Imprint of gravitational waves in models dominated by a dynamical cosmic scalar field

R. R. Caldwell and Paul J. Steinhardt

Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104

An alternative to the standard cold dark matter model has been recently proposed in which a significant fraction of the energy density of the universe is due to a dynamical scalar field (Q) whose effective equation of state differs from that of matter, radiation or cosmological constant (Λ). In this paper, we determine how the Q component modifies the primordial inflation gravitational wave (tensor metric) contribution to the cosmic microwave background anisotropy and, thereby, one of the key tests of inflation. [S0556-2821(98)06110-4]

I. INTRODUCTION

A key prediction of inflationary cosmology is a nearly scale-invariant spectrum of initial fluctuations composed of energy density (scalar metric) [1–4] and gravitational wave (tensor metric) [5–7] perturbations. The two components leave imprints on the cosmic microwave background (CMB) anisotropy that are potentially detectable and distinguishable. The spectrum of each component is characterized by a spectral index (n) which determines how the perturbation amplitude varies with wavelength. Each spectrum is also predicted to be nearly scale invariant; using the standard notational convention, this corresponds to $n_S \approx 1$ for the scalar metric fluctuations and $n_T \approx 0$ for the tensor metric fluctuations. The spectral indices and the ratio of tensor to scalar amplitudes on any given scale are determined by the equation of state during inflation or, equivalently, the inflaton potential. A prediction of inflation is a simple relation between n_S, n_T, and the ratio of tensor to scalar amplitudes that applies to nearly all inflationary models [8]. Unlike other features of inflation, such as flatness or a nearly Harrison-Zel'dovich spectra, the predicted relation between the spectral indices and amplitudes was unanticipated prior to the development of the inflationary model and is, in this sense, a unique stamp of inflation.

It is hoped that forthcoming measurements of CMB anisotropy [9,10], complemented by measurements of CMB polarization [11], can be used to detect a tensor component and to test the inflationary scenario. The CMB temperature anisotropy power spectrum can be expressed in multipole moments C_ℓ, where each C_ℓ can be divided into a sum of independent tensor ($C_\ell^{(T)}$) and scalar ($C_\ell^{(S)}$) subcomponents. While the individual tensor and scalar contribution to the temperature anisotropy cannot be observed independently, the presence of a strong tensor component may be measured through correlations of polarization and temperature [11–13]. The amplitude of these correlations, characterized by the ratio of tensor-to-scalar multipole moments $r_\ell \equiv C_\ell^{(T)}/C_\ell^{(S)}$, can be used to test the inflationary scenario. However, the moments do not depend on the primordial relation between spectral indices and amplitudes alone. The power spectrum also depends on the evolutionary history since inflation ended. For example, in the standard cold dark matter (SCDM) model in which $\Omega_m = 1$, the inflationary prediction [8,14] is well known to be

$$r_2 \approx 7(1 - n_S).$$

The transfer from primordial to present-day spectra is made simple by the absence of a late-time, integrated Sachs-Wolfe (ISW) effect for the scalar perturbations. In scenarios for which the time variation of the scalar and tensor perturbation amplitudes generates substantial CMB anisotropy due to the ISW effect, the ratio r_ℓ is modified, as occurs for open CDM [15] and cosmological constant ΛCDM [16,17].

The purpose of this paper is to discuss the gravitational wave contribution to the CMB anisotropy in a new class of cosmological models where a significant fraction of the energy density of the universe takes the form of a cosmic scalar field (Q) with an equation of state different from that of matter, radiation or cosmological constant [18]. The scalar field component of the cosmic fluid has been dubbed "quintessence" and cosmological models based on a combination of quintessence and cold dark matter components are known as QCDM models. While there have been other, detailed studies of the CMB anisotropy spectrum in CDM-based scenarios in which a substantial fraction of the cosmological energy density is in the form of a scalar field [19], none have examined the behavior of tensor, gravitational wave perturbations. In a recent paper [18] (henceforth, referred to as paper I), we computed the background evolution, the CMB power spectrum, and the mass spectrum for QCDM models. We showed that certain models of this type result in a significantly better fit than SCDM to the CMB temperature power spectrum, the mass power spectrum, early structure formation, and distant supernovae and gravitational lens count measurements. In paper I we restricted our attention to the case of strictly scale-invariant ($n_S = 1$) spectra of purely scalar metric perturbations. In the present paper, we discuss how the tensor contribution to the CMB anisotropy is modified by the presence of a Q component. We compute the full, scalar plus tensor CMB power spectrum for QCDM models based on inflationary initial conditions and determine how the relation between primordial spectral indices and amplitudes predicted by inflation is modified in QCDM models. The result is a generalization of a key test of inflationary cosmology.

