

Critical study of B decays to light pseudoscalars

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Motivated by the large branching ratios observed for the process $B \rightarrow \eta' K$, we examine critically all the ingredients that go into estimates of B decays into two light pseudoscalars. Within the factorization approximation, we examine several assumptions on the input parameters that could have a strong bearing on the predictions. Among these are (i) the QCD scale μ , (ii) value of the form factors, (iii) value of the light quark masses, and in particular m_s , (iv) the value $\xi = 1/N_c$, and (v) charm content of η' . It is possible to account for all the data without invoking new physics, though future experiments will provide tighter constraints on the parameter space. We find that CP violating asymmetries are in the observable range for some modes. [S0556-2821(98)00609-2]

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I. INTRODUCTION

The recent CLEO measurement for the branching ratio of $B \rightarrow \eta' K$ [1] is larger than expected. This result has initiated numerous investigations, with some even suggesting new physics. In this paper we attempt to explain the whole set of known results on two body decays of B mesons into light pseudoscalars within the context of standard model (SM) using the generalized factorization technique. This technique is very successful in decays of B mesons to D mesons [2]. If this approximation is able to explain all two body B decays, we will have a powerful tool to extract various parameters such as the Cabibbo-Kobayashi-Maskawa (CKM) elements.

Present attempts to explain the large branching ratio $B(B^\pm \rightarrow \eta' K^\pm)$ involve different assumptions [3–6]. Some [3,4] explain it on the basis of large form factors, but SU(3) constraints on the form factors have been ignored. For example, in the flavor SU(3) limit, there are relations among the form factors: $F^{B \rightarrow \eta'}(0) = (\sin \theta / \sqrt{6} + \cos \theta / \sqrt{3}) F^{B \rightarrow \pi^-}(0)$ and $F^{B \rightarrow \pi^-}(0) = F^{B \rightarrow K^-}(0)$ (where θ denotes the $\eta - \eta'$ mixing angle). Taking $F^{B \rightarrow \eta'}$ large could have the undesirable effect of increasing $B \rightarrow \pi K$ and $B \rightarrow \pi \pi$ rates above the present bound. Others have invoked charm for η' , with a contribution arising from $b \rightarrow s(\bar{c}c) \rightarrow s \eta'(\eta)$. Explanations have been proposed with large $|f_{\eta'}^{(c)}| \approx 50$ MeV [6] and a relatively smaller value of $|f_{\eta'}^{(c)}| \approx 6$ MeV [5]. The effect of a low strange quark mass in enhancing the rate has been noted [3,5]. In an interesting paper [7], the consequences of a large $B \rightarrow \eta' K$ branching ratio from the purely SU(3) viewpoint has been studied. We shall focus our attention on a more dynamical analysis based on generalized factorization in the spirit of Ali and Greub [5].

The branching ratio of $B \rightarrow \eta' K$ depends on a number of parameters. These parameters include the value of the strange quark mass m_s , the possibility of QCD scale dependence μ , the size of the form factors, the value of the param-

eter $\xi \equiv 1/N_c$ which arises in the generalized factorization model, the $\eta - \eta'$ mixing angle θ , and the value of the CKM elements and weak phases. We approach the problem by first studying $B \rightarrow \pi \pi$ decays. These decays have only slight μ dependence, and already limit the size of the form factors. By studying $B \rightarrow K \pi$ next, we again see the μ dependence in Wilson coefficients (WC's) is offset by the scale dependence of m_s , and the branching ratios have very slight μ dependence. It is possible to enhance $B \rightarrow \eta' K$ by choosing a small value ξ . Study of the ratio of $B \rightarrow \eta' K$ to $B \rightarrow \pi K$, which is independent of the form factors, reveals that a small value of γ , the weak phase, is preferred. We are able to account for all data without assuming the charm content of η' . With the parameter space obtained, we look at the CP asymmetries as a function of γ , and point out that $B \rightarrow \pi K$ and $B \rightarrow \eta K$ provide two interesting modes with significant asymmetries.

We organize this work as follows. In Sec. II we obtain the Wilson coefficients and the strange quark mass at the scale m_b and $m_b/2$. In Sec. III we discuss the factorization approximation. In Sec. IV, first we discuss the decays of B into $\pi \pi$ modes, then we discuss πK , $\eta' K$, and ηK and show the parameter space where the calculated branching ratio of $B \rightarrow \eta' K$ is experimentally allowed. In Sec. V, we discuss the CP asymmetries in the B decay modes. Finally, in Sec. VI we summarize our results.

II. DETERMINATION OF THE EFFECTIVE WILSON COEFFICIENTS

The effective weak Hamiltonian for hadronic B decays can be written as

$$H_{\Delta B=1} = \frac{4G_F}{\sqrt{2}} \left[V_{ub} V_{uq}^* (c_1 O_1^u + c_2 O_2^u) + V_{cb} V_{cq}^* (c_1 O_1^c + c_2 O_2^c) - V_{tb} V_{tq}^* \sum_{i=3}^{12} c_i O_i \right] + \text{H.c.}, \quad (1)$$

where O_i 's are defined as

$$\begin{aligned}
O_1^f &= \bar{q}_\alpha \gamma_\mu L f \bar{f}_\beta \gamma^\mu L b_\alpha, \\
O_2^f &= \bar{q} \gamma_\mu L f \bar{f} \gamma^\mu L b, \\
O_{3(5)} &= \bar{q} \gamma_\mu L b \Sigma \bar{q}' \gamma^\mu L(R) q', \\
O_{4(6)} &= \bar{q}_\alpha \gamma_\mu L b \Sigma \bar{q}'_\beta \gamma^\mu L(R) q'_\alpha, \\
O_{7(9)} &= \frac{3}{2} \bar{q} \gamma_\mu L b \Sigma e_{q'} \bar{q}' \gamma^\mu R(L) q', \\
O_{8(10)} &= \frac{3}{2} \bar{q}_\alpha \gamma_\mu L b \Sigma e_{q'} \bar{q}'_\beta \gamma^\mu R(L) q'_\alpha, \\
O_{11} &= \frac{g_s}{32\pi^2} m_b \bar{q} \sigma_{\mu\nu} R T_a b G_a^{\mu\nu}, \\
O_{12} &= \frac{e}{32\pi^2} m_b \bar{q} \sigma_{\mu\nu} R b F^{\mu\nu},
\end{aligned} \tag{2}$$

where $L(R) = (1 \mp \gamma_5)/2$, f can be u or c quark, q can be d or s quark, and q' is summed over u, d, s , and c quarks. α and β are the color indices. T^a is the SU(3) generator with the normalization $Tr(T^a T^b) = \delta^{ab}/2$. $G_a^{\mu\nu}$ and $F_{\mu\nu}$ are the gluon and photon field strength. c_i 's are the WC's. O_1, O_2 are the tree level and QCD corrected operators. O_{3-6} are the gluon induced strong penguin operators. O_{7-10} are the electroweak penguin operators due to γ and Z exchange, and "box" diagrams at loop level. In this work we shall take into account the chromomagnetic operator O_{11} but neglect the extremely small contribution from O_{12} .

