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Inclusive J/ψ production in Υ decay via color-octet mechanisms

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Calculations for inclusive ψ production in Υ decay through the color-octet mechanisms $b\bar{b}[{}^3S_1,\underline{1}]\to c\bar{c}[{}^{2s+1}L_J,\underline{8}]+g$ are presented. It is shown that these $\mathcal{O}(\alpha_s^5 v_c^4)$ contributions compete with other color-octet mechanisms considered in the literature. A critical numerical analysis of the color-octet contributions to $\Upsilon\to\psi+X$ shows that further work in this channel, both theoretical and experimental, is necessary in order to clearly understand the significance of the color-octet component of the $c\bar{c}$ component inside the ψ system. [S0556-2821(98)05809-3]

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I. INTRODUCTION

The unexpected large rates for ψ and ψ' prompt production at the Fermilab Tevatron [1] gave rise to two interesting theoretical developments. On the one hand it was realized that, at high energies, the most important contributions to ψ production come from processes with a fragmentation interpretation [2]. On the other hand it was necessary to develop new ideas for heavy quarkonium summarized in an effective theory called nonrelativistic QCD (NRQCD) [3]. This theory describes the interactions of nonrelativistic quarks and is typically applied to $Q\bar{Q}$ bound states such as upsilon or ψ. The NRQCD Lagrangian is made precisely equivalent to full QCD through the addition of local interactions that systematically incorporate relativistic corrections through any given order in the heavy quark velocity v. The theory has a simultaneous expansion in α_s and in the velocity of the heavy quark. The size of a term in the NRQCD Lagrangian can be estimated using well defined velocity counting rules [4].

Creation and annihilation of heavy quarkonium are naturally described within this theory which incorporates the factorization ideas. In fact, these processes, which necessarily occur at very small distances $\approx 1/m_Q$ (m_Q denotes the heavy quark mass) as compared with the size of the meson $\approx 1/m_Q v$, are described by the short-distance coefficients entering in the relativistic corrections to the leading order NRQCD Lagrangian. The evolution of the quark pair into physical quarkonium involves nonperturbative effects which are encoded within matrix elements of higher order operators with a well defined hierarchy in the v expansion.

As a consequence of NRQCD, the possibility arises for the formation of physical quarkonium through higher Fock states. In particular, we have the possibility for the formation of physical quarkonium from a $Q\bar{Q}$ pair in a color-octet configuration. This is the most attractive explanation to the surplus ψ and ψ' production at the Tevatron [5].

Unfortunately the NRQCD matrix elements can only be computed from lattice calculations which are still in their infancy. The way people proceed is to fit these parameters to some experimental data and use their values to make predictions for other processes where they are involved. Using the values for the NRQCD matrix elements as extracted from the fits to charmonium hadroproduction [6,17], predictions have been made for color-octet signals in prompt quarkonium production in e^+e^- colliders [7], Z_0 decays [8], photoproduction at fixed target experiments [9], hadroproduction experiments [10], and B-meson decays [11].

Inclusive ψ production in Υ decay is a very good place to test the NRQCD ideas [12]. Unlike other ψ production processes, the short-distance coefficients highly suppress color singlet contributions to $\Upsilon\to\psi+X$. In fact, color-singlet contributions start at $\mathcal{O}(\alpha_s^6)$ through the $b\bar{b}[{}^3S_1,\underline{1}]\to c\bar{c}[{}^3S_1,\underline{1}]+gg$ and $b\bar{b}[{}^3S_1,\underline{1}]\to c\bar{c}[{}^3S_1,\underline{1}]+gggg$ mechanisms whereas color-octet contributions to the same process start at $\mathcal{O}(\alpha_s^4 v_c^4)$ through the $b\bar{b}[{}^3S_1,\underline{1}]\to c\bar{c}[{}^3S_1,\underline{8}]+gg$ mechanism. Hence, the short distance factors of the color-octet contributions are enhanced by factors of $1/\alpha_s^2$ as compared with the color-singlet ones. This enhancement easily overcomes the v_c^4 suppression of the long distance factors.

In principle we also have $\mathcal{O}(\alpha_s^4 v_b^4)$ contributions from the $b\bar{b}$ in a color-octet configuration. However, such contributions are naively suppressed by $(v_b/v_c)^4$ as compared with contributions from a $c\bar{c}$ in a color-octet configuration. Actual calculations reflect this naive counting suppression [12].