The organization of this paper is as follows. In Sec. II we briefly review the predictions of inflation for the spectrum of initial tensor and scalar perturbations and the resultant contributions to the CMB anisotropy. In Sec. III we discuss how

the computation of the tensor contribution is modified in QCDM models. (The computation of the scalar spectrum has been described in paper I.) We then present the computed tensor contribution to the CMB power spectrum for QCDM models and compare with an SCDM model. In Sec. IV we present our key result, the generalization of the inflationary relation between spectral index and amplitude for QCDM models.

II. REVIEW OF INFLATIONARY PREDICTIONS FOR THE SCDM MODEL

Inflation predicts a nearly scale-invariant power spectrum of energy density (scalar metric) and gravitational wave (tensor metric) fluctuations. For both subcomponents, the power spectrum as a function of Fourier mode k can be expressed to lowest order in terms of the Hubble constant and its time derivatives evaluated when the Fourier mode k was stretched beyond the horizon during inflation, at $k=aH$. Here a is the Friedmann-Robertson-Walker scale factor and $H=H(\phi)$ is the Hubble parameter, which depends on the expectation value of the inflaton field ϕ. Expanding around some given wave number k_0, the power spectrum can be parametrized by spectral amplitudes $A_{(S,T)}$ and spectral indices $n_{(S,T)}$:

$$P_S(k)=A_S^2\left(\frac{k}{k_0}\right)^{n_S-1}=4\left(\frac{H^2}{m_p^2|H'|}\right)^2\Bigg|_{k=aH}, \quad (2.1)$$

$$P_T(k)=A_T^2\left(\frac{k}{k_0}\right)^{n_T}=\frac{16}{\pi}\left(\frac{H}{m_p}\right)^2\Bigg|_{k=aH}, \quad (2.2)$$

where $H'=dH/d\phi$ and m_p is the Planck mass.

A signature of inflation is the series of relations between spectral indices and amplitudes [8]. The tensor spectral index $H(\phi)$ and the power spectrum amplitudes obey the relation [8,14]

$$n_T=-\frac{m_p^2}{2\pi}\left(\frac{H'}{H}\right)^2=-\frac{1}{8}\frac{A_T^2}{A_S^2}. \quad (2.3)$$

The ratio of power spectrum amplitudes can be related to the ratio of tensor-to-scalar contributions to the CMB anisotropy quadrupole moment

$$r_2\equiv\frac{C_2^{(T)}}{C_2^{(S)}}\approx-7n_T. \quad (2.4)$$

We will comment on the coefficient below. The scalar spectral index satisfies a more complicated relation

$$n_S-1=n_T-\frac{m_p^2}{2\pi}\left(\frac{H'}{H}\right)'. \quad (2.5)$$

However, for all but an exceptional set of inflaton potentials, the Hubble parameter evolves so slowly during inflation that the second term is negligible and $n_S-1\approx n_T$. In this case,

$$r_2\approx7(1-n_S). \quad (2.6)$$

Hence, the relations arise because all parameters associated with the perturbation spectrum are ultimately determined only by H and its derivatives during inflation.

The coefficient in Eqs. (2.4) and (2.6) is crucial for the purposes of this paper. In the SCDM model, the coefficient is set largely by the Sachs-Wolfe effect, the variation of the gravitational potential on the surface of last scattering, which is only very weakly dependent on model parameters such as the Hubble constant or the baryon density. Hence, Eq. (2.6) has been presented in the literature [8,14] as a robust prediction of inflation. However, this prediction is valid *only* if $\Omega_m=1$. In models in which the matter density is less than unity, such as the QCDM models considered in this paper or models with a cosmological constant, there is a large, integrated Sachs-Wolfe contribution to the large angle CMB anisotropy, which changes the predicted relationship between the spectral indices and the tensor-to-scalar ratio. Because the integrated Sachs-Wolfe effect depends on the time variation of the gravitational potential along the line-of-site to the last scattering surface, the amplitude of the effect is sensitive to the present value of $\Omega_m=1-\Omega_Q$, the equation of state w, and the variation of w with time. Consequently, the coefficient in Eq. (2.6) is modified by a model-dependent function (Sec. IV), the central result of this paper.

III. TENSOR CONTRIBUTION TO THE CMB ANISOTROPY IN QCDM MODELS

In this section we discuss the QCDM models for which we have computed the gravitational wave contribution to the CMB anisotropy. We present the equations necessary to evolve the background and tensor perturbations. We discuss the properties of the Q matter and their dependence on the effective potential for Q, $V(Q)$, and identify two broad categories of models. This classification simplifies our survey, in the following section, of the imprint of gravitational waves in QCDM models. Finally, we present some sample results of our computation of the tensor contribution to the CMB anisotropy spectrum.

