Time-reversal odd distribution functions in leptoproduction

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We consider the various asymmetries, notably single spin asymmetries, that appear in leptoproduction as a consequence of the presence of time-reversal odd distribution functions. This could facilitate experimental searches for time-reversal odd phenomena. [S0556-2821(98)02709-X]

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In this paper we study the effects of the possible presence of time-reversal (T) odd distribution functions in leptoproduction. We limit ourselves to the production of hadrons in the current fragmentation region, for which we assume that the cross section factorizes into a product of a distribution function and a fragmentation function. Including the effects of transverse momenta, the cross section is assumed to factorize into a convolution of distribution and fragmentation functions which not only depends on the light-cone momentum fractions of quark and hadron, but also on the transverse momentum of quark with respect to hadron or vice versa [1].

Starting with the expressions of the soft parts in hard scattering processes as quark-quark light-front correlation functions, i.e. matrix elements of nonlocal combinations of quark fields, one can analyze the various possible distribution and fragmentation functions. Constraints arise from Lorentz invariance, Hermiticity, parity invariance and time-reversal invariance. The latter, however, cannot be used as a constraint on fragmentation functions, because the produced hadron can interact with the debris of the fragmenting quark, a well-known phenomenon in any decay process [2]. This allows so-called T-odd quantities, although it is hard to say something about their magnitude. In Ref. [3] it was even conjectured that final state interaction phases average to zero for single hadron production after summation over unobserved final states.

Without considering transverse momenta of quarks, the T-odd effects are higher twist, appearing at order $1/Q$ [4]. Including transverse momenta of quarks, there are leading order effects. One can have fragmentation of transversely polarized quarks into unpolarized or spin zero hadrons or production of transversely polarizated hadrons in the fragmentation of unpolarized quarks [1]. For the distribution functions, it has been conjectured that T-odd quantities also might appear without violating time-reversal invariance [5–8]. This might be due to soft initial state interactions or, as suggested recently [8], be a consequence of chiral symmetry breaking. Within QCD a possible description of the effects may come from gluonic poles [9].

It is convenient to use the hadron momenta in the process $lH \to l' hX$ to define two lightlike vectors n_+ and n_-, satisfying $n_+ \cdot n_- = 1$. These vectors then define the light-cone components of a vector as $a^{\pm} \equiv a \cdot n_{\mp}$. Up to mass terms the momentum P of the target hadron (H) is along n_+, that of the outgoing hadron along n_-. We assume here that we are discussing current fragmentation, for which one requires $P \cdot P_h \sim Q^2$, where $q^2 = -Q^2$ is the momentum transfer squared. In leading order in $1/Q$ the process factorizes into a product of two soft parts. For the description of the quark content of the target the following quantity (given in the light-cone gauge $A^+ = 0$) is relevant,

$$\Phi(x,\boldsymbol{p}_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P,S|\overline{\psi}(0)\psi(\xi)|P,S\rangle\Big|_{\xi^+=0}, \tag{1}$$

depending on the light-cone fraction of the quark momentum, $x = p^+/P^+$ and the transverse momentum component $\boldsymbol{p}_T$. Using Lorentz invariance, Hermiticity, and parity invariance one finds that the Dirac projections that will appear in a calculation up to leading order in $1/Q$ can be expressed in a number of distribution functions

$$\Phi(x,\boldsymbol{p}_T) = \frac{1}{2}\Bigg\{ f_1 \not{n}_+ + f_{1T}^{\perp} \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu p_T^\rho S_T^\sigma}{M} + g_{1s}\gamma_5 \not{n}_+ + h_{1T} i\sigma_{\mu\nu}\gamma_5 n_+^\mu S_T^\nu + h_{1s}^{\perp} \frac{i\sigma_{\mu\nu}\gamma_5 n_+^\mu p_T^\nu}{M} + h_1^{\perp} \frac{\sigma_{\mu\nu} p_T^\mu n_+^\nu}{M} \Bigg\}, \tag{2}$$

with arguments $f_1 = f_1(x,\boldsymbol{p}_T^2)$ etc. The quantity g_{1s} (and similarly $h_{1s}^{\perp}$) is shorthand for

$$g_{1s}(x,\boldsymbol{p}_T) = \lambda g_{1L}(x,\boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M} g_{1T}(x,\boldsymbol{p}_T^2), \tag{3}$$

with M the mass, $\lambda = MS^+/P^+$ the light-cone helicity, and $\boldsymbol{S}_T$ the transverse spin of the target hadron. Note that the difference with the analysis in Ref. [1], in which the time-reversal constraint has been imposed, is the appearance of the functions $f_{1T}^{\perp}$ and $h_1^{\perp}$. The function $f_{1T}^{\perp}$ is interpreted as the unpolarized quark distribution in a transversely polarized nucleon, while $h_1^{\perp}$ is interpreted as the quark transverse spin distribution in an unpolarized hadron. We have used and followed the naming convention of Ref. [1]. The function $f_{1T}^{\perp}$ is proportional to the function $\Delta^N f$ used in Refs. [6,8]. In this paper we simply want to investigate where the functions $f_{1T}^{\perp}$ and $h_1^{\perp}$ show up in leptoproduction. We do not discuss the possible mechanisms leading to them, but point out in which observables their existence can be checked experimentally.