In Ref. [12] $\mathcal{O}(\alpha_s^4 v_c^4)$ and leading electromagnetic contributions to $\Upsilon\to\psi+X$ were calculated within NRQCD. The most important mechanism turned out to be $b\bar{b}[{}^3S_1,\underline{1}]\to c\bar{c}[{}^3S_1,\underline{8}]+gg$, which gives a branching ratio $B\simeq 2.5\times 10^{-4}$ when the value $\alpha_s(2m_c)=0.253$ is used in the numerical calculations as suggested by the fragmentation limit for the $\Upsilon\to\psi+X$ decay width normalized to the $\Upsilon\to ggg$ decay width. The next largest contribution is the indirect production through χ_{cJ} decay considered by Trottier [13]. When the values for matrix elements extracted from the Tevatron data [6] are used, this contribution gives $\Sigma_J B(\Upsilon\to\chi_{cJ}+X\to\psi+X)\simeq 5.7\times 10^{-5}$. There also exist a comparable indirect contribution from ψ' having a branching ratio $\simeq 5\times 10^{-5}$ [12]. Adding up all these and other even smaller contributions a $B\simeq 4\times 10^{-4}$ is obtained [12]. This result is within 2σ of the CLEO data: $B_{\text{exp}}=1.1\pm 0.4\times 10^{-3}$ [14] which suggest other contributions need to be

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computed in order to bring theoretical predictions closer to experimental data.

Below I compute the $\mathcal{O}(\alpha_s^5 v_c^4)$ one-loop contributions to inclusive ψ production in Υ decay through the color-octet mechanisms $b\bar{b}[{}^3S_1,\underline{1}]\to c\bar{c}[{}^{2s+1}L_J,\underline{8}]+g$. The importance of these mechanisms can be easily understood from the calculation for the electromagnetic contribution to the same processes [12]. This is an $\mathcal{O}(\alpha_{em}^2\alpha_s v_c^4)$ contribution and gives a $B\simeq 1.6\times 10^{-5}$. An $\mathcal{O}(\alpha_s^5)$ contribution should give roughly a $B\simeq 10^{-4}$ or even larger if we consider the $e_b^2 e_c^2 \simeq 5\times 10^{-3}$ factor coming from the quark electromagnetic couplings. The color-octet processes studied in this paper are also interesting due to the distinctive signature that the ψ has a sharply peaked energy distribution described by a δ function with a peak at 5.2 GeV and is recoiled by a gluon jet. The δ function must be understood in the sense of NRQCD calculations, i.e., as a narrow distribution whose width is of order $m_c v_c^2$ in the ψ rest frame. When boosted to the Υ rest frame the width decreases by a factor $m_\psi/m_\Upsilon\simeq 0.3$, therefore giving narrow distribution peaked at 5.2 GeV whose width is $\simeq 150$ MeV.

II. THE $b\bar{b}[{}^3S_1]\to c\bar{c}[{}^{2s+1}L_J,\underline{8}]+g$ MECHANISMS

The factorization formalism developed in Ref. [3] can be easily generalized to the case of inclusive charmonium production from bottomonium decay [13]. For the process under consideration in this work we have the factorization formula

$$d\Gamma(\Upsilon\to\psi+X)=\sum_{m,n} d\Gamma(m,n)\langle\Upsilon|O(m)|\Upsilon\rangle\langle O^{\psi}(n)\rangle, \tag{1}$$

where $d\Gamma(m,n)$ is the short-distance factor for a $b\bar{b}$ pair in the state m to decay into a $c\bar{c}$ in the state n plus anything. The subscript m,n denote collectively the color and angular momentum quantum numbers of the heavy quark pairs. Contributions that are sensitive to the quarkonium scales (Bohr radius or larger) and to $\Lambda_{\rm QCD}$ can be absorbed into the NRQCD matrix elements $\langle\Upsilon|O(m)|\Upsilon\rangle$, $\langle O^{\psi}(n)\rangle$. The relative importance of the various terms in the double factorization formula (1) depend on the order of v in the NRQCD matrix elements and on the order of α_s in the short-distance factors.

The Feynman diagrams for the $\mathcal{O}(\alpha_s^5 v^4)$ contributions to the inclusive ψ production in Υ decay coming from the color-octet mechanisms $b\bar{b}[{}^3S_1,\underline{1}]\to c\bar{c}[{}^{2s+1}L_J,\underline{8}]+g$ are shown in Fig. 1. Calculations for these processes are very similar to the calculations for the radiative decays of quarkonium $Q\bar{Q}[{}^3S_1,\underline{1}]\to q\bar{q}[{}^{2s+1}L_J,\underline{1}]+\gamma$ studied in Ref. [15].

