Cooling modes of neutron stars

Chueng-Ryong Ji
Department of Physics, North Carolina State University, Raleigh, North Carolina 27695-8202

Dong-Pil Min
Department of Physics, North Carolina State University, Raleigh, North Carolina 27695-8202
and Department of Physics, Seoul National University, Seoul, Korea
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The direct Urca process is studied in the presence of kaon condensation. We consider the contributions from both strange and nonstrange hadrons. Our calculations indicate that the role of hyperons is as important as the role of nonstrange nucleons in the cooling mechanism of neutron stars. We also find that the temperature dependence of the weak neutral current contribution is given by T^7, which intermediates between the direct Urca process of T^6 and the modified Urca process of T^8. [S0556-2821(98)01910-9]

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I. INTRODUCTION

Neutron stars are made in the aftermath of type II supernovae explosions which result from the gravitational core collapse of massive stars. While the masses of type II supernovae are greater than $8M_\Theta$, where the solar mass $M_\Theta=2\times10^{33}$ g, the masses of neutron stars are between one and two solar masses [1]. The radius of a neutron star [2] is only about 10 km, while the radius of our sun is 6.96×10^5 km. Thus, the central density could reach as high as five to ten times the normal nuclear density, $\rho_0=2.65\times10^{14}$ g cm^{-3}. For comparison, the solar density is only 1.4 g cm^{-3}.

Neutron stars are born with interior temperatures of the order 20–50 MeV, but rapidly cool via neutrino emission to temperatures of less than 1 MeV within minutes. Then, the long-term cooling processes take place for about 10^5 to 10^6 years until the interior temperatures reach 10^6 K and photons are emitted from the surface. The chief mechanism of long-term cooling processes is the emission of neutrinos and antineutrinos from matter in the interior of the neutron star. Which reactions emitting neutrinos are most effective in removing energy from the neutron star has been the subject of numerous studies [3–6,8–11]. The standard model of the long-term cooling is the modified Urca process, even though the direct Urca process has a potential to be more effective in cooling neutron stars than the modified Urca process because the temperature dependence of the direct Urca process is T^6, while that of the modified Urca process is T^8. The reason why the direct Urca process is not feasible in the ordinary nuclear matter is because of the small proton fraction $x=n_p/(n_p+n_n)<0.03$ [2], while the energy-momentum conservation of the direct Urca process requires the minimum proton fraction to be $x_c=1/9$. The minimum proton fraction is determined by the fact that at temperatures well below typical Fermi temperatures ($T_F\sim10^{12}$ K), fermions participating in the process must have momenta close to the Fermi momenta p_{Fi}, where subscripts $i=n$, p, and e correspond to neutrons, protons, and electrons, respectively. Since neutrino and antineutrino momenta are $\sim kT/c\ll p_{Fi}$, the condition for momentum conservation is $p_{Fp}+p_{Fe}>p_{Fn}$. If the matter consists only of neutrons, protons, and electrons, the charge neutrality requires that $n_p=n_e$, where $n_i\sim p_{Fi}^3$ are the particle densities and thus the condition becomes $2p_{Fp}>p_{Fn}$, or $n_n<8n_p$, and the proton fraction at threshold ($n_n=8n_p$) is $x_c=1/9$. However, this threshold is not difficult to be reached if there exist boson condensations such as π^- [12] or K^- [13]. While the pion condensation due to p-wave interactions is not forthcoming because of the enhanced many body effects, it may be still feasible to consider the kaon condensation due to s-wave interactions. In this review, we thus present the calculations of the direct Urca processes in the presence of kaon condensation. We consider the contributions from both strange and nonstrange hadrons. Our calculations indicate that the role of hyperons is not negligible compare to the role of nonstrange nucleons. We also consider the weak neutral current contribution and find that the temperature dependence of the weak neutral current contribution is given by T^7, which intermediates between the direct Urca process of T^6 and the modified Urca process of T^8.

In the next section, Sec. II, the weak interaction for Urca process is reviewed in the presence of kaon condensation. In Sec. III, we calculate the neutrino emissivity (or luminosity). The calculations of the averaged amplitude square and the phase space integration are shown in detail and the results are presented for both strange and nonstrange hadrons. Then, the results on the weak neutral current are presented in Sec. IV. The conclusions and discussions are followed in Sec. V.