A. Background equations

The QCDM models are constructed from spatially flat, Friedmann-Robertson-Walker (FRW) space-times containing baryons, cold dark matter, neutrinos, radiation and a cosmic scalar field or Q component. The space-time metric is given by $ds^2=-dt^2+a^2(t)d\vec{x}^2$, where a is the expansion scale factor and t is the cosmological time. The background equation of motion, the energy density, and the pressure for the Q matter are

$$\ddot{Q}+3H\dot{Q}=-\frac{\partial V}{\partial Q}, \quad (3.1)$$

$$\rho_Q=\tfrac{1}{2}\dot{Q}^2+V, \quad p_Q=\tfrac{1}{2}\dot{Q}^2-V, \quad (3.2)$$

where the overdot represents $\partial/\partial t$. Hence, the relations in Eqs. (3.1),(3.2) supplement the usual background equations to specify the evolution of all components of the cosmological fluid.
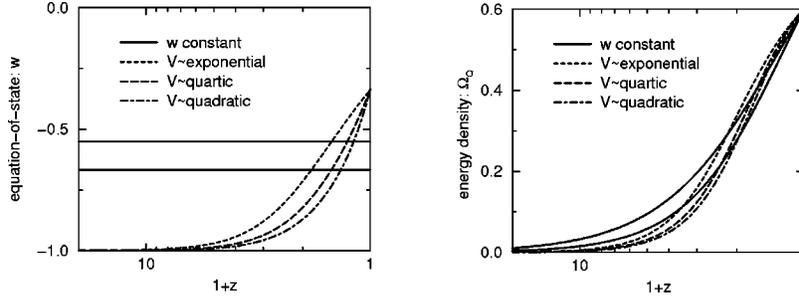
FIG. 1. For the monotonic class of QCDM models discussed in the text, the evolution of the equation of state, w, and energy density Ω_Q are shown in the left and right panels. The examples shown are for exponential, quartic, and quadratic potentials which reach $w(t_0) = -1/3$ and $\Omega_Q = 0.6$ by the present day. Constant equation of state models $w = -0.55$ chosen, according to the expression for w_{eff}, to mimic the monotonic exponential potential, and $w = -0.66$ for the quartic and quadratic potentials are also shown for comparison. In both panels, the $w = -0.55$ case is the upper, solid, black curve.

B. Gravitational wave equations

Fluctuations about the background space-time can be represented in the synchronous gauge, where the linearized space-time metric is

$$ds^2 = -dt^2 + a^2(t)(\gamma_{ij} + h_{ij})dx^i dx^j. \tag{3.3}$$

Here, γ_{ij} is the unperturbed spatial metric and $h_{ij} = h_{ij}^S + h_{ij}^T$ is the metric perturbation, which includes scalar (S) and tensor (T) perturbations. The equations of motion for the scalar perturbations and the fluctuations in the Q component have been described elsewhere [20]. The transverse, traceless gauge constraints and equations of motion of the gravitational wave perturbation are

$$\widetilde{\nabla}^i h^T{}_{ij} = \gamma^{ij} h^T{}_{ij} = 0, \quad \left[\frac{\partial^2}{\partial t^2} + 3H\frac{\partial}{\partial t} - \frac{1}{a^2}\widetilde{\nabla}^m \widetilde{\nabla}_m\right] h^{Ti}{}_j = 0, \tag{3.4}$$

where $\widetilde{\nabla}_j$ denotes the covariant derivative with respect to the spatial metric γ_{ij}. Because we have modeled the Q matter as a scalar field in a regime in which only the linearized fluctuations to first order are important, the explicit form of the equation of motion for the tensor perturbations is unchanged; there are no additional inhomogeneous tensor fields or anisotropic tensor sources (as would occur if the Q component were modeled as a tangled web of nonintercommuting cosmic strings [21], for example). Hence, the only effect on the evolution of the gravitational wave amplitude is through the background expansion. It is then straightforward to adopt the

standard algorithms for evolving the Boltzmann equations with tensor fluctuations for the problem at hand. We have modified a series of standard Boltzmann CMB codes based on the synchronous gauge, maintaining a fixed relationship between the initial amplitudes of the scalar and tensor power spectra, for the computations described below [22–24].

C. Classification of QCDM models

The CMB anisotropy spectrum, both scalar and tensor subcomponents, is sensitive to the detailed time evolution of the Hubble parameter H since last scattering. In the case of the QCDM models, these subcomponents are affected by the time evolution of the equation of state of the Q component $w \equiv p_Q/\rho_Q$ which in turn reflects the form of the potential $V(Q)$. We find that for a wide range of potentials, however, there are two broad categories by which we may classify the behavior of the models. Hence, we will focus our attention on representative models from each of these categories.

