Decuplet baryon magnetic moments in the chiral quark model

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We present calculations of the decuplet baryon magnetic moments in the chiral quark model. As input we use parameters obtained in qualitatively accurate fits to the octet baryon magnetic moments studied previously. The values found for the magnetic moments of Δ^{++} and Ω^- are in good agreement with experiments. We finally calculate the total quark spin polarizations of the decuplet baryons and find that they are considerably smaller than what is expected from the non-relativistic quark model. [S0556-2821(98)03309-8]

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I. INTRODUCTION

Hadron structure, showing up in experiments such as deep inelastic scattering (DIS) on nucleons, flavor asymmetry measurements in Drell-Yan production, and measurements of magnetic moments and axial-vector form factors, does not always fit quantitatively very well with simple non-relativistic quark model (NQM) predictions.

It has been difficult, though, to understand these features from the QCD Lagrangian alone. Since, at low energies, phenomena related to chiral symmetry breaking play a major role in QCD, Manohar and Georgi [1] have suggested that these phenomena can be described by a system of Goldstone bosons (GBs) interacting with the valence quarks of the NQM. The GBs will give rise to a polarized quark sea and modify the spin polarizations of the quarks by the creation of correlated quark-antiquark pairs. The relevant scale on which this takes place is $\Lambda_{\chi\mathrm{SB}}$, which is assumed to be around 1 GeV. This is much higher than the confinement scale Λ_{QCD}, which is of the order of 200 MeV. Thus inside hadrons we have a system of quarks, gluons and GBs interacting with each other. This theory, sometimes denoted the chiral quark model (χQM), has been used recently to calculate the spin polarization of the quarks in the proton in DIS [2–5] and octet baryon magnetic moments [5]. Lately, also SU(3) symmetry breaking in the χQM Lagrangian has been included in the calculations [6–9].

Since the χQM is quite successful in describing both octet magnetic moments, the $\overline{u}$-$\overline{d}$ asymmetry, and the spin polarizations, it is of interest to examine its performance also for other baryonic systems, such as the spin 3/2 decuplet, using the same approximation as for the octet baryons.

The decuplet baryon magnetic moments have been calculated in several models, e.g. in quenched lattice gauge theory [10], quark models [11,12], the chiral bag model [13], chiral perturbation theory [14], chiral quark-soliton model [15], and QCD sum rules [16,17].

Here we present a calculation of the decuplet baryon magnetic moments and quark spin polarizations using the χQM in the same approximation as for the octet baryons [9].

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II. THE CHIRAL QUARK MODEL

The GBs of the χQM are pseudoscalars and will be denoted by the 0^- meson names π, K, η, η', as is usually done. For convenience we will follow closely the notation of Refs. [3,9]. The Lagrangian of the interaction is to lowest order $\widetilde{\mathcal{L}} = g_8 \overline{\mathbf{q}} \widetilde{\Phi} \gamma^5 \mathbf{q}$ with

$$\widetilde{\Phi} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} & \alpha K^0 \\ \alpha K^- & \alpha \overline{K^0} & -\beta\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \tag{1}$$

We have here introduced two SU(3) symmetry breaking parameters, α and β, which allow for different strengths of the production of GBs containing strange quarks.

In addition to the octet of GBs there is also an SU(3) singlet of η' bosons. These are coupled to the quarks with different strength, since the theory would otherwise be U(3) symmetric (when $\alpha=1$ and $\beta=1$), something that does not agree with the measurements of the flavor asymmetry by the New Muon Collaboration (NMC) [18,19] in DIS and the NA51 Collaboration [20] in Drell-Yan production. The SU(3) scalar interaction has the form $\mathcal{L}' = g_0 \overline{\mathbf{q}} \eta' \gamma^5 \mathbf{q}/\sqrt{3}$, so the total Lagrangian of interaction is $\mathcal{L}_I = \widetilde{\mathcal{L}} + \mathcal{L}'$.

