Mass spectrum of orbitally and radially excited heavy-light mesons in the relativistic quark model

D. Ebert and V. O. Galkin*
Institut für Physik, Humboldt-Universität zu Berlin, Invalidenstrasse 110, D-10115 Berlin, Germany

R. N. Faustov
Russian Academy of Sciences, Scientific Council for Cybernetics, Vavilov Street 40, Moscow 117333, Russia
(Received 10 December 1997; published 1 April 1998)

The mass spectrum of orbitally and radially excited states of B and D mesons is calculated in the framework of the relativistic quark model. The expansion in inverse powers of the heavy quark mass is carried out up to first order, while the light quark is treated without expansion. We find that the relativistic treatment of the light quark plays an important role. Different patterns of P level inversion are discussed. The obtained masses of orbitally and radially excited states are in accord with available experimental data and heavy quark symmetry relations. [S0556-2821(98)06009-3]

PACS number(s): 12.40.Yx, 12.39.Ki, 14.40.Lb, 14.40.Nd

I. INTRODUCTION

The investigation of the properties of excited heavy-light mesons represents an interesting and important problem. The experimental data on the orbitally [1] and radially [2] excited B and D meson states are becoming available now. The B and $c-\tau$ factories will provide more accurate and comprehensive data for these states. On the other hand, the presence of the heavy quark in these systems considerably simplifies the theoretical description of heavy-light mesons. Heavy quark symmetry [3], arising in the limit of an infinitely heavy quark mass m_Q, imposes strict constraints on the mass spectrum of heavy-light mesons [4]. In this limit the heavy quark mass and spin decouples and all the meson properties are determined by light degrees of freedom alone. As a result the heavy quark spin degeneracy of the levels emerges. The light quark's spin couples with its orbital momentum, resulting for S-wave mesons in two degenerate $j=1/2$ states, corresponding to 1^- and 0^-.[1] For P-wave mesons two degenerate $j=3/2$ states $(1^+,2^+)$ and two degenerate $j=1/2$ states $(0^+,1^+)$ arise. The heavy quark symmetry constrains also the strong decays of these states. The decay rates of the P-wave states in degenerate doublets should be the same. The states with $j=1/2$ are expected to be wide, because they decay in an S wave, while $j=3/2$ mesons are narrow, because they decay in a D wave. Since the real b and c quarks are not infinitely heavy, the corrections in inverse powers of the heavy quark mass turn out to be important. These corrections break down the degeneracy of the levels. Heavy quark effective theory (HQET) (see, e.g., [5] and references therein) provides a systematic method for treating $1/m_Q$ corrections. However, in order to obtain quantitative predictions it is necessary to combine it with some dynamical nonperturbative approaches.

Many different approaches have been used for the calculation of orbital and radial excitations of heavy-light mesons [6–9]. However, in almost all of them the expansion in inverse powers not only of the heavy quark mass (m_Q) but also in inverse powers of the light quark mass (m_q) is carried out. The estimates of the light quark velocity in these mesons show that the light quark is highly relativistic $(v/c \sim 0.7-0.8)$. Thus the nonrelativistic approximation is not adequate for the light quark and one cannot guarantee the numerical accuracy of the expansion in inverse powers of the light quark mass. In this paper we use the relativistic quark model [10–12] for the calculation of the masses of orbitally and radially excited B and D mesons without employing the expansion in $1/m_q$. Thus the light quark is treated fully relativistically. Concerning the heavy quark we apply the expansion in $1/m_Q$ up to the first order. Our relativistic quark model is based on the quasipotential approach in quantum field theory. It has been used for the calculation of heavy quarkonia mass spectrum [10] and electroweak decays of heavy mesons [11,12]. Recent nonperturbative investigations indicate that the confining potential cannot be simply scalar. This is in agreement with our model assumptions on the Lorentz structure of quark confinement. The comparison of our model results with model independent constraints of HQET is given in [13].

The paper is organized as follows. In Sec. II we describe our relativistic quark model giving special emphasis to the role of the Lorentz structure of quark confinement. In Sec. III we construct the quasipotential of the interaction of a light quark with a heavy antiquark. We use the heavy quark $1/m_Q$ expansion to simplify the construction. The light quark is treated relativistically. First we consider the $m_Q \to \infty$ limit and then the corrections of the first order in $1/m_Q$. We present the predictions of our model for orbitally and radially excited states of D, D_s, B, and B_s mesons. In Sec. IV we compare our results with the heavy quark symmetry and other quark model predictions as well as available experimental data. Section V contains our conclusions.