The computation of the leading order leptoproduction cross sections requires in addition to the quark distribution functions, also fragmentation functions, contained in a soft part which (in the light-cone gauge $A^-=0$) is of the form

$$\Delta(z,\boldsymbol{k}_T)=\sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{2z(2\pi)^3}\, e^{ik\cdot\xi}\langle 0|\psi(\xi)|X;P_h,S_h\rangle \times\langle X;P_h,S_h|\overline{\psi}(0)|0\rangle\bigg|_{\xi^-=0}, \tag{4}$$

where $z=P_h^-/k^-$ is the light-cone fraction of the produced hadron and $\boldsymbol{k}_T$ is the quark transverse momentum with respect to the produced hadron, which implies a transverse momentum $\boldsymbol{k}_T'=-z\boldsymbol{k}_T$ of the produced hadron with respect to the fragmenting quark. At leading order the following expansion in fragmentation functions can be written

$$\Delta(z,\boldsymbol{k}_T)=\frac{1}{2}\bigg\{D_1\not{n}_- + D_{1T}^{\perp}\,\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^{\mu}n_-^{\nu}k_T^{\rho}S_{hT}^{\sigma}}{M_h}+G_{1s}\gamma_5\not{n}_- + H_{1T}i\sigma_{\mu\nu}\gamma_5 n_-^{\mu}S_{hT}^{\nu}+H_{1s}^{\perp}\,\frac{i\sigma_{\mu\nu}\gamma_5 n_-^{\mu}k_T^{\nu}}{M_h} + H_1^{\perp}\,\frac{\sigma_{\mu\nu}k_T^{\mu}n_-^{\nu}}{M_h}\bigg\}, \tag{5}$$

with arguments $D_1=D_1(z,z^2\boldsymbol{k}_T^2)$ etc. The quantity G_{1s} (and similarly $H_{1s}^{\perp}$) is shorthand for

$$G_{1s}(z,-z\boldsymbol{k}_T)=\lambda_h G_{1L}(z,z^2\boldsymbol{k}_T^2)+\frac{\boldsymbol{k}_T\cdot\boldsymbol{S}_{hT}}{M_h}\,G_{1T}(z,z^2\boldsymbol{k}_T^2), \tag{6}$$

with M_h the mass, $\lambda_h=M_h S_h^-/P_h^-$ the light-cone helicity, and $\boldsymbol{S}_{hT}$ the transverse spin of the produced hadron. The functions $D_{1T}^{\perp}$ and $H_1^{\perp}$ are the T-odd ones in the fragmentation part. Although written down for spin-1/2 hadrons, all results will include also target hadrons and produced hadrons with spin zero (putting $S=0$ or $S_h=0$).

The T-odd functions appear in pairs in the unpolarized leptoproduction cross section or in double spin asymmetries and they appear singly in single spin asymmetries. The hadron tensor in leading order in $1/Q$ (thus also neglecting all mass corrections) is given by

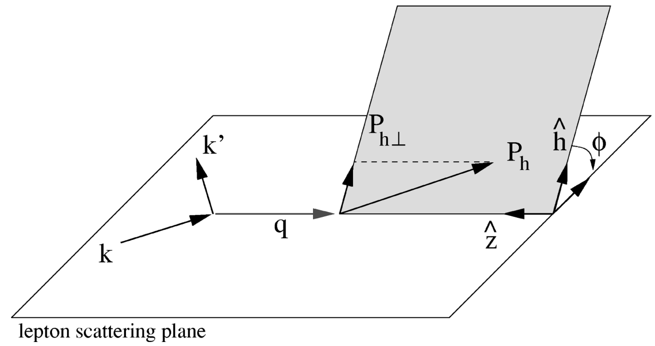

FIG. 1. Kinematics for one-particle inclusive leptoproduction. The lepton scattering plane is determined by the momenta k, k' and P.

$$2M\mathcal{W}_{\mu\nu}(q,P,P_h)=\int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\,\delta^2(\boldsymbol{p}_T+\boldsymbol{q}_T-\boldsymbol{k}_T)\times\frac{1}{4}\,\mathrm{Tr}(\Phi(x_B,p_T)\gamma_\mu\Delta(z_h,k_T)\gamma_\nu)+\begin{pmatrix} q\leftrightarrow -q\\ \mu\leftrightarrow\nu\end{pmatrix}, \tag{7}$$

where $x_B=Q^2/2P\cdot q$ and $z_h=P\cdot P_h/P\cdot q$. The momentum q_T^{μ} is the transverse momentum of the exchanged photon in the frame where P and P_h do not have transverse momenta, which is proportional to the transverse component of the produced hadron, $P_{h\perp}^{\mu}$, in the frame where P and q have no transverse components. In general we will indicate transverse momenta in the first frame with a subscript T (thus $P_T=0$ and $P_{hT}=0$) and those in the second frame with a subscript $\perp$ (thus $P_\perp=0$ and $q_\perp=0$). The kinematics for one-particle inclusive leptoproduction in the second frame have been shown in Fig. 1.

