Radiative leptonic B decays in the light front model

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Within the light front framework, we calculate the form factors for $B\rightarrow\gamma$ transitions directly in the entire physical range of momentum transfer. Using these form factors, we study the radiative decays of $B\rightarrow l\nu_l\gamma$ and $B_{s(d)}\rightarrow\nu\bar{\nu}\gamma$. We show that the decay rates of $B\rightarrow l\nu_l\gamma$ ($l=e,\mu$) and $B\rightarrow\nu\bar{\nu}\gamma$ are larger than that of the corresponding purely leptonic modes. Explicitly, in the standard model, we find that the branching ratios of $B\rightarrow\mu\nu_\mu\gamma$ and $B_s\rightarrow\nu\bar{\nu}\gamma$ are 1.3×10^{-6} and 2.0×10^{-8}, in contrast with 2.3×10^{-7} and 0 for $B\rightarrow\mu\nu_\mu$ and $B\rightarrow\nu\bar{\nu}$, respectively. [S0556-2821(98)04209-X]

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I. INTRODUCTION

It is known that the purely leptonic B decays of $B\rightarrow l\nu_l$ could be used to determine the weak mixing element of $|V_{ub}|$ in the Cabibbo-Kobayashi-Maskawa matrix [1] as well as the value of the B meson decay constant f_B [2]. The decay rates of these purely leptonic modes are given by

$$\Gamma(B\rightarrow l\bar{\nu_l})=\frac{G_F^2}{8\pi}|V_{ub}|^2 f_B^2\left(\frac{m_l^2}{m_B^2}\right)m_B^3\left(1-\frac{m_l^2}{m_B^2}\right)^2. \quad (1)$$

However, the rates for $B\rightarrow e\bar{\nu_e}$ and $\mu\bar{\nu_\mu}$ in Eq. (1) are helicity suppressed with the suppression factors of m_l^2/m_B^2 with $l=e$ and μ, respectively, and one has that $B(B^-\rightarrow e^-\bar{\nu_e},\mu^-\bar{\nu_\mu})\simeq(5.1\times10^{-12},2.3\times10^{-7})$ by taking $|V_{ub}|=3\times10^{-3}$, $f_B=180$ MeV and $\tau_{B^-}\simeq1.62\times10^{-12}$ s [3]. Clearly, it is difficult to measure these decays, especially for the light charged lepton mode. Although there is no suppression for the τ channel, it is hard to observe the decay experimentally because of the low efficiency. A similar helicity suppression effect is also expected in the flavor changing neutral current (FCNC) processes of $B_{s(d)}\rightarrow l^+l^-$, which are sensitive to new physics beyond the standard model [4]. Furthermore, to persevere the helicity conservation, the decays of $B_{s(d)}\rightarrow\nu\bar{\nu}$ are forbidden in the standard model.

Recently, there has been a considerable amount of theoretical attention [5–11] to the class of the radiative B decays, such as, $B\rightarrow l\nu_l\gamma$, $B_{s(d)}\rightarrow l^+l^-\gamma$ and $B_{s(d)}\rightarrow\nu\bar{\nu}\gamma$. These decays receive two types of contributions: internal bremsstrahlung (IB) and structure-dependent (SD) [12]. The IB contributions are still helicity suppressed [5], while the SD ones contain the electromagnetic coupling constant α, but they are free of the helicity suppression. Therefore, the radiative decay rates of $B\rightarrow l_i\bar{l}_j\gamma$ ($l_{i,j}=l,\nu_l$) could have an enhancement with respect to the purely leptonic modes of $B\rightarrow l_i\bar{l}_j$ due to the SD contributions. Indeed, it has been shown that, for example, the branching ratios of $B\rightarrow\mu\nu_\mu\gamma$ [5–8] and $B_s\rightarrow\nu\bar{\nu}\gamma$ [9,11] are $O(10^{-6})$ and $O(10^{-9})$, in contrast with that of $O(10^{-7})$ and 0 for the corresponding purely leptonic modes, respectively, in the standard model. The measurements of the above decays in future B factories provide an alternative way of knowing the B decay constants and the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [2].

