Primordial black hole formation in a double inflation model in supergravity

M. Kawasaki
Institute for Cosmic Ray Research, University of Tokyo, Tanashi 188-8502, Japan

Naoshi Sugiyama
Department of Physics, Kyoto University, Kyoto 606-8502, Japan

T. Yanagida
Department of Physics, University of Tokyo, Tokyo 113-0033, Japan

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It has been recently pointed out that the initial value problem in new inflation models is naturally solved by supergravity effects if there exists a preinflation before the new inflation. We study this double inflation model in detail and find that density fluctuations on small cosmological scales are much larger than those on large scales due to the peculiar property of the new inflation. We show that this results in the production of primordial black holes which have $\sim 1 M_{\odot}$ masses in a certain parameter region of the double inflation model. We stress that these black holes may be identified with massive compact halo objects observed in the halo of our galaxy. [S0556-2821(98)05610-0]

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I. INTRODUCTION

The new inflation model [1] is the most interesting among various inflation models proposed so far, since its reheating temperature T_R is naturally low to avoid the overproduction of gravitinos in supergravity [2]. It has been shown [3] that the upper bounds of the reheating temperature T_R should be less than 10^2–10^6 GeV in gauge-mediated supersymmetry-(SUSY-)breaking models [4–6] since the mass of the gravitino $m_{3/2}$ is predicted in a range of 10^2 keV–1 GeV [7]. In hidden sector SUSY-breaking models [8], on the other hand, $m_{3/2} \simeq 100$ GeV–1 TeV. The reheating temperature is also constrained as $T_R \lesssim 10^{6-9}$ GeV [2] even in this case. These constraints on T_R are easily satisfied in a large class of new inflation models.

The new inflation model, however, has two serious drawbacks. One is the fine-tuning problem of the initial condition. Namely, the universe has to have a large region over horizons at the beginning where the inflaton field ϕ is smooth and its average value is very close to a local maximum of the potential $V(\phi)$. Since the inflaton potential $V(\phi)$ should be very flat to satisfy the slow-roll condition, there is no dynamical reason for the universe to choose such a specific initial value of the ϕ. Another problem is related to the fact that in the new inflation model the Hubble parameter is much smaller than the gravitational scale. Thus, the new inflation itself does not give a full explanation for why our universe lived for a long time [9].

In a recent work [10], Izawa and two of us (M.K. and T.Y.) have shown that the above serious problems are simultaneously solved if there exists a preinflation with a sufficiently large Hubble parameter before the new inflation. In this double inflation model the inflation dynamics in a small scale region could be different from those in a large scale region in general. If the horizon scale at the turning epoch from one to another inflation is cosmologically relevant, one may expect that density fluctuations on the scales smaller than this horizon scale are significantly larger than fluctuations on the larger scales due to nature of the new inflation. Such an overpowered density spectrum on small scales may produce primordial black holes. Recent discovery of massive compact halo objects (MACHOs) by gravitational lensing effects [11] has revived the interest in the primordial black holes. Since the observed masses of MACHOs, which have not been directly observed yet, are about $0.5 \sim 0.6 M_{\odot}$, MACHOs are very unlikely to be standard stars such as white dwarfs or red dwarfs. In this paper we show that a large amount of black holes whose mass scales are about $1 M_{\odot}$ are formed in a certain parameter region of our double inflation model. We find that these black holes are considered as possible candidates for MACHOs.[1]

II. A NEW INFLATION MODEL

We adopt the new inflation model proposed in Ref. [10]. The inflaton superfield $\phi(x,\theta)$ is assumed to have an R charge $2/(n+1)$ so that the following tree-level superpotential is allowed:

$$W_0(\phi) = -\frac{g}{n+1}\phi^{n+1}, \tag{1}$$

where n is a positive integer and g denotes a coupling constant of order 1. Here and hereafter, we set the gravitational scale $M \simeq 2.4\times 10^{18}$ GeV equal to unity and regard it as a plausible cutoff in supergravity. We further assume that the continuous $U(1)_R$ symmetry is dynamically broken down to a discrete Z_{2nR} at a scale v generating an effective superpotential:

[1]Different models for the primordial black hole formation have been studied in Refs. [12] and [13].

$$W(\phi)=v^2\phi-\frac{g}{n+1}\phi^{n+1}. \tag{2}$$

We may consider that the scale v^2 is induced by some nonperturbative dynamics as shown in Ref. [10].