The effect of this coupling is that the emission of the GBs will create quark-antiquark pairs from the vacuum with quantum numbers of the pseudoscalar mesons. Since the GBs are pseudoscalars, the quark-antiquark fluctuations leave a quark with a spin opposite to that of the initial quark, which was absorbed into the GB. This leads naturally to a spin flip for the quarks. The interaction of the GBs is weak enough to be treated by perturbation theory. This means that on long enough time scales for the low energy parameters to develop we have

$$u^{\uparrow} \rightleftharpoons (d^{\downarrow} + \pi^+) + (s^{\downarrow} + K^+) + (u^{\downarrow} + \pi^0, \eta, \eta'), \tag{2a}$$

$$d^{\uparrow} \rightleftharpoons (u^{\downarrow} + \pi^-) + (s^{\downarrow} + K^0) + (d^{\downarrow} + \pi^0, \eta, \eta'), \tag{2b}$$

$$s^{\uparrow} \rightleftharpoons (u^{\downarrow} + K^-) + (d^{\downarrow} + \overline{K^0}) + (s^{\downarrow} + \eta, \eta'). \tag{2c}$$

The probability of transforming a quark with spin up by one interaction can be expressed by the functions

$$|\psi(u^{\uparrow})|^2=\tfrac{1}{6}a\,(3+\beta^2+2\zeta^2)\hat{u}^{\downarrow}+a\hat{d}^{\downarrow}+a\alpha^2\hat{s}^{\downarrow}, \qquad (3a)$$

$$|\psi(d^{\uparrow})|^2=a\hat{u}^{\downarrow}+\tfrac{1}{6}a\,(3+\beta^2+2\zeta^2)\hat{d}^{\downarrow}+a\alpha^2\hat{s}^{\downarrow}, \qquad (3b)$$

$$|\psi(s^{\uparrow})|^2=a\alpha^2\hat{u}^{\downarrow}+a\alpha^2\hat{d}^{\downarrow}+\tfrac{1}{3}a\,(2\beta^2+\zeta^2)\hat{s}^{\downarrow}, \qquad (3c)$$

where the parameter $\zeta \equiv g_0/g_8$. The coefficient of a quark $\hat{q}^{\downarrow}$ is the transition probability to $q^{\downarrow}$. For example, in Eq. (3a), $a(3+\beta^2+2\zeta^2)/6$ is the probability for $u^{\uparrow}\rightarrow u^{\downarrow}$. The parameter a is proportional to $|g_8|^2$ and measures the probability of emission of a GB from a quark.

The total probabilities of no GB emission P_q, where $q=u,d,s$, are

$$P_u=P_d=1-a[(9+\beta^2+2\zeta^2)/6+\alpha^2], \qquad (4)$$

$$P_s=1-a[(2\beta^2+\zeta^2)/3+2\alpha^2]. \qquad (5)$$

The spin structure of a baryon B is described by the function

$$\hat{B}(xyz)=n_{x\uparrow}\hat{x}^{\uparrow}+n_{x\downarrow}\hat{x}^{\downarrow}+n_{y\uparrow}\hat{y}^{\uparrow}+n_{y\downarrow}\hat{y}^{\downarrow}+n_{z\uparrow}\hat{z}^{\uparrow}+n_{z\downarrow}\hat{z}^{\downarrow}. \qquad (6)$$

The coefficient $n_{q\uparrow\downarrow}$ of each symbol $\hat{q}^{\uparrow\downarrow}$ is the number of $q^{\uparrow\downarrow}$ quarks. The spin polarization for a quark q is then defined as

$$\Delta q^B=n_{q\uparrow}(B)-n_{q\downarrow}(B). \qquad (7)$$

The magnetic moment of a baryon is now obtained as

$$\mu(B)=\Delta u^B\mu_u+\Delta d^B\mu_d+\Delta s^B\mu_s, \qquad (8)$$

where μ_q is the magnetic moment of the quark of flavor q and Δq^B is the corresponding quark spin polarization for the baryon B. The total quark spin polarization of a baryon B is given by the expression

$$\Delta\Sigma^B=\Delta u^B+\Delta d^B+\Delta s^B. \qquad (9)$$

The expressions for the quark spin polarizations for the octet baryons can be found in Ref. [9].