In this paper, we concentrate on the radiative decays of $B\rightarrow l\nu_l\gamma$ and $B\rightarrow\nu\bar{\nu}\gamma$. We will use the light front formulation [13,14] to evaluate the hadronic matrix elements. These decays have been studied in various quark models [5–11]. It is known that as the recoil momentum increases, we have to start considering relativistic effects seriously. In particular, at the maximum recoil point, there is no reason to expect that the nonrelativistic quark model is still applicable. A consistent treatment of the relativistic effect of the quark motion and spin in a bound state is a main issue of the relativistic quark model. The light front quark model [15,16] is the widely accepted relativistic quark model in which a consistent and relativistic treatment of quark spins and the center-of-mass motion can be carried out. In this paper we calculate the $P\rightarrow\gamma$ (P: pseudoscalar meson) form factors directly at the timelike momentum transfers for the first time. We will give their dependence on the momentum transfer p^2 in the whole kinematic region of $0\leqslant p^2\leqslant p^2_{\max}$.

The paper is organized as follows. In Sec. II, we present the relevant effective Hamiltonians for the radiative decays of $B\rightarrow l\bar{\nu_l}\gamma$ and $B_{s(d)}\rightarrow\nu\bar{\nu}\gamma$, respectively. In Sec. III, we study the form factors in the $B\rightarrow\gamma$ transition within the light front framework. We calculate the decay branching ratios in Sec. IV. We give our conclusions in Sec. V.

II. EFFECTIVE HAMILTONIAN

To study the decays of $B\rightarrow l\nu_l\gamma$, we start with the effective Hamiltonian for $b\rightarrow ul\nu_l$ at the quark level in the standard model, which is given by

$$H_{eff}(b\rightarrow ul\nu_l)=\frac{G_F}{\sqrt{2}}V_{ub}\bar{u}\gamma_\mu(1-\gamma_5)b\bar{\nu}\gamma_\mu(1-\gamma_5)l. \quad (2)$$

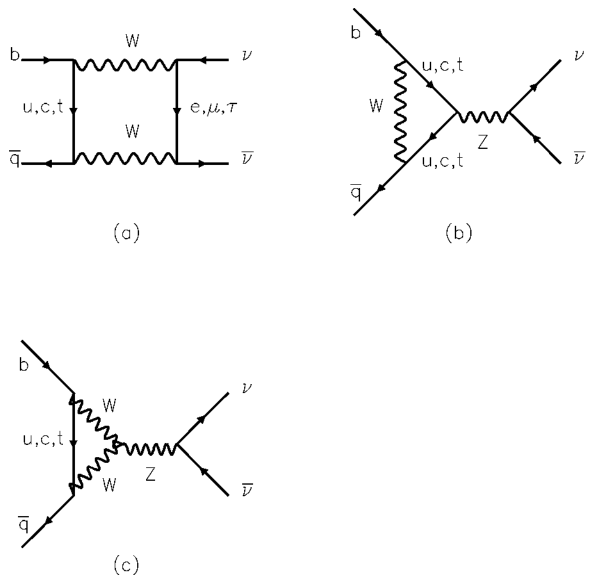

FIG. 1. Loop diagrams that contribute to $b\to q\nu\bar{\nu}$.

For the radiative B decays, if we neglect the helicity suppressed photon emission from the final lepton, from Eq. (2) we get

$$H_{eff}(B\to l\nu_l\gamma)=\frac{G_F}{\sqrt{2}}V_{ub}\langle\gamma|\bar{u}\gamma_\mu(1-\gamma_5)b|B\rangle \times\bar{\nu_l}\gamma_\mu(1-\gamma_5)l. \tag{3}$$

For the processes of $B_q\to\nu_l\bar{\nu_l}\gamma$ $(q=s,d;l=e,\mu,\tau)$, at the quark level, they arise from the box and Z-penguin diagrams, as shown in Fig. 1, that contribute to $b\to q\nu_l\bar{\nu_l}$ with the photon emitting from the charged particles in the diagrams. However, when the photon line is attached to the internal charge lines as the W boson and t quark lines, there is a suppression factor of m_b^2/M_W^2 in the Wilson coefficient in comparison with those in $b\to q\nu_l\bar{\nu_l}$ [5]. Thus, we need only consider the diagrams with the photon from the external quarks. From the effective interactions for $b\to q\nu_l\bar{\nu_l}$, we obtain the effective Hamiltonians for $B_q\to\nu_l\bar{\nu_l}\gamma$ as follows:

