 - \delta[a(u)+b(v)]\}, \\ \Psi_2 &= -\frac{1}{2} C_{\mu\nu\lambda\delta}[l^\mu n^\nu l^\lambda n^\delta - l^\mu n^\nu m^\lambda \overline{m}^\delta] \\ &= -\frac{\delta^2 e^M}{4} a'(u) b'(v), \\ \Psi_4 &= -C_{\mu\nu\lambda\delta} n^\mu \overline{m}^\nu n^\lambda \overline{m}^\delta \\ &= \frac{\delta^2 a'(u)^2 e^M}{4}\{3 + \delta[a(u)+b(v)]\}. \end{aligned} \tag{3.5}$$

The reason to project the Weyl tensor to the null tetrad is that now all the components Ψ_A have their direct physical interpretations [9,10]: Ψ_0 represents the transverse gravitational wave component along the null direction l_μ, Ψ_2 the Coulomb-like component, and Ψ_4 the transverse gravitational wave component along the null direction n_μ. Since l_μ (n_μ) defines the outgoing (ingoing) null geodesics [11], Ψ_0 (Ψ_4) now represents the outgoing (ingoing) cylindrical gravitational wave component.

To use solutions (3.2) as the perturbations of gravitational waves to the Gregory solution, we have to recover them under certain limits. To find such limits, let us study the three cases $b(t) = \ln[\cosh(\beta t)]$, $+\beta t$, $-\beta t$ separately.

(a) $b(t) = \beta t$. In this case, if we choose

$$b(v) = C_0 e^{\sqrt{2}\beta v}, \tag{3.6}$$

and replace the null coordinate u by u', where $du' = e^{-\sqrt{2}\beta u} du$, it can be shown that the solutions given by Eq. (3.2) reduce to the corresponding Gregory solution, as $\delta, a(u') \to 0$. Submitting Eq. (3.6) into Eqs. (3.3) and (3.5), we find that the Kretschmann scalar and the Ψ_A's are all finites as $t \to -\infty$, while near the CEH where $r \to r_0^-$, the Kretschmann scalar $\mathcal{R}$ and the components Ψ_0 and Ψ_2 are finite, but Ψ_4 becomes infinite. It can be shown that now all the fourteen scalars built from the Riemann tensor are finite as $r \to r_0^-$. Therefore, the perturbations of the gravitational waves in this case do not turn the CEH into a scalar singularity, although they do turn it into a nonscalar one. The latter can be seen by considering the tidal forces, represented by the tetrad components of the Riemann tensor in a free-falling frame (PPON). For example, the component $R_{(1)(2)(1)(2)}$ in the PPON frame defined by Eqs. (A3) and (A4) in paper I diverges as

$$R_{(1)(2)(1)(2)} \to (r - r_0^-)^{-2}, \tag{3.7}$$

as $r \to r_0^-$. Therefore, the perturbations due to the gravitational waves turn the CEHs into nonscalar curvature singularities. However, different from the perturbations of null dust fluids [6], now the singularity is weak in the sense that the distortion, which is equal to the twice integral of the tidal forces, is finite as $r \to r_0^-$,

$$\int \int R_{(1)(2)(1)(2)} d\tau d\tau \to (\tau_0 - \tau)\ln(\tau_0 - \tau), \tag{3.8}$$

where τ_0 is a constant and chosen such that $\tau \to \tau_0$ as $r \to r_0^-$.

It should be noted that in using the PPON frame defined in paper I to obtain the above expressions, we have assumed that the gravitational wave perturbations are weak, so the PPON frame of the perturbed solutions can be replaced by the one of nonperturbed solutions. This is the case when $\delta, a(u')$ and its derivatives are all very small. In the following, whenever we use this frame, we always assume that the corresponding conditions hold.

(b) $b(t)=-\beta t$. In this case, to have the solutions given by Eq. (3.2) reduce to the corresponding Gregory solution, as $\delta, b(v)\rightarrow 0$, we have to choose

$$a(u)=C_0 e^{-\sqrt{2}\beta u}. \tag{3.9}$$

Once this is done, it can be shown from Eqs. (3.3) and (3.5) that now the CEH is also not stable and turned into a nonscalar singularity in a manner quite similar to that in the last case. In particular, the project of the Riemann tensor onto the PPON frame defined in paper I diverges, for example, the component $R_{(1)(2)(1)(2)}$ diverges exactly as that of Eq. (3.7), while the twice integral of it is given by Eq. (3.8). Thus, the nonscalar singularity is also weak.