The R-invariant effective Kähler potential is given by

$$K(\phi,\chi)=|\phi|^2+\frac{\kappa}{4}|\phi|^4+\cdots, \tag{3}$$

where κ is a constant of order one and the ellipsis denotes higher-order terms, which we neglect in the present analysis.

The effective potential of a scalar component of the superfield $\phi(x,\theta)$ in supergravity is given by

$$V=e^{K(\phi)}\left\{\left(\frac{\partial^2 K}{\partial\phi\partial\phi^*}\right)^{-1}|D_\phi W|^2-3|W|^2\right\}, \tag{4}$$

with

$$D_\phi W=\frac{\partial W}{\partial\phi}+\frac{\partial K}{\partial\phi}W. \tag{5}$$

This potential yields a vacuum

$$\langle\phi\rangle\simeq\left(\frac{v^2}{g}\right)^{1/n}. \tag{6}$$

We have negative energy as

$$\langle V\rangle\simeq-3e^{\langle K\rangle}|\langle W\rangle|^2\simeq-3\left(\frac{n}{n+1}\right)^2|v|^4|\langle\phi\rangle|^2. \tag{7}$$

The negative vacuum energy (7) is assumed to be canceled out by a SUSY-breaking effect [14] which gives a positive contribution $\Lambda_{\rm SUSY}^4$ to the vacuum energy. Namely, we impose

$$-3\left(\frac{n}{n+1}\right)^2|v|^4\left|\frac{v^2}{g}\right|^{2/n}+\Lambda_{\rm SUSY}^4=0. \tag{8}$$

In supergravity the gravitino acquires a mass

$$m_{3/2}\simeq\frac{\Lambda_{\rm SUSY}^2}{\sqrt{3}}=\left(\frac{n}{n+1}\right)|v|^2\left|\frac{v^2}{g}\right|^{1/n}. \tag{9}$$

The inflaton ϕ has a mass m_ϕ in the vacuum with

$$m_\phi\simeq n|g|^{1/n}|v|^{2-2/n}. \tag{10}$$

The inflaton ϕ may decay into ordinary particles through gravitationally suppressed interactions,[2] which yields reheating temperature T_R given by

$$T_R\simeq 0.1 m_\phi^{3/2}\simeq 0.1 n^{3/2}|g|^{3/2n}|v|^{3-3/n}. \tag{11}$$

[2]For example, ϕ decays into a pair of Higgs doublets $(H,\overline{H})$ if R charges of H and $\overline{H}$ are $1/(n+1)$ and the Kähler potential contains a term $h\phi^* H\overline{H}$ (h: coupling constant) [10].

For example, the reheating temperature T_R is as low as $2-6\times10^4$ GeV for $v\simeq10^{-8}-10^{-6}$ ($m_{3/2}\simeq0.02$ GeV-2 TeV), $n=4$, and $g\simeq1$, and hence the present model is consistent even with the existence of light gravitino ($m_{3/2}\lesssim1$ GeV) in the gauge-mediated SUSY-breaking model.

Let us discuss the dynamics of the new inflation. From Eq. (4) the effective potential for $\phi<\langle\phi\rangle$ is written approximately as

$$V\simeq|v^2-g\phi^n|^2-\kappa v^4|\phi|^2. \tag{12}$$

Then, identifying the inflaton field $\varphi(x)/\sqrt{2}$ with the real part of the field $\phi(x)$, we obtain a potential for the inflaton:

$$V(\varphi)\simeq v^4-\frac{\kappa}{2}v^4\varphi^2-\frac{g}{2^{n/2-1}}v^2\varphi^n+\frac{g^2}{2^n}\varphi^{2n}. \tag{13}$$

It has been shown in Ref. [14] that the slow-roll condition for the inflaton is satisfied for $\kappa<1$ and $\varphi\lesssim\varphi_f$, where

$$\varphi_f\simeq\sqrt{2}\left(\frac{(1-\kappa)v^2}{gn(n-1)}\right)^{1/(n-2)}. \tag{14}$$