Using the standard techniques for heavy quarkonium calculations [6,16,21] the following amplitude is obtained for a $b\bar{b}$ pair annihilating into three off-shell gluons:

$$\mathcal{M}\{b\bar{b}[{}^3S_1]\to g_1(a)g_2(b)g_3(c)\}=T_{abc}A, \tag{2}$$

where $T_{abc}=d_{abc}/4\sqrt{N_c}$ is the color factor

$$A=ig_s^3\sqrt{\frac{32}{\pi}}\,m_b\,\frac{1}{k_1\cdot(k_2+k_3)k_2\cdot(k_3+k_1)k_3\cdot(k_1+k_2)}\,\mathrm{LI},$$

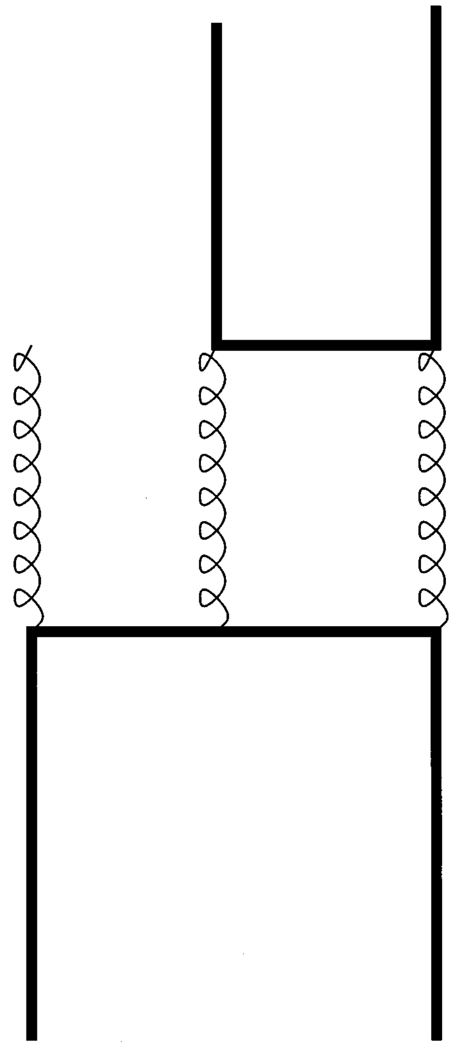

FIG. 1. One of the six diagrams for the short-distance processes $b\bar{b}[{}^3S_1,\underline{1}]\to c\bar{c}[{}^{2s+1}L_J,\underline{8}]+g$.

where LI stand for the Lorentz invariant structure

$$\begin{aligned}\mathrm{LI}=\,&\epsilon_1\cdot\epsilon_2(k_1\cdot k_3\epsilon_3\cdot k_2\epsilon\cdot k_1+k_2\cdot k_3\epsilon_3\cdot k_1\epsilon\cdot k_2\\&+k_1\cdot k_3 k_2\cdot k_3\epsilon\cdot\epsilon_3)+\epsilon\cdot\epsilon_3(k_1\cdot k_2\epsilon_1\cdot k_3\epsilon_2\cdot k_3\\&-k_1\cdot k_3\epsilon_1\cdot k_2\epsilon_2\cdot k_3-k_2\cdot k_3\epsilon_2\cdot k_1\epsilon_1\cdot k_3)\\&+(\epsilon_1,k_1\leftrightarrow\epsilon_3,k_3)+(\epsilon_2,k_2\leftrightarrow\epsilon_3,k_3);\end{aligned} \tag{3}$$

here, ϵ denotes the $b\bar{b}[{}^3S_1]$ polarization vector and ϵ_i (k_i) stand for the polarization vector (momentum) of g_i.

The amplitudes quoted in this paper are obtained by projecting the free quarks' amplitudes over definite angular momentum (${}^{2s+1}L_J$) and color ($\underline{1}$,$\underline{8}$) quantum numbers in the usual way [6,16].