The two categories correspond to cases where w increases monotonically versus cases where w has begun to oscillate about an asymptotic value by the present epoch. In either case, the initial value of the scalar field Q is assumed to be set at a particular value by some conditions (e.g., inflation) in the early universe. So long as $V''(Q) \ll H^2$, Q remains fixed because the Hubble red shift term ($3H\dot{Q}$) dominates the equation of motion. Consequently, the energy density is nearly constant during the early history of the universe so that $w \approx -1$. Once H reduces sufficiently that Q begins to evolve down the potential, the balance of kinetic and poten-
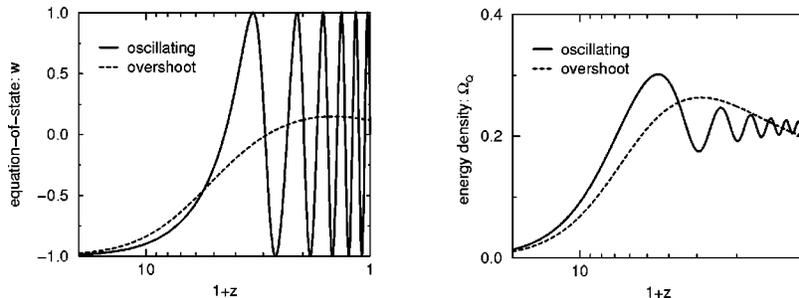

FIG. 2. For the oscillatory class of QCDM models discussed in the text, the evolution of the equation of state w and energy density Ω_Q are shown in the left and right panels, for oscillatory quadratic and overshooting exponential potentials. In each case, $\Omega_Q \approx 0.2$ at the present day.
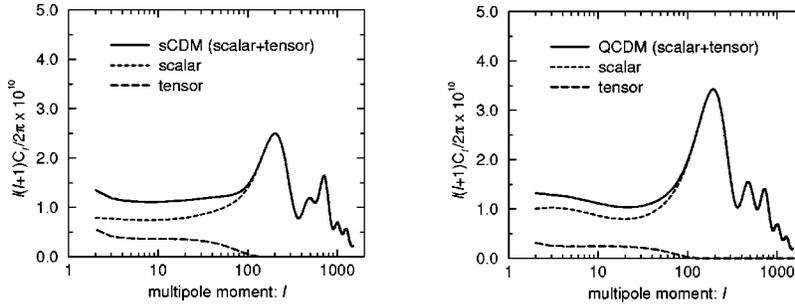
FIG. 3. The CMB anisotropy spectrum, decomposed into the scalar and tensor subcomponents, for a tilted SCDM model and a tilted QCDM model with a constant equation of state $w=-1/3$ and $\Omega_Q=0.7$. In each case, $n_S=0.9$, and the relationship between the primordial power spectra as predicted by inflation has been maintained.

tial energy in Q changes and w begins to grow. Depending on $V(Q)$ and the initial conditions, w may continue to grow up to the present epoch or it may begin to oscillate around some asymptotic value. The first possibility corresponds to the "monotonic" class of models. Here we find that the imprint of Q on the CMB anisotropy and the mass power spectrum is well approximated by the result obtained if w is held constant at roughly the mean value during the period when Ω_Q is non-negligible. We have found that the expression

$$w_{\mathrm{eff}}= \int da \; w(a)\Omega_Q(a)/ \int da \; \Omega_Q(a) \qquad (3.5)$$

provides a reasonable value for the effective w. To convert from a general potential $V(Q)$ with monotonic behavior, Eqs. (3.1),(3.2) must be used in Eq. (3.5). In Fig. 1 we show two examples where we compare the evolution of the equation of state and energy density of a model with monotonically increasing w to a constant w model where w is chosen according to Eq. (3.5). One can see that while the equation of state as function of z is very different, the evolution of the energy density is very similar. Later, in Fig. 6, we show that the CMB anisotropy spectra are similar as well. The oscillatory class of models corresponds to cases where Q evolves to a point in its potential where w begins to oscillate around some asymptotic value. One example is a potential in which Q evolves towards, and then begins to oscillate about, a minimum of $V(Q)$ before the present epoch. Correspondingly, w begins to oscillate around a mean value determined by the shape of the potential, e.g., $w\rightarrow 0$ for a quadratic potential $(V\propto Q^2)$, $w\rightarrow 1/3$ for a quartic potential $(V$

$\propto Q^4)$. See Fig. 2. Another example is an exponential potential of the form $V=m^4\exp(-\beta Q)$; as the potential energy begins to dominate the universe, the equation of state first overshoots, then relaxes towards $w\rightarrow(\beta^2/24\pi)-1$ for $\beta \leq \sqrt{48\pi}$. We will use this exponential example to represent the oscillatory-w category.

Note that a given potential can belong to either the monotonic or oscillatory class depending on parameters and the initial conditions for Q. Namely, for some choices of the initial value of Q, w may have begun to oscillate around its asymptotic value by the present epoch. For other choices, Q may not have evolved so far by the present epoch and w has been increasing monotonically. For example, while an exponential potential is illustrated both in Fig. 1 (under monotonic) and Fig. 2 (under oscillatory), different initial conditions, and values of m and β, have been selected in the two cases.