II. WEAK INTERACTIONS FOR URCA PROCESSES IN KAON CONDENSATION

A. Weak interactions for Urca processes

We start from the quark-based standard model weak interaction [14]. For the process $d'\to ue\bar{\nu}$, the invariant amplitude is given by

$$M=\frac{g}{2\sqrt{2}}\overline{\psi}_u\gamma^\mu(1-\gamma_5)\psi_{d'}\frac{1}{M_W^2}\frac{g}{2\sqrt{2}}\overline{\psi}_e\gamma^\mu(1-\gamma_5)\psi_\nu$$
$$=\left(\frac{g}{2\sqrt{2}M_W}\right)^2\left(\overline{\psi}_q\frac{\lambda_1+i\lambda_2}{2}\gamma^\mu(1-\gamma_5)\psi_q\cos\theta_c+\overline{\psi}_q\frac{\lambda_4+i\lambda_5}{2}\gamma^\mu(1-\gamma_5)\psi_q\sin\theta_c\right)\overline{\psi}_e\gamma^\mu(1-\gamma_5)\psi_\nu, \tag{2.1}$$

where the Cabbibo mixing is given by

$$\begin{pmatrix} d' \\ s' \end{pmatrix}=\begin{pmatrix} \cos\,\theta_c & \sin\,\theta_c \\ -\sin\,\theta_c & \cos\,\theta_c \end{pmatrix}\begin{pmatrix} d \\ s \end{pmatrix} \tag{2.2}$$

and the quark triplet ψ_q and antitriplet $\overline{\psi_q}$ are given by

$$\psi_q=\begin{pmatrix} u \\ d \\ s \end{pmatrix},\quad \overline{\psi_q}=(\overline{u},\overline{d},\overline{s}). \tag{2.3}$$

Now, if we define the following currents:

$$V_\alpha^\mu=\overline{\psi_q}\frac{\lambda_\alpha}{2}\gamma^\mu\psi_q, \tag{2.4}$$

$$A_\alpha^\mu=\overline{\psi_q}\frac{\lambda_\alpha}{2}\gamma^\mu\gamma_5\psi_q, \tag{2.5}$$

$$l^\mu=\overline{\psi_e}\gamma^\mu(1-\gamma_5)\psi_\nu, \tag{2.6}$$

and the weak interaction coupling constant

$$\left(\frac{g}{2\sqrt{2}M_W}\right)^2=\frac{G_W}{\sqrt{2}}, \tag{2.7}$$

then the above invariant amplitude becomes

$$M=\frac{G_W}{\sqrt{2}}J^\mu l_\mu, \tag{2.8}$$

where

$$J^\mu=(V_{1+2i}^\mu-A_{1+2i}^\mu)\cos\theta_c+(V_{4+5i}^\mu-A_{4+5i}^\mu)\sin\theta_c. \tag{2.9}$$

The time reversed reaction is also given by the Hermitian conjugate and thus the effective Lagrangian for the direct Urca process is given by

$$L=\frac{G_W}{\sqrt{2}}(J^\mu l_\mu+\mathrm{H.c.}). \tag{2.10}$$

B. Kaon condensation

As it is well known in $SU(2)_L\times SU(2)_R$ chiral theory, the spontaneous symmetry breaking of chiral symmetry is described by the degenerate ground state of the Mexican-hat shape potential as a function of σ and π fields [15]. Similarly, in $SU(3)_L\times SU(3)_R$ space, the kaon condensation can be described by the unitary transformation given by [5,16]

$$U_K(\theta,\mu)=\exp(i\mu t\hat{Q}_{\mathrm{em}})\exp(i\theta F_4^5), \tag{2.11}$$

where $F_\alpha(F_\alpha^5), \alpha=1\sim8$ are the vector (axial-vector) charges which satisfy the current algebra and $\hat{Q}_{\mathrm{em}}$ is the electromagnetic charge $\hat{Q}_{\mathrm{em}}=F_3+\sqrt{1/3}F_8$. Here the chiral angle θ represents the amount of the kaon condensation and the factor μ is its chemical potential. As one can easily see from the meson octet representation,

$$\psi_q\overline{\psi_q}=\begin{pmatrix} u\overline{u} & u\overline{d} & u\overline{s} \\ d\overline{u} & d\overline{d} & d\overline{s} \\ s\overline{u} & s\overline{d} & s\overline{s} \end{pmatrix}$$
$$=\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}, \tag{2.12}$$

the SU(3) generator

$$\lambda_4=\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{2.13}$$

yields both K^+ and K^-. Thus, the vacuum expectation value of kaon condensation v_K is given by

$$v_K=\langle 0|e^{-i\theta F_4^5}K^\pm e^{i\theta F_4^5}|\rangle=\frac{f\sin\theta}{\sqrt{2}}, \tag{2.14}$$

where the factor $\sqrt{2}$ comes from the normalization of two degrees of freedom K^+ and K^-. Since the kaon condensed vacuum $|K\rangle$ is related to the ordinary meson vacuum $|0\rangle$ by

$$|K\rangle=U_K(\theta,\mu)|0\rangle, \tag{2.15}$$

one obtains $\langle K|K^\pm|K\rangle$ as follows:

$$\langle K|K^\pm|K\rangle=e^{\mp i\mu t}\langle 0|e^{-i\theta F_4^5}K^\pm e^{i\theta F_4^5}|0\rangle=\frac{f\sin\theta}{\sqrt{2}}e^{\mp i\mu t}, \tag{2.16}$$

where we calculate the factor $e^{\mp i\mu t}$ from

$$\exp\left(-i\mu t\frac{\lambda_3+\sqrt{1/3}\lambda_8}{2}\right)\frac{\lambda_4\pm i\lambda_5}{2}\exp\left(i\mu t\frac{\lambda_3+\sqrt{1/3}\lambda_8}{2}\right)$$
$$=e^{\mp i\mu t}\frac{\lambda_4\pm i\lambda_5}{2} \tag{2.17}$$

for $K^\pm$, respectively.