One has

$$q_T^{\mu}=(g^{\mu\nu}-n_+^{\{\mu}n_-^{\nu\}})q_\nu=q^{\mu}+x_B P^{\mu}-\frac{P_h^{\mu}}{z_h}=-\frac{P_{h\perp}}{z_h} \equiv -Q_T\hat{h}^{\mu}. \tag{8}$$

It is convenient to introduce the tensors $g_\perp^{\mu\nu}$ and $\epsilon_\perp^{\mu\nu}$ given by

$$g_\perp^{\mu\nu}=g^{\mu\nu}-\frac{q^{\mu}q^{\nu}}{q^2}+\frac{\tilde{P}^{\mu}\tilde{P}^{\nu}}{\tilde{P}^2}, \tag{9}$$

$$\epsilon_\perp^{\mu\nu}=\frac{\epsilon^{\mu\nu\rho\sigma}P_\rho q_\sigma}{P\cdot q}, \tag{10}$$

where $\tilde{P}^{\mu}=P^{\mu}-(P\cdot q/q^2)q^{\mu}$. The tensors act in the transverse space orthogonal to P and q. If $Q_T\ll Q$ one has $(p_T)_\perp\approx p_T$, $(k_T)_\perp\approx k_T$, $(S_T)_\perp\approx S_T$, and $(S_{hT})_\perp\approx S_{hT}$. Azimuthal angles will be defined in this space with respect to the lepton scattering plane (see Fig. 1), e.g. $\phi_h^l=\phi_h-\phi^l$ is the angle between the hadron production plane (defined by P_h and q) and the lepton scattering plane.

TABLE I. Azimuthal asymmetries $\langle W\rangle_{ABC}$ [see Eq. (11)] for the case of fully unpolarized leptoproduction. The last column indicates the time-reversal behavior of the distribution and fragmentation function, respectively (e=even, o=odd).

ABC	W	$\langle W\rangle_{ABC}\cdot[4\pi\alpha^2 s/Q^4]^{-1}$	T
OOO	1	$(1-y+\frac{1}{2}y^2)\Sigma_{a,\bar{a}}e_a^2 x_B f_1^a(x_B)D_1^a(z_h)$	ee
OOO	$(Q_T^2/4MM_h)\cos(2\phi_h^l)$	$(1-y)\Sigma_{a,\bar{a}}e_a^2 x_B h_1^{\perp(1)a}(x_B)H_1^{\perp(1)a}(z_h)$	oo

We will next discuss the explicit results for the cross sections. At leading order they are obtained from the contraction of the lepton tensor with the hadron tensor $\mathcal{W}^{\mu\nu}$. We will in general consider cross sections integrated over the transverse momentum of the produced hadron (i.e. over $\mathbf{q}_T$) and depending on the weight denote them by

$$\langle W\rangle_{ABC}=\int d\phi^l d^2\boldsymbol{q}_T W\,\frac{d\sigma_{ABC}^{[\vec{e}\vec{H}\to e\vec{h}X]}}{dx_B dy dz_h d\phi^l d^2\boldsymbol{q}_T}, \tag{11}$$

where $W=W(Q_T,\phi_h^l,\phi_S^l,\phi_{S_h}^l)$. In order to see in a glance which polarizations are involved, we have added the subscripts ABC for polarizations of lepton, target hadron and produced hadron, respectively. We use O for unpolarized, L for longitudinally polarized ($\lambda\neq 0$) and T for transversely polarized ($|\boldsymbol{S}_T|\neq 0$) particles.

Starting (as a reference) with the cross section for *unpolarized leptons scattering off an unpolarized hadron producing a spin zero particle or summing over spin in the final state*, one finds

$$\langle 1\rangle_{OOO}=\frac{4\pi\alpha^2 s}{Q^4}\left(1-y+\frac{y^2}{2}\right)\sum_{a,\bar{a}} e_a^2 x_B f_1^a(x_B)D_1^a(z_h). \tag{12}$$

The above is the well-known unpolarized result containing a sum over flavors of quarks and antiquarks with in each term the product of the unpolarized distribution function f_1^a (quarks a in hadron H) and the unpolarized quark fragmentation function D_1^a (quark a fragmenting into hadron h). Only considering T-even distribution functions, this is the only nonvanishing averaged unpolarized cross section at leading order. At subleading ($1/Q$) order one has a nonvanishing $\cos\phi_h^l$-asymmetry originating from kinematical [10] and dynamical [11] effects, while a $\cos 2\phi_h^l$ asymmetry only appears at order $1/Q^2$ [12,13]. Allowing for T-odd functions, however, the following weighted cross section projects out a *leading* azimuthal $\cos 2\phi_h^l$ asymmetry,

$$\left\langle \frac{Q_T^2}{MM_h}\cos(2\phi_h^l)\right\rangle_{OOO}=\frac{16\pi\alpha^2 s}{Q^4}(1-y)\times\sum_{a,\bar{a}} e_a^2 x_B h_1^{\perp(1)a}(x_B)H_1^{\perp(1)a}(z_h). \tag{13}$$