We obtain the $c_i(\mu)$'s by solving the following renormalization group equation (RGE):

$$\left(-\frac{\partial}{\partial t} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) \mathbf{C}(m_W^2/\mu^2, g^2) = \frac{\hat{\gamma}^T(g^2)}{2} \mathbf{C}(t, \alpha_s(\mu), \alpha_e), \tag{3}$$

where $t \equiv \ln(M_W^2/\mu^2)$ and \mathbf{C} is the column vector that consists of (c_i) 's. The beta and the gamma are given by

$$\begin{aligned}
\beta(\alpha_s) &= -\left(11 - \frac{2}{3} n_f \right) \frac{\alpha_s^2}{16\pi^2} \\
&\quad - \left(102 - \frac{38}{3} n_f \right) \frac{\alpha_s^4}{(16\pi^2)^2} + \dots, \\
\hat{\gamma}(\alpha_s) &= \left(\gamma_s^{(0)} + \gamma_{se}^{(1)} \frac{\alpha_{em}}{4\pi} \right) \frac{\alpha_s}{4\pi} + \gamma_e^{(0)} \frac{\alpha_{em}}{4\pi} + \gamma_s^{(1)} \frac{\alpha_s^2}{(4\pi)^2} + \dots,
\end{aligned} \tag{4}$$

where α_{em} is the electromagnetic coupling and n_f is the number of active quark flavors.

The anomalous-dimension matrices $\gamma_s^{(0)}$ and $\gamma_e^{(0)}$ determine the leading log corrections and they are renormalization scheme independent. The next to leading order correc-

tions which are determined by $\gamma_{se}^{(1)}$ and $\gamma_s^{(1)}$ are renormalization scheme dependent. The γ 's have been determined in Refs. [8,9].

We can express $C(\mu)$ (where μ lies between M_W and m_b) in terms of the initial conditions for the evolution equations :

$$C(\mu) = U(\mu, M_W) C(M_W). \tag{5}$$

$C(M_W)$'s are obtained from matching the full theory to the effective theory at the M_W scale [9,10]. The WC's so far obtained are renormalization scheme dependent. In order to make them scheme independent we need to use a suitable matrix T [9]. The WC's at the scale $\mu = m_b$ are given by

$$\bar{C}(\mu) = T U(m_b, M_W) C(M_W). \tag{6}$$

The matrix T is given by

$$T = \mathbf{1} + \hat{r}_s^T \frac{\alpha_s}{4\pi} + \hat{r}_e^T \frac{\alpha_e}{4\pi}, \tag{7}$$

where \hat{r} depends on the number of up-type quarks and the down type quarks, respectively. The r 's are given in Ref. [9]. In order to determine the coefficients at the scale $\mu < m_b$, we need to use the matching of the evolutions between the scales larger and smaller than the threshold. In that case in the expression for T we need to use $\delta\hat{r}$ instead of \hat{r} , where $\delta\hat{r} = r_{u,d} - r_{u,d-1}$, and u and d are the number of up type quarks and the number of down type quarks, respectively. The matrix elements (O_i 's) are also needed to have a one loop correction. The procedure is to write the one loop matrix element in terms of the tree level matrix element and to generate the effective Wilson coefficients [11]:

$$\langle c_i O_i \rangle = \sum_{ij} c_j(\mu) \left[\delta_{ij} + \frac{\alpha_s}{4\pi} m_{ij}^s + \frac{\alpha_{em}}{4\pi} m_{ij}^e \right] \langle O_j \rangle^{tree}, \tag{8}$$

$$\begin{pmatrix} c_1^{eff} \\ c_2^{eff} \\ c_3^{eff} \\ c_4^{eff} \\ c_5^{eff} \\ c_6^{eff} \\ c_7^{eff} \\ c_8^{eff} \\ c_9^{eff} \\ c_{10}^{eff} \end{pmatrix} = \begin{pmatrix} \bar{c}_1 \\ \bar{c}_2 \\ \bar{c}_3 - P_s/3 \\ \bar{c}_4 + P_s \\ \bar{c}_5 - P_s/3 \\ \bar{c}_6 + P_s \\ \bar{c}_7 + P_e \\ \bar{c}_8 \\ \bar{c}_9 + P_e \\ \bar{c}_{10} \end{pmatrix} \tag{9}$$

where

TABLE I. Effective Wilson coefficients for the $b \rightarrow s$ transition at the scale m_b and $m_b/2$ are shown.

WC's	$\mu = \frac{m_b}{2}$	$\mu = m_b$
C_1^{eff}	-0.282	-0.3209
C_2^{eff}	1.135	1.149
C_3^{eff}	0.0228718+i0.004689	0.02175-i0.0041396
C_4^{eff}	-0.051144-i0.004689	-0.04906-i0.0124188
C_5^{eff}	0.0162153+i0.004689	0.015601+i0.0041396
C_6^{eff}	-0.0653549-i0.0140673	-0.060632-i0.0124188
C_7^{eff}	0.00122773+i0.00005724	-0.000859+i0.0000728
C_8^{eff}	-0.0000953211	0.00143303
C_9^{eff}	-0.0120155+i0.0000572433	-0.011487+i0.0000727
C_{10}^{eff}	0.00218628	0.00317436
C_{11}^{eff}	-0.334	-0.295

$$P_s = (\alpha_s/8\pi) \bar{c}_2 \left[\frac{V_{cb}V_{cq}^*}{V_{tb}V_{tq}^*} (10/9 + G(m_c, \mu, q^2)) + \frac{V_{ub}V_{uq}^*}{V_{tb}V_{tq}^*} (10/9 + G(m_u, \mu, q^2)) \right]$$

and

$$P_e = (\alpha_{em}/9\pi) (3\bar{c}_1 + \bar{c}_2) \left[\frac{V_{cb}V_{cq}^*}{V_{tb}V_{tq}^*} \times (10/9 + G(m_c, \mu, q^2)) + \frac{V_{ub}V_{uq}^*}{V_{tb}V_{tq}^*} \times (10/9 + G(m_u, \mu, q^2)) \right].$$

$V_{i,j}$ are the elements of the CKM matrix. m_c is the charm quark mass and m_u is the up quark mass. The function $G(m, \mu, q^2)$ is given by

$$G(m, \mu, q^2) = 4 \int_0^1 x(1-x) dx \ln \frac{m^2 - x(1-x)q^2}{\mu^2}. \quad (10)$$

In the numerical calculation, we will use $q^2 = m_b^2/2$ which represents the average value and the full expressions for $P_{s,e}$. In Table I we show the values of the effective Wilson coefficients at the scale m_b and $m_b/2$ for the process $b \rightarrow sq\bar{q}$. Values for $b \rightarrow dq\bar{q}$ can be similarly obtained. These coefficients are scheme independent and gauge invariant.

