

## Long-distance final-state interactions and $J/\psi$ decay

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To understand the short- versus long-distance final-state interactions, we have performed a detailed amplitude analysis for the two-body decay  $J/\psi \rightarrow 1^-0^-$ . The current data favor a large relative phase nearly  $90^\circ$  between the three-gluon and the one-photon decay amplitudes. The source of the phase is apparently the long-distance final-state interaction. Nothing anomalous is found in the magnitudes of the three-gluon and one-photon decay amplitudes. We discuss the implications of this large phase in the weak decay of heavy particles. [S0556-2821(98)05009-7]

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### I. INTRODUCTION

Though the final-state interaction phases are important to the observability of  $CP$  violating decays, it is very difficult to compute them or to extract them from data. Only the short-distance contribution has been computed in the quark-gluon picture [1,2]. Any attempt to estimate the long-distance contribution in the hadron picture has so far been limited to elastic or quasielastic rescattering, which is presumably only a small portion of the long-distance effect, particularly in the heavy particle decays such as  $B$  decay. Some argue that the long-distance contribution is negligible at least in some decay modes [3], while others identify a specific long-distance contribution and show that it is actually much larger than the short-distance effect [4]. Some entertain the idea that many long-distance contributions might average out to a small effect after being summed up.

In this paper we try to test whether or not the short-distance final-state interaction dominates over the long-distance one in the  $J/\psi$  decay. Though the  $J/\psi$  decay proceeds with strong and electromagnetic interactions, its narrow width allows us to treat the decay just as we do weak decays, namely, a short-distance decay followed by long-distance rescattering. In the quark-gluon picture  $J/\psi$  decays either directly into three gluons or into a quark and an anti-quark through one photon. Both processes acquire a short-distance QCD rescattering phase of  $O(\alpha_s/\pi)$ . If this is the dominant source of the final-state interaction phase, the relative phase ought to be very small for the amplitudes of all decay modes. On the other hand, if long-distance processes are important to generating the phase, the decay amplitudes can have large relative phases to each other. Since there are sufficient data on the decay  $J/\psi \rightarrow 1^-0^-$ , we are able to perform an amplitude analysis and extract the relative phase between the three-gluon decay and the one-photon decay amplitudes.

In Sec. II we attempt a detailed numerical analysis of the decay amplitudes in the framework of the broken flavor  $SU(3)$  symmetry. We include all first-order symmetry breakings and some of second order effects. The result of our analysis shows that the three-gluon and one-photon amplitudes have a large relative phase of rescattering, nearly  $90^\circ$  off phase to each other. It indicates that the major source of

the final-state interaction phases is in the long-distance hadronic rescattering.

The decay  $J/\psi \rightarrow 1^-0^-$  has been a subject of discussion in connection with the abnormally small yield of the decay  $\psi' \rightarrow 1^-0^-$ . A few exotic models were proposed to resolve this puzzle [5–7]. They suggested that the dominant process of  $J/\psi \rightarrow 1^-0^-$  is not a perturbative three-gluon decay. However, the result of our analysis shows that the magnitude of the  $I=0$  decay amplitudes is consistent with the three-gluon decay. We can also show a serious shortcoming of those models. We shall discuss this point in Sec. III. Finally in Sec. IV, we discuss the implications of the present analysis in the  $B$  decay.

### II. AMPLITUDE ANALYSIS IN BROKEN FLAVOR $SU(3)$ SYMMETRY

Before entering our amplitude analysis, we read off one relevant piece of information from the current data. The lepton-pair decay branching fraction  $B_l$  for  $J/\psi \rightarrow \gamma \rightarrow l^+l^- (= e^+e^- + \mu^+\mu^-)$  has been measured with a high accuracy. In contrast, the inclusive one-photon annihilation into hadrons for  $J/\psi \rightarrow \gamma \rightarrow q\bar{q} (= u\bar{u} + d\bar{d} + s\bar{s})$  was obtained only indirectly from the nonresonant background cross section  $\sigma(e^+e^- \rightarrow q\bar{q})$  interpolated to the  $J/\psi$  mass. This two decade old value [9] quoted by the Particle Data Group (PDG) [8] has a large uncertainty. Actually we can obtain a more accurate value for  $B_\gamma \equiv B(J/\psi \rightarrow \gamma \rightarrow q\bar{q})$  from the leptonic branching fraction  $B_l$  by making the perturbative correction of  $(1 + \bar{\alpha}_s/\pi)$ . With the help of this theory, we can compute the inclusive three-gluon decay branching fraction  $B_{ggg}$  with  $B_{ggg} = 1 - B_l - B_\gamma$ . We thus obtain for the inclusive three-gluon and one-photon annihilations into hadrons