C. Weak interactions in the kaon condensation

The weak interactions in the presence of the kaon condensation is given by the similarity transformation of the original weak interaction Lagrangian given by Eq. (2.10) [5,16]. Thus, the hadronic current in the weak interaction J^μ given by Eq. (2.9) is now changed to the new current $\widetilde{J}^\mu$,

$$\widetilde{J}^\mu = U_K^{-1}(\theta,\mu) J^\mu U_K(\theta,\mu). \tag{2.18}$$

Substituting Eq. (2.11) to Eq. (2.18) and using the SU(3) algebra, we obtain [17], modulo $e^{\mp i\mu t}$,

$$\begin{aligned}\widetilde{J}^\mu = {} & \cos\theta_c \cos\frac{\theta}{2}(V^\mu_{1+2i} - A^\mu_{1+2i}) \\ & + \sin\theta_c (V^\mu_4 - A^\mu_4) + i\sin\theta_c\cos\theta(V^\mu_5 - A^\mu_5) \\ & - i\cos\theta_c \sin\frac{\theta}{2}(V^\mu_{6-i7} - A^\mu_{6-i7}) \\ & + \frac{i}{2}\sin\theta_c\sin\theta[V^\mu_3 - A^\mu_3 + \sqrt{3}(V^\mu_8 - A^\mu_8)].\end{aligned} \tag{2.19}$$

Each term in the current $\widetilde{J}^\mu$ is responsible for a specific channel of direct Urca process. The following summarizes which current governs which channel:

$$\begin{aligned} & V^\mu_{1+2i} - A^\mu_{1+2i}; \quad n \to p + e + \overline{\nu}, \\ & V^\mu_4 - A^\mu_4, \quad V^\mu_5 - A^\mu_5; \quad n \to \Sigma^- + e^+ + \nu, \\ & V^\mu_{6-i7} - A^\mu_{6-i7}; \quad n + \langle K^- \rangle \to \begin{pmatrix} \Sigma^0 \\ \Lambda \end{pmatrix} + e + \overline{\nu}, \\ & V^\mu_3 - A^\mu_3 + \sqrt{3}(V^\mu_8 - A^\mu_8); \quad n + \langle K^- \rangle \to n + e + \overline{\nu}, \\ & \qquad p + \langle K^- \rangle \to p + e + \overline{\nu}. \end{aligned} \tag{2.20}$$

Here, it should be understood that all the processes are under the influence of the $\langle K^- \rangle$ condensation even though we explicitly write $\langle K^- \rangle$ only for the processes requiring $\langle K^- \rangle$ in the initial state for the charge conservation. Thus far, only the relevant terms for the nonstrange nucleonic matter such as the first and the last terms in Eq. (2.19) have been studied extensively [16]. In this work, we consider the rest terms in Eq. (2.19) as well. As we shall see, it turns out that the terms responsible for the cooling modes due to strangeness are not negligible compared to the ones considered only for the nonstrange nucleonic matter.

III. NEUTRINO EMISSIVITY

Since we have obtained the effective weak interaction Lagrangian for the Urca process in the presence of kaon condensation, we now apply this to calculate the neutrino emissivity or luminosity. From the Fermi's "golden rule" [18], the neutrino emissivity in the presence of K^- condensation is given by [3]

$$\begin{aligned}\epsilon_{\overline{\nu}} = {} & \int \frac{d^3\vec{p}}{(2\pi)^3}\int \frac{d^3\vec{p}\,'}{(2\pi)^3}\int \frac{d^3\vec{p}_e}{(2\pi)^3 2E_e}\int \frac{d^3\vec{p}_{\overline{\nu}}}{(2\pi)^3 2E_{\overline{\nu}}} \\ & \times E_{\overline{\nu}}(2\pi)\delta(E' + E_e + E_{\overline{\nu}} - E - \mu)(2\pi)^3 \\ & \times \delta^3(\vec{p}\,' - \vec{p} - \vec{p}_e - \vec{p}_{\overline{\nu}})|M_{\alpha\beta}|^2 f_\alpha(E)[1 - f_\beta(E')] \\ & \times [1 - f_e(E_e)],\end{aligned} \tag{3.1}$$

where the Fermi-Dirac distribution functions for the initial and final baryons, B_α and B_β, respectively, and the electron are given by ($i = B_\alpha, B_\beta, e$)