III. MATRIX ELEMENTS IN FACTORIZATION APPROXIMATION

The generalized factorizable approximation has been quite successfully used in two body D decays as well as $B \rightarrow D$ decays. The method includes color octet nonfactorizable contribution by treating $\xi \equiv 1/N_c$ as an adjustable parameter [12]. Justification for this process has been recently discussed from QCD considerations [13,14]. In general ξ is process dependent, but using SU(3) flavor symmetry, it should be the same for the $B \rightarrow \pi\pi, K\pi, K\eta'(\eta)$ system. Es-

tablishing the range of value of ξ for the best fit will be one of our goals.

The technique of parametrizing a two body decay amplitude in factorization approximation is well known. Here we shall do it for the $B \rightarrow K\eta'(\eta)$ process to establish our notation and discuss some special issues relating to this process.

We define the decay constants and the form factors as

$$\langle 0 | A_\mu | M(p) \rangle = i f_M p_\mu, \quad (11)$$

$$\langle M(p') | V_\mu | B(p) \rangle = \left[(p' + p)_\mu - \frac{m_B^2 - m_K^2}{q^2} q_\mu \right] F_1^{B \rightarrow M}(q^2) + \frac{m_B^2 - m_K^2}{q^2} q_\mu F_0^{B \rightarrow M}(q^2), \quad (12)$$

where M , V_μ , and A_μ denote a pseudoscalar meson, a vector current, and an axial-vector current, respectively, and $q = p - p'$. Note that $F_1(0) = F_0(0)$ and we can set $F_{0,1}^{B \rightarrow M}(q^2 = m_M^2) \approx F_{0,1}^{B \rightarrow M}(0)$ since these form factors are pole dominated by mesons at scale m_B^2 .

The physical states η and η' are mixtures of SU(3) singlet state η_1 and octet state η_8 :

$$\eta = \eta_8 \cos \theta - \eta_1 \sin \theta, \quad \eta' = \eta_8 \sin \theta + \eta_1 \cos \theta, \quad (13)$$

with

$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}), \quad \eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}), \quad (14)$$

The decay constants f_η^u and f_η^s , which are similarly defined as Eqs. (17) and (18), have the relations similar to Eq. (19):

$$f_\eta^u = \frac{f_8}{\sqrt{6}} \cos \theta - \frac{f_1}{\sqrt{3}} \sin \theta, \quad f_\eta^s = -2 \frac{f_8}{\sqrt{6}} \cos \theta - \frac{f_1}{\sqrt{3}} \sin \theta. \quad (15)$$

In the SU(3) limit, $f_K = f_\pi = f_8$. However from light quark meson decays their values can be obtained. In particular the values of f_8 and f_1 can be obtained from $\eta \rightarrow \gamma\gamma$ and $\eta' \rightarrow \gamma\gamma$ provided the mixing angle θ is known. We shall see later that a larger magnitude of θ enhances the η' decays. We shall thus use the value $\theta = -25^\circ$ which leads to $f_8 \sim 1.75 f_\pi$ and $f_8 \sim f_\pi$ [15] and we use $f_\pi = 132$ MeV and $f_K = 158$ MeV. A technical point is to note that when we evaluate $\langle 0 | \bar{s}i\gamma_5 s | \eta \rangle$ or $\langle 0 | \bar{s}i\gamma_5 s | \eta' \rangle$, because of anomalies in the corresponding current $\bar{s}\gamma_\mu\gamma_5 s$, we use anomaly free currents and neglect terms corresponding to light quark masses as discussed in Ref. [15]. We then have

$$\begin{aligned}\langle 0|\bar{s}i\gamma_5s|\eta'\rangle &= -\frac{\sqrt{3}f_8\sin\theta m_{\eta'}^2}{\sqrt{2}2m_s}, \\ \langle 0|\bar{s}i\gamma_5s|\eta\rangle &= -\frac{\sqrt{3}f_8\cos\theta m_{\eta'}^2}{\sqrt{2}2m_s}.\end{aligned}\quad (16)$$

The decay constants $f_{\eta'}^u$ and $f_{\eta'}^s$ are defined as

$$\langle 0|\bar{u}\gamma_\mu\gamma_5u|\eta'\rangle = if_{\eta'}^u p_\mu, \quad (17)$$

$$\langle 0|\bar{s}\gamma_\mu\gamma_5s|\eta'\rangle = if_{\eta'}^s p_\mu. \quad (18)$$

Due to $\eta-\eta'$ mixing, $f_{\eta'}^u$ and $f_{\eta'}^s$ are related to f_8 and f_1 by

$$f_{\eta'}^u = \frac{f_8}{\sqrt{6}}\sin\theta + \frac{f_1}{\sqrt{3}}\cos\theta, \quad f_{\eta'}^s = -2\frac{f_8}{\sqrt{6}}\sin\theta + \frac{f_1}{\sqrt{3}}\cos\theta, \quad (19)$$

where f_8 and f_1 are defined as

$$\langle 0|\bar{u}\gamma_\mu\gamma_5u|\eta_8\rangle = i\frac{f_8}{\sqrt{6}}p_\mu, \quad \langle 0|\bar{u}\gamma_\mu\gamma_5u|\eta_1\rangle = i\frac{f_1}{\sqrt{3}}p_\mu. \quad (20)$$

We shall assume that form factors are related by nonet symmetry. For a current $V_\mu = \bar{u}\gamma_\mu b$ this implies

$$\begin{aligned}F_0^{B\rightarrow K} &= F_0^{B\rightarrow\pi^-} \\ &= \sqrt{2}F_0^{B\rightarrow\pi^0} \\ &= \sqrt{6}F_0^{B\rightarrow\eta_8} \\ &= \sqrt{3}F_0^{B\rightarrow\eta_0}.\end{aligned}\quad (21)$$

We expect the SU(3) breaking effect to be $O(15)\%$. In particular $F_0^{B\rightarrow\eta_0}$ could be smaller if η_0 has significant glue content. Form factors $F_0^{B\rightarrow\eta}$ and $F_0^{B\rightarrow\eta'}$ are then

$$\begin{aligned}F_0^{B\rightarrow\eta} &= F_0^{B\rightarrow\pi^-}(\cos\theta/\sqrt{6} - \sin\theta/\sqrt{3}), \\ F_0^{B\rightarrow\eta'} &= F_0^{B\rightarrow\pi^-}(\sin\theta/\sqrt{6} + \cos\theta/\sqrt{3}).\end{aligned}\quad (22)$$

There seems to be considerable variation in the range of $F_0^{B\rightarrow\pi^-}$ estimated in the literature. Bauer *et al.* [16] estimate it at 0.33 while Deandrea *et al.* [17] obtain 0.5. Since the rate is proportional to the $|F_0|^2$, this can be a source of considerable error. We find that data on the $B\rightarrow\pi^+\pi^-$ mode places rather stringent constraint on the magnitude of the form factors with values near Bauer *et al.* being preferred.