The weighted cross section involves $\boldsymbol{p}_T^2$ moments of the distribution and fragmentation functions $h_1^\perp$ and $H_1^\perp$, defined as

$$h_1^{\perp(n)}(x)\equiv\int d^2\boldsymbol{p}_T\left(\frac{\boldsymbol{p}_T^2}{2M^2}\right)^n h_1^\perp(x,\boldsymbol{p}_T), \tag{14}$$

$$H_1^{\perp(n)}(z)\equiv z^2\int d^2\boldsymbol{k}_T\left(\frac{\boldsymbol{k}_T^2}{2M_h^2}\right)^n H_1^\perp(z,-z\boldsymbol{k}_T). \tag{15}$$

While the $\boldsymbol{k}_T$-dependent function are light-front correlation functions (i.e. $\xi^+=0$), the integrated functions and $\boldsymbol{k}_T^2$ moments are light-cone correlation functions (i.e. $\xi^+=\xi_T=0$ in the matrix elements) for which we expect factorization to remain valid, although this has not yet been proven. The above two cases are summarized in Table I.

We note that a similar $\cos 2\phi$ asymmetry involving the azimuthal angle of two hadrons in opposite jets appears in electron-positron annihilation [14]. In that case only T-odd fragmentation functions $H_1^{\perp(1)}$ and $\bar{H}_1^{\perp(1)}$ are involved. A similar asymmetry in Drell-Yan would involve only T-odd distribution functions.

Next, we consider leading order single spin asymmetries, which we separate in single spin asymmetries for lepton, target hadron and produced hadron, respectively. There are no leading order lepton spin asymmetries. The one lepton spin asymmetry that is possible in one-particle inclusive leptoproduction is a $\sin\phi_h^l$ asymmetry. It is, however, subleading, i.e. order $1/Q$ (see Ref. [15]).

There are four leading order single spin asymmetries involving the spin of the target hadron, given in Table II. The

TABLE II. Leading order single spin asymmetries for the case of leptoproduction into unpolarized final states.

ABC	W	$\langle W\rangle_{ABC}\cdot[4\pi\alpha^2 s/Q^4]^{-1}$	T
OLO	$(Q_T^2/4MM_h)\sin(2\phi_h^l)$	$-\lambda(1-y)\Sigma_{a,\bar{a}}e_a^2 x_B h_{1L}^{\perp(1)a}(x_B)H_1^{\perp(1)a}(z_h)$	eo
OTO	$(Q_T/M_h)\sin(\phi_h^l+\phi_S^l)$	$\lvert\boldsymbol{S}_T\rvert(1-y)\Sigma_{a,\bar{a}}e_a^2 x_B h_1^a(x_B)H_1^{\perp(1)a}(z_h)$	eo
OTO	$(Q_T^3/6M^2M_h)\sin(3\phi_h^l-\phi_S^l)$	$\lvert\boldsymbol{S}_T\rvert(1-y)\Sigma_{a,\bar{a}}e_a^2 x_B h_{1T}^{\perp(2)a}(x_B)H_1^{\perp(1)a}(z_h)$	eo
OTO	$(Q_T/M)\sin(\phi_h^l-\phi_S^l)$	$\lvert\boldsymbol{S}_T\rvert(1-y+\frac{1}{2}y^2)\Sigma_{a,\bar{a}}e_a^2 x_B f_{1T}^{\perp(1)a}(x_B)D_1^a(z_h)$	oe

TABLE III. Leading order single spin asymmetries due to T-odd distribution functions that require polarimetry.

ABC	W	$\langle W\rangle_{ABC}\cdot[4\pi\alpha^2 s/Q^4]^{-1}$	T
OOT	$(Q_T/M_h)\sin(\phi_h^l-\phi_{S_h}^l)$	$-\lvert\boldsymbol{S}_{hT}\rvert(1-y+\frac{1}{2}y^2)\Sigma_{a,\bar{a}}e_a^2 x_B f_1^a(x_B)D_{1T}^{\perp(1)a}(z_h)$	eo
OOL	$(Q_T^2/4MM_h)\sin(2\phi_h^l)$	$-\lambda_h(1-y)\Sigma_{a,\bar{a}}e_a^2 x_B h_1^{\perp(1)a}(x_B)H_{1L}^{\perp(1)a}(z_h)$	oe
OOT	$(Q_T/M)\sin(\phi_h^l+\phi_{S_h}^l)$	$-\lvert\boldsymbol{S}_{hT}\rvert(1-y)\Sigma_{a,\bar{a}}e_a^2 x_B h_1^{\perp(1)a}(x_B)H_1^a(z_h)$	oe
OOT	$(Q_T^3/6MM_h^2)\sin(3\phi_h^l-\phi_{S_h}^l)$	$-\lvert\boldsymbol{S}_{hT}\rvert(1-y)\Sigma_{a,\bar{a}}e_a^2 x_B h_1^{\perp(1)a}(x_B)H_{1T}^{\perp(2)a}(z_h)$	oe

first three involve T-even distribution functions. The fourth one involves a T-odd distribution function.