$$f_i(E_i) = \frac{1}{1 + \exp[(E_i - \mu_i)/T]}. \tag{3.2}$$

In Eq. (3.1), $\vec{p}$, $\vec{p}\,'$, $\vec{p}_e$, and $\vec{p}_{\overline{\nu}}$ are the momenta of B_α, B_β, e, and $\overline{\nu}$ in the process of $B_\alpha \to B_\beta + e + \overline{\nu}$ and E, E', E_e, and $E_{\overline{\nu}}$ are the corresponding energies. It should be noted that the energy conservation is modified by the chemical potential μ, which stems from the time dependence of the K^- condensation. For example, the process $n + \langle K^- \rangle \to n + e^- + \overline{\nu}$ requires this modification due to the appearance of the K^- condensation in the initial state. However, this modification should also be understood in accordance with the charge conservation. For instance, the process $n \to p + e + \overline{\nu}$ involves the K^- condensation in both initial and final states due to the charge conservation and thus the cancellation of the factor μ in the energy conservation of this process is implicit in Eq. (3.1). Before we show explicitly in the subsection III B how to do the phase space integration of Eq. (3.1), we will present first the calculations of the relevant amplitude square, $|M_{\alpha\beta}|^2$, for various baryons B_α and B_β.

A. Calculation of amplitude square

In order to calculate the relevant amplitude $M_{\alpha\beta}$, one should know the rules to calculate the matrix elements of vector and axial currents, V^μ_a and A^μ_a, for $a = 1,2,\ldots,8$. From SU(3) algebra, these are given by [16]

$$\langle B_\beta | V^\mu_a | B_\alpha \rangle = \overline{\psi}_\beta \gamma^\mu \psi_\alpha i\mathrm{Tr}\left(\frac{\lambda_a}{2}[B_\alpha, B^\dagger_\beta]\right), \tag{3.3}$$

$$\begin{aligned}\langle B_\beta | A^\mu_a | B_\alpha \rangle = {} & \overline{\psi}_\beta \gamma^\mu \gamma_5 \psi_\alpha \left[D\mathrm{Tr}\left(\frac{\lambda_a}{2}\{B_\alpha, B^\dagger_\beta\}\right) \right. \\ & \left. + F\mathrm{Tr}\left(\frac{\lambda_a}{2}[B_\alpha, B^\dagger_\beta]\right)\right],\end{aligned} \tag{3.4}$$

where $F + D = g_A$ and $F - D/3 = \widetilde{g}_A$. With these rules at hand, we now show explicitly how to calculate the relevant amplitude square. To do this, we have to consider a specific process. As an example, we consider here the process $n \to p + e + \overline{\nu}$. Then, the matrix element $\langle p | \widetilde{J}^\mu | n \rangle$ is obtained from Eq. (2.20) as

$$\langle p | \widetilde{J}^\mu | n \rangle = \cos\theta_c \cos\frac{\theta}{2}\langle p | V^\mu_{1+i2} - A^\mu_{1+i2} | n \rangle. \tag{3.5}$$

Since the baryon octet is given by

$$B=\begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \overline{\Xi^0} & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}, \tag{3.6}$$

it is straightforward to calculate the V-A current matrix element:

$$\begin{aligned}\langle p|V^{\mu}_{1+i2}-A^{\mu}_{1+i2}|n\rangle &= \bar{\psi}_p\gamma^{\mu}\psi_n i\mathrm{Tr}\left(\frac{\lambda_1+i\lambda_2}{2}\left[\frac{\lambda_6+i\lambda_7}{2},\frac{\lambda_4-i\lambda_5}{2}\right]\right)\\ &\quad -\bar{\psi}_p\gamma^{\mu}\gamma_5\psi_n\left\{D\mathrm{Tr}\left(\frac{\lambda_1+i\lambda_2}{2}\,\frac{\lambda_6+i\lambda_7}{2},\frac{\lambda_4-i\lambda_5}{2}\right)\right.\\ &\quad \left.+F\mathrm{Tr}\left(\frac{\lambda_1+i\lambda_2}{2}\left[\frac{\lambda_6+i\lambda_7}{2},\frac{\lambda_4-i\lambda_5}{2}\right]\right)\right\}\\ &= i\bar{\psi}_p\gamma^{\mu}\psi_n-(D+F)\bar{\psi}_p\gamma^{\mu}\gamma_5\psi_n\,. \end{aligned}\tag{3.7}$$

In order to calculate the square of the relevant amplitude $M_{\alpha\beta}$, the hadronic current tensor $H^{\mu\nu}$,

$$H^{\mu\nu}=\langle n|\tilde{J}^{\mu\dagger}|p\rangle\langle p|\tilde{J}^{\nu}|n\rangle \tag{3.8}$$

is then multiplied by the leptonic current tensor

$$L_{\mu\nu}=2\{p_e^{\nu}p_{\bar{\nu}}^{\mu}+p_e^{\mu}p_{\bar{\nu}}^{\nu}-(p_e\cdot p_{\bar{\nu}})g^{\mu\nu}\}/(p_e^0p_{\bar{\nu}}^0), \tag{3.9}$$

where we took into account the factors $2E_e$ and $2E_{\bar{\nu}}$ from the Lorentz invariant phase space integration of electron and antineutrino in Eq. (3.1). Since the baryons can be treated as nonrelativistic particles, the momentum dependence of the reduced-squared-matrix element $H^{\mu\nu}L_{\mu\nu}$ turns out to be so small that the square of $M_{\alpha\beta}$ can be replaced by the average value $|\overline{M}_{\alpha\beta}|^2$ that is averaged over the momentum directions of electron and the antineutrino. We then obtain, modulo the factor $G_W^2/2$,

$$|\overline{M}_{np}|^2=4\cos^2\theta_c\cos^2\frac{\theta}{2}(1+3g_A^2). \tag{3.10}$$