The decay amplitude for $B^-\rightarrow\eta'K^-$ is now found to be

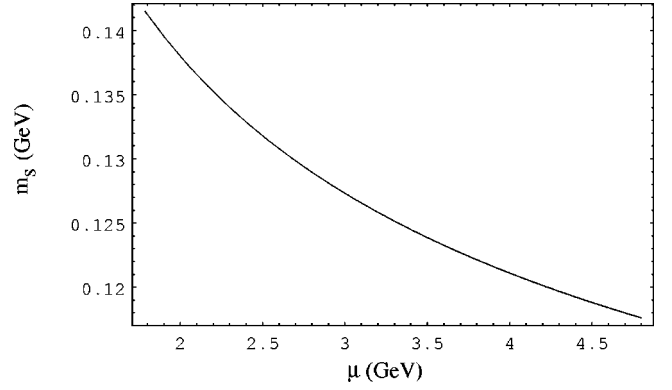


FIG. 1. Strange quark mass m_s is plotted as a function of μ . μ has been varied between the m_τ mass and the m_b mass.

$$\begin{aligned}A(B^-\rightarrow\eta'K^-) &= \frac{G_F}{\sqrt{2}}\left\{V_{ub}V_{us}^*[(c_1+\xi c_2)C^u+(c_1+c_2)T] \right. \\ &\quad - V_{tb}V_{ts}^*\left[(c_3+\xi c_4-c_5-\xi c_6) \right. \\ &\quad \times(2C^u+C^s)+(c_3+c_4)(C^s+T) \\ &\quad +2(\xi c_5+c_6)(X\bar{C}^s+YT) \\ &\quad -\frac{1}{2}(c_7+\xi c_8-c_9-\xi c_{10})(C^u-C^s) \\ &\quad -(\xi c_7+c_8)X\bar{C}^s+2(\xi c_7+c_8)YT \\ &\quad \left. \left. -\frac{1}{2}(\xi c_9+c_{10})C^s+(\xi c_9+c_{10})T\right]\right\}+A_{11},\end{aligned}\quad (23)$$

where

$$C^u = if_{\eta'}^u F_0^{B\rightarrow K}(m_B^2 - m_K^2),$$

$$C^s = if_{\eta'}^s F_0^{B\rightarrow K}(m_B^2 - m_K^2),$$

$$\bar{C}^s = -i\frac{\sqrt{3}}{\sqrt{2}}f_8\sin\theta F_0^{B\rightarrow K}(m_B^2 - m_K^2),$$

$$T = if_K F_0^{B\rightarrow\eta'}(m_B^2 - m_{\eta'}^2),$$

$$X = \frac{m_{\eta'}^2}{2m_s(m_b - m_s)},$$

$$Y = \frac{m_K^2}{(m_s + m_u)(m_b - m_u)}. \quad (24)$$

Here we have neglected a small contribution of the annihilation term which is proportional to f_B . A_{11} represents a contribution from chromomagnetic operator O_{11} , and is evaluated as in Refs. [3,18].

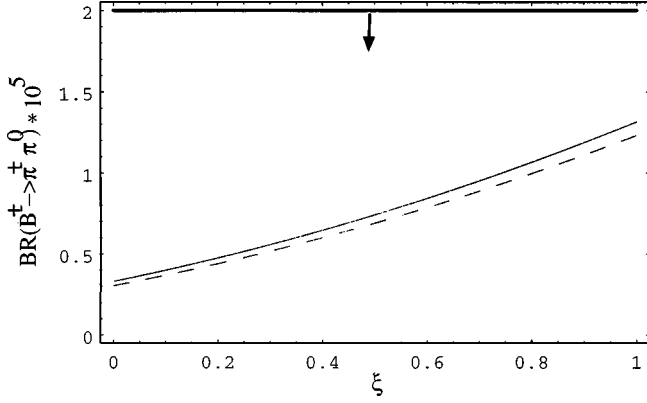


FIG. 2. Branching ratio for the average of $B^\pm \rightarrow \pi^\pm \pi^0$ as a function of ξ . The solid curve is for $\mu = m_b/2$ and the dashed curve is for $\mu = m_b$. The direction of the thick arrows indicate the regions allowed by the available experimental data.

The amplitude for $B \rightarrow \eta K$ can be deduced by appropriate replacement of η' by η . In amplitudes where penguin contributions dominate, we observe that X and Y contributions are very sensitive to the value of light quark contributions m_s . Depending on the scale μ , we have to employ the corresponding value of m_s . We show a plot of m_s as a function of μ in Fig. 1. We use $m_s = 165$ GeV at $\mu = 1$ GeV. This leads to $m_s(m_b/2) = 121$ MeV and $m_s(m_b) = 118$ MeV. If a smaller value of m_s (1 GeV) is used, processes involving K mesons are enhanced. Although this will enhance $B \rightarrow K \eta'$, we will then have too large a value for $B \rightarrow K^+ \pi^-$. We find the choice of 165 MeV is optimal. We shall show later that the μ dependence of the rate is quite weak because of the compensating effect of μ dependence of m_s and μ dependence of WC's. Ali and Greub have advocated that η' and η might contain a considerable amount of $c\bar{c}$ contribution, and this enhances $B \rightarrow \eta' K$. They have argued that if

$$\langle 0 | \bar{c} \gamma_\mu \gamma_5 c | \eta'(p) \rangle = i f_{\eta'}^c p_\mu, \quad (25)$$

$$\langle 0 | \bar{c} \gamma_\mu \gamma_5 c | \eta(p) \rangle = i f_\eta^c p_\mu,$$

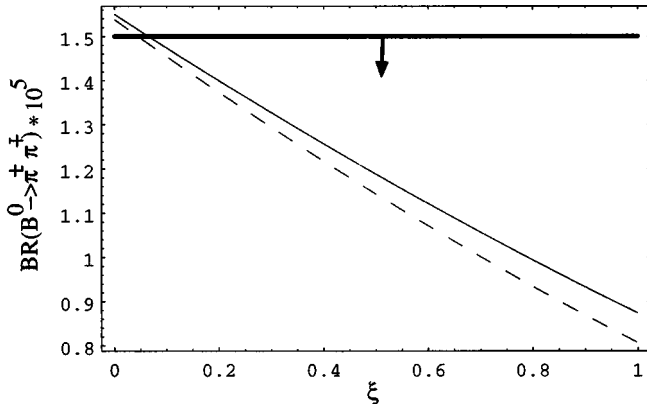


FIG. 3. Branching ratio for the average of $B^0(\bar{B}^0) \rightarrow \pi^+ \pi^-$ as a function of ξ . The solid curve is for $\mu = m_b/2$ and the dashed curve is for $\mu = m_b$.