The first three asymmetries are T-odd in the fragmentation part, in particular they all feature the fragmentation function $H_1^{\perp(1)}$. The second of the three asymmetries was first discussed by Collins [16]. The existence of the other two was pointed out in Refs. [17,18]. The third asymmetry in Table II involves the second $\boldsymbol{p}_T^2/2M^2$ moment of the function $h_{1T}^{\perp}$ and appears also as an OTO asymmetry with slightly different azimuthal angle dependence. For a detailed discussion of the asymmetries we refer to Refs. [19,20]. The fourth entry in Table II is again an OTO single spin asymmetry containing the T-odd distribution function $f_{1T}^{\perp(1)}$. It appears in *scattering of unpolarized leptons off transversely polarized targets*. This asymmetry is probably the easiest way to look for the function $f_{1T}^{\perp}$ as it just requires searching for a correlation between the azimuthal angles of the produced hadron and the target transverse spin, e.g., in $ep^{\uparrow}\to e\pi X$ or $ep^{\uparrow}\to eKX$. This possibility was pointed out in Ref. [7] (measurement a).

Next we consider the single spin asymmetries related to the spin of the produced hadron. At leading order there are four single spin asymmetries, given in Table III, three of which contain the T-odd distribution function $h_1^{\perp(1)}$.

The latter three odd-even asymmetries appear in *unpolarized lepton scattering off an unpolarized target hadron*. They require polarimetry in the final state and are the direct counterparts of the three even-odd asymmetries in Table II with the role of distribution and fragmentation functions being reversed. These asymmetries can for instance be measured in $ep\to e\Lambda^{\uparrow}X$, by determining the Λ polarization and its orientation from the $p\pi^-$ final state. At this point it may be good to reiterate the interpretation of the single spin asymmetries. In all cases a T-odd effect is needed in either the distribution or in the fragmentation part. The asymmetries in Table II are due to

$$\text{eo:}\quad \text{polarized target}\xrightarrow{\text{T-even}}\text{quark}^{\uparrow}\xrightarrow{\text{T-odd}}\text{unpolarized hadron},$$

$$\text{oe:}\quad \text{target}^{\uparrow}\xrightarrow{\text{T-odd}}\text{unpolarized quark}\xrightarrow{\text{T-even}}\text{unpolarized hadron},$$

those in Table III are due to

$$\text{eo:}\quad \text{unpolarized target}\xrightarrow{\text{T-even}}\text{unpolarized quark}\xrightarrow{\text{T-odd}}\text{hadron}^{\uparrow},$$

$$\text{oe:}\quad \text{unpolarized target}\xrightarrow{\text{T-even}}\text{quark}^{\uparrow}\xrightarrow{\text{T-even}}\text{polarized hadron},$$

where the up arrow denotes transversely polarized quarks or hadrons.

Next we turn to double spin asymmetries. These contain either both T-even or both T-odd distribution and fragmentation functions. In Table IV we have repeated only those even-even combinations from Ref. [1] that do not involve azimuthal angles in combination with azimuthal spin angle of the produced hadron. There exists only one odd-odd asymmetry at leading order. This is the last entry in Table IV.

The even-even asymmetries include LLO, LOL and OLL asymmetries where compared with the $\langle 1\rangle_{OOO}$ result pairs of unpolarized particles are replaced by longitudinally polarized particles. The LTO asymmetry is the even-even equivalent of the odd-even OTO single spin asymmetry in Table II and is probably the easiest way to obtain the function $g_{1T}^{\perp(1)}$, e.g., in

TABLE IV. Some leading order even-even double spin asymmetries in leptoproduction and the only leading order odd-odd double spin asymmetry.

ABC	W	$\langle W\rangle_{ABC}\cdot[4\pi\alpha^2 s/Q^4]^{-1}$	T
LLO	1	$\lambda_e\lambda y(1-\frac{1}{2}y)\Sigma_{a,\bar{a}}e_a^2 x_B g_1^a(x_B)D_1^a(z_h)$	ee
LTO	$(Q_T/M)\cos(\phi_h^l-\phi_S^l)$	$\lambda_e\lvert\boldsymbol{S}_T\rvert y(1-\frac{1}{2}y)\Sigma_{a,\bar{a}}e_a^2 x_B g_{1T}^{(1)a}(x_B)D_1^a(z_h)$	ee
LOL	1	$\lambda_e\lambda_h y(1-\frac{1}{2}y)\Sigma_{a,\bar{a}}e_a^2 x_B f_1^a(x_B)G_1^a(z_h)$	ee
OLL	1	$\lambda\lambda_h(1-y+\frac{1}{2}y^2)\Sigma_{a,\bar{a}}e_a^2 x_B g_1^a(x_B)G_1^a(z_h)$	ee
OTT	$\cos(\phi_S^l+\phi_{S_h}^l)$	$-\lvert\boldsymbol{S}_T\rvert\lvert\boldsymbol{S}_{hT}\rvert\frac{1}{2}(1-y)\Sigma_{a,\bar{a}}e_a^2 x_B h_1^a(x_B)H_1^a(z_h)$	ee
OTT	$(Q_T^2/MM_h)\cos(\phi_S^l-\phi_{S_h}^l)$	$\lvert\boldsymbol{S}_T\rvert\lvert\boldsymbol{S}_{hT}\rvert(1-y+\frac{1}{2}y^2)\Sigma_{a,\bar{a}}e_a^2 x_B f_{1T}^{\perp(1)a}(x_B)D_{1T}^{\perp(1)a}(z_h)$	oo