The same result was obtained in Ref. [16]. Likewise, we have computed $|\overline{M}_{nn}|^2$ and $|\overline{M}_{pp}|^2$ and verified that our results are same with the ones given in Ref. [16]:

$$\begin{aligned}|\overline{M}_{nn}|^2&=\tfrac{1}{4}\sin^2\theta_c\sin^2\theta[4+3(g_A-3\tilde{g}_A)^2],\\ |\overline{M}_{pp}|^2&=\tfrac{1}{4}\sin^2\theta_c\sin^2\theta[16+3(g_A+3\tilde{g}_A)^2].\end{aligned}\tag{3.11}$$

With the nominal values of $g_A\approx1.25$, $\tilde{g}_A\approx0.15$, and $\sin\theta_c\approx0.22$, the results of $|\overline{M}_{np}|^2$, $|\overline{M}_{nn}|^2$, and $|\overline{M}_{pp}|^2$ are estimated as

$$|\overline{M}_{np}|^2\approx22\cos^2\frac{\theta}{2},\quad |\overline{M}_{nn}|^2\approx0.07\sin^2\theta,\quad |\overline{M}_{pp}|^2\approx0.3\sin^2\theta. \tag{3.12}$$

As we mentioned earlier, we have also considered the strange sectors from Eq. (2.20) and found the results for $|\overline{M}_{n\Sigma^-}|^2$, $|\overline{M}_{n\Sigma^0}|^2$, and $|\overline{M}_{n\Lambda}|^2$ as follows;

$$\begin{aligned}|\overline{M}_{n\Sigma^-}|^2&=\tfrac{1}{4}\sin^2\theta_c(1-\cos\theta)^2[4+3(g_A-3\tilde{g}_A)^2],\\ |\overline{M}_{n\Sigma^0}|^2&=\tfrac{1}{2}\cos^2\theta_c\sin^2\frac{\theta}{2}[4+3(g_A-3\tilde{g}_A)^2],\\ |\overline{M}_{n\Lambda}|^2&=\tfrac{3}{2}\cos^2\theta_c\sin^2\frac{\theta}{2}[4+3(g_A+\tilde{g}_A)^2].\end{aligned}\tag{3.13}$$

Thus, we find that the ratio of $|\overline{M}_{n\Sigma^0}|^2/|\overline{M}_{nn}|^2$ is independent of the values of g_A and $\tilde{g}_A$ and if we just use $\sin\theta_c\approx0.22$, then in the limit of $\theta\to0$ the ratio becomes

$$\frac{|\overline{M}_{n\Sigma^0}|^2}{|\overline{M}_{nn}|^2}\approx10. \tag{3.14}$$

If we now use the values of $g_A\approx1.25$ and $\tilde{g}_A\approx0.15$, then we also obtain

$$\frac{|\overline{M}_{n\Lambda}|^2}{|\overline{M}_{n\Sigma^0}|^2}\approx5. \tag{3.15}$$

Therefore, the strange baryon contribution is not suppressed at all in the cooling processes compare to the nonstrange contribution even though $n\to p+e+\bar{\nu}$ process should still be regarded as the dominant cooling mechanism.

B. Phase space integration

Now, let us describe in detail how to do the phase space integration in Eq. (3.1). First, we separate the angular integration defined as

$$A=\int d\Omega_p d\Omega_{p'}d\Omega_e d\Omega_{\bar{\nu}}\delta^3(\vec{p}\,'-\vec{p}-\vec{p}_e-\vec{p}_{\bar{\nu}}), \tag{3.16}$$

so that the Eq. (3.1) becomes

$$\begin{aligned}\epsilon_{\bar{\nu}}=\int & p^2dp\,p'^2dp'\,p_e^2dp_e\,p_{\bar{\nu}}^2dp_{\bar{\nu}}E_{\bar{\nu}}\\ &\times\delta(E'+E_e+E_{\bar{\nu}}-E-\mu)\\ &\times f_p(1-f_{p'})(1-f_e)|\overline{M}|^2\frac{A}{(2\pi)^8},\end{aligned}\tag{3.17}$$