III. OCTET BARYONS

First we have to specify the values of the parameters used in the model.

The constants a and ζ are estimated from the $\bar{u}-\bar{d}$ asymmetry ($\bar{u}-\bar{d}=-0.15\pm0.04$ and $\bar{u}/\bar{d}=0.51\pm0.09$). When $\beta=1$, the value of ζ is found to be in the interval $-4.3<\zeta<-0.7$. Following Cheng and Li [3], we use the value $\zeta=-1.2$, which gives $a\approx0.10$. ($a\approx0.10$ is in good agreement with Ref. [2].) However, when β is a free parameter in the calculations, the relation between a, ζ, and β is

$$a\simeq0.45/(3-2\zeta-\beta). \qquad (10)$$

This means that we have to use the relation $2\zeta+\beta\simeq-1.4$, in order to keep $a\approx0.10$, and therefore $\zeta=-0.7-\beta/2$.

We also use the relations $\mu_u=-2\mu_d$ and $\mu_s=2\mu_d/3$, typically used in the NQM.

TABLE I. Parameter values obtained in the different fits. The values in columns 3, 4, and 6 can be found in Ref. [9]. Hyphen (-) indicates that the parameter was not defined in the fit. The magnetic moment of the d quark, μ_d, is given in units of the nuclear magneton, μ_N.

Parameter/ Quantity	Experimental value	NQM	χQM	χQM with SU(3) symmetry breaking	
ζ			-1.2	-0.5	-1.2
μ_d		-0.91	-1.35	-1.30	-1.23
a		-	0.10[a]	0.15[a]	0.10[a]
α		-	1[a]	0.67	0.52
β		-	1[a]	1[a]	0.99
$\bar{u}-\bar{d}$	-0.15 ± 0.04	-	-0.15	-0.15	-0.15
$\bar{u}/\bar{d}$	0.51 ± 0.09	-	0.53	0.64	0.53
g_A	1.26 ± 0.01	$\frac{5}{3}$	1.12	1.18	1.24
$\Delta\Sigma^p$	0.30 ± 0.06	1	0.37	0.34	0.52

[a]Fixed in fit.

The remaining three parameters α, β, and μ_d are determined by a fit to the magnetic moments of the octet baryons [9]. As a result from this fit, we obtained $\alpha\approx0.52$, $\beta\approx0.99$, and $\mu_d\approx-1.23\mu_N$.

Recently, other choices of parameters have also been attempted [7,8]. One such choice is $\zeta=-0.3$. For $\alpha=\beta=0.6$ this simplifies Eq. (10), so that $a=0.15\pm0.04$. However, the value of $\bar{u}/\bar{d}$ becomes quite high, 0.68.

Since our fits affirm that $\beta\approx1$, we have fixed the value of this parameter to $\beta=1$ in a fit with $\zeta=-0.5$, which is presented for comparison. This value corresponds to no suppression of the η contribution ($\beta=1$) and large ($\zeta=-0.5$) suppression of the η' contribution. The price to be paid is that the value for $\bar{u}/\bar{d}$ goes out of the experimental range and becomes 0.64. Since this measurement is made only for $x=0.18$, this might nevertheless be a possibility to keep in mind.

Our parameters α and β correspond to the parameters ϵ and δ, respectively, in Ref. [8], whereas the parameter ϵ used in Ref. [7] is related to both our parameters α and β. The value $\epsilon=0.2$ used by [7] corresponds to our parameter values $\alpha\approx0.77$ and $\beta\approx1.86$. The high value for β, when $a=0.12$, which keeps the proton quark spin polarization down, destroys the agreement with the magnetic moments (which prefer $\beta\approx1$). We have therefore kept the value $\beta=1$, and fixed a to 0.15.

One could try to fix $\zeta=0$, corresponding to complete suppression of the η' contribution, and let $\beta=1$, corresponding to no η suppression. However, the value of $\bar{u}/\bar{d}$ then becomes 0.75 with $\bar{u}-\bar{d}\approx-0.08$. We have therefore not included this case in the tables.