$$H_{eff}(B_q\to\nu_l\bar{\nu_l}\gamma)=\frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\theta_W}V_{tb}V_{tq}^*D(x_t) \times\langle\gamma|\bar{q}\gamma_\mu(1-\gamma_5)b|B\rangle\bar{\nu_l}\gamma_\mu(1-\gamma_5)\nu_l, \tag{4}$$

where $x_t=m_t^2/M_W^2$ and

$$D(x_t)=\frac{x_t}{8}\left[-\frac{2+x_t}{1-x_t}+\frac{3x_t-6}{(1-x_t)^2}\ln x_t\right]. \tag{5}$$

We note that in Eqs. (4) and (5), only the leading contributions have been included and the additional $1/m_b^2$ and α_s corrections to the result, which are small, can be found in Ref. [17].

III. FORM FACTORS ON THE LIGHT FRONT

From the effective Hamiltonians in Eqs. (3) and (4), we see that to find the decay rates, we have to evaluate the hadronic matrix elements: $\langle\gamma|J_\mu|B\rangle$, where $J_\mu=\bar{u}\gamma_\mu(1-\gamma_5)b$ with u representing the light quarks of up, down and strange, respectively. The elements can be parametrized as follows:

$$\langle\gamma(q)|\bar{u}\gamma^\mu\gamma_5 b|B(p+q)\rangle=ie\frac{F_A}{M_B}[\epsilon^{*\mu}(p\cdot q)-(\epsilon^*\cdot p)q^\mu],$$

$$\langle\gamma(q)|\bar{u}\gamma^\mu b|B(p+q)\rangle=ie\frac{F_V}{M_B}\epsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha p_\beta q_\gamma, \tag{6}$$

where q and $p+q$ are photon and B meson four momenta, F_A and F_V are form factors of axial-vector and vector, respectively, and ϵ is the photon polarization vector.

The form factors in Eq. (6) will be calculated in the light front quark model at the timelike momentum transfers in which the physically accessible kinematic region is $0\leqslant p^2\leqslant p^2_{\max}$. We consider that a meson bound state consists of a quark q_1 and an antiquark $\bar{q_2}$ with total momentum $(p+q)$. For the B meson bound state we use the Gaussian type wave function, given by [14,18,19]

$$|B(p+q)\rangle=\sum_{\lambda_1\lambda_2}\int[dk_1][dk_2]2(2\pi)^3\delta^3(p+q-k_1-k_2) \times\Phi_B^{\lambda_1\lambda_2}(x,k_\perp)b_b^+(k_1,\lambda_1)d_u^+(k_2,\lambda_2)|0\rangle, \tag{7}$$

where $k_{1(2)}$ is the on-mass shell light front momentum of the internal quark $b(\bar{u})$. The light front relative momentum variables $(x,k_\perp)$ are defined by

$$k_1^+=x(p+q)^+,\quad k_{1\perp}=x(p+q)_\perp+k_\perp. \tag{8}$$

The normalization conditions can be written as

$$\langle B(p)|B(p')\rangle=2(2\pi)^3p^+\delta^3(p-p'), \tag{9}$$

which leads to

$$\sum_{\lambda_1\lambda_2}\int\frac{dx\,d^2k_\perp}{2(2\pi)^3}|\Phi_B^{\lambda_1\lambda_2}(x,k_\perp)|^2=1. \tag{10}$$

The B meson wave function $\Phi_B^{\lambda_1\lambda_2}(x,k_\perp)$ is chosen to be a Gaussian type momentum distribution:

$$\Phi_B^{\lambda_1\lambda_2}(x,k_\perp)=N\left[\frac{2k_1^+k_2^+}{M_0^2-(m_u-m_b)^2}\right]^{1/2} \times\bar{u}(k_1,\lambda_1)\gamma^5 v(k_2,\lambda_2)\sqrt{\frac{dk_z}{dx}}\exp\left(-\frac{\vec{k}^2}{2\omega_B^2}\right), \tag{11}$$