where p, p', p_e, and $p_{\bar{\nu}}$ are the magnitudes of the momenta $\vec{p}$, $\vec{p}\,'$, $\vec{p}_e$, and $\vec{p}_{\bar{\nu}}$, respectively, and the factors $2E_e$ and $2E_{\bar{\nu}}$ from the Lorentz invariant phase space integration of e and $\bar{\nu}$ are now absorbed in $|\overline{M}|^2$ as we discussed in the last subsection [see Eq. (3.9)]. Since the momentum of antineutrino is negligible compared to the Fermi momenta of fermions, we can safely neglect $\vec{p}_{\bar{\nu}}$ inside the momentum delta function in Eq. (3.16) and obtain

$$\delta^3(\vec{p}\,'-\vec{p}-\vec{p}_e)=\frac{1}{p_e^2}\,\delta(p_e-|\vec{p}\,'-\vec{p}|)\,\delta(\Omega_e-\Omega_e^0)$$
$$=\frac{1}{p_e p p'}\,\delta(\cos\theta_p-\cos\theta_p^0)\,\delta(\Omega_e-\Omega_e^0), \tag{3.18}$$

where Ω_e^0 and $\cos\theta_p^0$ are given by the products of $\vec{p}$, $\vec{p}\,'$, and $\vec{p}_e$. Then the rest of the angular integration is simply reduced to

$$A=\frac{32\pi^3}{p_e p p'}. \tag{3.19}$$

After we substitute A in Eq. (3.18), we obtain

$$\epsilon_{\bar{\nu}}=\frac{32\pi^3}{(2\pi)^8}\int p\,dp\,p'\,dp'\,p_e dp_e p_{\bar{\nu}}^2 dp_{\bar{\nu}} E_{\bar{\nu}}$$
$$\times\,\delta(E'+E_e+E_{\bar{\nu}}-\mu)f_p(1-f_{p'})(1-f_e)|\overline{M}|^2. \tag{3.20}$$

Here, the nonrelativisitic treatment of the baryons leads to the change of integration variables as

$$p\,dp=m\,dE,\quad p'\,dp'=m'\,dE', \tag{3.21}$$

where m and m' are the masses of the initial and final baryons, respectively, and the relativistic treatment of the leptons yields,

$$p_e=E_e,\quad p_{\bar{\nu}}=E_{\bar{\nu}}. \tag{3.22}$$

Also, the Pauli factors for the fermions are given by

$$f_p=\frac{1}{1+\exp[(E-\mu_p)/T]},$$
$$1-f_{p'}=\frac{1}{1+\exp[-(E'-\mu_{p'})/T]},$$
$$1-f_e=\frac{1}{1+\exp[-(E_e-\mu_e)/T]}. \tag{3.23}$$

Thus, the Eq. (3.18) becomes

$$\epsilon_{\bar{\nu}}=\frac{32\pi^3 mm'}{(2\pi)^8}|\overline{M}|^2\int dE\,dE'\,E_e dE_e E_{\bar{\nu}}^3 dE_{\bar{\nu}}\,\delta(E'+E_e+E_{\bar{\nu}}-E-\mu)$$
$$\times\frac{1}{1+\exp[(E-\mu_p)/T]}\;\frac{1}{1+\exp[-(E'-\mu_{p'})/T]}\;\frac{1}{1+\exp[-(E_e-\mu_e)/T]}. \tag{3.24}$$

Now, let us define the dimensionless variables as follows:

$$x_1=\frac{E-\mu_p}{T},\quad x_2=-\frac{E'-\mu_{p'}}{T},\quad x_3=-\frac{E_e-\mu_e}{T},\quad x_4=\frac{E_{\bar{\nu}}}{T}. \tag{3.25}$$

Then, by substituting the energies by these dimensionless variables and using the β-equilibrium condition $\mu_{p'}+\mu_e=\mu_p+\mu$ (μ is the chemical potential of the K^- condensation.), the phase space integration is reduced to

$$\epsilon_{\bar{\nu}}=\frac{32\pi^3 mm'\mu_e}{(2\pi)^8}|\overline{M}|^2T^6\int_{-\infty}^{+\infty}dx_1\int_{-\infty}^{+\infty}dx_2\int_{-(x_1+x_2)}^{\infty}dx_3$$
$$\times\frac{(x_1+x_2+x_3)^3}{(1+e^{x_1})(1+e^{x_2})(1+e^{x_3})}=\frac{457\pi mm'\mu_e}{16\times 2520}|\overline{M}|^2T^6, \tag{3.26}$$

where we neglected the temperature compare to the chemical potentials of the fermions. This result is consistent with what others obtained in the literature [4–6].

IV. THE WEAK NEUTRAL CURRENT

The procedure that we took for the weak interactions with $W^{\pm}$ exchanges can be applied to the weak neutral current mediated by Z^0 similarly. One of the reasons why it is interesting to investigate the weak neutral current is because the usual standard model without any kaon condensation suppresses the flavor changing neutral current due to the GIM mechanism [7]. It is worth checking if indeed some channel of flavor changing neutral current is open via the kaon condensation.