$$\frac{B_{ggg}}{B_\gamma} = 5.8 \pm 0.3(4.2 \pm 0.6), \quad (1)$$

where the number in the brackets is the *experimental* value quoted by the PDG. Except for the number in Eq. (1), we take the numbers tabulated by the PDG as the *current* data throughout this paper.

If branching fractions of the  $J/\psi \rightarrow 1^-0^-$  modes scale with those of the inclusive hadron decay processes, we ex-

TABLE I. The SU(3) parametrization of the  $1^-0^-$  decay amplitudes. Listed are the coefficients of the amplitudes written on the top of each column. For instance, the coefficient of  $a_\gamma$  for  $K^{*0}\bar{K}^0$  is  $-2 \times (1/3)$ . The  $\eta$ - $\eta'$  mixing and the  $\rho$ - $\omega$  mixing are introduced as explained in the text.

Decay modes	$a$	$\epsilon$	$\frac{1}{3}a_\gamma$	$\frac{1}{3}\epsilon_{\gamma 1}$	$\frac{1}{3}\epsilon_{\gamma 2}$	$a'$	$\epsilon'$	$\frac{1}{3}a'_\gamma$
$\rho^+\pi^- (= \rho^-\pi^+)$	1	0	1	0	0	0	0	0
$\rho^0\pi^0$	1	0	1	0	0	0	0	0
$K^{*+}K^- (= K^{*-}K^+)$	1	1	1	2	0	0	0	0
$K^{*0}\bar{K}^0 (= \bar{K}^{*0}K^0)$	1	1	-2	-1	0	0	0	0
$\omega\pi^0$	0	0	3	0	0	0	0	0
$\rho^0\eta_8$	0	0	$\sqrt{3}$	0	$-\sqrt{3}$	0	0	0
$\omega\eta_8$	$\sqrt{1/3}$	0	$\sqrt{1/3}$	0	$-\sqrt{1/3}$	0	0	0
$\phi\eta_8$	$\sqrt{2/3}$	$\sqrt{8/3}$	$-\sqrt{8/3}$	$-\sqrt{8/3}$	$-\sqrt{2/3}$	0	0	0
$\rho^0\eta_1$	0	0	0	0	0	0	0	$\sqrt{3/2}$
$\omega\eta_1$	0	0	0	0	0	$\sqrt{2/3}$	0	$\sqrt{1/6}$
$\phi\eta_1$	0	0	0	0	0	$-\sqrt{1/3}$	$-\sqrt{1/3}$	$\sqrt{1/3}$
$\phi\pi^0$	0	0	0	0	0	0	0	0

pect that the amplitude analysis should give us the ratio of the three-gluon to one-photon decay amplitudes into the exclusive channels  $1^-0^-$  somewhere between 2 and 3 ( $\approx \sqrt{5.8}$ ). Later we shall compare the results of our amplitude analysis with this number.

#### A. Fit to $J/\psi \rightarrow V_9 P_8$

We first study the two-body decay modes of  $J/\psi \rightarrow 1^-0^-$  in which the pseudoscalar meson belongs to an SU(3) octet. The singlet  $\eta'$  will be included later with the  $\eta$ - $\eta'$  mixing. For the vector mesons, we study the singlet and the octet together as a nonet. We parametrize the decay amplitudes as follows.

(1) The  $1^-$  mesons form the ideally mixed nonet [10], namely  $\phi = -\bar{s}s$ . Therefore the SU(3) symmetric coupling is given by

$$L_{\text{int}} = a \text{tr}(V_9 P_8), \quad (2)$$

where  $V_9$  and  $P_8$  are represented in the  $3 \times 3$  matrices.