leptoproduction of pions $\vec{e}p^\uparrow\to e\pi^+X$ [19]. The even-even OTT asymmetry has been suggested by Artru as the way to obtain the transverse spin distribution h_1 in the proton via e.g. $ep^\uparrow\to e\Lambda^\uparrow X$ [21]. In the same process the odd-odd asymmetry can be investigated. While in the even-even Artru asymmetry a (longitudinal) virtual photon scatters off a transversely polarized quark, one has in the odd-odd asymmetry a (transverse) virtual photon scattering off an unpolarized quark.

It is useful at this point to mention that the asymmetries with a fragmentation function D_1 can also be obtained by looking at the asymmetry in jet production. In that case one needs the fragmentation of a quark into a quark, which (at tree level) is given by $D_1(z)=\delta(1-z)$. One then can perform the z_h integration. Thus the azimuthal asymmetry $\cos(\phi_{jet}^l-\phi_S^l)$ is a way to probe g_{1T}. The first result in Table IV even survives after full integration over the final states, giving the ordinary double spin asymmetry in inclusive leptoproduction in terms of the polarized quark distribution function g_1.

Finally, for completeness, we give in Table V the two possible leading order triple spin asymmetries with a T-odd distribution function for *polarized leptons scattering off a transversely polarized target* leading to a spin asymmetry in the final state. The first asymmetry is the analogue of the OTO single spin asymmetry in Table II with the unpolarized particles replaced by longitudinally polarized particles.

At leading order the T-odd distribution functions only appear in azimuthal asymmetries. At subleading $(1/Q)$ order T-odd distribution functions appear also in the simple $\boldsymbol{q}_T$-integrated cross sections, i.e., the ones that do not involve powers of Q_T and azimuthal angle ϕ_h^l in the weight function, but at most the azimuthal spin angles (ϕ_S^l or $\phi_{S_h}^l$). In that case all that is needed are the soft parts integrated over transverse momenta. At subleading order, however, one needs to include parts proportional to M/P^+ in Φ and parts proportional to M_h/P_h^- in Δ. In the cross sections these factors give rise to a suppression factor $1/Q$. The quantity needed in the calculation is

$$\Phi(x)\equiv\int d^2\boldsymbol{p}_T\Phi(x,\boldsymbol{p}_T)=\frac{1}{2}\Bigg\{f_1\not{n}_+ +\lambda g_1\gamma_5\not{n}_+ + h_1\frac{[\not{S}_T,\not{n}_+]\gamma_5}{2}\Bigg\}+\frac{M}{2P^+}\Bigg\{f_T\epsilon_T^{\rho\sigma}S_{T\rho}\gamma_\sigma+e\mathbf{1} -i\lambda e_L\gamma_5+g_T\gamma_5\not{S}_T+\lambda h_L\frac{[\not{n}_+,\not{n}_-]\gamma_5}{2} +ih\frac{[\not{n}_+,\not{n}_-]}{2}\Bigg\}, \tag{16}$$

in terms of distribution functions with arguments $f_1=f_1(x)$ etc. All the leading twist (twist-two) functions are T-even. Of the twist-three functions [multiplying (M/P^+)] the functions f_T, e_L and h are T-odd ones. The function f_T is also discussed in Ref. [22] ($f_T\propto c_V$). Noteworthy is the relation between some of the $\boldsymbol{p}_T$-integrated twist-three functions and $\boldsymbol{p}_T^2/M^2$ moments of leading $\boldsymbol{p}_T$-dependent distribution functions [1],

$$g_T(x)=g_1(x)+\frac{d}{dx}\,g_{1T}^{(1)}, \tag{17}$$

$$h_L(x)=h_1(x)-\frac{d}{dx}\,h_{1L}^{\perp(1)}, \tag{18}$$

$$f_T(x)=-\frac{d}{dx}\,f_{1T}^{\perp(1)}, \tag{19}$$

$$h(x)=-\frac{d}{dx}\,h_1^{\perp(1)}. \tag{20}$$

For the first case this relation appears in a slightly different form (using quark-quark-gluon correlation functions) in Ref. [23]. For the fragmentation part one needs at order $1/Q$ the quantity

TABLE V. The leading order triple spin asymmetries with T-odd distribution functions in leptoproduction.