with

$$[dk]=\frac{dk^+dk_\perp}{2(2\pi)^3},\quad N=4\left(\frac{\pi}{\omega_B^2}\right)^{3/4},$$

$$k_z=\left(x-\frac{1}{2}\right)M_0+\frac{m_b^2-m_u^2}{2M_0}, \quad M_0^2=\frac{k_\perp^2+m_u^2}{x}+\frac{k_\perp^2+m_b^2}{1-x},$$

$$\sum_\lambda u(k,\lambda)\overline{u}(k,\lambda)=\frac{m+\not k}{k^+}, \quad \sum_\lambda v(k,\lambda)\overline{v}(k,\lambda)=-\frac{m-\not k}{k^+}, \tag{12}$$

where the ω is a parameter related to the physical size of the meson, which is of order Λ_{QCD}. The value of ω ranges from 0.3 to 0.6 [20]. The spinors in Eq. (11) approximately take care of the relativistic spin kinematics of quarks inside the B mesons.

The gauged photon state with momentum p and spin λ can be described by

$$|p\lambda\rangle=N'\Bigg\{a^+(p,\lambda)+\sum_{\lambda_1,\lambda_2}\int[dk_1]\times[dk_2]\Phi_{q\underline{q}}^{\lambda_1\lambda_2\lambda}(p,k_1,k_2)2(2\pi)^3\delta^3(p-k_1-k_2)\times b^+(k_1,\lambda_1)d^+(k_2,\lambda_2)\Bigg\}|0\rangle. \tag{13}$$

The second term in Eq. (13) corresponds to the photon state in QED in terms of quark pairs. Equation (13) satisfies the light front bound state

$$H_{LF}|p,\lambda\rangle=\frac{p_\perp^2}{p^+}|p,\lambda\rangle, \tag{14}$$

with

$$H_{LF}=H_0+H_I, \tag{15}$$

where H_0 is the free-energy Hamiltonian of quarks and photons, and H_I is the QED interacting part between quarks and photons in the light front gauge $A^+=0$, given by

$$H_I=e_q\int q_+^+\Bigg\{-2\frac{1}{\partial^+}\partial^i A_\perp^i-\gamma\cdot A_\perp\frac{1}{\partial^+}(\gamma\cdot\partial_\perp-im)-\frac{1}{\partial^+}(\gamma_\perp\cdot\partial_\perp+im)\gamma\cdot A_\perp\Bigg\}q_+\frac{dx^+d^2k_\perp}{2}, \tag{16}$$

and e_q is the quarks' electric change, q_+ is the dynamical component of the quark field on the light front: $q(x)=q_+(x)+q_-(x)$, with $q_\pm(x)=\frac{1}{2}\gamma^0\gamma^\pm q(x)$, and $A_\perp$ is the transverse component of the gauge field in the light front gauge.

From Eqs. (14)–(16), we find the distribution of $\Phi_{q\underline{q}}^{\lambda_1\lambda_2\lambda}$ as

$$\Phi_{q\underline{q}}^{\lambda_3\lambda_4\lambda}(q,k_1,k_2)=\frac{e_q}{ED}\chi_{-\lambda_2}^+\Bigg\{-2\frac{q_\perp\cdot\epsilon_\perp}{q^+}-\gamma_\perp\cdot\epsilon_\perp\frac{\gamma_\perp\cdot k_{2_\perp}-m_2}{k_2^+}-\frac{\gamma_\perp\cdot k_{1_\perp}-m_1}{k_1^+}\gamma_\perp\cdot\epsilon_\perp\Bigg\}\chi_{\lambda_1}, \tag{17}$$

with

$$ED=\frac{q_\perp^2}{q^+}-\frac{k_{1_\perp}^2+m_1^2}{k_1^+}-\frac{k_{2_\perp}^2+m_2^2}{k_2^+}. \tag{18}$$

Thus the gauge boson state wave function in Eq. (13) can be rewritten as

$$|\gamma(q)\rangle=N'\Bigg\{a^+(q,\lambda)+\sum_{\lambda_1\lambda_2}\int[dk_1][dk_2]2(2\pi)^3\times\delta^3(q-k_1-k_2)\Phi_{q\underline{q}}^{\lambda_1\lambda_2\lambda}(q,k_1,k_2)\times b_q^+(k_1,\lambda_1)d_{\bar q}^+(k_2,\lambda_2)\Bigg\}|0\rangle. \tag{19}$$