(2) The strong SU(3) breaking of  $\lambda_8$  is included in the three-gluon decay

$$L_{\text{int}} = \epsilon \text{tr}(\{V_9, P_8\} + T_3^3), \quad (3)$$

where we use  $T_3^3$  instead of  $\lambda_8$  to simplify the numerical coefficients of parametrization. The symmetrization of  $V_9$  and  $P_8$  is required by charge conjugation invariance.

(3) The one-photon annihilation amplitudes transform like  $\lambda_E = (\lambda_3 + \lambda_8/\sqrt{3})/2$ . Therefore, they are parametrized as

$$L_{\text{int}} = a_\gamma \text{tr}(\{V_9, P_8\} + \lambda_E). \quad (4)$$

(4) The phases of the amplitudes  $a$ ,  $\epsilon$ , and  $a_\gamma$  are group theoretically independent. Therefore, we introduce two relative phases  $\delta_\gamma$  and  $\delta_\epsilon$  defined as

$$\arg(a_\gamma a^*) = \delta_\gamma, \quad \arg(\epsilon a^*) = \delta_\epsilon. \quad (5)$$

(5) The  $\rho$ - $\omega$  [11] mixing can be potentially important to the processes of  $\Delta I = 1$ . For instance, the process  $J/\psi \rightarrow \omega \eta$

$\rightarrow \rho \eta$  interferes with the direct process  $J/\psi \rightarrow \gamma \rightarrow \rho \eta$  and is counted as part of the branching to  $J/\psi \rightarrow \rho \eta$ . The decay  $J/\psi \rightarrow \omega \eta$  can proceed through three gluons while the direct process is the one-photon process. Therefore, the  $\rho$ - $\omega$  mixing may not be negligible in this mode. Since the  $\omega$  width is much narrower than the  $\rho$  width, the major contribution of the  $\rho$ - $\omega$  transition to  $J/\psi \rightarrow \rho \eta$  occurs at the  $\omega$  resonance peak of  $\pi^+\pi^-$ . In comparison, in a process such as  $J/\psi \rightarrow \omega \pi^0$ , the effect of the  $\rho$ - $\omega$  transition is less significant because the large  $\rho$  width suppresses it kinematically. In the processes of  $\Delta I = 0$ , the  $\rho$ - $\omega$  transition is negligible.

We have included the  $\rho$ - $\omega$  mixing with the effective transition coupling

$$L_{\text{int}} = m_\omega^2 f_{\rho\omega} \rho_\mu \omega^\mu, \quad (6)$$

with  $f_{\rho\omega} = -6.8 \times 10^{-3}$ , as extracted from the process  $e^+e^- \rightarrow \pi^+\pi^-$  around the  $\omega$  mass.

(6) The  $p$ -wave phase space correction is made with  $p^3$  for all decay branching fractions. If the flavor symmetry applies best to the dimensionless decay couplings, the phase space factor  $p^3$  should be divided by some quantity having the dimension of squared mass that may be subject to symmetry breaking. However, this uncertainty has been incorporated through the  $\lambda_8$  breaking of Eq. (3) above, at least, to first order.

In Table I, we have tabulated the parametrization of seven  $V_9 P_8$  decay amplitudes with the SU(3) amplitudes

$$a, \quad \epsilon, \quad a_\gamma. \quad (7)$$

If the  $1^-0^-$  decay branchings scale more or less with the inclusive ones [cf. Eq. (1)], we expect  $|a_\gamma/a| \approx 0.67$  in the normalization used in Table I. For the strong symmetry breaking,  $|\epsilon/a| \leq 0.3$  is a reasonable range. In addition, we have two relative phases as free parameters,  $\delta_\epsilon$  and  $\delta_\gamma$ .

The best fit with these five parameters is obtained for

$$a = 1, \quad \epsilon = -0.22, \quad a_\gamma = 0.34, \quad \delta_\epsilon = -22.5^\circ, \quad \delta_\gamma = 80.3^\circ, \quad (8)$$

TABLE II. The observed branching fractions and our fits with and without  $\eta'$  included. ‘‘No phase’’ means a fit assuming that all amplitudes be real. The bottom row lists  $\chi^2$  for each fit. All numbers are in percent except for  $\chi^2$ .