The two gluon fusion into a $c\bar{c}[{}^{2s+1}L_J,\underline{8}]$ process are described by

$$\begin{aligned}\mathcal{M}[g_1(a)g_2(b)\to{}^1S_0,8c]=\,&\frac{d^{abc}}{2\sqrt{2}}\,g_s^2\sqrt{\frac{2}{\pi}}\,\frac{1}{k_1\cdot k_2}\\&\times\varepsilon(\epsilon_1,k_1,\epsilon_2,k_2),\end{aligned} \tag{4}$$

$$\mathcal{M}[g_1(a)g_2(b)\to{}^3P_J,8c]=\frac{d^{abc}}{2\sqrt{2}}\,g_s^2\,\frac{4}{\sqrt{m_c^3}}\left(\frac{1}{k_1\cdot k_2}\right)^2 A_J, \tag{5}$$

where

$$A_0=\sqrt{\frac{1}{6}}\,[(k_1\cdot k_2+4m_c^2)(\epsilon_1\cdot\epsilon_2 k_1\cdot k_2-\epsilon_1\cdot k_2\epsilon_2\cdot k_1)$$
$$+k_1^2\epsilon_1\cdot k_2\epsilon_2\cdot k_2+k_2^2\epsilon_1\cdot k_1\epsilon_2\cdot k_1-k_1^2k_2^2\epsilon_1\cdot\epsilon_2$$
$$-\epsilon_1\cdot k_1\epsilon_2\cdot k_2 k_1\cdot k_2],$$

$$A_1=m_c[k_1^2\varepsilon(e^*,\epsilon_1,\epsilon_2,k_2)+\epsilon_1\cdot k_1\varepsilon(e^*,\epsilon_2,k_1,k_2)$$
$$+\{k_1,\epsilon_1\}\leftrightarrow\{k_2,\epsilon_2\}],$$

$$A_2=\sqrt{8}m_c^2[k_1\cdot k_2\epsilon_{1\mu}\epsilon_{2\nu}+k_{2\mu}k_{1\nu}\epsilon_1\cdot\epsilon_2-k_{1\mu}\epsilon_{2\nu}\epsilon_1\cdot k_2$$
$$-k_{2\mu}\epsilon_{1\nu}\epsilon_2\cdot k_1]e^{*\mu\nu},$$

where $e^*(e^{*\mu\nu})$ denote the polarization vector (tensor) for the outgoing spin-1 (2) bound state and ε stands for the Levi-Civita tensor.

Using the helicity projector techniques described in Ref. [15], the helicity amplitudes can be readily calculated for the different processes. A $\frac{1}{2}$ factor is inserted in the amplitudes for the full processes in order to avoid double counting of the one-loop diagrams as the amplitudes in Eqs. (2), (4), and (5) are symmetric in the two gluons involved in the loop. Performing the loop integration and matching the theories by the procedure described in Ref. [6] (alternatively the cross section for free quarks annihilation can be computed and matched with the NRQCD results by expanding in the quarks relative momentum and identifying the different terms by a procedure described in Ref. [21]) the following results are obtained for the short-distance coefficients:

$$\Gamma\{b\overline{b}[{}^3S_1,\underline{1}]\to c\overline{c}[{}^1S_0,\underline{8}],g\}=Fr(1-r)|\hat{H}^s(r)|^2, \tag{6}$$

$$\Gamma\{b\overline{b}[{}^3S_1,\underline{1}]\to c\overline{c}[{}^3P_J,\underline{8}],g\}$$
$$=\frac{2F}{(2J+1)m_c^2}\,r(1-r)\sum_{i=0}^{J}|\hat{H}_i^{(J)}(r)|^2, \tag{7}$$

where

$$F=\frac{5}{32}\,\frac{5}{486}\,\frac{\alpha_s^5\langle\Upsilon|O_1({}^3S_1)|\Upsilon\rangle}{m_b^2m_c^3},\quad r=(m_c/m_b)^2.$$

The explicit formulae for the helicity functions $\hat{H}_i^{(J)}$ are rather lengthy and are deferred to the Appendix.