Since the transfer momenta in the decay processes are timelike, it is convenient to choose the light front coordinate: $p^+\geqslant 0$ and $p_\perp=0$. By considering the ''+'' component in the weak current the matrix elements in Eq. (6) become

$$\langle\gamma(q)|u_+^+\gamma_5 b_+|B(p+q)\rangle=-ie\frac{F_A}{2M_B}(\epsilon_\perp^*\cdot q_\perp)p^+,$$
$$\langle\gamma(q)|u_+^+b_+|B(p+q)\rangle=e\frac{F_V}{2M_B}\epsilon^{ij}\epsilon_i^*q_jp^+. \tag{20}$$

The form factors of F_A and F_V in Eq. (20) are found to be

$$F_A(p^2)=i4M_B\int\frac{dx'd^2k_\perp}{2(2\pi)^3}\Phi(x,k_\perp^2)\frac{x'-x}{x(1-x)}\Bigg\{\frac{1}{3}\frac{-m_b+Bk_\perp^2\Theta}{m_b^2+k_\perp^2}-\frac{2}{3}\frac{m_u-Ak_\perp^2\Theta}{m_u^2+k_\perp^2}\Bigg\}, \tag{21}$$

$$F_V(p^2)=i4M_B\int\frac{dx'd^2k_\perp}{2(2\pi)^3}\Phi(x,k_\perp^2)\frac{x'-x}{x(1-x)}\Bigg\{\frac{1}{3}\frac{-m_b-(1-x)(m_b-m_u)k_\perp^2\Theta}{m_b^2+k_\perp^2}-\frac{2}{3}\frac{m_u-x(m_b-m_u)k_\perp^2\Theta}{m_u^2+k_\perp^2}\Bigg\}, \tag{22}$$

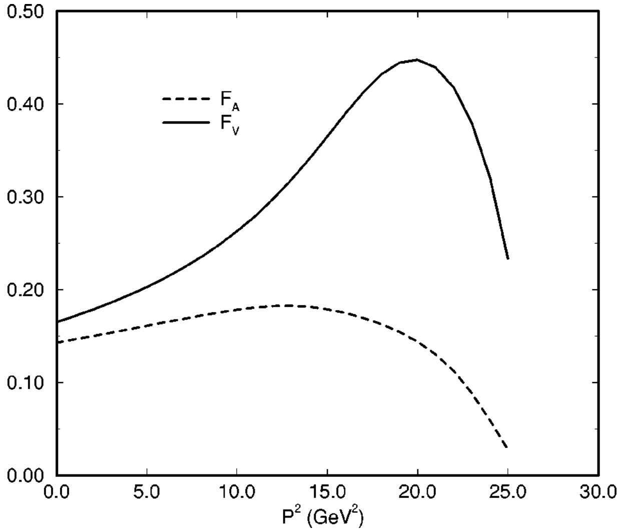

FIG. 2. The values of the form factors F_V (solid curve) and F_A (dashed curve) as functions of the momentum transfer p^2.

where

$$A=(1-2x')x(m_b-m_u)-2x'm_u,$$

$$B=[(1-2x')x-1]m_b+(1-2x')(1-x)m_u,$$

$$\Phi(x,k_\perp^2)=N\left(\frac{2x(1-x)}{M_0^2-(m_u-m_b)^2}\right)^{1/2}\sqrt{\frac{dk_z}{dx}}\times\exp\left(-\frac{\vec{k}^2}{2\omega_B^2}\right),$$

$$\Theta=\frac{1}{\Phi(x,k_\perp^2)}\frac{d\Phi(x,k_\perp^2)}{dk_\perp^2},$$

$$x=x'\left(1-\frac{p^2}{M_B^2}\right),\quad \vec{k}=(\vec{k}_\perp,\vec{k}_z). \tag{23}$$

To illustrate the form factors, we input the values of $m_u=0.3$, $m_b=4.5$, $M_B=5.2$, and $\omega=0.57$ in GeV to integral whole range of p^2. The results of F_A and F_V in the entire range of momentum transfer p^2 are shown in Fig. 2. We note that the reason that the tails of $F_{V,A}$ at the large momentum transfer go down may be because the light front model does not include the long-distance contribution associated with B-B^*-γ vertex, etc. However, we expect that this contribution should not be essentially important.