Decay modes	Observed (in percent)	$V_9 P_8$			$V_9 P_{8,1}$	
		Best fit	No phase I	No phase II	Best fit	No phase
$\rho^+ \pi^- (= \rho^- \pi^+)$	$0.43 \pm 0.03$	0.44	0.44	0.47	0.43	0.42
$\rho^0 \pi^0$	$0.42 \pm 0.05$	0.44	0.44	0.47	0.43	0.42
$K^{*+} K^- (= K^{*-} K^+)$	$0.25 \pm 0.02$	0.25	0.30	0.25	0.25	0.31
$K^{*0} \bar{K}^0 (= \bar{K}^{*0} K^0)$	$0.21 \pm 0.02$	0.21	0.14	0.16	0.21	0.16
$\omega \pi^0$	$0.042 \pm 0.006$	0.047	0.034	0.33	0.052	0.030
$\phi \pi^0$	$< 6.8 \times 10^{-4}$	0	0	0	0	0
$\rho^0 \eta$	$0.0193 \pm 0.0023$	0.0174	0.0104	0.0186	0.0140	0.0144
$\omega \eta$	$0.158 \pm 0.016$	0.131	0.129	0.146	0.146	0.150
$\phi \eta$	$0.065 \pm 0.007$	0.065	0.062	0.076	0.064	0.058
$\rho^0 \eta'$	$0.0105 \pm 0.0018$				0.0098	0.0112
$\omega \eta'$	$0.0168 \pm 0.0025$				0.167	0.0169
$\phi \eta'$	$0.033 \pm 0.004$				0.033	0.032
$\chi^2$		4.8	57	21	8.2	43

where the magnitude of the amplitude  $a$  has been normalized to unity. The fitted values to the data are tabulated in the third column of Table II next to the observed values. The  $\chi^2$  is 4.8 for this fit. Though the ratio of  $a_\gamma/a$  in Eq. (8) is a half of the scaled value  $\approx 0.67$ , it can hardly be called an enhancement of the three-gluon decay. The magnitudes of  $a$ ,  $a_\gamma$ , and  $\epsilon$  are in line with the expectation from the inclusive branching ratios: There is no sign of significant enhancement of the three-gluon processes relative to the one-photon processes.

If we fit the data without the phases, the best  $\chi^2$  is 57. When there is no rescattering phase, the  $\rho$ - $\omega$  mixing is unimportant because the  $\rho$ - $\omega$  transition amplitudes are  $90^\circ$  off phase to the main amplitudes. The fitted values are listed in the fourth column (no phase I) of Table II. In order to make a quantitative comparison of the fits with and without the phases, it is more appropriate to fit the data with the same number of free parameters, namely, five real amplitudes. We may add the amplitudes of the  $\lambda_8$  breaking to  $a_\gamma$  as the second-order small quantities. We may also include breakdown of the ideal nonet coupling ansatz. Actually, there is a subtlety between breakdown of the nonet and the  $O(\lambda_8 e)$  correction. In the nonet scheme, which is realized in the non-relativistic quark model,  $V_1$  does not form an SU(3) coupling without accompanying  $V_8$ . For instance, a term such as  $\text{tr} V_9 \propto V_1$  is not allowed in an SU(3) coupling. When a  $\lambda_8$  breaking is taken into account, however, one may include  $\text{tr}(\lambda_8 V_9)$  or  $\text{tr}(T_3^3 V_9)$  among others. Normally it would not matter which of these is included since their difference has the same SU(3) property as the term of the symmetry limit. In the case of the nonet, the difference  $\sim \text{tr} V_9$  is a term forbidden by the nonet coupling ansatz. Therefore we must choose between them either from the observed  $V_9$  parameters or from some theoretical reasoning. Since the  $\lambda_8$  breaking is caused by the  $s$ -quark mass term and the origin of the nonet is in the nonrelativistic quark model, we feel that  $T_3^3$  is more appropriate than  $\lambda_8$  in the case of the nonet. We include the strong SU(3) breaking to  $a_\gamma$  along this line to see an outcome of the fit.