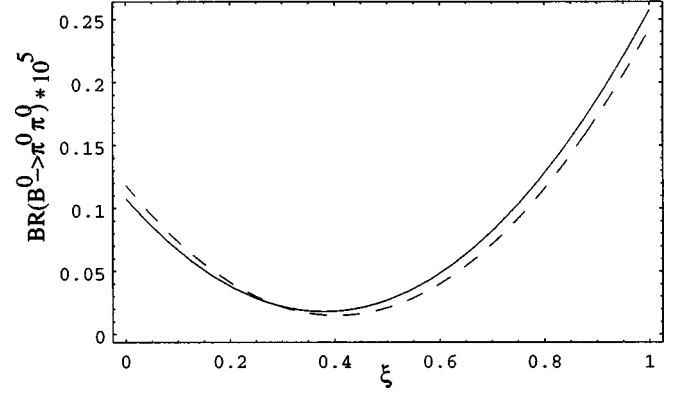


FIG. 4. Branching ratio for the average of $B^0(\bar{B}^0) \rightarrow \pi^0 \pi^0$ as a function of ξ . The solid curve is for $\mu = m_b/2$ and the dashed curve is for $\mu = m_b$.

then $f_{\eta'}^c \approx 6$ MeV and $f_\eta^c \approx 2.3$ MeV. This should be compared to $f_{\eta'}^u = 50$ MeV and $f_\eta^u = 100$ MeV. We shall show that it is possible to fit data without the charm content within 1σ of the experimental error. If further experiments were to narrow the rate for $B \rightarrow \eta' K$ at the upper end of the present range, this would be a strong argument for the charm content. With the inclusion of charm, the amplitude in Eq. (23) has to include the term

$$A' = -\frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (c_1 + \xi c_2) (f_{\eta'}^c / f_{\eta'}^u) C^u. \quad (26)$$

For $B \rightarrow \eta K$ we must include a similar term with $f_{\eta'}^c$ replaced by f_η^c .

IV. DECAYS OF B INTO PSEUDOSCALARS

A. Process $B \rightarrow \pi\pi$

Here we consider the decays $B^\pm \rightarrow \pi^\pm \pi^0$, $B^0(\bar{B}^0) \rightarrow \pi^+ \pi^-$, and $B^0(\bar{B}^0) \rightarrow \pi^0 \pi^0$. The recent measurement at CLEO [19] yielded the following bound at 90% C.L.:

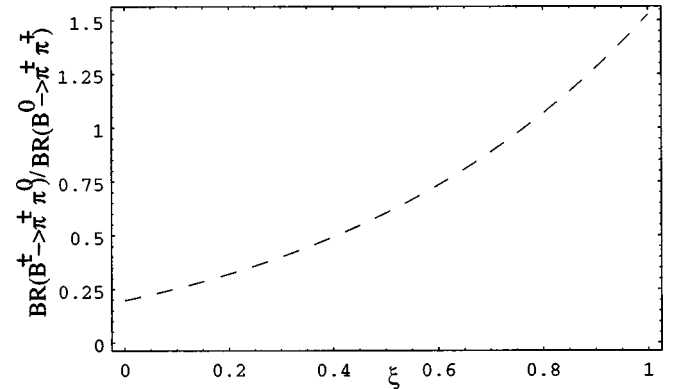


FIG. 5. Ratio of the branching ratio of $B^\pm \rightarrow \pi^\pm \pi^0$ and $B^0(\bar{B}^0) \rightarrow \pi^+ \pi^-$ as a function of ξ . The curve is drawn for $\mu = m_b$.

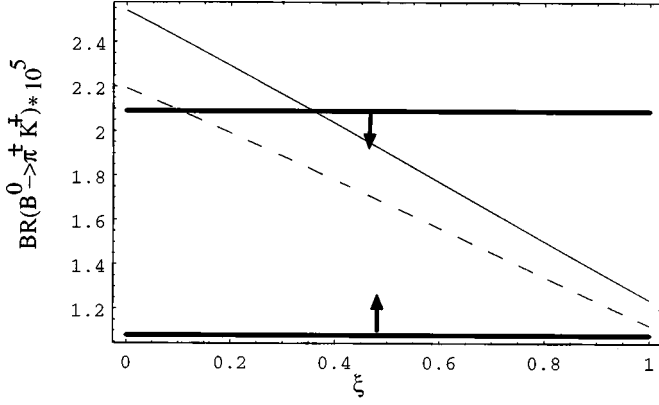


FIG. 6. Branching ratio for the average of $B^0(\bar{B}^0) \rightarrow \pi^+ \pi^-$ as a function of ξ . The solid curve is for $\mu = m_b/2$ and the dashed curve is for $\mu = m_b$.

$$B(B^+ \rightarrow \pi^0 \pi^+) < 2 \times 10^{-5}, \quad B(B^0 \rightarrow \pi^+ \pi^-) < 1.5 \times 10^{-5}.$$

The decay rates scale as $|F_0^{B \rightarrow \pi}(0)|^2$ and since the tree diagram dominates the processes $B^\pm \rightarrow \pi^\pm \pi^0$ and $B^0(\bar{B}^0) \rightarrow \pi^+ \pi^-$, these two decays also scale as $|V_{ub}/V_{cb}|^2$. Dependence on the QCD scale μ for these two decays is rather mild because the tree amplitude depends on the Wilson coefficients c_1 and c_2 , and these are weakly dependent on μ . Further, the light quark masses in the matrix elements also scale with μ , partially offsetting the μ dependence from $c_{1,2}$. The partial width for $B^+ \rightarrow \pi^+ \pi^0$ for example is obtained from

$$\Gamma(B^+ \rightarrow \pi^+ \pi^0) = \frac{1}{8\pi} \frac{|p|}{m_B^2} |A(B^+ \rightarrow \pi^+ \pi^0)|^2 \quad (27)$$

where $|p|$ is the pion momentum and the branching ratio is calculated by multiplying by the total rate $\tau_B = 1.49$ ps. In Figs. 2–4 we plot branching ratios averaged over particle and antiparticle for the modes $\pi^\pm \pi^0$, $\pi^+ \pi^-$, and $\pi^0 \pi^0$, as a function of $\xi \equiv 1/N_c$ for two different values of the scale μ , $\mu = m_b$, and $\mu = m_b/2$. We have assumed $|V_{ub}/V_{cb}| = 0.07$, $\gamma = 35^\circ$, and the form factor $F_0^{B \rightarrow \pi} = 0.36$. We see

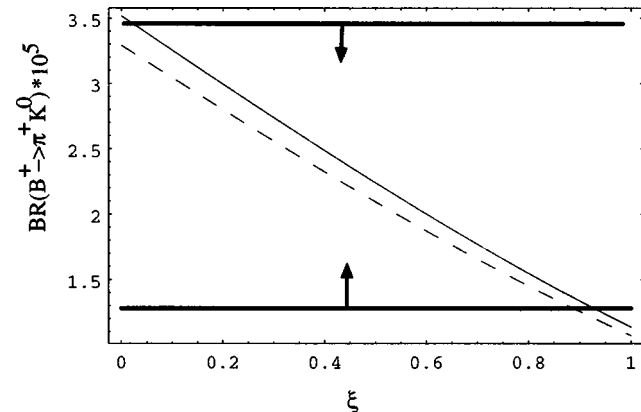


FIG. 7. Branching ratio for the average of $B^+ \rightarrow \pi^+ K^0$ as a function of ξ . The solid curve is for $\mu = m_b/2$ and the dashed curve is for $\mu = m_b$.