This provides the value of φ at the end of inflation. The Hubble parameter during the new inflation is given by

$$H\simeq\frac{v^2}{\sqrt{3}}. \tag{15}$$

The scale factor of the universe increases by a factor of e^N when the inflaton φ rolls slowly down the potential from φ_N to φ_f. The e-fold number N is given by

$$N\simeq\int_{\varphi_f}^{\varphi_N}d\varphi\frac{V}{V'}=\begin{cases}\dfrac{1}{\kappa}\ln\left(\dfrac{\tilde{\varphi}}{\varphi_N}\right)+\dfrac{1-n\kappa}{(n-2)\kappa(1-\kappa)} & \left(\kappa\leqslant\dfrac{1}{n}\right),\\[2ex] \dfrac{1}{\kappa}\ln\left(\dfrac{\varphi_f}{\varphi_N}\right) & \left(\kappa>\dfrac{1}{n}\right),\end{cases} \tag{16}$$

where

$$\tilde{\varphi}=\sqrt{2}\left(\frac{\kappa v^2}{gn}\right)^{1/(n-2)}. \tag{17}$$

The amplitude of primordial density fluctuations $\delta\rho/\rho$ due to the new inflation is written as

$$\frac{\delta\rho}{\rho}\simeq\frac{1}{5\sqrt{3}\pi}\frac{V^{3/2}(\varphi_N)}{|V'(\varphi_N)|}=\frac{1}{5\sqrt{3}\pi}\frac{v^2}{\kappa\varphi_N}. \tag{18}$$

Notice here that we have large density fluctuations for small φ_N. Another interesting point on the above density fluctuations is that it results in the tilted spectrum whose spectrum index n_s is given by [10,14]

$$n_s\simeq1-2\kappa. \tag{19}$$

As is shown later, we assume $\kappa\sim0.3$ and $n_s\sim0.4$. This tilted power spectrum is crucial for suppressing the formation of small primordial black holes.

The e-fold number N is related to the present cosmological scale L by

$$N \simeq 60 + \ln\left(\frac{L}{3000\ \mathrm{Mpc}}\right). \tag{20}$$

Thus, if the total e-fold number N_{tot} of the new inflation is larger than about 60, the new inflation accounts for all cosmological scales of the present universe and the COBE normalization [15] gives

$$\frac{V^{3/2}(\varphi_{60})}{|V'(\varphi_{60})|} \simeq 5.3 \times 10^{-4}. \tag{21}$$

On the other hand, if $N_{\mathrm{tot}} < 60$, the new inflation can only provide density fluctuations corresponding to small scales of the universe and the preinflation discussed in the next section must account for density fluctuations on the large scales of the present universe.[3]

III. A PREINFLATION MODEL

In this section we discuss preinflation which occurs before the new inflation. In Ref. [10] it has been pointed out that the initial value $\varphi(x)$ required for the new inflation is dynamically tuned by the preinflation. Here we adopt a hybrid inflation model in Ref. [16] as the preinflation.

The hybrid inflation model contains two kinds of superfields: one is $S(x,\theta)$ and the others are $\Psi(x,\theta)$ and $\overline{\Psi}(x,\theta)$. The model is also based on the $U(1)_R$ symmetry. The superpotential is given by [17,16][4]

$$W = -\mu^2 S + \lambda S \overline{\Psi} \Psi. \tag{22}$$

The R-invariant Kähler potential is given by

$$K(S,\Psi,\overline{\Psi}) = |S|^2 + |\Psi|^2 + |\overline{\Psi}|^2 - \frac{\zeta}{4}|S|^4 + \cdots. \tag{23}$$

The potential in supergravity is given by

$$V \simeq |\mu^2 - \lambda \overline{\Psi}\Psi|^2 + |\lambda \Psi S|^2 + |\lambda \overline{\Psi} S|^2 + \zeta \mu^4 |S|^2 + \cdots. \tag{24}$$

The real part of $S(x)$ is identified with the inflaton field $\sigma/\sqrt{2}$. The potential is written as

$$V \simeq |\mu^2 - \lambda \overline{\Psi}\Psi|^2 + \frac{|\lambda|^2}{2}\sigma^2(|\Psi|^2 + |\overline{\Psi}|^2) + \frac{\zeta}{2}\mu^4\sigma^2 + \cdots. \tag{25}$$

We readily see that if the universe starts with sufficiently large value of σ, the inflation occurs for $0 < \zeta < 1$ and continues until $\sigma \simeq \sigma_c = \sqrt{2}\mu/\sqrt{\lambda}$.