(c) $b(t)=\ln[\cosh\beta t]$. In this case, it is easy to show that as $\delta\rightarrow 0$, the solutions given by Eq. (3.2) reduce to the corresponding Gregory solution, provided that the functions $a(u)$ and $b(v)$ are chosen such that

$$a(u)=\frac{C_0}{2}e^{-\sqrt{2}\beta u}, \quad b(v)=\frac{C_0}{2}e^{\sqrt{2}\beta v}. \tag{3.10}$$

Inserting the above expressions into Eqs. (3.3) and (3.5) we find that all of the fourteen scalars built from the Riemann tensor are finite both at the initial $t=-\infty$ and as $r\rightarrow r_0^-$. Thus, similar to the last two cases, the perturbations of the gravitational waves do not turn the CEH into a scalar singularity. To see whether or not they produce non-scalar singularities, we can project the Riemann tensor onto the PPON frame defined in paper I. After doing so, we find that some of the tetrad components indeed diverge, for example, the component $R_{(1)(2)(1)(2)}$ diverges exactly as that given by Eq. (3.7).

Therefore, in all the three cases the gravitational perturbations turn the CEHs into spacetime singularities, and the singularities are nonscalar ones, and are weak in the sense that although the tidal forces diverge, the distortion is finite.

IV. PERTURBATIONS OF MASSLESS SCALAR FIELD

To study perturbations of massless scalar fields, we shall use a theorem given in [12], which is described as follows: If the solutions $\{M_g, U_g, V_g\}$ is a solution of the Einstein *vacuum* field equations for the metric (3.1) then the solution

$$\{M,V,U,\phi\}=\{M_g-\Omega_g, V_g, U_g, \lambda V_g/\sqrt{2}\} \tag{4.1}$$

is a solution of the Einstein-scalar field equations $G_{\mu\nu}=\phi_{,\mu}\phi_{,\nu}-g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}/2$, where λ is a constant, and

$$\Omega_g(u,v)=\lambda^2\left\{\frac{3}{2}U_g-\ln|2U_{g,u}U_{g,v}|-M_g\right\}. \tag{4.2}$$

For more details we refer readers to [12].

In order to use this theorem, the condition $U_{g,u}U_{g,v}\neq 0$ has to be true. However, from Eq. (2.6) we can see that this is the case only for $b(t)=\ln[\cosh(\beta t)]$. To overcome this problem, we shall use the solutions given by Eq. (3.2) with $\delta=0$ as the vacuum solutions for the cases $b(t)=\pm\beta t$. It can be shown that in these two cases the corresponding solutions are flat, and can be brought to the forms that the corresponding Gregory solutions take by some coordinate transformations. Once this is clear, we take the solutions given by Eq. (3.2) with $\delta=0$ as the vacuum solution $\{M_g, U_g, V_g\}$ of the Einstein field equations. Submitting them into Eq. (4.1), we find

$$\begin{aligned} M&=(1+\lambda^2)M_g+\lambda^2\ln 2|a'(u)b'(v)| \\ &\quad -\frac{\lambda^2}{2}\ln[a(u)+b(v)], \\ V&=\ln[a(u)+b(v)]-2\ln C_0, \\ U&=-\ln[a(u)+b(v)], \\ \phi&=\frac{\lambda}{\sqrt{2}}\{\ln[a(u)+b(v)]-2\ln C_0\}, \end{aligned} \tag{4.3}$$

where for the case $b(t)=+\beta t$, the function $b(v)$ is given by Eq. (3.6), while the function $a(u)$ is arbitrary. For the case $b(t)=-\beta t$, the function $a(u)$ is given by Eq. (3.9), while the function $b(v)$ is arbitrary. For the case $b(t)=\ln[\cosh(\beta t)]$, the two functions $a(u)$ and $b(v)$ are all fixed and given by Eq. (3.10). To consider the solutions given by Eq. (4.3) as perturbations of the corresponding Gregory solutions, we require that the constant λ, the arbitrary function $a(u)$, and its derivatives in the case $b(t)=+\beta t$, and the arbitrary function $b(v)$ and its derivatives in the case $b(t)=-\beta t$, are all small. In particular, when $\lambda,\ a(u)\rightarrow 0$, these solutions reduce to the Gregory solution for $b(t)=+\beta t$, and when $\lambda,\ b(v)\rightarrow 0$, they reduce to the Gregory solution for $b(t)=-\beta t$.