II. RELATIVISTIC QUARK MODEL

In the quasipotential approach a meson is described by the wave function of the bound quark-antiquark state, which sat-

*On leave of absence from Russian Academy of Sciences, Scientific Council for Cybernetics, Vavilov Street 40, Moscow 117333, Russia.

[1]We use the J^P notation, where $J=j\pm 1/2$ and P are the total angular momentum and parity of the meson.

isfies the quasipotential equation [14] of the Schrödinger type [15]

$$\left(\frac{b^2(M)}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p})=\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R=\frac{E_qE_Q}{E_q+E_Q}=\frac{M^4-(m_q^2-m_Q^2)^2}{4M^3}, \tag{2}$$

and E_q, E_Q are given by

$$E_q=\frac{M^2-m_Q^2+m_q^2}{2M},\quad E_Q=\frac{M^2-m_q^2+m_Q^2}{2M}, \tag{3}$$

where $M=E_q+E_Q$ is the meson mass, $m_{q,Q}$ are the masses of light and heavy quarks, and $\mathbf{p}$ is their relative momentum. In the center of mass system the relative momentum squared on mass shell reads

$$b^2(M)=\frac{[M^2-(m_q+m_Q)^2][M^2-(m_q-m_Q)^2]}{4M^2}. \tag{4}$$

The kernel $V(\mathbf{p},\mathbf{q};M)$ in Eq. (1) is the quasipotential operator of the quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. An important role in this construction is played by the Lorentz structure of the confining quark-antiquark interaction in the meson. In the literature there is no consent on this item. For a long time the scalar confining kernel has been considered to be the most appropriate one [16]. The main argument in favor of this choice is based on the nature of the heavy quark spin-orbit potential. The scalar potential gives a vanishing long-range magnetic contribution, which is in agreement with the picture of quark confinement of Ref. [17], and allows one to get the fine structure for heavy quarkonia in accord with experimental data. However, the calculations of electroweak decay rates of heavy mesons with a scalar confining potential alone yield results which are in worse agreement with data than for a vector potential [18,19]. The radiative $M1$ transitions in quarkonia such as, e.g., $J/\psi\to\eta_c\gamma$ are the most sensitive for the Lorentz structure of the confining potential. The relativistic corrections for these decays arising from vector and scalar potentials have different signs [18,19]. In particular, as it has been shown in Ref. [19], agreement with experiments for these decays can be achieved only for a mixture of vector and scalar potentials. In this context, it is worth remarking that the recent study of the $q\bar{q}$ interaction in the Wilson loop approach [20] indicates that it cannot be considered as simply a scalar. Moreover, the found structure of spin-independent relativistic corrections is not compatible with a scalar potential. A similar conclusion has been obtained in Ref. [21] on the basis of a Foldy-Wouthuysen reduction of the full Coulomb gauge Hamiltonian of QCD. There, the Lorentz structure of confinement has been found to be of vector nature. The scalar character of spin splittings in heavy quarkonia in this approach is dynamically generated through the interaction with collective gluonic degrees of freedom.

All these new results are in agreement with the assumptions of the relativistic quark model approach [19,10]. Indeed, in constructing the quasipotential of quark-antiquark interaction we have assumed that the effective interaction is the sum of the usual one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli interaction. The quasipotential is then defined by [10]

$$V(\mathbf{p},\mathbf{q};M)=\bar{u}_q(p)\bar{u}_Q(-p)\Big\{\frac{4}{3}\alpha_sD_{\mu\nu}(\mathbf{k})\gamma_q^\mu\gamma_Q^\nu + V_{\rm conf}^V(\mathbf{k})\Gamma_q^\mu\Gamma_{Q;\mu}+V_{\rm conf}^S(\mathbf{k})\Big\}u_q(q)u_Q(-q), \tag{5}$$

where α_s is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge

$$D^{00}(\mathbf{k})=-\frac{4\pi}{\mathbf{k}^2},\quad D^{ij}(\mathbf{k})=-\frac{4\pi}{k^2}\left(\delta^{ij}-\frac{k^ik^j}{\mathbf{k}^2}\right),\quad D^{0i}=D^{i0}=0, \tag{6}$$