In a region of small σ ($\sigma_c \lesssim \sigma \lesssim \lambda/\sqrt{8\pi^2\zeta}$) radiative corrections become important for the inflation dynamics as shown by Dvali *et al*. [18]. Including one-loop corrections, the potential for the inflaton σ is given by

$$V \simeq \mu^4 \left[1 + \frac{\zeta}{2}\sigma^2 + \frac{\lambda^2}{8\pi^2}\ln\left(\frac{\sigma}{\sigma_c}\right)\right]. \tag{26}$$

The Hubble parameter and e-folding factor N' are given by

$$H \simeq \frac{\mu^2}{\sqrt{3}}, \tag{27}$$

and

$$N' \simeq \begin{cases} \dfrac{1}{2\zeta} + \dfrac{1}{\zeta}\ln\dfrac{\sigma_{N'}}{\tilde{\sigma}} & (\sigma_{N'} > \tilde{\sigma}), \\ \dfrac{4\pi^2\sigma_{N'}^2}{\lambda^2} & (\sigma_{N'} < \tilde{\sigma}), \end{cases} \tag{28}$$

where

$$\tilde{\sigma} \simeq \frac{\lambda}{2\sqrt{2\zeta}\pi}. \tag{29}$$

The crucial point observed in Ref. [10] is that the preinflation sets dynamically the initial condition for the new inflation. The inflaton field $\varphi(x)$ for the new inflation gets an effective mass $\sim \mu^2$ from the $e^K V$ term [19] during the preinflation. The precise value of the effective mass depends on the details of the Kähler potential. For example, if the Kähler potential contains $-k|\phi|^2|S|^2$, the effective mass is equal to $\sqrt{1+k}\mu^2$. Thus, taking account of this uncertainty we write the effective mass m_{eff} as

$$m_{\mathrm{eff}} = c\mu^2 = \sqrt{3}cH, \tag{30}$$

where c is a free parameter. For $c \gtrsim 1$ this effective mass is larger than the Hubble parameter for the preinflation. Therefore, the φ oscillates during the preinflation and its amplitude decreases as $a^{-3/2}$ where a denotes the scale factor of the universe. Thus, at the end of the preinflation the φ takes a value

$$\varphi \simeq \varphi_i e^{-(3/2)N'_{\mathrm{tot}}}, \tag{31}$$

where φ_i is the value of φ at the beginning of the preinflation and N'_{tot} the total e-fold number of the preinflation.

As pointed in Ref. [10], the minimum of the potential for φ deviates from zero through the effect of $|D_S W|^2 + |D_\phi W|^2 - 3|W|^2$ term [see Eq. (4)] . The effective potential for φ during the preinflation is written as

$$V(\varphi) \simeq \frac{1}{2}c^2\mu^4\varphi^2 + \frac{\sqrt{2}}{\sqrt{\lambda}}v^2\mu^3\varphi + \cdots. \tag{32}$$

This potential has a minimum

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[3] In the case of $N_{\mathrm{tot}} \lesssim 60$, we have a domain wall problem since the discrete Z_{2nR} symmetry is spontaneously broken. However, we can avoid this problem by introducing a tiny Z_{2nR} breaking term without affecting the inflation dynamics.

[4] Symmetries of this model are discussed in Refs. [17,16].

$$\varphi_{\min} \simeq -\frac{\sqrt{2}}{c^2\sqrt{\lambda}} v\left(\frac{v}{\mu}\right). \tag{33}$$

This determines the mean initial value φ_b of the inflaton for the new inflation as

$$\varphi_b \simeq \frac{\sqrt{2}}{c^2\sqrt{\lambda}} v\left(\frac{v}{\mu}\right). \tag{34}$$

We now discuss quantum effects during the preinflation. It is known that in the de Sitter universe massless fields have quantum fluctuations whose amplitude is given by $H/(2\pi)(H/m_{\rm eff})^{1/2}$. If the fluctuations of the inflaton φ were large, we would yet have the initial value problem. Fortunately, the quantum fluctuations for φ are strongly suppressed [20] since the mass of φ during the preinflation is not less than the Hubble parameter for $c \gtrsim 1$. In fact, the amplitude of fluctuations with wavelength corresponding to the horizon scale at the beginning of the new inflation is given by

$$\delta\varphi \simeq \frac{H}{2\pi}\left(\frac{H}{m_{\rm eff}}\right)^{1/2} \exp[-(3/2)\ln(\mu/v)] \simeq \frac{H}{3^{1/4}c^{1/2}2\pi}\left(\frac{v}{\mu}\right)^{3/2}. \tag{35}$$