It is interesting to note that the formulas in Eqs. (21) and (22) can be used for other pseudoscalar and photon transitions as well once we put in the corresponding masses. For example, for $K^+\to\gamma$ at $p^2=0$, we get that $(F_A(0),F_V(0))|_{K^+\to\gamma}=(0.0429,0.0915)$, in comparison with $(0.0425,0.0945)$ found in the chiral perturbation theory at the one-loop level [21], which agrees with the experiments.

IV. DECAY BRANCHING RATIOS

A. $B^+\to l^+\nu_l\gamma$

For the radiative decays of $B^+\to l^+\nu_l\gamma$, we will only consider the cases of $l=e$ and μ. From the effective Hamiltonian in Eq. (3) and the matrix element in Eq. (20), we find that the amplitude of $B^+\to l^+\nu_l\gamma$ is

$$M_{B^+\to l^+\nu\gamma}=-\frac{ieG_FV_{ub}}{\sqrt{2}}\epsilon_\mu^* H^{\mu\nu}\overline{u(p_\nu)}\gamma_\mu(1-\gamma_5)v(p_l), \tag{24}$$

with

$$H_{\mu\nu}=\frac{F_A}{M_B}(-p'\cdot qg_{\mu\nu}+p'_\mu q_\nu)+i\epsilon_{\mu\nu\alpha\beta}\frac{F_V}{M_B}q^\alpha p'^\beta, \tag{25}$$

where p' and q are B meson and photon four momenta, respectively, and ϵ_μ is a photon polarization vector. Since the form factors $F_{V,A}$ depend on the transfer momentum p^2, we need to replace p^2 into (p',q). In the physical allowed region of $B^+\to l^+\nu_l\gamma$, one has that

$$m_l^2\leqslant p^2\leqslant M_B^2. \tag{26}$$

To describe the kinematic of the decay, two variables are needed. For convention, we defined $x''=2E_\gamma/M_B$ and $y=2E_l/M_B$ in the B meson rest frame in order to easily write down momentum p^2 in terms of x'', which has the form

$$p^2=M_B^2(1-x''). \tag{27}$$

We get the differential decay rate

$$\frac{d^2\Gamma^l}{dx''d\lambda}=\frac{M_B}{256\pi^3}|M|^2=C\rho(x'',\lambda), \tag{28}$$

where $\lambda=(x''+y-1-r)/x''$,

$$C=\frac{\alpha}{32\pi^2}G_F^2M_B^5|V_{ub}|^2, \tag{29}$$

and

$$\rho(x,\lambda)=\rho_+(x'',\lambda)+\rho_-(x'',\lambda), \tag{30}$$

with

$$\rho_+=\frac{1}{2}|F_A+F_V|^2x''\lambda[(\lambda x''+r)(1-x'')-r],$$

$$\rho_-=\frac{1}{2}|F_A-F_V|^2x''(1-\lambda)\{(x''-1)[r+x''(\lambda-1)]+r\},$$

$$r=\frac{m_l^2}{M_B^2}. \tag{31}$$

We write the physical region for x'' and λ as

$$0\leqslant x''\leqslant 1-r,$$

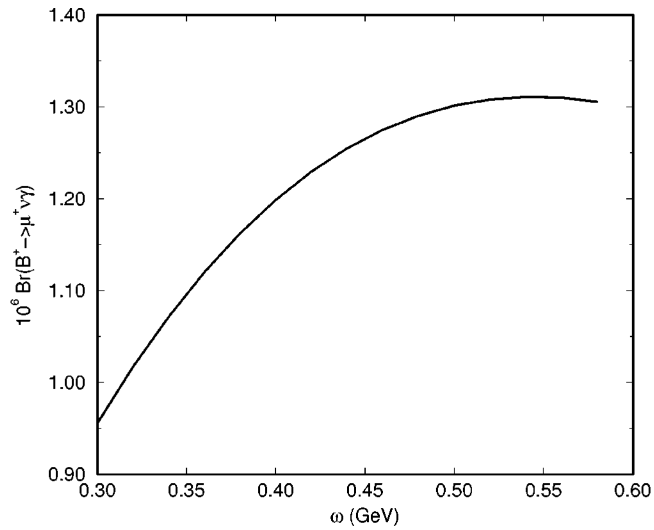

FIG. 3. The branching ratio of $B^+ \to \mu^+ \nu_\mu \gamma$ as a function of the parameter ω.