D. Scalar and tensor anisotropy spectra in QCDM models

We have computed the CMB anisotropy spectrum due to the scalar and tensor subcomponents for a number of QCDM models, which we now present. Figure 3 shows the full CMB power spectrum predicted by inflation for tilted SCDM and a QCDM model with a constant $w=-1/3$ equation of state and $\Omega_Q=0.7$, each with $n_S=0.9$. In both panels the total spectrum is broken down into scalar and tensor components. It it noticeable that in the range $2\leq \ell \leq 10$ the fractional contribution of the tensor spectrum to the total power is different in the two models.

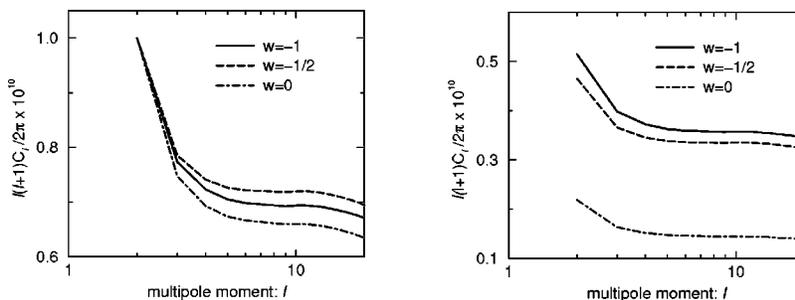Figure 4 indicates how the Q component changes the shape of the tensor subcomponent of the power spectrum

FIG. 4. The tensor subcomponent of the CMB anisotropy spectrum for a series of three QCDM models with $\Omega_Q=0.5$ and $n_S=0.9$. In the left panel, the tensor quadrupoles have been artificially set to $C_2^{(T)}=1$ in order to compare the shapes. In the right panel, the COBE-normalized amplitudes have been restored.
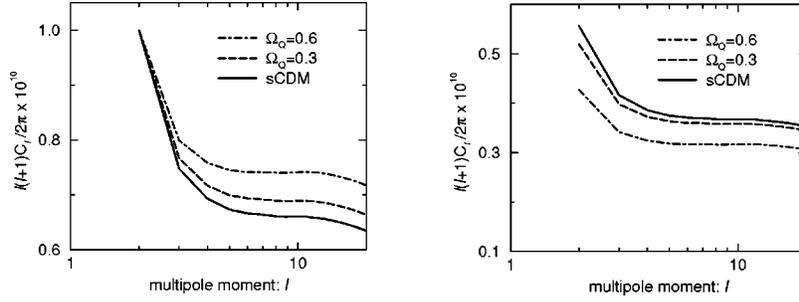
FIG. 5. The tensor subcomponent of the CMB anisotropy spectrum for a pair of QCDM models with $w=-1/2$, and tilted SCDM, all with $n_S=0.9$. In the left panel, the tensor quadrupoles have been artificially set to $C_2^{(T)}=1$ in order to compare the shapes. In the right panel, the COBE-normalized amplitudes have been restored.

depending on w for a set of constant equation of state models. The tensor power spectra in the first panel have been artificially normalized to $C_2^{(T)}=1$ to compare the shapes, demonstrating that the shape is not strongly affected by the change in w. In the second panel, the curves have been properly normalized with respect to COBE. Here we see that the main difference is in the overall amplitude; for increasing w, the fractional contribution of the tensor spectrum decreases. In Fig. 5, a similar set of panels demonstrates the effect of changes in Ω_Q on the tensor subcomponent. While the shape is not strongly affected, we see that for increasing Ω_Q, the fractional contribution of the tensor spectrum decreases.

In paper I, it was already observed that, even for purely scalar metric fluctuations, the CMB power spectrum at large angular scales (low ℓ) in QCDM models exhibits unusual features that do not occur in SCDM or other conventional models. This is owing to a combination of the modification of cosmic expansion caused by Q (that is, an integrated Sachs-Wolfe effect) and the direct effect of fluctuations in Q. Adding a tensor component can add to further features at low ℓ.

IV. GENERALIZATION OF THE INFLATIONARY PREDICTION FOR QCDM MODELS

We have discussed in Sec. III how a very large class of QCDM models can be divided into two categories: (1) models in which w is constant or monotonically increasing and

(2) models in which w overshoots and then approaches or oscillates about an asymptotic value. We have computed the scalar and tensor components of the CMB anisotropy for representative models of each type for a wide range of parameters. We have used the numerical results to obtain a revision of the inflationary relation between spectral amplitudes and spectral indices.