Here we have assumed that the reheating takes place soon after the preinflation.[5,6] When the total e-fold of the new inflation is less than 60, this amplitude should be much less than ϕ_b, otherwise the present universe becomes very inhomogeneous.[7] Thus, we require the ratio R of $\delta\varphi$ to φ_b should be much less than 1:

$$R \equiv \frac{\delta\varphi}{\varphi_b} \ll 1. \tag{36}$$

The preinflation also produces the density fluctuations $\delta\rho/\rho$ with amplitude given by

$$\frac{\delta\rho}{\rho} \simeq \frac{1}{5\sqrt{3}\pi}\frac{V^{3/2}(\sigma_{N'})}{|V'(\sigma_{N'})|} = \begin{cases} \dfrac{1}{5\sqrt{3}\pi}\dfrac{\mu^2}{\zeta\sigma_{N'}} & (\sigma_{N'} > \tilde{\sigma}), \\ \dfrac{1}{5\sqrt{3}\pi}\dfrac{8\pi^2\mu^2\sigma_{N'}}{\lambda^2} & (\sigma_{N'} < \tilde{\sigma}). \end{cases} \tag{37}$$

The spectral index n_s is almost 1 for $\sigma_{N'} < \tilde{\sigma}$ and $1+2\zeta$ for $\sigma_{N'} > \tilde{\sigma}$.

[5] ψ and $\bar{\psi}$ can decay into quarks (q) if the Kähler potential contains a term $\psi\bar{\psi}qq^*$. S also quickly decays into Goldstone multiplets associated with the breaking of U(1) symmetry by ψ and $\bar{\psi}$ condensations.

[6] Since the reheating temperature after the preinflation is high, gravitinos are produced at the reheating epoch. However, these gravitinos are sufficiently diluted by the new inflation if $N_{\rm tot} \gtrsim 7$.

[7] We thank J. Yokoyama for this point.

As mentioned in the previous section, if the new inflation provides sufficiently large e-fold number (i.e., $N_{\rm tot} > 60$) the density fluctuations produced in the preinflation are cosmologically irrelevant. However, in the case of $N_{\rm tot} < 60$, density fluctuations of the preinflation should account for the large scale structure of the universe, while the new inflation gives density fluctuations relevant only for formation of small scale structures. In this case the amplitude of the density fluctuations produced in the new inflation is free from the Cosmic Background Explorer (COBE) normalization and hence much larger density fluctuations may be produced. Such large density fluctuations may yield a large number of primordial black holes whose mass depends on $N_{\rm tot}$. The most interesting case is that the black holes have mass $\sim 1M_\odot$ and explain the MACHOs in our galaxy. Therefore, in the following sections, we study the formation of $\sim 1M_\odot$ black holes in our double inflation model.

IV. BLACK HOLE FORMATION

In a radiation dominated universe,[8] primordial black holes are formed if the density fluctuations δ at horizon crossing satisfy a condition $1/3 \leqslant \delta \leqslant 1$ [21,22]. Masses of the black holes $M_{\rm BH}$ are roughly equal to the horizon mass:

$$M_{\rm BH} \simeq 4\sqrt{3}\pi\frac{1}{\sqrt{\rho}} \simeq 0.066 M_\odot\left(\frac{T}{\rm GeV}\right)^{-2}, \tag{38}$$

where ρ and T are the total density and temperature of the universe, respectively. Thus, the black holes with mass $\sim 1M_\odot$ can be formed at temperature ~ 0.26 GeV. Since we are interested in the black holes to be identified with the MACHOs, we assume hereafter that the temperature of the black hole formation is $T_* \simeq 0.26$ GeV.