After taking into account the GIM mechanism, the Cabbibo mixing angles do not appear in the effective Lagrangian for the Z^0 interaction;

$$L_{Z^0}=\frac{G_W}{\sqrt{2}}\{(J^{\mu}_{Z^0,u}+J^{\mu}_{Z^0,d}+J^{\mu}_{Z^0,s})l_{Z^0,\mu}+\text{H.c.}\}, \tag{4.1}$$

where the current $J^{\mu}_{Z^0,u}$, $J^{\mu}_{Z^0,d}$, and $J^{\mu}_{Z^0,s}$ are given by

$$J^{\mu}_{Z^0,u}=\frac{1}{3}\left[\left(1-\frac{8}{3}\sin^2\theta_W\right)V_0^{\mu}-A_0^{\mu}\right] + \frac{\sqrt{3}}{6}\left[\left(1-\frac{8}{3}\sin^2\theta_W\right)V_8^{\mu}-A_8^{\mu}\right] + \frac{1}{2}\left[\left(1-\frac{8}{3}\sin^2\theta_W\right)V_3^{\mu}-A_3^{\mu}\right], \tag{4.2}$$

$$J^{\mu}_{Z^0,d}=\frac{1}{3}\left[\left(1-\frac{4}{3}\sin^2\theta_W\right)V_0^{\mu}-A_0^{\mu}\right] + \frac{\sqrt{3}}{6}\left[\left(1-\frac{4}{3}\sin^2\theta_W\right)V_8^{\mu}-A_8^{\mu}\right] - \frac{1}{2}\left[\left(1-\frac{4}{3}\sin^2\theta_W\right)V_3^{\mu}-A_3^{\mu}\right], \tag{4.3}$$

and

$$J^{\mu}_{Z^0,s}=\frac{1}{3}\left[\left(1-\frac{4}{3}\sin^2\theta_W\right)V_0^{\mu}-A_0^{\mu}\right] + \frac{1}{\sqrt{3}}\left[\left(1-\frac{4}{3}\sin^2\theta_W\right)V_8^{\mu}-A_8^{\mu}\right]. \tag{4.4}$$

In Eq. (4.1), the leptonic current $l_{Z^0,\mu}$ is given by

$$l_{Z^0,\mu}=\overline{\psi}_{\nu}\gamma_{\mu}(1-\gamma_5)\psi_{\nu}, \tag{4.5}$$

and θ_W is the Weinberg angle in Eqs. (4.2)–(4.4). The effect of the kaon condensation to the neutral currents $J_{Z^0,u}$,$J_{Z^0,d}$, and $J_{Z^0,s}$ can be obtained by the similarity transformation as shown in Eq. (2.19). Interestingly, $J_{Z^0,d}$ is invariant under the similarity transformation, while $J_{Z^0,s}$ is transformed to

$$\begin{aligned}\widetilde{J}^{\mu}_{Z^0,s}&=U_K^{-1}(\theta,\mu)J^{\mu}_{Z^0,s}U_K(\theta,\mu)\\ &=\frac{1}{3}(\widetilde{V}_0^{\mu}-A_0^{\mu})+\frac{1}{2}\sin^2\frac{\theta}{2}(\widetilde{V}_3^{\mu}-A_3^{\mu})\\ &\quad+\frac{\sqrt{3}}{6}\left(1-3\cos^2\frac{\theta}{2}\right)(\widetilde{V}_8^{\mu}-A_8^{\mu})\\ &\quad+\sin\frac{\theta}{2}\cos\frac{\theta}{2}(\widetilde{V}_5^{\mu}-A_5^{\mu}),\end{aligned} \tag{4.6}$$

where $\widetilde{V}_i^{\mu}=(1-\frac{4}{3}\sin^2\theta_W)V_i^{\mu}$. The invariance of $J_{Z^0,d}$ under the similarity transformation leads to the absence of $n\to\Lambda+\nu+\overline{\nu}$ and $n\to\Sigma^0+\nu+\overline{\nu}$ processes among the flavor neutral changing currents even if the kaon condensation is turned on. However, $J_{Z^0,u}$ is transformed as follows:

$$\begin{aligned}\widetilde{J}^{\mu}_{Z^0,u}&=U_K^{-1}(\theta,\mu)J^{\mu}_{Z^0,u}U_K(\theta,\mu)\\ &=\frac{1}{3}(\overline{V}_0^{\mu}-A_0^{\mu})+\frac{1}{2}\cos^2\frac{\theta}{2}(\overline{V}_3^{\mu}-A_3^{\mu})\\ &\quad+\frac{\sqrt{3}}{6}\left(3\cos^2\frac{\theta}{2}-2\right)(\overline{V}_8^{\mu}-A_8^{\mu})\\ &\quad-\sin\frac{\theta}{2}\cos\frac{\theta}{2}(\overline{V}_5^{\mu}-A_5^{\mu}),\end{aligned} \tag{4.7}$$

where $\overline{V}_i^{\mu}=(1-\frac{8}{3}\sin^2\theta_W)V_i^{\mu}$. Thus, we find that the channel of $n+\langle K^-\rangle\to\Sigma^-+\nu+\overline{\nu}$ is indeed open in the presence of $\langle K^-\rangle$ condensation. While the charge of the final state is obtained from the kaon condensation, this channel is one of the flavor changing process for the Z^0 interaction. The relevant amplitude square averaged over the initial and final spins is given by

$$|\overline{M}_{n\Sigma^-}|^2=\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\left[\left(1-\frac{8}{3}\sin^2\theta_W\right)+\frac{3}{4}(g_A-3\widetilde{g}_A)^2\right]. \tag{4.8}$$