From Eq. (4.3), we find that the corresponding physical quantities are given by

$$\begin{aligned} T&=T^{\lambda}_{\lambda}=-\frac{c_1}{[a(u)+b(v)]^{2+\lambda^2/2}}, \\ \mathcal{R}&=\frac{3c_1^2}{\lambda^2[a(u)+b(v)]^{\lambda^2/4}}, \\ \Psi_0&=\frac{c_1 b'(v)}{4a'(u)[a(u)+b(v)]^{2+\lambda^2/2}}, \\ \Psi_2&=\frac{c_1}{12[a(u)+b(v)]^{2+\lambda^2/2}}, \\ \Psi_4&=\frac{c_1 a'(u)}{4b'(v)[a(u)+b(v)]^{2+\lambda^2/2}}, \end{aligned} \tag{4.4}$$

where $c_1=\lambda^2 2^{\lambda^2}e^{M_c(1+\lambda^2)}$. To study the asymptotic behavior of these quantities, let us consider the three cases separately.

(a) $b(t)=\beta t$. In this case, the function $a(u)$ is arbitrary but small, and the function $b(v)$ is given by Eq. (3.6), from which we find that

$$b(v), \quad b'(v) \sim \beta(r_0 - r)e^{\beta t}. \tag{4.5}$$

Submitting the above expression into Eq. (4.4), we find that all these quantities are finite, except for Ψ_4 which diverges as $e^{-\beta t}/(r_0-r)$ both as $t \to -\infty$ and as $r \to r_0^-$. Note that the amplitude of the gravitational wave components Ψ_0 and Ψ_4 is not completely fixed.[1] Thus, the divergence of Ψ_4 does not really mean that the spacetime is singular. To clarify this point, let us first consider the fourteen scalars built from the Riemann tensor, which are found finite for $t=-\infty$ and $r=r_0$. Therefore, in this case the spacetime is free of scalar curvature singularities both at the initial and on the CEH. To see if there exist nonscalar singularities, let consider the tidal forces. Projecting the corresponding Riemann tensor onto the PPON frame defined in paper I, we find that some of its components are diverge, for example, the component $R_{(1)(2)(1)(2)}$ diverges as Eq. (3.7), while the corresponding distortion vanishes similar to that of Eq. (3.8). Therefore, in this case the perturbations of the massless scalar field turn the CEH into a weak and nonscalar spacetime singularity.

(b) $b(t)=-\beta t$. In this case, the function $b(v)$ is arbitrary and small, and the function $a(u)$ is given by Eq. (3.8), from which we find that

$$a(u), \quad a'(u) \sim \beta(r_0 - r)e^{-\beta t} \to \infty, \tag{4.6}$$

as $t \to -\infty$, and

$$a(u), \quad a'(u) \sim \beta(r_0 - r)e^{-\beta t} \to 0, \tag{4.7}$$

as $r \to r_0^-$. Substituting Eq. (4.6) into Eq. (4.4) we find that all these quantities are finite as $t \to -\infty$. That is, the spacetime is initially free of spacetime singularities. However, the combination of Eqs. (4.4) and (4.7) shows that the spacetime may be singular on the CEH $r=r_0$. A closer study shows that this is indeed the case, and similar to the last case, the nature of singularity is a weak and nonscalar one.

(d) $b(t)=\ln[\cosh(\beta t)]$. In this case the two functions $a(u)$ and $b(v)$ are given by Eq. (3.10). Combining this equation with Eq. (4.4) we find that all the quantities given by Eq. (4.4) are finite at the initial, and diverge as $r \to r_0$. That is, in this case the CEH is turned into a scalar curvature singularity.

V. CONCLUSIONS

The stability of the CEHs appearing in Gregory's nonstatic global cosmic strings [5] have been studied. It has been shown that the gravitational wave perturbations always turn the CEHs into nonscalar weak spacetime curvature singularities, where ''weak'' means that although the tidal forces become unbounded, the distortion remains finite as these singularities approach. It has been also shown that the CEHs are not stable against perturbations of massless scalar fields, and are turned into nonscalar weak singularities for the cases $b(t)=\pm\beta t$ and into scalar ones for the case $b(t)=\ln[\cosh(\beta t)]$. These results are not consistent with the ones obtained by studying the perturbations of the test massless scalar field. In particular, in the last case the latter predicted that the singularity should be a nonscalar one. Thus, to generalize the HK conjecture [7] to the study of the stability of CEHs, more labor is required.

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[1] The amplitude of the two null vectors l^μ and n^μ defined in Eq. (3.4) are not completely fixed by the conditions $l^\lambda l_\lambda = n^\lambda n_\lambda = 0$ and $n^\lambda l_\lambda = 1$. In [10], it was shown that in general they take the form $l^\mu = B\delta^\mu_v$ and $n^\mu = A\delta^\mu_u$, where $AB=e^M$. Then, it was found that $\Psi_0 = B^2\Psi_0^{(0)}$, $\Psi_2 = AB\Psi_2^{(0)}$, and $\Psi_4 = A^2\Psi_4^{(0)}$, where $\Psi_A^{(0)}$'s are independent of the choice of the functions A and B.

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