The mass fraction $\beta_*(=\rho_{\rm BH}/\rho)$ of the primordial black holes with mass $M_* \sim 1M_\odot$ is given by [22]

$$\beta_*(M_*) = \int_{1/3}^{1}\frac{d\delta}{\sqrt{2\pi}\bar{\delta}(M_*)}\exp\left(-\frac{\delta^2}{2\bar{\delta}^2(M_*)}\right) \simeq \bar{\delta}(M_*)\exp\left(-\frac{1}{18\bar{\delta}^2(M_*)}\right), \tag{39}$$

where $\bar{\delta}(M_*)$ is the mass variance at horizon crossing. Assuming that only black holes with mass M_* are formed (this assumption is justified later), the density of the black holes $\rho_{\rm BH}$ is given by

$$\frac{\rho_{\rm BH}}{s} \simeq \frac{3}{4}\beta_* T_*, \tag{40}$$

[8] There exists a long period of matter dominance after the end of the new inflation and before the reheating. During this matter dominated epoch, the small density fluctuations of the inflaton fields grow and may form gravitationally bound systems. However, these systems are destroyed at the reheating epoch because the inflaton decays produce a tremendous amount of radiations.

TABLE I. $\mu, \sigma_{\rm COBE}$, c, R, $\lambda_{\rm max}$, and $\lambda_{\rm min}$ for $\kappa=0.3$ and $\zeta=0.02,0.04,0.1,0.2$.

ζ	0.02	0.04	0.1	0.2
μ	$3.0\times10^{-3}\lambda^{1/2}$	$4.8\times10^{-3}\lambda^{1/2}$	$1.5\times10^{-2}\lambda^{1/2}$	$8.9\times10^{-2}\lambda^{1/2}$
$\sigma_{\rm COBE}$	0.86λ	1.1λ	4.3λ	75λ
c	$6.5\lambda^{-1/2}$	$5.1\lambda^{-1/2}$	$2.9\lambda^{-1/2}$	$1.2\lambda^{-1/2}$
R	$0.18\lambda^{1/2}$	$0.25\lambda^{1/2}$	$0.60\lambda^{1/2}$	$2.2\lambda^{1/2}$
$\lambda_{\rm max}$	2.8×10^{-2}	1.4×10^{-2}	2.5×10^{-3}	1.8×10^{-4}
$\lambda_{\rm min}$	5.3×10^{-3}	1.2×10^{-2}	6.0×10^{-2}	4.4×10^{-1}

where s is the entropy density. Since $\rho_{\rm BH}/s$ is constant at $T<T_*$, we can write the density parameter $\Omega_{\rm BH}$ of the black holes in the present universe as

$$\Omega_{\rm BH}h^2\simeq5.6\times10^7\beta_*, \qquad (41)$$

where we have used the present entropy density 2.9×10^3 $\rm cm^{-3}$ and h is the present Hubble constant in units of 100 km/sec/Mpc. Requiring that the black holes (=MACHOs) are dark matter of the universe, i.e., $\Omega_{\rm BH}h^2\sim0.25$, we obtain $\beta_*\sim5\times10^{-9}$ which leads to

$$\overline{\delta}(M_*)\simeq0.06. \qquad (42)$$

This mass variance suggests that the amplitude of the density fluctuations at the mass scale M_* are given by

$$\frac{\delta\rho}{\rho}\simeq2\Phi\simeq0.01, \qquad (43)$$

where Φ is the gauge-invariant fluctuations of the gravitational potential [9]. We will show later that such large density fluctuations are naturally produced during the new inflation.

Since only fluctuations produced during the new inflation have amplitudes large enough to form the primordial black holes, the maximum mass of the black holes is determined by the fluctuations with wavelength equal to the horizon at the beginning of the new inflation. We require that the maximum mass is $\sim1M_\odot$. On the other hand, the formation of black holes with smaller masses is suppressed since the spectrum of the density fluctuations predicted by the new inflation is tilted [see Eq. (19)]: the amplitude of the fluctuations with smaller wavelength is smaller. The tiny decrease of $\overline{\delta}(M)$ results in large suppression of the black hole formation rate as is seen from Eq. (39). Therefore, only black holes with mass in a narrow range are formed in the present model.