The decay width for $\Upsilon\to\psi+X$ is obtained by adding up the short-distance coefficients listed above multiplied by $\langle\Upsilon|O_1({}^3S_1)|\Upsilon\rangle$ and their respective $\langle O_8^\psi({}^{2s+1}L_J)\rangle$ long-distance matrix elements:

$$\Gamma(\Upsilon\to c\overline{c}[{}^{2s+1}L_J,\underline{8}]g\to\psi+X)$$
$$=\frac{5\alpha_s^5}{486m_c^3m_b^2}\,\langle\Upsilon|O_1({}^3S_1)|\Upsilon\rangle D, \tag{8}$$

where

$$D=\frac{5}{32}\left[f^s\langle O_8^\psi({}^1S_0)\rangle+\sum_{J=0}^{2}f^{(J)}\,\frac{\langle O_8^\psi({}^3P_J)\rangle}{(2J+1)m_c^2}\right]. \tag{9}$$

The f factors appearing in Eq. (9) are obtained by evaluating $\hat{H}_i^{(J)}$ at $r=(m_c/m_b)^2=0.118$:

$$f^s=r(1-r)|\hat{H}^s(r)|^2=5.79,\quad f^{(0)}=2r(1-r)\hat{H}^{(0)}(r)$$
$$=6.54,$$

$$f^{(1)}=2r(1-r)\hat{H}^{(1)}(r)=7.32,$$

$$f^{(2)}=2r(1-r)\hat{H}^{(2)}(r)=8.34,$$

where

$$\hat{H}^{(J)}(r)=\sum_i|\hat{H}_i^{(J)}(r)|^2. \tag{10}$$

Using the heavy quark symmetry relation $\langle O_8^\psi({}^3P_J)\rangle=(2J+1)\langle O_8^\psi({}^3P_0)\rangle$, Eq. (9) can be written in the compact form

$$B=\frac{5}{32}\left[f^s\langle O_8^\psi({}^1S_0)\rangle+f^p\,\frac{\langle O_8^\psi({}^3P_0)\rangle}{m_c^2}\right], \tag{11}$$

where

$$f^p=\sum_{J=0}^{2}f^{(J)}=22.20.$$

For numerical analysis purposes I list the decay width for inclusive ψ production in Υ decay through the main mechanism studied in Ref. [12]:

$$\Gamma(\Upsilon\to c\overline{c}[{}^3S_1,\underline{8}]gg\to\psi+X)$$
$$=\frac{5\pi\alpha_s^4}{486m_c^3m_b^2}\,\langle\Upsilon|O_1({}^3S_1)|\Upsilon\rangle\langle O_8^\psi({}^3S_1)\rangle(0.571). \tag{12}$$

III. NUMERICAL ANALYSIS

The decay width for $\Upsilon\to\psi+X$ in Eqs. (8) and (12) depends on α_s and the NRQCD matrix elements $\langle\Upsilon|O_1({}^3S_1)|\Upsilon\rangle$, $\langle O_8^\psi({}^3P_0)\rangle$, $\langle O_8^\psi({}^3S_1)\rangle$, and $\langle O_8^\psi({}^1S_0)\rangle$. In particular it is very sensitive to the chosen values for α_s, $\langle O_8^\psi({}^3P_0)\rangle$ and $\langle O_8^\psi({}^3S_1)\rangle$.

The values for the color-octet matrix elements have been mainly extracted from hadroproduction and photoproduction data [6,11,17,18], these values being largely affected by NLO and perhaps by higher-twist corrections [17,19,20]. For the color-octet matrix element $\langle O_8^\psi({}^3S_1)\rangle$ a fit to Collider Detector at Fermilab (CDF) data gives [6] $\langle O_8^\psi({}^3S_1)\rangle\in[0.0045,0.0087]$ GeV^3. The analysis of the same data including higher order QCD effects such as initial state radiation of gluons by the interacting partons [19] gives somewhat smaller values: $\langle O_8^{\psi}(^3S_1)\rangle \in [0.0033,0.0046]$ GeV^3. As to $\langle O_8^{\psi}(^3P_0)\rangle$ and $\langle O_8^{\psi}(^1S_0)\rangle$, different combinations of these matrix elements have been extracted from different hadronic processes [6,11,17–19]. The values extracted for the same combination entering in different processes differ roughly by a factor of 2. In a recent analysis [20] which summarizes fixed target hadroproduction data [17], Tevatron data [6,19], and photoproduction data [18], the following range was obtained for the combination of these matrix elements entering in the inclusive ψ production at Tevatron: $\langle O_8^{\psi}(^1S_0)\rangle + 3\langle O_8^{\psi}(^3P_0)\rangle/m_c^2 \in [0.01,0.06]$ GeV^3.