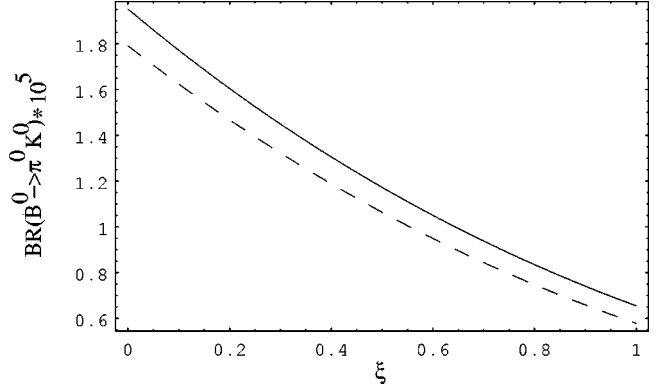


FIG. 8. Branching ratio for the average of $B^0(\bar{B}^0) \rightarrow \pi^0 K^0$ as a function of ξ . The solid curve is for $\mu = m_b/2$ and the dashed curve is for $\mu = m_b$.

the weak dependence on the scale μ , but strong dependence on ξ . We shall see later that to enhance $B \rightarrow \eta' K$ values of $\xi \sim 0$ are preferred. In the range where ξ is small, the present bounds on the $\pi^+ \pi^-$ branching ratio of 1.5×10^{-5} already tells us that the product $|V_{ub}F/V_{cb}| \leq 0.024$. To enhance $B \rightarrow \eta' K$ a large form factor is preferred. Since $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$, we see that we are forced into a region of small $|V_{ub}|$ if we wish to explain $B \rightarrow \eta' K$ without invoking new physics. Further, the form factor cannot be taken larger than 0.4 without violating the present bounds on $|V_{ub}/V_{cb}|$. The value of γ used does not alter the above conclusions; however, it will be important when we consider CP violating effects. The ratio of $B^\pm \rightarrow \pi^\pm \pi^0$ and $B^0(\bar{B}^0) \rightarrow \pi^+ \pi^-$ is not sensitive to the form factor or V_{ub} , but is very sensitive to values of ξ . In Fig. 5 we plot this ratio for $\mu = m_b$. Future measurements of this ratio will constrain the value of ξ . We shall see later that a larger form factor, although favorable in increasing $B \rightarrow \eta' K$, also enhances $B \rightarrow \pi K$, resulting in conflict with the experiment. We find that the form factor $|F_0^{B \rightarrow \pi^-}| = 0.36$ and $|V_{ub}/V_{cb}| \sim 0.07$ are the best compromise. In summary, bounds on $B \rightarrow \pi^+ \pi^-$ already provide strong constraints on the size of the form factors and the value of $|V_{ub}/V_{cb}|$.

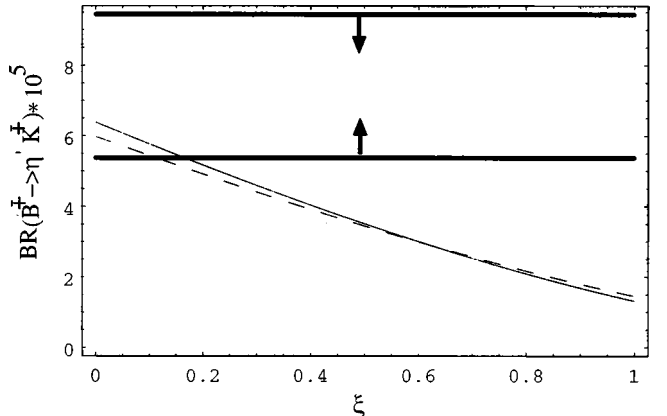


FIG. 9. Branching ratio for the average of $B^+ \rightarrow \eta' K^+$ as a function of ξ . The solid curve is for $\mu = m_b/2$ and the dashed curve is for $\mu = m_b$.

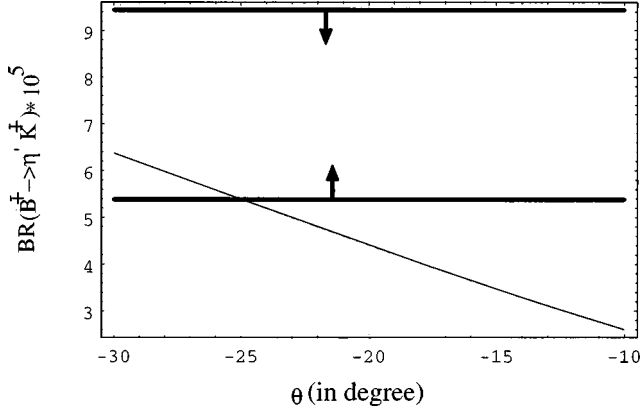


FIG. 10. Branching ratio for the average of $B^\pm \rightarrow \eta' K^\pm$ as a function of θ . The curve is drawn at $\mu = m_b$.

B. Processes $B \rightarrow \pi K$, $B^\pm \rightarrow \eta' K^\pm$, and $B^\pm \rightarrow \eta K^\pm$

We now examine the two body processes involving kaons. The recent measurement at CLEO [1,19] yielded the following bound:

$$B(B^\pm \rightarrow \pi^\pm K) = (2.3_{-1.0-0.2}^{+1.1+0.2} \pm 0.2) \times 10^{-5},$$

$$B(B^0 \rightarrow \pi^\pm K^\mp) = (1.5_{-0.4-0.1}^{+0.5+0.1} \pm 0.1) \times 10^{-5},$$

$$B(B^\pm \rightarrow \eta' K^\pm) = (7.8_{-2.2}^{+2.7} \pm 1.0) \times 10^{-5}.$$

We again choose the value of the form factor $F_0^{B \rightarrow K} = 0.36$, and weak phase $\gamma \sim 35^\circ$. In Figs. 6 and 7, we have plotted the branching ratio for $B^+ \rightarrow \pi^+ K^0$ and $B^0 \rightarrow \pi^- K^+$, averaged over particle-antiparticle decays as a function of ξ for $\mu = m_b$ and $\mu = m_b/2$. There is only a slight μ dependence with $B^+ \rightarrow \pi^+ K^0$ being slightly larger for $\mu = m_b$. Both rates are enhanced at small ξ . In particular the observed branching ratio of $B^0 \rightarrow \pi^- K^+$ already constrains $\xi > 0.1$. If a smaller value of the form factor is used, the rate for B