The horizon length at the black hole formation epoch ($T=T_*$) corresponds to scale L_* in the present universe given by

$$L_*\simeq\frac{a(T_0)}{a(T_*)}H^{-1}(T_*)\simeq0.25\ \ {\rm pc}, \qquad (44)$$

where T_0 is the temperature of the present universe. From Eq. (20), the density fluctuations corresponding to L_* are produced when $N=N_*\simeq40$ during the new inflation. Since the initial value φ_b for the new inflation is given by $\varphi_b=\varphi_{N_*}$, we obtain

$$\frac{V^{3/2}}{V}\simeq\frac{v^2}{\kappa\varphi_b}\simeq0.3, \qquad (45)$$

where φ_b is given by Eq. (34) and we have used Eq. (43). Equations (14) and (16) lead to

$$v\simeq0.3\kappa\exp(-\kappa N_*), \qquad (46)$$

where we have taken $n=4$ and $\sqrt{(1-\kappa)/(6g)}\simeq1$. Then the gravitino mass $m_{3/2}$ and the reheating temperature T_R are given by

$$m_{3/2}\simeq4.0\times10^{-2}\kappa^{2.5}\exp(-2.5\kappa N_*), \qquad (47)$$

$$T_R\simeq5.3\times10^{-2}\kappa^{2.25}\exp(-2.25\kappa N_*), \qquad (48)$$

which give $m_{3/2}\simeq(0.041-400)$ GeV and $T_R\simeq(3.8-1.6\times10^4)$ GeV for $\kappa\simeq0.3-0.4$.[9] Thus the present model does not have the gravitino problem as mentioned before. Using Eqs. (34) and (45) we write the scale of the preinflation μ as

$$\mu\simeq0.42\frac{\kappa}{\lambda^{1/2}c^2}. \qquad (49)$$

The density fluctuations produced in the preinflation should be normalized by the COBE data. For this we must take into account the fact that the fluctuations produced at the late stage of the preinflation reenter the horizon before the beginning of the new inflation. Such fluctuations are cosmologically irrelevant since the new inflation produces much larger fluctuations. Thus, the COBE scale corresponds to the e-fold number of the preinflation given by

$$N_{\rm COBE}=60-N_*+\ln\left(\frac{\mu}{v}\right). \qquad (50)$$

Then, from Eqs. (28) and (37), we get

$$\mu\simeq6.5\times10^{-3}\lambda^{1/2}N_{\rm COBE}^{-1/4}, \qquad (51)$$

for $\sigma_{N_{\rm COBE}}\equiv\sigma_{\rm COBE}<\tilde{\sigma}$, and

$$\mu\simeq6.0\times10^{-3}\zeta^{1/4}\lambda^{1/2}\exp(\zeta N_{\rm COBE}/2), \qquad (52)$$

for $\sigma_{\rm COBE}>\tilde{\sigma}$.

[9] Since the mass of the gravitino becomes larger than ~1 TeV for $\kappa\lesssim0.29$, we take $\kappa\gtrsim0.29$.

First let us consider the case where the one-loop corrections control the preinflation dynamics (i.e., $\sigma_{\rm COBE}<\tilde{\sigma}$). This corresponds to a small ζ region ($\zeta\lesssim 0.017$). We will show that there exists a parameter region where the black hole MACHOs are produced. For this purpose, we take $\kappa=0.3$ as an example. Using Eqs. (28), (46), (49), and (51) we obtain

$$v\simeq 5.5\times 10^{-7}, \tag{53}$$

$$\mu\simeq 2.8\times 10^{-3}\lambda^{1/2}, \tag{54}$$

$$\sigma_{\rm COBE}\simeq 0.85\lambda, \tag{55}$$

$$c\simeq 6.7\lambda^{-1/2}. \tag{56}$$

Here we have neglected $\ln\lambda$ corrections. The ratio R [see Eq. (36)] is written as

$$R\simeq 0.049\frac{\lambda^{1/2}\mu^{3/2}c^{3/2}}{v^{1/2}}\simeq 0.17\lambda^{1/2}. \tag{57}$$

We require $R\lesssim 0.03$ for the fluctuations of φ at the beginning of the new inflation to be negligible,[10] which leads to $\lambda\lesssim 3.1\times 10^{-2}$. The lower limit on λ is obtained from the condition $\sigma_c\lesssim\sigma_{\rm COBE}$, which leads to $\lambda\gtrsim 4.6\times 10^{-3}$. Therefore, for $4.6\times 10^{-3}\lesssim\lambda\lesssim 3.1\times 10^{-2}$, our double inflation model can produce the black holes which may be identified with MACHOs.