ABC	W	$\langle W\rangle_{ABC}\cdot[4\pi\alpha^2 s/Q^4]^{-1}$	T
LTL	$(Q_T/M)\sin(\phi_h^l-\phi_S^l)$	$\lambda_e\lvert\boldsymbol{S}_T\rvert\lambda_h y(1-\frac{1}{2}y)\Sigma_{a,\bar{a}}e_a^2 x_B f_{1T}^{\perp(1)a}(x_B)G_1^a(z_h)$	oe
LTT	$(Q_T^2/2MM_h)\sin(\phi_S^l-\phi_{S_h}^l)$	$\lambda_e\lvert\boldsymbol{S}_T\rvert\lvert\boldsymbol{S}_{hT}\rvert y(1-\frac{1}{2}y)\Sigma_{a,\bar{a}}e_a^2 x_B f_{1T}^{\perp(1)a}(x_B)G_{1T}^{(1)a}(z_h)$	oe

TABLE VI. The subleading order spin asymmetries in leptoproduction.

ABC	W	$\langle W\rangle_{ABC}\cdot[4\pi\alpha^2 s/Q^4]^{-1}$	T
LTO	$\cos\phi_S^l$	$-\lambda_e\vert\boldsymbol{S}_T\vert y\sqrt{1-y}\sum_{a,\bar a} e_a^2\left[\frac{Mx_B^2}{Q}\, g_T^a(x_B)D_1^a(z_h)+\frac{M_h x_B}{z_h Q}\, h_1^a(x_B)\widetilde{E}^a(z_h)\right]$	ee/ee
OTO	$\sin\phi_S^l$	$\vert\boldsymbol{S}_T\vert(2-y)\sqrt{1-y}\sum_{a,\bar a} e_a^2\left[\frac{M_h x_B}{z_h Q}\, h_1^a(x_B)\widetilde{H}^a(z_h)-\frac{Mx_B^2}{Q}\, f_T^a(x_B)D_1^a(z_h)\right]$	eo/oe
LOT	$\cos\phi_{S_h}^l$	$-\lambda_e\vert\boldsymbol{S}_{hT}\vert y\sqrt{1-y}\sum_{a,\bar a} e_a^2\left[\frac{Mx_B^2}{Q}\, e^a(x_B)H_1^a(z_h)+\frac{M_h x_B}{z_h Q}\, f_1^a(x_B)\widetilde{G}^a(z_h)\right]$	ee/ee
OOT	$\sin\phi_{S_h}^l$	$\vert\boldsymbol{S}_{hT}\vert(2-y)\sqrt{1-y}\sum_{a,\bar a} e_a^2\left[\frac{M_h x_B}{z_h Q}\, f_1^a(x_B)\widetilde{D}_T^a(z_h)-\frac{Mx_B^2}{Q}\, h^a(x_B)H_1^a(z_h)\right]$	eo/oe
OTL	$\cos\phi_S^l$	$-\vert\boldsymbol{S}_T\vert\lambda_h(2-y)\sqrt{1-y}\sum_{a,\bar a} e_a^2\left[\frac{Mx_B^2}{Q}\, g_T^a(x_B)G_1^a(z_h)+\frac{M_h x_B}{z_h Q}\, h_1^a(x_B)\widetilde{H}_L^a(z_h)\right]$	ee/ee
LTL	$\sin\phi_S^l$	$-\lambda_e\vert\boldsymbol{S}_T\vert\lambda_h y\sqrt{1-y}\sum_{a,\bar a} e_a^2\left[\frac{M_h x_B}{z_h Q}\, h_1^a(x_B)\widetilde{E}_L(z_h)+\frac{Mx_B^2}{Q}\, f_T^a(x_B)G_1^a(z_h)\right]$	eo/oe
OLT	$\cos\phi_{S_h}^l$	$-\lambda\vert\boldsymbol{S}_{hT}\vert(2-y)\sqrt{1-y}\sum_{a,\bar a} e_a^2\left[\frac{Mx_B^2}{Q}\, h_L^a(x_B)H_1^a(z_h)+\frac{M_h x_B}{z_h Q}\, g_1^a(x_B)\widetilde{G}_T^a(z_h)\right]$	ee/ee
LLT	$\sin\phi_{S_h}^l$	$\lambda_e\lambda\vert\boldsymbol{S}_{hT}\vert y\sqrt{1-y}\sum_{a,\bar a} e_a^2\left[\frac{M_h x_B}{z_h Q}\, g_1^a(x_B)\widetilde{D}_T(z_h)+\frac{Mx_B^2}{Q}\, e_L^a(x_B)H_1^a(z_h)\right]$	eo/oe