After charge conjugation invariance is taken into account, there are three independent amplitudes of  $O(T_3^3 e)$  that have different group structures from the amplitudes in Eq. (7). One of them [ $\propto \text{tr}(T_3^3 V_9)$ ] contributes only to the  $\phi \pi^0$  and  $\phi \eta$  modes. Meanwhile a severe upper bound has been set on  $B(J/\psi \rightarrow \phi \pi^0)$  by experiment. With this upper bound, we find that this SU(3) amplitude contributes no more than 6% to  $\phi \eta$ , smaller than normally expected for strong SU(3) breakings and below the level of our concern. We therefore drop this small amplitude and retain only the remaining two amplitudes as the  $O(T_3^3 e)$  corrections:

$$L_{\text{int}} = \epsilon_{\gamma 1} [\text{tr}(V_9 \lambda_E P_8 T_3^3) + \text{tr}(P_8 \lambda_E V_9 T_3^3)] + \epsilon_{\gamma 2} \text{tr}(P_8 T_3^3) \text{tr}(V_9 \lambda_E). \quad (9)$$

Now we have five real amplitudes. The best fit attains  $\chi^2 = 21$  in this case. The fitted values are

$$a = 1, \quad \epsilon = -0.14, \quad a_\gamma = 0.30, \quad \epsilon_{\gamma 1} = -0.12, \quad \epsilon_{\gamma 2} = -0.11. \quad (10)$$

The fitted branching fractions are listed in the fifth column (no phase II) of Table II.

A simple qualitative explanation can be given as to why the best fit needs the phases. Refer to the Table I and the observed branching fractions listed in Table II for the following discussion.

(1) First of all, the significant difference between the  $\rho \pi$  and  $K^{*+} K^-$  branchings must be explained by the SU(3) breaking  $\epsilon$  amplitude. The  $\epsilon$  amplitude of a right magnitude ( $|\epsilon/a| = 0.22$ ) produces this difference.

(2) Next, we need a sizable  $a_\gamma$  amplitude in order to account for the  $\omega \pi^0$  mode.

(3) The  $a_\gamma$  amplitude contributes to splitting the branching fractions of  $K^{*+} K^-$  and  $K^{*0} \bar{K}^0$  too. However, if  $a_\gamma$  and  $a$  substantially interfered, this splitting would be much too large. To keep the splitting between  $K^{*+} K^-$  and  $K^{*0} \bar{K}^0$  small,  $a_\gamma$  and  $a$  must be largely off phase to each other.

The relative phase between  $\epsilon$  and  $a$  turns out to be small. One interpretation for the smallness of this relative phase is that the main source of  $\epsilon$  is the kinematical SU(3) breaking due to mass splitting in the phase space and decay coupling.

### B. Including $\eta'$

Once  $\eta'$  is included, the number of independent parameters suddenly increases since unlike the vector meson couplings, the singlet  $0^-$  couplings are not related to the octet  $0^-$  couplings. Complication grows further when we include the  $\eta$ - $\eta'$  mixing.

Here we present a relatively simple sample of analysis instead of the most general one in order to show that need of the relative phase between the three-gluon and the one-photon amplitudes persists. We add all SU(3) independent amplitudes involving  $\eta'$  that correspond to  $a$ ,  $\epsilon$ , and  $a_\gamma$  of  $P_8$ :

$$L_{\text{int}} = a' \text{tr}(V_9 P_1) + \epsilon' \text{tr}(V_9 P_1 T_3^2) + a'_\gamma \text{tr}(V_9 P_1 \lambda_E). \quad (11)$$

In addition to the  $\rho$ - $\omega$  mixing, we include the  $\eta$ - $\eta'$  mixing

$$\eta = \eta_8 \cos \theta_p - \eta_1 \sin \theta_p, \quad \eta' = \eta_8 \sin \theta_p + \eta_1 \cos \theta_p, \quad (12)$$

where  $\theta_p$  is  $-10^\circ \sim -20^\circ$  [12]. For the relative phase, we put a common phase  $\delta$  between all one-photon amplitudes and all three-gluon amplitudes (e.g.,  $\delta_\epsilon = 0$ ).