Our results are expressed in terms of an empirical relation between the scalar spectral index n_S and the ratios $r_2 \equiv C_2^{(T)}/C_2^{(S)}$ and $r_{10}\equiv C_{10}^{(T)}/C_{10}^{(S)}$. The quadrupole is a conventional choice; we also chosen $\ell=10$ because C_{10} is only weakly dependent on the cosmological model (compared to C_2) when the predicted spectra are COBE normalized, and so our relations can be applied more simply. These differences between QCDM and SCDM results have been expressed in terms of correction factors f_2 and f_{10} multiplying the known SCDM relations [8,14]

$$r_2 \approx [7(1-n_S)] \times f_2(\Omega_Q, n_S, w),$$
$$r_{10} \approx [4.8(1-n_S)] \times f_{10}(\Omega_Q, n_S, w). \quad (4.1)$$

We have not included the higher order corrections, such as those proportional to $dn_S/d\ln k$, which are negligibly small for most models [25]. The functions $f_{2,10}$ are defined in the following subsections. In all cases, $f \to 1$ as $\Omega_Q \to 0$. The dependence on cosmological parameters h, Ω_b is very weak
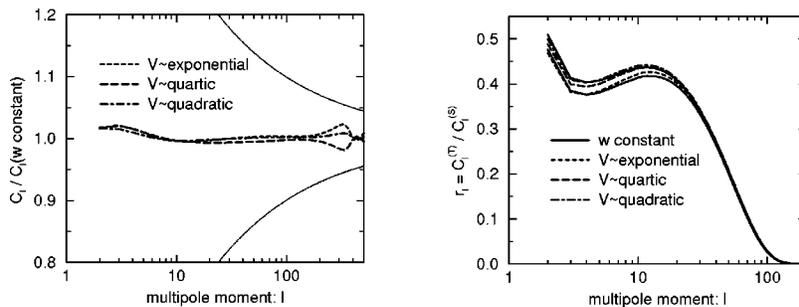
FIG. 6. In the first panel, the CMB anisotropy spectra are shown for same QCDM models as presented in Fig. 1, with exponential, quartic, and quadratic potentials, relative to the case of a constant equation of state. The exponential potential spectrum is shown relative to $w=-0.55$. The quartic and quadratic potential spectra are shown relative to $w=-0.66$. The light dotted lines show the 68% probability region due to cosmic variance. That the ratios are close to unity demonstrates the quality of the approximation of monotonic models by a constant, effective w. In the second panel, the tensor-to-scalar ratio $r_\ell=C_\ell^{(T)}/C_\ell^{(S)}$ is shown for the same set of models. The lower, solid, black curve, which lies closer to the exponential potential curve, represents the $w=-0.55$ case. In these models, the tilt is $n_s=0.9$, and the tensor component has been included.
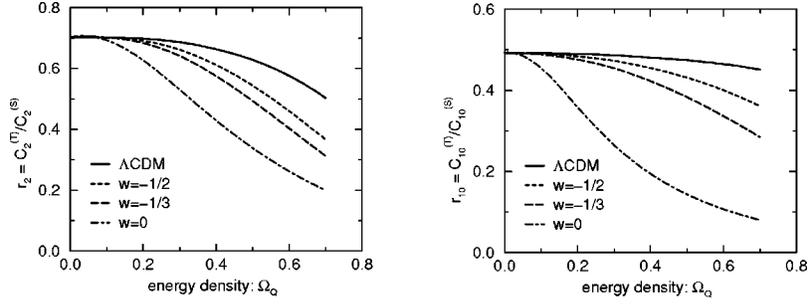
FIG. 7. The ratios r_2 and r_{10} for a series of QCDM models with constant equation of state w and spectral index $n_S = 0.9$, as a function of Ω_Q.

(which is why the relations are considered to be model-independent tests of inflation) and has been ignored.

A. Monotonic evolution of w

Models in which w evolves monotonically leave an imprint on the CMB that is well-approximated by the constant w models in which w is set to the average value during which Ω_Q is non-negligible. As a demonstration, in Fig. 6 we compare the CMB spectra for several QCDM models, specified by a particular form of the potential, with constant, effective w models. The ratios r_2 and r_{10} for the constant equation of state models are shown in Fig. 7 for the case $n_S = 0.9$. Based on plots of this sort for a range of n_S, we have obtained the following correction factors for QCDM models. For strictly constant w models, empirical fits are valid to within 10% for $-1 \leq w \leq 0$, $0.7 \leq n_S \leq 1$, and $0 \leq \Omega_Q \leq 0.7$:

$$f_2 = \left[1 + \frac{10}{9}(1 - n_S)(2 + w)\Omega_Q^2\right](1 - \Omega_Q/x)^{g_2(\Omega_Q/x)},$$

$$g_2(y) = -0.21 + 2.35y - 1.03y^2,$$

$$x = \frac{8}{5}\left[1 + \frac{2}{3}(1 + w) - \frac{2}{3}(1 + w)^{10}\right]^{-1}, \quad (4.2)$$