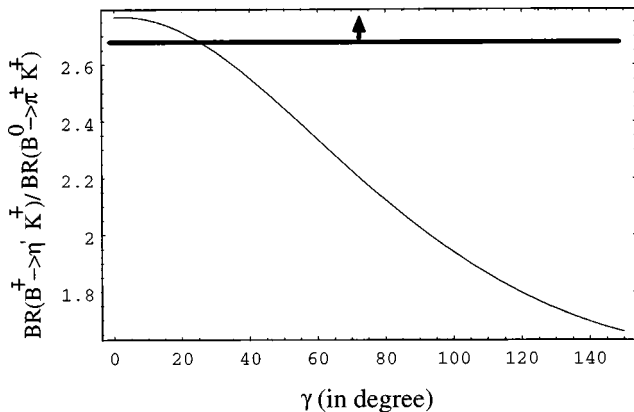


FIG. 11. Ratio of the branching ratio for $B^\pm \rightarrow \eta' K^\pm$ and $B^0(\bar{B}^0) \rightarrow \pi^\pm K^\mp$ as a function of γ . The curve is drawn at $\mu = m_b$.

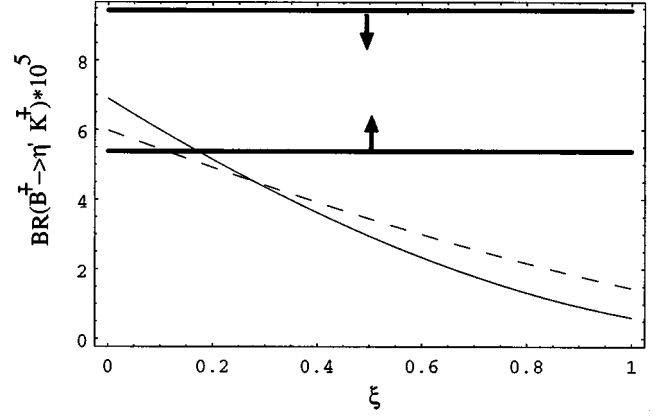


FIG. 12. Branching ratio for the average of $B^\pm \rightarrow \eta' K^\pm$ as a function of ξ (dashed line). Same branching ratio but with the assumption that η' has charm content (solid line) with $f_{\eta'}^c = 5.8$ MeV. Both lines are drawn for $\mu = m_b$.

$\rightarrow \eta' K$ will go down correspondingly. We have also plotted the average value of $B^0 \rightarrow \pi^0 K^0$ as a function of ξ for two different μ in Fig. 8.

Turning to $B \rightarrow \eta' K$, we first examine the ξ dependence for two different values of μ . We again see in Fig. 9 an enhancement for small ξ and slightly larger values for $\mu = m_b$. This figure is based on $\eta - \eta'$ mixing of $\theta = -25^\circ$. Clearly, values of $\xi = 0.2$ are consistent with data at 1σ . It is not possible to enhance the rate by increasing the form factor, because $B^0 \rightarrow \pi^- K^+$ will then become too large.

We have examined the branching ratio dependence of $B \rightarrow \eta' K$ on the mixing angle θ . In Fig. 10 we plot the branching ratio for $B \rightarrow \eta' K$ as a function of θ , and find that the branching ratio increases as θ becomes more negative.

From experiment we also have the following bound at 1σ :

$$R = \frac{B(B^+ \rightarrow \eta' K^+)}{B(B^0 \rightarrow \pi^- K^+)} \geq 2.7. \quad (28)$$

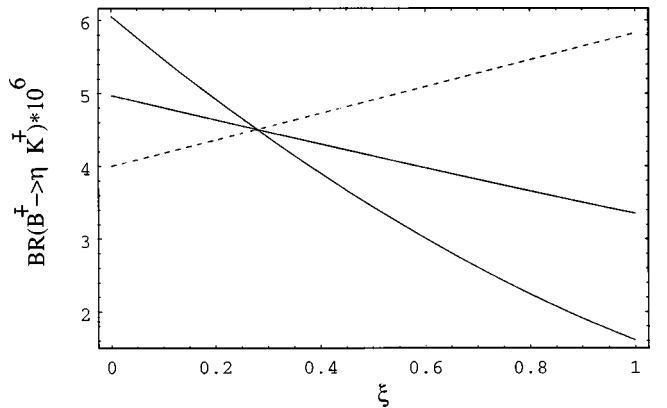


FIG. 13. Branching ratio for the average of $B^\pm \rightarrow \eta' K^\pm$ as a function of ξ (dashed line). Same branching ratio but with the assumption that η has charm content (solid line and small dashed line). The solid line and the small dashed line have different combination of signs for $f_{\eta'}^c$. We have used $f_{\eta'}^c = 2.3$ MeV. All the contours are drawn for $\mu = m_b$.

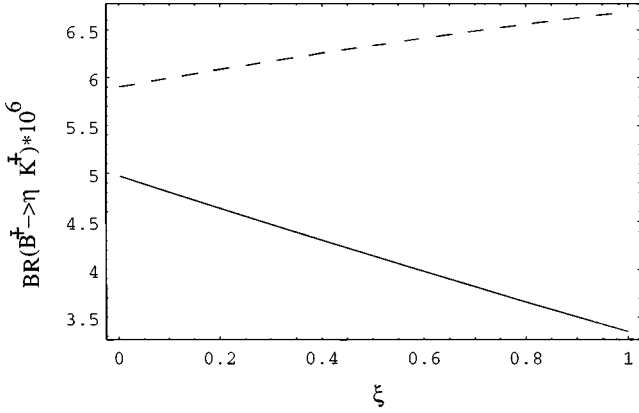


FIG. 14. Branching ratio for the average of $B^\pm \rightarrow \eta K^\pm$ as a function of ξ (solid line). Same branching ratio but without the electroweak contribution (dashed line). Both the lines are drawn for $\mu = m_b$.

In Fig. 11, we plot this ratio as a function of the weak phase γ for $\xi = 0.1$. Since this ratio does not depend on the size of the form factors, or the value of the $|V_{cb}|$, we see that there is a strong preference for the values of small γ . We therefore have chosen a small value of $\gamma \sim 35^\circ$ for our plots.

If further experiments reduce the error on the $B \rightarrow \eta' K$ branching ratio, and it turns out to be a larger number, one may have to consider the Ali-Greub suggestion of including the charm content. With values of $f_{\eta'}^c \approx 5.8$ MeV, and sign so chosen to give a constructive interference, we plot the branching ratio of $B \rightarrow K \eta'$ as a function of ξ in Fig. 12. As we see, there is about a 15% enhancement in the rate at $\xi = 0$.