As one can expect from the standard model, this process is suppressed in the limit $\theta\to 0$. The usual $n\to n+\nu+\overline{\nu}$ process depends also on the kaon condensation and its spin averaged amplitude square $|\overline{M}_{nn}|^2$ is given by

$$|\overline{M}_{nn}|^2=4\left\{\left[\left(1-\frac{1}{2}\sin^2\frac{\theta}{2}\right)-\frac{4}{3}\sin^2\theta_W\cos^2\frac{\theta}{2}\right]^2 +3\left(g_A+\frac{1}{4}(g_A-3\widetilde{g}_A)\sin^2\frac{\theta}{2}\right)^2\right\}. \tag{4.9}$$

Following the procedure presented in Sec. III B, we also calculated the neutrino emissivity from the interactions of weak neutral current and obtained

$$\epsilon_{\nu\overline{\nu}}=\frac{16\pi^3 mm'}{3(2\pi)^8}|\overline{M}_{\alpha\beta}|^2T^7 \times\int_{-\infty}^{+\infty}dx_1\int_{-x_1}^{+\infty}dx_2\frac{(x_1+x_2)^5}{(1+e^{x_1})(1+e^{x_2})}. \tag{4.10}$$

By noting

$$\int_{-\infty}^{+\infty}dx_1\int_{-x_1}^{+\infty}dx_2\frac{(x_1+x_2)^5}{(1+e^{x_1})(1+e^{x_2})}=720\zeta(7), \tag{4.11}$$

where $\zeta(7)\approx 1.00835$, we find

$$\epsilon_{\nu\overline{\nu}}=\frac{15\zeta(7)mm'}{\pi^5}|\overline{M}_{\alpha\beta}|^2T^7. \tag{4.12}$$

It is rather natural to expect that the temperature dependence in this case is T^7 because, compared to the process $B_\alpha\to B_\beta+e+\overline{\nu}$ discussed in the last two sections, the electron is replaced by the neutrino and thus the chemical potential of electron, μ_e, is replaced by the temperature T, in the process

$B_\alpha \rightarrow B_\beta + \nu + \bar{\nu}$. Thus, we observe that the temperature behavior of the weak neutral current process intermediates between the direct Urca process and the modified Urca process.

V. CONCLUSION AND DISCUSSION

In this paper, we have presented the derivation of the effective Lagrangian for the Urca processes in the presence of kaon condensation and applied to the calculations of the neutrino emissivity. We have shown explicitly how to do the phase space integration in the calculation of neutrino emissivity and computed the relevant average amplitude square $|\overline{M}_{n\beta}|^2$ for the neutron decay to various baryon octet members, B_β, including both nonstrange and strange sectors. We found that the ratio $|\overline{M}_{n\Sigma^0}|^2/|\overline{M}_{nn}|^2 \approx 10$ independent from the values of g_A and $\widetilde{g_A}$ in the limit of zero chiral angle θ. This leads us to estimate the importance of the hyperons in the cooling mechanism with the kaon condensation, which is comparable with the nonstrange nucleon contributions. We have also extended our calculations to include the weak neutral current contributions generated by Z^0. We find that the kaon condensation influences the current $J^\mu_{Z^0,u}$ for $u \rightarrow u\nu\bar{\nu}$, while the current $J^\mu_{Z^0,d}$ for $d \rightarrow d\nu\bar{\nu}$ is not affected by the kaon condensation. Thus, the $n + \langle K^- \rangle \rightarrow \Sigma^- + \nu + \bar{\nu}$ channel is indeed open in the presence of kaon condensation even though this channel is absent in the standard model due to the GIM mechanism. The results of the spin averaged invariant amplitude squares for $n + \langle K^- \rangle \rightarrow \Sigma^- + \nu + \bar{\nu}$ and $n \rightarrow n + \nu\bar{\nu}$ processes are presented in Eqs. (4.8) and (4.9). However, the most interesting observation in the weak neutral current contribution is that the temperature dependence of neutrino emissivity, $\epsilon_{\nu\bar{\nu}}$, is given by T^7. One may easily understand this behavior by noting that the chemical potential of the electron in the direct Urca process is replaced by the temperature due to the replacement of electron by the neutrino in the weak neutral current. In any case, we thus observe that the temperature behavior of the weak neutral current contribution intermediates between the direct Urca process of T^6 and the modified Urca process of T^8. Further detailed numerical computations would be necessary to give a more definite assessment of which cooling modes are most efficient in the neutron stars.

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