$$f_{10} = \left[1 + \frac{1}{10}(1 - n_S)[(8 + 7w)\Omega_Q^2 + 3]\right]$$

$$\times (1 - \Omega_Q/x)^{g_{10}(\Omega_Q/x)},$$
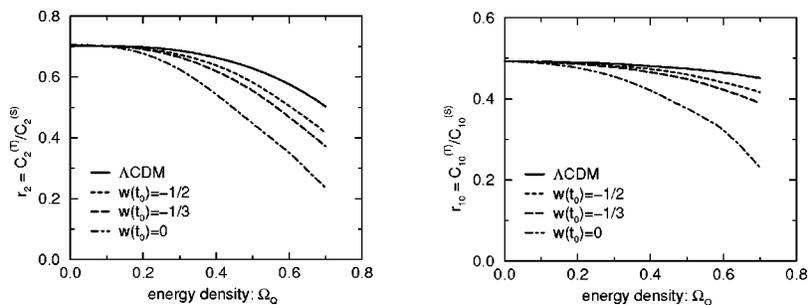
$$g_{10}(y) = 0.18 + 0.84y^2,$$

$$x = \frac{3}{4}\left[1 - \frac{2}{3}w + \frac{5}{3}w^2 - \frac{1}{2}(1 + w)^5\right]. \quad (4.3)$$

Simpler and more precise expressions can be found for $w \gtrsim -1/3$ models alone where the variation with parameters is more trivial (see Fig. 7), but it is more useful to have a single fitting formula that extends all the way to $w = 0$. In the case of ΛCDM, where $w = -1$, the function f_2 agrees with the ΛCDM result [16,17].

B. Oscillatory evolution of w

Models in which w overshoots and then approaches or oscillates about an asymptotic value are well-represented by exponential potentials in which $V(Q)$ dominates the energy density of the universe by the present epoch. The ratios r_2 and r_{10} for the exponential potential models are shown in Fig. 8. Empirical fits to the correction factors, for the same range of models and with the same accuracy as described in the previous subsection, are given below:

$$f_2 = \left[1 + \frac{10}{9}(1 - n_S)(2 + w)\Omega_Q^2\right](1 - \Omega_Q/x)^{g_2(\Omega_Q/x)},$$

$$g_2(y) = -0.21 + 2.35y - 1.03y^2, \quad x = 1 - \frac{3}{5}w, \quad (4.4)$$

$$f_{10} = \left[1 + \frac{1}{10}(1 - n_S)[(8 + 7w)\Omega_Q^2 + 3]\right]$$

$$\times (1 - \Omega_Q/x)^{g_{10}(\Omega_Q/x)},$$

FIG. 8. The ratios r_2 and r_{10} for a series of monotonic exponential potential QCDM models with spectral index $n_S = 0.9$, as a function of Ω_Q. For these models, the equation of state w is time varying and $w(t_0)$ is the present value of the equation of state.

$$g_{10}(y) = 0.18 + 0.84y^2, \quad x = 1 - \frac{3}{2}w. \tag{4.5}$$

In these expressions, w represents the present value of the equation of state $w = w(t_0)$. In all cases, the initial conditions correspond to $w \to -1$. In the expressions above, the limit $w = -1$ corresponds to $w = -1$ throughout, which coincides with the standard ΛCDM result [16]. We note that these equations do not apply to cases in which the oscillations in the Q matter are strong enough to leave a distinct feature, such as a sharp peak, in the large angle CMB spectrum. The scalar and tensor subcomponents for such a model depend sensitively on the detailed evolution of the Q component, so that a general correction factor is not possible.

V. CONCLUSION

We have described how Boltzmann codes to compute the CMB anisotropy power spectrum and the mass power spectrum in QCDM models can be simply modified to incorporate the contribution of tensor metric fluctuations, as predicted by inflationary cosmology. We have demonstrated that a Q component has two important effects on the tensor component. First, by modifying the expansion history of the universe and, hence, producing an integrated Sachs-Wolfe contribution, a Q component changes the shape of the tensor anisotropy power spectrum (see Fig. 3). Secondly, since the same effect modifies the scalar component, but by a different factor, the ratio of tensor-to-scalar contributions to the CMB anisotropy is changed. The net result for any given tilt is to reduce the tensor contribution compared to SCDM when spectra are COBE normalized (see Fig. 4). Since the mass power spectrum is normalized by the scalar contribution to the CMB anisotropy, a consequence is that, for a given tilt, including the tensor contribution does not reduce the COBE normalization of the mass power spectrum as much as in SCDM models. (However, as pointed out in paper I, the shape of the mass power spectrum in QCDM is changed and

the small-scale power is reduced by other effects that do not occur in SCDM models.)