We consider $B \rightarrow \eta K$ as a function of ξ with or without inclusion of charm in Fig. 13. For $\xi = 0.1$, the branching ratio is of the order of 5×10^{-6} and the process will not be hard to observe. Inclusion of the electroweak penguin contribution actually suppresses this decay significantly. In Fig. 14, we show the branching ratio without electroweak penguin and with electroweak penguin.

V. CP ASYMMETRY IN THE DECAY MODES

So far we have found that the branching ratio of $B^\pm \rightarrow \eta' K^\pm$ is large if we go to a parameter space where ξ is

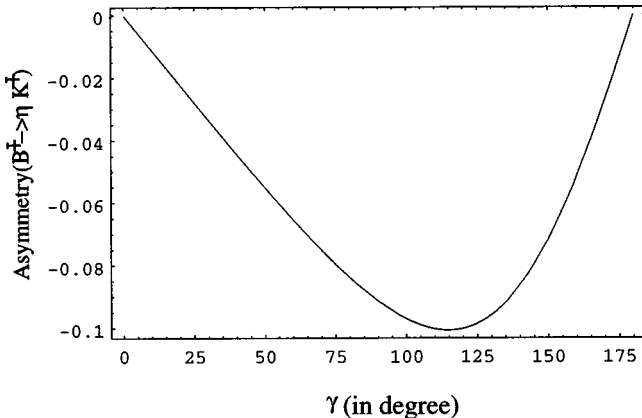


FIG. 15. CP asymmetry for the mode $B^\pm \rightarrow \eta' K^\pm$ as a function of γ .

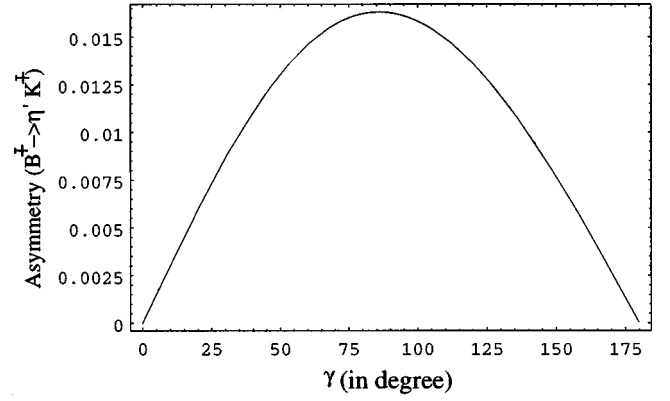


FIG. 16. CP asymmetry for the mode $B^\pm \rightarrow \eta K^\pm$ as a function of γ .

small, the form factor is large, the weak phase γ is small, and the $\eta - \eta'$ mixing angle θ is $\approx -25^\circ$. We now calculate the rate asymmetry for the $B \rightarrow \eta' K$, $B \rightarrow \eta K$, and $B^0 \rightarrow \pi^- K^+$ modes in this parameter space. Interestingly enough we find the rate asymmetry to be 10% for $B \rightarrow \eta K$ when γ is around 110° and $\xi = 0.1$. In Fig. 15 we plot the rate asymmetry for $B \rightarrow \eta K$ against different values of γ for a fixed $\xi = 0.1$. If we include the charm content the rate asymmetry is slightly higher or lower depending on the sign of $f_{\eta'}^c$. In Fig. 16, we show the rate asymmetry for $B \rightarrow \eta' K$ as a function of γ . The asymmetry in the $B \rightarrow \eta' K$ mode is largest, about 2%, when γ is large, $\sim 85^\circ$. In Fig. 17 we show the rate asymmetry for the $B^0 \rightarrow \pi^- K^+$ mode as a function of γ . For γ of about 35° and $\xi = 0.2$ the rate asymmetry in this mode is about 5%. The asymmetry maximizes for γ around 70° for $\xi = 0.2$.

VI. CONCLUSION

We have shown that it is possible to understand the decays of the B meson to light pseudoscalar mesons, i.e., $\pi\pi$, πK , and ηK , $\eta' K$ within factorization approximation. No new physics is needed. We can have a large branching ratio for $B \rightarrow \eta' K$ as seen by CLEO, in the parameter space which is not excluded by the other experimental observations. This region is found for $0.1 < \xi < 0.2$. The parameters

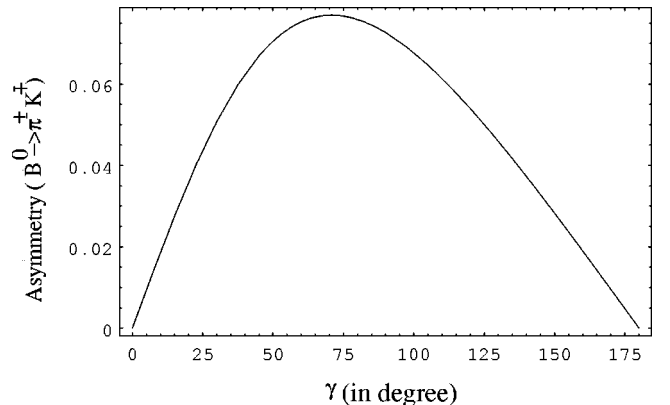


FIG. 17. CP asymmetry for the mode $B^0 \rightarrow \pi^\pm K^\pm$ as a function of γ .

which we have varied to fit all the data are the form factors, the QCD scale (μ), $\xi(\equiv 1/N_c)$, the absolute values of the CKM elements, the weak phases, and the $\eta-\eta'$ mixing angle θ . We found that the large form factor helps to increase the branching ratio of $B\rightarrow\eta'K$. But $B\rightarrow\pi\pi$ and $B\rightarrow\pi K$ decays restrict the size of these form factors. We also have found that smaller values of ξ enhance the branching ratio of $B\rightarrow\eta'K$. In order to find the dependence on γ , we have studied the ratio of the branching ratio of $B\rightarrow\eta'K$ and the branching ratio of $B\rightarrow\pi K$. The ratio does not depend on the form factors and we have found that the small value of the weak phase $\gamma\approx 35^\circ$ is preferred. We have found that the smaller $|V_{ub}/V_{cb}|$ is preferred. Our investigation is closest in spirit to Ali and Greub [5]. They choose the QCD scale $\mu=m_b/2$. We have found that μ dependence introduces only a small effect on decay rates. We have included electroweak

penguin contributions; these are important for $B\rightarrow\eta K$. We have examined dependence on mixing θ and the weak phase γ . We agree on preference for small γ . We also agree that small values of ξ are preferred. We do not find the need for charm in η' compelling. We have also looked at CP asymmetries in the allowed parameter space and have found two modes where it may be possible to measure this asymmetry in the future. The CP asymmetries for these two modes are (i) $B^\pm\rightarrow\eta K^\pm$ about 10% and (ii) $B^0\rightarrow\pi^\pm K^\mp$ about 5%.

ACKNOWLEDGMENTS

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