The best fit to the data is obtained with the following values of the parameters:

$$\begin{aligned} a &= 1, & \epsilon &= -0.18, & a_\gamma &= 0.36, \\ a' &= 0.44, & \epsilon' &= 0.051, & a'_\gamma &= -0.40, \\ \delta &= 75.2^\circ, \end{aligned} \quad (13)$$

The value of  $\chi^2$  is 8.2 for fitting eleven data with seven parameters. The relative magnitudes of the parameters are *normal*, namely, in line with the expectation from the inclusive branchings and strong SU(3) breakings. On the other hand, if we attempt to fit the data without the phase  $\delta$ , the  $\chi^2$  jumps to 43. In this case, sum of the one-photon branchings is close to that of the three-gluon branchings for  $V_9 P_1$ . The tendency of deterioration of the fit without a phase persists as we have seen in the case without  $\eta'$ . The fitted values of the branching fractions are tabulated with and without the phase in the last two columns of Table II. The phase is unimportant in fitting to the decay modes involving  $\eta'$ .

We conclude that as long as the currently listed data are taken at their face value, the three-gluon and one-photon amplitudes have a large relative phase to each other. Apart from this unexpected result, our amplitude analysis shows that the magnitudes of all decay amplitudes are within the range of what we expect.

### III. IS THE DECAY $J/\psi \rightarrow 1^- 0^-$ ANOMALOUS?

We have chosen the  $J/\psi \rightarrow 1^- 0^-$  decay modes for study of the final-state interaction phases since they are the most

extensively measured decay modes. No similar analysis can be made for other modes at present.

Meanwhile, there was one disturbing twist related to these decay modes. That is, the  $\psi' \rightarrow 1^- 0^-$  decay modes are severely suppressed in comparison with the corresponding  $J/\psi$  modes [13]. For  $\rho\pi$ , the upper bound on the branching fractions normalized to the  $e^+e^-$  branching fraction obey the inequality [14]:

$$\frac{B(\psi' \rightarrow \rho\pi)}{B(\psi' \rightarrow e^+e^-)} < 2.1 \times 10^{-2} \times \frac{B(J/\psi \rightarrow \rho\pi)}{B(J/\psi \rightarrow e^+e^-)}. \quad (14)$$

This vast difference between  $J/\psi$  and  $\psi'$  has stimulated many speculations on the pure QCD decay of  $J/\psi$  and  $\psi'$ . The argument goes as follows: Normally  $J/\psi(\psi') \rightarrow 1^- 0^-$  would be highly suppressed by chirality mismatch of perturbative QCD. However, this suppression is compensated by an enhancement in the  $I=0$  channels of  $J/\psi$ . The enhancement brings  $\Gamma(J/\psi \rightarrow 1^- 0^-)$  back to the value predicted by the perturbative three-gluon annihilation [6]. The cause of enhancement may be either a vector gluonium state nearly degenerate with  $J/\psi$  [5,6] or a hidden charm pair in the light  $1^-$  mesons [7].  $\Gamma(\psi' \rightarrow 1^- 0^-)$  is small because it suffers from chirality mismatch but receives no enhancement.

However, our amplitude analysis raises a doubt about such an explanation. We have seen that both the  $I=0$  and the  $I=1$  amplitudes are as normal as we expect from the inclusive three-gluon and one-photon annihilations. If the observed magnitude of the  $I=0$  amplitudes were actually the result of the compensation between a chirality suppression and a dynamical  $I=0$  enhancement, we would expect that the one-photon annihilation amplitude for  $\omega\pi(I=1)$  should be suppressed by chirality without a compensating enhancement. If their models are correct, we can read off the chirality suppression factor from Eq. (14). With the chirality suppression,  $B(J/\psi \rightarrow \omega\pi^0)$  would have to be