On the other hand, ψ production in e^+e^- annihilation was recently used to extract the same combination of matrix elements [22]. This process is not sensitive to $\langle O_8^{\psi}(^1S_0)\rangle$ but is highly sensitive to $\langle O_8^{\psi}(^3P_0)\rangle$. The values obtained for $\langle O_8^{\psi}(^3P_0)\rangle$ are very stable under changes in the input values for α_s, charm quark mass, and the color-singlet matrix element $\langle O_1^{\psi}(^3S_1)\rangle$ entering in the fit, thus allowing us to strongly constrain the values for this matrix element: $\langle O_8^{\psi}(^3P_0)\rangle/m_c^2 \in [0.72,0.76]\times 10^{-2}$ GeV^3.

In summary the only color-octet matrix element which seems to be firmly established is $\langle O_8^{\psi}(^3P_0)\rangle$ and Eqs. (8) and (12) should be analyzed as a function of the remaining color-octet matrix elements.

The values $\alpha_s(2m_c)=0.253$, $\langle O_8^{\psi}(^3S_1)\rangle = 0.014$ GeV^3, $\langle \Upsilon|O_1(^3S_1)|\Upsilon\rangle = 2.3$ GeV^3 were used in [12] in the numerical evaluation of Eq. (12). The value for $\langle \Upsilon|O_1(^3S_1)|\Upsilon\rangle$ is extracted from the Υ leptonic width and is very similar to potential model calculations, thus reflecting the suitability of the nonrelativistic description for bottomonium system. The value for α_s is suggested by the fragmentation limit of Eq. (12) normalized to $\Gamma(\Upsilon\to ggg)$ which gives three times the fragmentation probability for a gluon fragmenting into a ψ, $P_{g\to\psi}$ [12,5]. However, fragmentation is not a good approximation for the process under consideration and the value used for $\langle O_8^{\psi}(^3S_1)\rangle$ seems to be too large in light of updated data [6,17,20].

A direct evaluation of Eq. (12) was not performed in Ref. [12]. Instead, authors evaluated this equation normalized to $\Gamma(\Upsilon\to ggg)$ and multiplied by $B(\Upsilon\to ggg)$ which was assumed to be $\simeq B(\Upsilon\to \text{light hadrons})=0.92$. This procedure reduces the direct evaluation of Eq. (12) by a factor of $\frac{1}{2.5}$, which is compensated by the large value used for $\langle O_8^{\psi}(^3S_1)\rangle$. A direct evaluation of Eq. (12) using $\alpha_s(2m_c)$ and the central value $\langle O_8^{\psi}(^3S_1)\rangle \simeq 0.0066$ GeV^3 obtained in Ref. [6] gives a branching ratio $\simeq 2.6\times 10^{-4}$. This B is reduced by a factor $\approx \frac{1}{2}$ if the central value quoted in Ref. [20] for $\langle O_8^{\psi}(^3S_1)\rangle$ is used in the numerical evaluations.

Using the values $\langle O_8^{\psi}(^3P_0)\rangle/m_c^2 = 0.0074$ GeV^3 and $\langle O_8^{\psi}(^1S_0)\rangle = 0.011$ GeV^3 as extracted from ψ production in e^+e^- annihilation [22] and $\alpha_s(2m_c)=0.253$ the contributions calculated in this work gives a branching ratio $B \simeq 2.1\times 10^{-4}$. The pseudoscalar process (6) accounts for $\approx 25\%$ of the contributions in Eq. (8), thus the branching ratio is less sensitive to $\langle O_8^{\psi}(^1S_0)\rangle$ than to $\langle O_8^{\psi}(^3P_0)\rangle$. Adding up the contributions from Eqs. (8) and (12) with other even smaller contributions (indirect psi production through χ_{cJ} and ψ', etc.) calculated in Ref. [12] a total branching ratio $B\simeq 6.2\times 10^{-4}$ is obtained when $\alpha_s(2m_c)$ is used in the numerical calculations. However, this B is very sensitive to the chosen value for α_s.