$$\frac{r}{1-x''} \leqslant \lambda \leqslant 1. \tag{32}$$

In Fig. 3, we show the branching ratio of $B^+ \to \mu^+ \nu_\mu \gamma$ as a function of the parameter ω, where we have used $m_u = 300$ MeV, $|V_{ub}| \simeq 3 \times 10^{-3}$, and $\tau_{B^+} \simeq 1.62 \times 10^{-12} s$ [3]. For $\omega = 0.57$ GeV, we get the integrated branching ratios of $B^+ \to l^+ \nu_l \gamma$ $(l = e, \mu)$ as

$$Br(B^+ \to l^+ \nu_\mu \gamma) \simeq 1.3 \times 10^{-6}. \tag{33}$$

From Eq. (33), we find that

$$R_B = \frac{\Gamma(B^+ \to \mu^+ \nu_\mu \gamma)}{\Gamma(B^+ \to \mu^+ \nu_\mu)} \simeq 5.6, \tag{34}$$

which is within the range of 1–30 as expected in Ref. [5]. Our results in Fig. 3 and the branching ratios in Eq. (33) agree well with that in Ref. [6] where the light cone QCD sum rules were used in their calculations. However, the value in Eq. (33) is about a factor of 2 smaller and larger than that in Ref. [7] and Ref. [8], respectively.

B. $B_{s(d)} \to \nu \bar{\nu} \gamma$

From the effective Hamiltonians for $B_q \to \nu_l \bar{\nu}_l \gamma$ in Eq. (4) and the form factors defined in Eq. (20), we can write the amplitude of $B_q \to \nu_l \bar{\nu}_l \gamma$ as

$$M = -ie \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} V_{tb} V_{tq}^* D(x_t) \epsilon_\mu^* H^{\mu\nu} \overline{u}(p_{\bar{\nu}}) \times \gamma_\mu (1-\gamma_5) v(p_\nu), \tag{35}$$

with

$$H_{\mu\nu} = \frac{F_A}{M_B}(-p' \cdot q g_{\mu\nu} + p'_\mu q_\nu) + i\epsilon_{\mu\nu\alpha\beta} \frac{F_V}{M_B} q^\alpha p'^\beta, \tag{36}$$

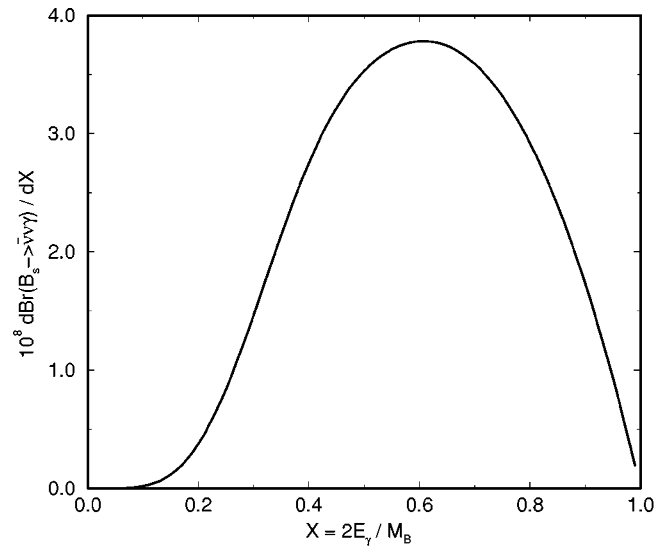

FIG. 4. The differential decay branching ratio $dB(B_s \to \nu \bar{\nu} \gamma)/dX$ as a function of $X = 2E_\gamma / M_B$.

where the form factors are given by Eqs. (21) and (22) with the replacement of the light quark (u) by s and d quarks, respectively.