$$\begin{aligned} B(J/\psi \rightarrow \omega\pi^0) &\approx (\text{chirality suppression}) \\ &\times \frac{B(J/\psi \rightarrow \gamma \rightarrow q\bar{q})}{B(J/\psi \rightarrow ggg)} B(J/\psi \rightarrow \rho\pi) \\ &< \frac{1}{270} \times B(J/\psi \rightarrow \rho^0\pi^0) \end{aligned} \quad (15)$$

in those models. The data violate this inequality by an order of magnitude. Therefore, the origin of the relative suppression of  $\psi' \rightarrow 1^- 0^-$  to  $J/\psi \rightarrow 1^- 0^-$  is not in  $J/\psi$  but in  $\psi'$ . The large relative phase between  $a$  and  $a_\gamma$  cannot be attributed to a resonance in the  $s$  channel of  $I=0$ . Since the  $I=0$  amplitudes receive no net enhancement, a contribution of an  $s$ -channel resonance, if any, would be a tiny fraction of the whole amplitude [5,14]. Then the  $I=0$  amplitudes could not have a large phase close to  $90^\circ$ . It should be pointed out that there exist other attempts to explain the relative suppression of  $\psi' \rightarrow 1^- 0^-$  with different dynamical assumptions or intricate dynamical coincidences [15–18]. The possibility of a destructive interference in  $\psi'$  [18], though it is fortuitous, cannot be ruled out in view of our finding of the large long-

distance final-state interaction in  $J/\psi$ . Whatever the cause of the  $\psi' \rightarrow 1^- 0^-$  suppression may be, the group theoretical parametrization of the amplitudes remains the same for  $J/\psi \rightarrow 1^- 0^-$ . Though we are unable to choose the solution to this  $\psi' \rightarrow \rho\pi$  puzzle among the existing models at present, we are confident that this puzzle does not interfere with our analysis in this paper.

#### IV. IMPLICATION OF THE LARGE RELATIVE PHASE

In our analysis we have found the evidence for a large final-state interaction phase in a heavy particle decay which is quite different in nature from the common subchannel resonant phases. What generates the large relative phase  $\delta_\gamma$  between  $a_\gamma$  and  $a$ ? It is obvious that it must arise from long-distance strong interactions. The short-distance final-state interaction phase difference can be evaluated in the quark-gluon picture of perturbative QCD. It is of  $O(\alpha_s/\pi)$  where  $\alpha_s/\pi$  is 0.1 or less. The large phase difference close to  $90^\circ$  found in our analysis cannot be produced with the perturbative QCD interaction. The source of  $\delta_\gamma$  must be in the long-distance part of strong interactions, namely, rescattering among hadrons in their inelastic energy region.

When many channels are open for strong interaction rescattering, the phase of a decay amplitude into a physically observed state is determined by the phase shifts of eigenchannels of the  $S$  matrix and the coupling to them. In the case of the  $J/\psi$  decay, the decay amplitude  $D(J/\psi \rightarrow h)$  into a final state  $h$  (e.g.,  $\rho^+ \pi^-$ ) is written in the form

$$D(J/\psi \rightarrow h) = \sum_{\alpha} X(J/\psi \rightarrow \alpha) e^{i\delta_{\alpha}} O_{\alpha h}, \quad (16)$$

where  $\alpha$  refers to the eigenchannels of the partial-wave  $S$  matrix of  $J^{PC} = 1^{--}$  at the energy of the  $J/\psi$  mass with  $\delta_{\alpha}$ 's being their eigenphase shifts. Time-reversal invariance requires that  $X(J/\psi \rightarrow \alpha)^* = X(J/\psi \rightarrow \alpha)$  and that  $O_{\alpha h}$  be an orthogonal matrix relating  $h$  to  $\alpha$  by  $\langle h | = \sum \langle \alpha | O_{\alpha h}$ . The partial-wave phase shifts contain much of long-distance physics no matter how high the energy is. One indication of substantial long-distance physics in the high-energy phase shifts was pointed out by making a partial-wave projection of the diffractive scattering amplitude [4]. Long-distance physics enters the eigenchannel matrix  $O_{\alpha h}$  as well. When we compare the three-gluon and one-photon decay amplitudes of

$J/\psi$  with Eq. (16), we see no simple relation between their phases in general: The eigenphase factors  $e^{i\delta_{\alpha}}$  are summed with the weights  $X(J/\psi \rightarrow \alpha)$  different for  $J/\psi \rightarrow ggg$  and  $J/\psi \rightarrow \gamma \rightarrow q\bar{q}$ , leading to two phases practically unrelated to each other. In this picture the final-state interaction phases of the  $J/\psi$  decay are generally not determined by short-distance physics alone even though perturbative QCD applies to the inclusive  $J/\psi$  decays. The analysis of this paper indicates that long-distance physics can be far more important in the exclusive decays.