IV. CONCLUSIONS

Summarizing, inclusive ψ production in Υ decay through the color-octet mechanisms $b\bar{b}[^3S_1,\underline{1}]\to c\bar{c}[^{2s+1}L_J,\underline{8}]+g$ is considered. A critical numerical analysis of these $\mathcal{O}(\alpha_s^5 v_c^4)$ contributions and other in the current literature is performed. The total branching ratio for $\Upsilon\to\psi+X$ and the size of the calculated contributions strongly depend on the values chosen for α_s and the color-octet matrix elements involved in the different mechanisms. Using $\alpha_s(2m_c)$ as suggested by the fragmentation interpretation of the $\mathcal{O}(\alpha_s v_c^4)$ term [12] and the central values $\langle O_8^{\psi}(^3P_0)\rangle/m_c^2 = 0.0074$ GeV^3, $\langle O_8^{\psi}(^1S_0)\rangle = 0.011$ GeV^3 as recently extracted from e^+e^- annihilation [22], the contributions considered in this paper give a $B\simeq 2.1\times 10^{-4}$. Adding up this branching ratio with the contributions calculated in Ref. [12] a total branching ratio $B\simeq 6.2\times 10^{-4}$ is obtained in good agreement with the CLEO data $B_{\text{exp}}=1.1\pm 4\times 10^{-3}$ and consistent with the ARGUS upper bound $B_{\text{exp}}<0.68\times 10^{-3}$ [14].

In light of the available experimental information, the calculations presented in this paper indicate that color-octet mechanisms account for most of the ψ production in Υ decay. However, further work, both theoretical and experimental, is necessary in order to have a clear idea on the role of the color-octet component of the $c\bar{c}$ pair inside the ψ system.

From the experimental point of view, it is necessary to remove the existing inconsistencies as the ARGUS Collaboration results are not confirmed by the CLEO Collaboration [14]. A measurement of the decay width and of the energy spectrum of the ψ would be desirable as the latter can discriminate between the different ψ production mechanisms. In particular, ψ's produced through the color-octet mechanisms considered in this paper have the distinctive signature that the ψ's are sharply peaked in energy and are recoiled by a gluon jet. This is in contrast with other mechanisms where a spread distribution in energy is expected.

From the theoretical point of view, the size of the calculated contributions strongly depends on the assumed value for α_s. A numerical evaluation of Eqs. (8) and (12) using $\alpha_s(2m_b)$ decreases the calculated B roughly by a factor of $\frac{1}{5}$. Hence, a calculation of the $\mathcal{O}(\alpha_s^5 v_c^4)$ color-octet mechanisms $b\bar{b}[^3S_1,\underline{1}]\to c\bar{c}[^{2S+1}L_J,\underline{8}]ggg$ and the $\mathcal{O}(\alpha_s^6)$ color-singlet mechanisms $b\bar{b}[^3S_1,\underline{1}]\to c\bar{c}[^3S_1,\underline{1}]gg$ and $b\bar{b}[^3S_1,\underline{1}]\to c\bar{c}[^3S_1,\underline{1}]gggg$ is necessary in order draw definitive conclusions. The color singlet contributions have been ''crudely estimated'' to give a branching ratio of a few $\times 10^{-4}$ [13].

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APPENDIX

$$\hat{H}^{s}(r)=\frac{4}{x}\left[\mathrm{Di}(2x)-\mathrm{Di}(0)-\frac{x}{(1-2x)}\ln(2x)-\frac{1-x}{2-x}\left(2\,\mathrm{Di}(x)-2\,\mathrm{Di}(0)+\frac{1}{2}\ln^{2}(1-x)\right)+i4\pi\,\frac{1-x}{x(2-x)}\ln(1-x)\right],$$

$$\begin{aligned}\hat{H}_{0}^{(0)}(r)=\sqrt{\frac{2}{3}}\Bigg[&\frac{2-3x}{x^{2}}+\left(10\,\frac{1-x}{x^{3}}+4\,\frac{1-2x}{x^{2}}\ln(2)\right)\ln(1-x)-3\,\frac{1-x}{x(2-x)}\ln^{2}(1-x)\\&+\left(\frac{8}{x^{2}}+2\,\frac{1-x}{x(1-2x)}\right)\ln(2x)+\frac{8-6x+x^{2}-6x^{3}}{x^{3}(2-x)}\mathrm{Di}(0)-\frac{4-5x+2x^{2}}{x^{3}}\mathrm{Di}(2x)\\&-4\,\frac{2-2x-x^{2}}{x^{2}(2-x)}\mathrm{Di}(x)+i6\pi\,\frac{1-x}{x(2-x)}\ln(1-x)\Bigg],\end{aligned}$$