and $\mathbf{k}=\mathbf{p}-\mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p)=\sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\begin{pmatrix}1\\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m}\end{pmatrix}\chi^\lambda, \tag{7}$$

with $\epsilon(p)=\sqrt{p^2+m^2}$. The effective long-range vector vertex is given by

$$\Gamma_\mu(\mathbf{k})=\gamma_\mu+\frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu, \tag{8}$$

where κ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V_{\rm conf}^V(r)=(1-\varepsilon)(Ar+B),\qquad V_{\rm conf}^S(r)=\varepsilon(Ar+B), \tag{9}$$

reproducing

$$V_{\rm conf}(r)=V_{\rm conf}^S(r)+V_{\rm conf}^V(r)=Ar+B, \tag{10}$$

where ε is the mixing coefficient.

The expression for the quasipotential for the heavy quarkonia, expanded in v^2/c^2, can be found in Ref. [10]. The structure of the spin-dependent interaction is in agreement with the parametrization of Eichten and Feinberg [22]. All the parameters of our model, such as quark masses, parameters of linear confining potential A and B, mixing coefficient ε and anomalous chromomagnetic quark moment κ, were fixed from the analysis of heavy quarkonia masses [10] and radiative decays [19]. The quark masses $m_b=4.88$ GeV, $m_c=1.55$ GeV, $m_s=0.50$ GeV, $m_{u,d}=0.33$ GeV and the parameters of the linear potential $A=0.18$ GeV^2 and $B=-0.30$ GeV have standard values of quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon=-1$ has been determined from the consideration of the heavy quark expansion [13] and meson radiative de-

cays [19]. Finally, the universal Pauli interaction constant $\kappa=-1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ^3P_J states [10]. Note that the long-range magnetic contribution to the potential in our model is proportional to $(1+\kappa)$ and thus vanishes for the chosen value of $\kappa=-1$.

III. QUASIPOTENTIAL OF THE INTERACTION OF A LIGHT QUARK WITH A HEAVY ANTIQUARK

The expression for the quasipotential (5) can, in principle, be used for arbitrary quark masses. The substitution of the Dirac spinors (7) into (5) results in an extremely nonlocal potential in the configuration space. Clearly, it is very hard to deal with such potentials without any simplifying expansion. Fortunately, in the case of heavy-light mesons, one can carry out (following HQET) the expansion in inverse powers of the heavy quark mass m_Q. The leading terms then follow in the limit $m_Q\to\infty$.

A. Infinitely heavy quark limit

In the limit $m_Q\to\infty$ the heavy quark Dirac spinor has only an upper component. Thus only the zeroth component of Dirac matrices in (5) contributes, and we get

$$V(\mathbf{p},\mathbf{q};M)=\bar{u}_q(p)\Bigg\{-\frac{4}{3}\alpha_s\frac{4\pi}{\mathbf{k}^2}\gamma_q^0+V_{\rm conf}^V(\mathbf{k})\times\left[\gamma_q^0+\frac{\kappa}{2m_q}\gamma_q^0(\boldsymbol{\gamma}\mathbf{k})\right]+V_{\rm conf}^S(\mathbf{k})\Bigg\}u_q(q). \quad (11)$$

The resulting quasipotential is still nonlocal in configuration space. However, taking into account that B and D mesons are weakly bound,[2] we can replace $\epsilon_q(p)\to E_q$ in the Dirac spinors (7). Such simplifying substitution is widely used in quantum electrodynamics [23–25] and introduces only minor corrections of the order of the ratio of the binding energy $\langle V\rangle$ to E_q. This substitution makes the Fourier transformation of the potential (11) local and works well for the confining part of the potential. However, it leads to a fictitious singularity $1/r^3$ at the origin arising from the one-gluon exchange part, which is absent in the initial potential. Note that this singularity is not important if the perturbation theory in $1/m_q$ is used. As we are not going to expand our potential in $1/m_q$, additional analysis is necessary. The explicit one-gluon contribution to (11) is given by

$$\bar{u}_q(p)\left(-\frac{4}{3}\alpha_s\frac{4\pi}{\mathbf{k}^2}\gamma_q^0\right)u_q(q)=\left(-\frac{4}{3}\alpha_s\frac{4\pi}{\mathbf{k}^2}\right)\sqrt{\frac{\epsilon_q(p)+m_q}{2\epsilon_q(p)}}\sqrt{\frac{\epsilon_q(q)+m_q}{2\epsilon_q(q)}}\times\left(1+\frac{\mathbf{pq}+i\boldsymbol{\sigma}_q[\mathbf{pq}]}{[\epsilon_q(p)+m_q][\epsilon_q(q)+m_q]}\right). \quad (12)$$

[2]The sum of constituent quark masses m_Q+m_q is very close to the ground state meson mass M.