Finally, we have also made a fit with only μ_d as a free parameter, i.e. when we have put $\alpha=\beta=1$ and $\zeta=-1.2$. See Table I for the result.

IV. DECUPLET BARYONS

The χQM can easily be applied to the decuplet. This will give us a further test of the predictability of the χQM and also a possibility to estimate the fraction of the spin carried by the quarks in the decuplet baryons. We will use the χQM in the same approximation for the decuplet as for the octet. The quark masses used in these fits should be understood as effective quark masses. They incorporate, together with the parameter a, describing the emission probability of the GBs, effects of relativistic corrections and possible exchange currents [21]. When relativistic corrections are included explicitly in the model, the fits become worse [22].

In order to calculate the spin polarization for the quarks in Δ^{++}, we must count the number of quarks of each flavor in the two polarizations states.

For example, the probability of transforming a $u^{\uparrow}$ quark with one interaction is given by Eq. (3a). After one interaction, all quarks that have participated in this interaction have their spins down. For a particle like $\Delta^{++}(J_z=+3/2)$, which has all valence u quarks with spin up, we can in this way understand that it will create a quite negatively polarized sea around it in the χQM.

The general expression for the spin structure of the decuplet baryons $B(xxy)$ is after one interaction given by

$$\hat{B}(xxy)=2P_x\hat{x}^{\uparrow}+P_y\hat{y}^{\uparrow}+2|\psi(x^{\uparrow})|^2+|\psi(y^{\uparrow})|^2. \quad (11)$$

Similarly, the spin structure for the Σ^{*0} is given by

$$\hat{\Sigma}^{*0}(uds)=P_u\hat{u}^{\uparrow}+P_d\hat{d}^{\uparrow}+P_s\hat{s}^{\uparrow}+|\psi(u^{\uparrow})|^2+|\psi(d^{\uparrow})|^2 +|\psi(s^{\uparrow})|^2. \quad (12)$$

The spin structures now lead to the following quark spin polarizations.

For the Δ^{++} the spin polarizations are given by

$$\Delta u^{\Delta^{++}}=3-a(6+3\alpha^2+\beta^2+2\zeta^2), \quad (13)$$

$$\Delta d^{\Delta^{++}}=-3a, \quad (14)$$

$$\Delta s^{\Delta^{++}}=-3a\alpha^2, \quad (15)$$

and for the Δ^{+} by

$$\Delta u^{\Delta^{+}}=2-a(5+2\alpha^2+\tfrac{2}{3}\beta^2+\tfrac{4}{3}\zeta^2), \quad (16)$$

$$\Delta d^{\Delta^{+}}=1-a(4+\alpha^2+\tfrac{1}{3}\beta^2+\tfrac{2}{3}\zeta^2), \quad (17)$$

$$\Delta s^{\Delta^{+}}=-3a\alpha^2, \quad (18)$$

and for the Σ^{*+} by

$$\Delta u^{\Sigma^{*+}}=2-a(4+3\alpha^2+\tfrac{2}{3}\beta^2+\tfrac{4}{3}\zeta^2), \quad (19)$$

$$\Delta d^{\Sigma^{*+}}=-a(2+\alpha^2), \quad (20)$$

$$\Delta s^{\Sigma^{*+}}=1-a(4\alpha^2+\tfrac{4}{3}\beta^2+\tfrac{2}{3}\zeta^2), \quad (21)$$

and for the Σ^{*0} by

$$\Delta u^{\Sigma^{*0}}=1-a(3+2\alpha^2+\tfrac{1}{3}\beta^2+\tfrac{2}{3}\zeta^2), \quad (22)$$

$$\Delta d^{\Sigma^{*0}}=1-a(3+2\alpha^2+\tfrac{1}{3}\beta^2+\tfrac{2}{3}\zeta^2), \quad (23)$$

TABLE II. Decuplet magnetic moments. The decuplet magnetic moments are given in units of the nuclear magneton, μ_N. In the Review of Particle Physics [23] one can find the averages $\mu(\Delta^{++})=(3.7\rightarrow 7.5)\mu_N$ and $\mu(\Omega^-)=(-2.02\pm 0.05)\mu_N$ obtained from all existing experiments.