$$\begin{aligned}\hat{H}_{0}^{(1)}(r)=\frac{4}{x^{2}}\Bigg[&\frac{1}{x}[\mathrm{Di}(0)-\mathrm{Di}(2x)]-6(1-x)[\mathrm{Di}(x)-\mathrm{Di}(2x)-\ln(2)\ln(1-x)]\\&+\frac{2}{x}(1-x)(1-2x)\ln(1-x)+\frac{2-8x+7x^{2}}{1-2x}\ln(2x)\Bigg],\end{aligned}$$

$$\begin{aligned}\hat{H}_{1}^{(1)}(r):=\frac{4\sqrt{1-x}}{x^{2}}\Bigg[&\frac{1}{x}[\mathrm{Di}(0)-\mathrm{Di}(2x)]-x-2x[\mathrm{Di}(x)-\mathrm{Di}(2x)-\ln(2)\ln(1-x)]+\frac{2-x-2x^{2}}{x}\ln(1-x)\\&+2(1+x)\ln(2x)\Bigg],\end{aligned}$$

$$\begin{aligned}\hat{H}_{0}^{(2)}(r)=\frac{2\sqrt{3}}{x^{3}}\Bigg[&(6-5x)x+\frac{2}{3}\,\frac{6-19x+18x^{2}}{x}(1-x)\ln(1-x)-\frac{1}{3}\,\frac{10-12x+5x^{2}}{2-x}[\mathrm{Di}(0)-\mathrm{Di}(2x)]\\&+\frac{2}{3}\,\frac{6-38x+71x^{2}-37x^{3}}{1-2x}\ln(2x)-8\,\frac{(1-x)^{2}}{x^{2}(2-x)}\left(\mathrm{Di}(2x)-2\,\mathrm{Di}(x)-\frac{1}{2}\ln^{2}(1-x)+\mathrm{Di}(0)+i\pi\ln(1-x)\right)\\&+\frac{4}{3}\,\frac{6-6x-x^{2}}{x}\left(\ln(2)-i\,\frac{\pi}{2}\right)-\frac{4}{3}(12-26x+13x^{2})[\mathrm{Di}(x)-\mathrm{Di}(2x)-\ln(2)\ln(1-x)]\Bigg],\end{aligned}$$

$$\begin{aligned}\hat{H}_{1}^{(2)}(r)=\frac{2\sqrt{1-x}}{x^{3}}\Bigg[&-\frac{1}{3}(38-9x)x-\frac{2}{x}(4-13x+16x^{2}-4x^{3})\ln(1-x)+2\,\frac{x(1-x)}{2-x}[\mathrm{Di}(0)-\mathrm{Di}(2x)]\\&-\frac{4}{1-2x}(2-11x+16x^{2}-4x^{3})\ln(2x)+8\,\frac{(1-x)(2-2x+x^{2})}{x^{2}(2-x)}\Bigg(\mathrm{Di}(2x)-2\,\mathrm{Di}(x)-\frac{1}{2}\ln^{2}(1-x)+\mathrm{Di}(0)\\&+i\pi\ln(1-x)\Bigg)-\frac{16}{3}\,\frac{3-3x+x^{2}}{x}\left(\ln(2)-i\,\frac{\pi}{2}\right)+4(8-12x+3x^{2})[\mathrm{Di}(x)-\mathrm{Di}(2x)-\ln(2)\ln(1-x)]\Bigg],\end{aligned}$$

$$\begin{aligned}\hat{H}_{2}^{(2)}(r)=\sqrt{2}\,\frac{1-x}{x^{3}}\Bigg[&\frac{16}{3}x+\frac{4}{x}(1-6x+6x^{2})\ln(1-x)-\frac{2}{2-x}(5-6x+2x^{2})[\mathrm{Di}(0)-\mathrm{Di}(2x)]\\&-4\,\frac{2-4x+6x^{2}-4x^{3}+x^{4}}{x^{2}(2-x)}\left[\mathrm{Di}(2x)-2\,\mathrm{Di}(x)-\frac{1}{2}\ln^{2}(1-x)+\mathrm{Di}(0)+i\pi\ln(1-x)\right]\\&+\frac{4}{3}\,\frac{6-6x+11x^{2}}{x}\left(\ln(2)-i\,\frac{\pi}{2}\right)-16(1-x)[\mathrm{Di}(x)-\mathrm{Di}(2x)-\ln(2)\ln(1-x)]+4(1-6x)\ln(2x)\Bigg],\end{aligned}$$

where $x=1-r$ and Di denotes the Dilog function $\mathrm{Di}(s)=\int_{1}^{s}[\ln(t)/(1-t)]dt$.

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