Next we consider the case of large ζ (i.e., $\zeta\gtrsim 0.017$). The value of ζ cannot be larger than about 0.2 because the spectral index n_s of the density fluctuations near the COBE scale becomes $1+2\zeta$ and the COBE data [15] give $n_s=1.2\pm 0.3$. From Eqs. (28), (46), (49), and (52), we can determine $\mu,\sigma_{\rm COBE}$, c, and R for various ζ's. The result is shown in Table I for $\kappa=0.3$. By requiring $R\lesssim 0.03$, we get upper bounds on λ ($\equiv\lambda_{\max}$). On the other hand, the lower bounds $\lambda_{\min}$ are obtained from the condition $\tilde{\sigma}<\sigma_{\rm COBE}$. $\lambda_{\max}$ and $\lambda_{\min}$ are also shown in Table I, from which it is seen that for $\zeta\lesssim 0.04$ there exists a consistent region ($\lambda_{\max}\geqslant\lambda_{\min}$).[11] Since the value of allowed ζ is small, the power spectrum on the COBE scale is almost scale invariant ($n_s\simeq 1-1.1$).

V. CONCLUSION

In this paper we have studied the recently proposed double inflation model in supergravity and discussed the formation of primordial black holes with mass about $1M_\odot$. We have shown that in a certain parameter space the primordial black holes are produced with mass $\sim 1M_\odot$ which may be identified with MACHOs in the halo of our galaxy. For successful formation of the black holes it is important for the inflaton φ to have a large effective mass ($c\gtrsim 30$) during the preinflation.[12] The allowed parameter space in the present model is very restricted. This may be due to our specific choice of the new and preinflation models. Therefore, this paper may be regarded as an existence proof of a double inflation model which accounts for the MACHOs as primordial black holes. If we relax the relation between the SUSY breaking and the new inflation scales, a wider parameter space may be allowed.

One may consider a very steep initial power spectrum with the power law index $n_s\simeq 1.4$ in order to have sufficient formation of primordial black holes under the COBE normalization of density fluctuations. However, models with such steep initial spectra overly produce black holes on smaller scales. The existence of these small black holes are severely constrained from the observation of γ rays. Moreover, these models are very difficult to use to explain the large scale structure of the universe.

On the other hand, our double inflation model can naturally provide the power spectrum which has high amplitude and shallow slope ($n_s<1$) on small scales and low amplitude and nearly scale free spectrum ($n_s\sim 1$) on large scales, which is favored for the structure formation of the universe [23]. This shallow slope on small scales and rapid jump at the horizon scale of the turning epoch from one to another inflation make the mass range of primordial black holes very narrow.

The primordial black holes play a role as a usual cold dark matter in the large scale structure formation. The scales of the fluctuations for the primordial black hole formation themselves are much smaller than the galactic scale and thus we cannot see any signals for such fluctuations in $\delta T/T$ measurements.

The primordial black holes are also attractive as a source of gravitational waves. If the primordial black holes dominate dark matter of the present universe, some of them likely form binaries. Such binary black holes coalesce and produce significant gravitational waves [24] which may be detected by future detectors.

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[10]The present universe contains e^{60} regions which were horizons at the beginning of the new inflation. For $R\lesssim 0.03$, the probability that the e-fold number of a region exceeds $N_*+1.3$ is less than e^{-60}. Thus, the effect of quantum fluctuations of φ is negligible for $R\lesssim 0.03$.

[11]The allowed range of ζ becomes narrower if we take κ larger than 0.3.

[12]In this paper we have used a large quartic coupling in the Kähler potential $-k|\phi|^2|S|^2$ to produce the large effective mass for the inflaton φ. An alternative is to introduce an extra field X which gives the effective mass during the preinflation. For example, consider a superpotential $W_X=\tilde{g}X\phi^2+mX\bar{X}$. If $v^2\ll m\lesssim\mu$, X may have a large value $X\sim 1$ during the preinflation, which gives a large effective mass $\tilde{g}X$. After the preinflation X and $\bar{X}$ take vacuum expectation values $X\simeq 0$ and $\bar{X}\simeq\tilde{g}\phi^2/m$, and hence they never affect the dynamics of the new inflation. In this alternative model it is sufficient to take $\tilde{g}\sim 10^{-4}$ for our purpose.

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