Similar to the decays discussed in the previous subsection, we also define $x'' = 2E_\gamma / M_B$ and $y = 2E_{\bar{\nu}} / M_B$ in the B meson rest frame in order to rescale the energies of the photon and antineutrino. By integrating the variable y in the phase space of variable y, we obtain the differential decay rate of $B \to \nu \bar{\nu} \gamma$ as

$$\frac{d\Gamma}{dx''} = 6\alpha \left(\frac{G_F \alpha}{16\pi^2 \sin^2\theta_W} \right)^2 (|F_A|^2 + |F_V|^2) |V_{tb} V_{tq}^*|^2 \times D^2(x_t) x''^3 (1-x'') M_B^5, \tag{37}$$

where we have included the three generations of neutrinos.

Using $m_d = 300$ MeV, $m_s = 400$ MeV, $m_t = 176$ GeV, $|V_{tb}| = 1$, $|V_{ts}| \simeq 0.04$, and $\omega = 0.57$, the differential decay branching ratio $dB(B_s \to \nu \bar{\nu} \gamma)/dx''$ as a function of x''

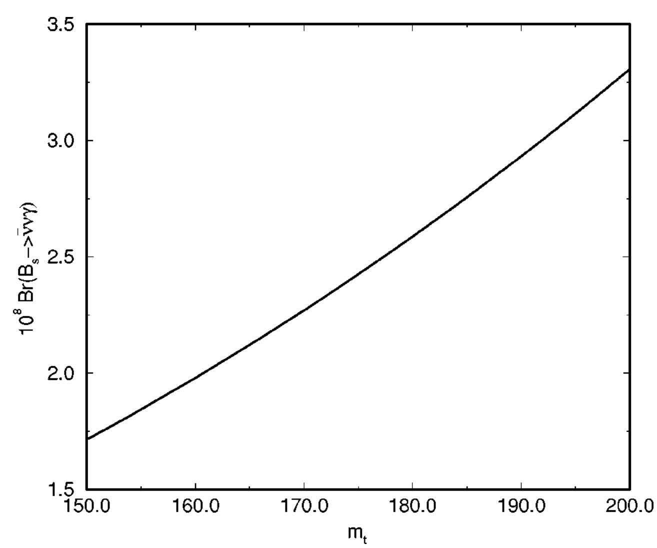

FIG. 5. The branching ratio of $B_s \to \nu \bar{\nu} \gamma$ as a function of m_t.

$=2E_\gamma/M_B$ is shown in Fig. 4. In Fig. 5, we give the branching ratio of $B_s\to\nu\bar{\nu}\gamma$ as a function of m_t. Here, we have used $\tau_{B_s}=1.61\times10^{-12}s$ and $\tau_{B_d}=1.56\times10^{-12}s$ [3], respectively. From the figure we find that, for $m_t=176$ GeV and $|V_{td}|\simeq0.01$,

$$B(B_s\to\nu\bar{\nu}\gamma)=2.0\times10^{-8},$$

$$B(B_d\to\nu\bar{\nu}\gamma)=1.4\times10^{-9}. \tag{38}$$

We note that the branching ratios in Eq. (38) are about the same as that in Ref. [9], but about a factor of 3 smaller than that in Ref. [11].

V. CONCLUSIONS

We have studied the form factors for $B\to\gamma$ transitions directly within the light front framework in the entire physical range of momentum transfer. Using these form factors, we have calculated the radiative decays of $B\to l\nu_l\gamma$ and $B_{s(d)}\to\nu\bar{\nu}\gamma$. We have shown that the decays of $B\to l\nu_l\gamma$ $(l=e,\mu)$ and $B\to\nu\bar{\nu}\gamma$ are dominated by the contributions from the diagrams with photon emission from the external quarks and thus overcome the helicity suppression effect. We have found that, in the standard model, the branching ratios of $B\to e\nu_e\gamma$, $B\to\mu\nu_\mu\gamma$ and $B_{s(d)}\to\nu\bar{\nu}\gamma$ are 1.3×10^{-6}, 1.3×10^{-6} and 2.0×10^{-8} (1.4×10^{-9}), respectively. Some of the modes are clearly accessible in the future B factories.

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