The conclusion of our amplitude analysis, if it is sustained, has a significant implication in a wide range of phenomena. For instance, when we evaluate the baryon asymmetry in the early Universe from  $CP$ -violating particle decays, we compute only the short-distance contribution of final-state interactions. Such a calculation makes sense only as an order-of-magnitude estimate at best. In the case of the baryon asymmetry we may not ask for a high precision after all. However, in the  $B$ -meson decay where knowledge of much higher precision will be needed for final-state interaction phases, we shall have to know the long-distance final-state interaction phases above the inelastic thresholds. It is nearly an impossible task to either compute them theoretically or extract them from scattering data. If this is the case, the parameters of the fundamental interactions can be extracted only from those data which are free from complications due to the final-state interaction. It will not be an easy task to look for meaningful physics in the rest of data.

To conclude this paper, we should emphasize that numerical conclusion of our analysis relies on the current data listed by the PDG, not only their central values but also the experimental uncertainties. We cannot rule out the possibility that a future change in the data may upset our conclusion, i.e., the need of a large rescattering phase. For this reason, high precision measurement of the  $J/\psi$  decay branchings, particularly for  $\rho\pi$ ,  $K^* \bar{K}$ , and  $\omega\pi^0$ , will be very important to our understanding of the final-state interactions in general.

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- [1] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **43**, 242 (1979).  
 [2] G. Kramer, W. F. Palmer, and H. Simma, Nucl. Phys. **B428**, 77 (1994); Z. Phys. C **66**, 429 (1995).  
 [3] J. D. Bjorken, in *Gauge Bosons and Heavy Quarks*, Proceedings of the 18th SLAC Summer Institute on Particle Physics, Stanford, California, 1990, edited by P. Hawthorne (SLAC Report No. 378, Stanford, 1991).  
 [4] J. F. Donoghue, E. Golowich, A. A. Petrov, and J. Soares, Phys. Rev. Lett. **77**, 2178 (1996).  
 [5] W.-S. Hou and A. Soni, Phys. Rev. Lett. **50**, 569 (1983); W.-S. Hou, Phys. Rev. D **55**, 6952 (1997), and earlier references therein.  
 [6] S. Brodsky, G. P. Lepage, and S. F. Tuan, Phys. Rev. Lett. **59**, 621 (1987).  
 [7] S. Brodsky and M. Karliner, Phys. Rev. Lett. **78**, 4682 (1997).  
 [8] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).  
 [9] Mark I Collaboration, A. M. Boyarski *et al.*, Phys. Rev. Lett. **34**, 1357 (1975).  
 [10] S. Okubo, Phys. Lett. **5**, 165 (1963).

- [11] Y. Nambu and J. J. Sakurai, Phys. Rev. Lett. **8**, 79 (1962).
- [12] J. Schwinger, Phys. Rev. **135**, B816 (1964); F. J. Gilman and R. Kauffman, Phys. Rev. D **36**, 2761 (1987).
- [13] Mark II Collaboration, M. Franklin *et al.*, Phys. Rev. Lett. **51**, 963 (1983).
- [14] J. Z. Bai *et al.*, Phys. Rev. D **54**, 1221 (1996).
- [15] G. Karl and W. Roberts, Phys. Lett. **144B**, 263 (1984).
- [16] M. Chaichian and N. A. Tornquist, Nucl. Phys. **B323**, 75 (1989).
- [17] S. S. Pinsky, Phys. Lett. B **236**, 476 (1990).
- [18] X.-Q. Li, D. V. Bugg, and B.-S. Zou, Phys. Rev. D **55**, 1421 (1997).