Quantity ζ	Experimental value	NQM	χQM	χQM with SU(3) symmetry breaking	
			-1.2	-0.5	-1.2
$\mu(\Delta^{++})$	4.52 ± 0.95[24]	5.43	5.30	5.11	5.21
$\mu(\Delta^{+})$	-	2.72	2.58	2.35	2.45
$\mu(\Delta^{0})$	-	0	-0.13	-0.41	-0.30
$\mu(\Delta^{-})$	-	-2.72	-2.85	-3.17	-3.06
$\mu(\Sigma^{*+})$	-	3.02	2.88	2.77	2.85
$\mu(\Sigma^{*})$	-	0.30	0.17	0.00	0.09
$\mu(\Sigma^{*-})$	-	-2.41	-2.55	-2.76	-2.66
$\mu(\Xi^{*0})$	-	0.60	0.47	0.42	0.49
$\mu(\Xi^{*-})$	-	-2.11	-2.25	-2.34	-2.27
$\mu(\Omega^{-})$	-1.94 ± 0.31[25] -2.02 ± 0.06[26]	-1.81	-1.95	-1.93	-1.87

$$\Delta s^{\Sigma^{*0}}=1-a(4\alpha^2+\tfrac{4}{3}\beta^2+\tfrac{2}{3}\zeta^2), \quad (24)$$

and for the Ξ^{*0} by

$$\Delta u^{\Xi^{*0}}=1-a(2+3\alpha^2+\tfrac{1}{3}\beta^2+\tfrac{2}{3}\zeta^2), \quad (25)$$

$$\Delta d^{\Xi^{*0}}=-a(1+2\alpha^2), \quad (26)$$

$$\Delta s^{\Xi^{*0}}=2-a(5\alpha^2+\tfrac{8}{3}\beta^2+\tfrac{4}{3}\zeta^2), \quad (27)$$

and for the Ω^{-} by

$$\Delta u^{\Omega^{-}}=-3a\alpha^2, \quad (28)$$

$$\Delta d^{\Omega^{-}}=-3a\alpha^2, \quad (29)$$

$$\Delta s^{\Omega^{-}}=3-a(6\alpha^2+4\beta^2+2\zeta^2). \quad (30)$$

The spin polarizations for the other decuplet baryons are found from isospin symmetry.

We now use the parameters found for the octet baryons to calculate the magnetic moments and the total quark spin polarizations of the decuplet baryons. The results are presented in Tables II and III.

TABLE III. Total quark spin polarizations of the decuplet baryons.

Quantity ζ	NQM	χQM	χQM with SU(3) symmetry breaking	
		-1.2	-0.5	-1.2
$\Delta\Sigma^{\Delta}$	3	1.11	1.03	1.55
$\Delta\Sigma^{\Sigma^*}$	3	1.11	1.19	1.70
$\Delta\Sigma^{\Xi^*}$	3	1.11	1.36	1.85
$\Delta\Sigma^{\Omega}$	3	1.11	1.53	2.00

In Table II we present the results obtained for the magnetic moments of the decuplet baryons. We observe that the value of $\mu(\Omega^-)$ in the χQM and the χQM with SU(3) symmetry breaking are closer to experiments than the NQM value, while the values for $\mu(\Delta^{++})$ are almost equivalent. This is quite non-trivial, since the quark spin polarizations in the χQM deviate substantially from the NQM values.

In Table III we list the total quark spin polarizations for the decuplet baryons. Using Eq. (9) together with Eqs. (13)–(30), we obtain the total quark spin polarization for a baryon B as

$$\Delta\Sigma^B = 3 - a(9 + 6\alpha^2 + \beta^2 + 2\zeta^2) + a(3 - 2\alpha^2 - \beta^2)x, \tag{31}$$

where x is the number of s quarks in the baryon B. Note that in both the NQM and the χQM with no SU(3) symmetry breaking, the $\Delta\Sigma$'s have the same value for all decuplet baryons, but when introducing SU(3) symmetry breaking, they vary linearly with the number of strange quarks.

For the octet baryons there is no formula similar to Eq. (31).