Finally, the inflationary relations linking the ratio of tensor-to-scalar multipole moments to the tilt (n_S) must be modified. The most important results presented here are the generalized relations, shown in the previous section, between the tensor and scalar contributions to the CMB anisotropy spectrum in inflationary models in which the primordial spectral indices are related by $n_S - 1 \approx n_T$. These relations provide the key test for inflation in future CMB anisotropy and polarization measurements, now extended to include QCDM models. Assuming inflation is correct, the relations must be used in the fitting procedure to determine the Hubble parameter, baryon density, and other cosmic parameters from the CMB anisotropy [10]. To determine which relations to use, one first looks for characteristic signatures of QCDM (or alternative models) in the small scale anisotropy spectrum. Large angular scale anistropy is then studied to determine the tensor component. For a given value of n_S, we have shown that in general the presence of a Q matter component lowers the ratio r_ℓ for increasing Ω_Q. This makes it somewhat more difficult to detect tensor perturbations in the CMB. For SCDM models, $r_2 \gtrsim 0.14$ is required to distinguish the gravitational wave contribution above cosmic variance [9], and a similar result is needed for QCDM. This threshold occurs only for substantial tilts in QCDM models. Clearly, any primordial signature in the large-angle CMB spectrum is also obscured by the additional degrees of freedom introduced in the QCDM scenarios. The determination of cosmic parameters from CMB anisotropy measurements in QCDM models, including the effect of the new relations between spectral amplitudes and tilt, will be discussed quantitatively in a forthcoming publication.

ACKNOWLEDGMENTS

---

[1] A. H. Guth and S.-Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982).

[2] S. W. Hawking, Phys. Lett. **115B**, 295 (1982).

[3] A. A. Starobinskii, Phys. Lett. **117B**, 175 (1982).

[4] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D **28**, 679 (1983).

[5] V. A. Rubakov, M. V. Sazhin, and A. V. Veryaskin, Phys. Lett. **115B**, 189 (1982).

[6] A. I. Starobinskii, Sov. Astron. Lett. **11**, 133 (1985).

[7] L. F. Abbott and M. Wise, Nucl. Phys. **B244**, 541 (1984).

[8] R. L. Davis, H. M. Hodges, G. F. Smoot, P. J. Steinhardt, and M. S. Turner, Phys. Rev. Lett. **69**, 1856 (1992).

[9] L. Knox and M. S. Turner, Phys. Rev. Lett. **73**, 3347 (1994).

[10] G. Jungman, M. Kamionkowski, A. Kosowsky, and D. N. Spergel, Phys. Rev. D **54**, 1332 (1996).

[11] David N. Spergel and Matias Zaldarriaga, Phys. Rev. Lett. **79**, 2180 (1997).

[12] U. Seljak and M. Zaldarriaga, Phys. Rev. Lett. **78**, 2054 (1997).

[13] M. Kamionkowski, A. Kosowsky, and A. Stebbins, Phys. Rev. Lett. **78**, 2058 (1997).

[14] F. Lucchin, S. Matarrese, and S. Mollerach, Astrophys. J. Lett. **401**, 49 (1992); D. Salopek, Phys. Rev. Lett. **69**, 3602 (1992); A. Liddle and D. Lyth, Phys. Lett. B **291**, 391 (1992); V. Sahni and T. Souradeep, Mod. Phys. Lett. A **7**, 3541i (1992); J. E. Lidsey and P. Coles, Mon. Not. R. Astron. Soc. **258**, 57P (1992); L. Krauss and M. White, Phys. Rev. Lett. **69**, 869 (1992).

[15] Wayne Hu and Martin White, Astrophys. J. Lett. **486**, L1 (1997).

[16] Lloyd Knox, Phys. Rev. D **52**, 4307 (1995).

[17] Michael S. Turner and Martin White, Phys. Rev. D **53**, 6822 (1996).

[18] R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1586 (1998).

[19] Detailed calculations of the CMB anisotropy spectrum for such scalar field cosmologies have been carried out by Kimberly

Coble, Scott Dodelson, and Joshua A. Frieman, Phys. Rev. D **55**, 1851 (1997); Pedro G. Ferreira and Michael Joyce, Phys. Rev. Lett. **79**, 4740 (1997); astro-ph/9711102.

[20] See paper I, and R. R. Caldwell, R. Dave, and P. J. Steinhardt (work in progress). In paper I, we noted that the CMB power spectrum is insensitive to the postinflationary spectrum of perturbations generated in the Q matter. The important fluctuations in the Q matter arise in response to the growing perturbations in the radiation and matter at late times. Similar conclusions were reached by P. T. P. Viana and A. R. Liddle,

Phys. Rev. D **57**, 674 (1998). The power spectrum P_S in Eq. (2.1) refers to the fluctuations in the radiation and matter.

[21] David N. Spergel and Ue-Li Pen, Astrophys. J. **491**, L67 (1997).

[22] R. Crittenden, J. R. Bond, R. L. Davis, G. Efstathiou, and P. J. Steinhardt, Phys. Rev. Lett. **71**, 324 (1993).

[23] C.-P. Ma and E. Bertschinger, Astrophys. J. **455**, 7 (1995).

[24] U. Seljak and M. Zaldarriaga, astro-ph/9603033.

[25] L. Wang, V. Mukhanov, and P. J. Steinhardt, Phys. Lett. B **414**, 18 (1997).