$$\Delta(z)\equiv z^2\int d^2\boldsymbol{k}_T\Delta(z,\boldsymbol{k}_T)=\frac{1}{2}\Bigg\{D_1\not n_- +\lambda_h G_1\gamma_5\not n_- +H_1\frac{[\not S_{hT},\not n_-]\gamma_5}{2}\Bigg\}+\frac{M_h}{2P_h^-}\Bigg\{D_T\epsilon_T^{\rho\sigma}\gamma_\rho S_{hT\sigma}+E\mathbf{1} -i\lambda_h E_L\gamma_5+G_T\gamma_5\not S_{hT}+\lambda_h H_L\frac{[\not n_-,\not n_+]\gamma_5}{2} +iH\frac{[\not n_-,\not n_+]}{2}\Bigg\}, \tag{21}$$

in terms of fragmentation functions with arguments $D_1=D_1(z)=\int d^2\boldsymbol{k}_T' D_1(z,\boldsymbol{k}_T^2)$, etc. For spin zero particles (e.g. pions) only the twist-two function D_1 and the twist-three functions E and H appear, the latter one being T-odd. Some of the $\boldsymbol{k}_T$-integrated twist-three functions were already mentioned in Ref. [4] ($E\propto\hat{e}_1$ and $H\propto\hat{e}_{\bar 1}$). The function D_T was also discussed in Refs. [24,14]. The relations between $\boldsymbol{k}_T$-integrated twist-three functions and $\boldsymbol{k}_T^2/2M_h^2$ moments of $\boldsymbol{k}_T$-dependent fragmentation functions are

$$\frac{G_T(z)}{z}=\frac{G_1(z)}{z}-z^2\frac{d}{dz}\left[\frac{G_{1T}^{(1)}(z)}{z}\right], \tag{22}$$

$$\frac{H_L(z)}{z}=\frac{H_1(z)}{z}+z^2\frac{d}{dz}\left[\frac{H_{1L}^{\perp(1)}(z)}{z}\right], \tag{23}$$

$$\frac{D_T(z)}{z}=z^2\frac{d}{dz}\left[\frac{D_{1T}^{\perp(1)}(z)}{z}\right], \tag{24}$$

$$\frac{H(z)}{z}=z^2\frac{d}{dz}\left[\frac{H_1^{\perp(1)}(z)}{z}\right]. \tag{25}$$

In the calculation of the hadron tensor not only the quark-quark correlation functions in Φ and Δ need to be considered, but as well quark-quark-gluon correlation functions, which contain transverse gluon fields. With the help of the equations of motion, however, the subleading contribution in the cross sections can be expressed in terms of the twist-three quark-quark correlation functions [25]. The fragmentation functions appear in specific combinations

$$\frac{\widetilde{D}_T(z)}{z}=\frac{D_T(z)}{z}+D_{1T}^{\perp(1)}(z) \tag{26}$$

$$\frac{\widetilde{E}(z)}{z}=\frac{E(z)}{z}-\frac{m}{M_h}D_1(z) \tag{27}$$

$$\frac{\widetilde{E}_L(z)}{z}=\frac{E_L(z)}{z} \tag{28}$$

$$\frac{\widetilde{G}_T(z)}{z}=\frac{G_T(z)}{z}-\frac{m}{M_h}H_1(z)-G_{1T}^{(1)} \tag{29}$$

$$\frac{\widetilde{H}_L(z)}{z}=\frac{H_L(z)}{z}-\frac{m}{M_h}G_1(z)+2H_{1L}^{\perp(1)}(z) \tag{30}$$

$$\frac{\widetilde{H}(z)}{z}=\frac{H(z)}{z}+2H_1^{\perp(1)}(z) \tag{31}$$

which are the truly interaction-dependent parts of the twist-three functions [1]. The results for the asymmetries at subleading order are given in Table VI.

There are two asymmetries for leptoproduction of spin zero particles (e.g., pions or kaons), the first two entries in Table VI. The first one is an LTO asymmetry which contains, like all asymmetries in the table, two terms. The first one is the one which survives in inclusive leptoproduction when one sums over all final states. The presence of the second term shows that using production of specific hadrons, e.g., strange ones to tag strange quarks cannot be used to disentangle different flavor contributions g_T^a.

At order $1/Q$ and integrating over the transverse momentum of the produced hadrons, there exist two single spin asymmetries, an OTO and an OOT asymmetry. They involve odd-even and even-odd combinations of distribution and fragmentation functions. Other such combinations lead to triple spin asymmetries. The ordinary double-spin asymmetries involve only even-even combinations. Since for the transverse momentum averaged correlation functions the T-odd functions only appear at the twist-three level, any odd-odd combination is of order $1/Q^2$.

In conclusion, we have presented leading order azimuthal asymmetries involving the azimuthal angles of the transverse momentum of the produced hadron or of the spin vectors of any of the hadrons involved, i.e. the target hadron or the produced hadron. Furthermore, for the transverse momentum integrated case we have given the results up to order $1/Q$. One of the reasons is the relations between the twist-three functions relevant at subleading order and the transverse momentum dependent functions, which exist for both distribution and fragmentation functions. Several isolated cases have been pointed out before, but we have presented a systematic overview including in particular a number of new asymmetries that could facilitate experimental searches for the recently much debated T-odd fragmentation functions.

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