V. SUMMARY AND CONCLUSIONS

Applying the results from calculations of the octet baryons to the decuplet baryons, we have found values for the magnetic moments of Δ^{++} and Ω^- in good agreement with experiments. This holds not only for the parameters used in the previous fits, but also for some other choices of the SU(3) breaking parameters.

The case $\zeta = -0.5$ represents a large η' suppression. However, one has to pay the price that the $\overline{u}/\overline{d}$ asymmetry becomes 0.64. Fixing the value of a to 0.15 brings the quark spin polarization of the proton down to 0.34. On the other hand, the octet magnetic moments are then not quite as good.

In our opinion, the quantities $\Delta\Sigma^p$, $\overline{u}/\overline{d}$, etc., often used to determine the parameters of the χQM are not better understood, and certainly less well measured, than the magnetic moments chosen here to fix the parameters, and therefore subject to at least as much uncertainty. To ask for complete agreement already at this level for each item might be to ask for too much.

The fraction of the baryon spin carried by the quarks in the decuplet baryons is in our analysis predicted to be about one third of the value from the NQM when there is no SU(3) symmetry breaking and to be slightly larger and increase with the number of strange quarks when SU(3) is broken.

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[1] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
[2] E. J. Eichten, I. Hinchliffe, and C. Quigg, Phys. Rev. D **45**, 2269 (1992).
[3] T. P. Cheng and L.-F. Li, Phys. Rev. Lett. **74**, 2872 (1995).
[4] L.-F. Li and T. P. Cheng, Report No. hep-ph/9506397.
[5] T. P. Cheng and L.-F. Li, in *International Europhysics Conference on High Energy Physics (HEP95)*, Proceedings, Brussels, Belgium, edited by J. Lemonne *et al.* (World Scientific, Singapore, 1996), hep-ph/9510236.
[6] X. Song, J. S. McCarthy, and H. J. Weber, Phys. Rev. D **55**, 2624 (1997).
[7] H. J. Weber, X. Song, and M. Kirchbach, Mod. Phys. Lett. A **12**, 729 (1997).
[8] T. P. Cheng and L.-F. Li, Phys. Rev. D **57**, 344 (1998).
[9] J. Linde, T. Ohlsson, and H. Snellman, Phys. Rev. D **57**, 452 (1998).
[10] D. B. Leinweber, T. Draper, and R. M. Woloshyn, Phys. Rev. D **46**, 3067 (1992).
[11] F. Schlumpf, Phys. Rev. D **48**, 4478 (1993).
[12] J. Linde and H. Snellman, Phys. Rev. D **53**, 2337 (1996).
[13] S.-T. Hong and G. E. Brown, Nucl. Phys. **A580**, 408 (1994).
[14] M. N. Butler, M. J. Savage, and R. P. Springer, Phys. Rev. D **49**, 3459 (1994).
[15] H.-C. Kim, M. Praszałowicz, and K. Goeke, Phys. Rev. D **57**, 2859 (1998).
[16] F. X. Lee, Report No. CU-NPL-1149, hep-ph/9707411.
[17] F. X. Lee, Phys. Rev. D **57**, 1801 (1998).
[18] New Muon Collaboration, P. Amaudruz *et al.*, Phys. Rev. Lett. **66**, 2712 (1991).
[19] New Muon Collaboration, M. Arneodo *et al.*, Phys. Rev. D **50**, R1 (1994).
[20] NA51 Collaboration, A. Baldit *et al.*, Phys. Lett. B **332**, 244 (1994).
[21] K. Dannbom, L. Ya. Glozman, C. Helminen, and D. O. Riska, Nucl. Phys. **A616**, 555 (1997).
[22] H. J. Weber and K. Bodoor, Int. J. Mod. Phys. E **6**, 693 (1997).
[23] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).
[24] A. Bosshard *et al.*, Phys. Rev. D **44**, 1962 (1991).
[25] H. T. Diehl *et al.*, Phys. Rev. Lett. **67**, 804 (1991).
[26] N. B. Wallace *et al.*, Phys. Rev. Lett. **74**, 3732 (1995).