The most interesting is the last term in this expression, which comes from the lower components of the spinors u_q. In configuration space, it has a $1/r$ asymptotics for $r\to 0$ and decreases as $1/r^3$ for $r\to\infty$. If we simply replace $\epsilon_q\to E_q$, then we get the asymptotics $1/r^3$, which is correct at infinity but too singular at the origin. To cure this asymptotics, let us notice that if binding effects are taken into account, it is necessary to replace $\epsilon_q\to E_q-V$, where V is the quark interaction potential. If we consider larger distances of the order of the hadron radius, the contributions of V are of the order of the (small) binding energy and can be neglected. But for $r\to 0$ the Coulomb singularity in V becomes important. Thus, we keep this contribution (up to the first order in the binding energy) in the last term in (12). As a result we find that the Coulomb-like one-gluon potential $V_{\rm Coul}(r)=-\frac{4}{3}(\alpha_s/r)$ in this term should be replaced by

$$\widetilde{V}_{\rm Coul}(r)=V_{\rm Coul}(r)\frac{1}{\left(1+\frac{4}{3}\frac{\alpha_s}{E_q}\frac{1}{r}\right)\left(1+\frac{4}{3}\frac{\alpha_s}{E_q+m_q}\frac{1}{r}\right)}, \quad (13)$$

leading to the correct asymptotics both at the origin and infinity.

The resulting local quark-antiquark potential for $m_Q\to\infty$ can be presented in configuration space in the following form:

$$V_{m_Q\to\infty}(r)=\frac{E_q+m_q}{2E_q}\Bigg[V_{\rm Coul}(r)+V_{\rm conf}(r)+\frac{1}{(E_q+m_q)^2}\times\Big\{\mathbf{p}[\widetilde{V}_{\rm Coul}(r)+V_{\rm conf}^V(r)-V_{\rm conf}^S(r)]\mathbf{p}-\Delta V_{\rm conf}^V(r)[1-(1+\kappa)]+\frac{2}{r}(\widetilde{V}'_{\rm Coul}(r)-V'_{\rm conf}{}^S(r)-V'_{\rm conf}{}^V(r)[1-(1+\kappa)])\mathbf{LS}_q\Big\}\Bigg]. \quad (14)$$

Here the prime denotes differentiation with respect to r, $\mathbf{L}$ is the orbital momentum, and $\mathbf{S}_q$ is the spin operator of the light quark. Note that the last term in (14) is of the same order as the first two terms and thus cannot be treated perturbatively.

In the infinitely heavy quark limit the quasipotential equation (1) in configuration space becomes

$$\left(\frac{E_q^2-m_q^2}{2E_q}-\frac{\mathbf{p}^2}{2E_q}\right)\Psi_M(r)=V_{m_Q\to\infty}(r)\Psi_M(r), \quad (15)$$

and the mass of the meson is given by $M=m_Q+E_q$.

Solving (15) numerically [26] we get the eigenvalues E_q and the wave functions Ψ_M. The obtained results are presented in Table I. We see that the heavy quark spin decouples in the limit $m_Q\to\infty$, and thus we get the number of degenerated states in accord with the heavy quark symmetry prediction.

TABLE I. The values of E_q and meson masses M in the limit $m_Q \to \infty$ (in GeV). We use the notation $(n^j L)$ for meson states, where n is the radial quantum number, and j is the total angular momentum of the light quark.

	D		D_s		B		B_s	
State	$E_{u,d}$	M	E_s	M	$E_{u,d}$	M	E_s	M
$1^{1/2}S$	0.497	2.047	0.607	2.157	0.514	5.394	0.628	5.508
$1^{3/2}P$	0.800	2.350	0.913	2.463	0.800	5.680	0.927	5.807
$1^{1/2}P$	0.886	2.436	0.985	2.535	0.898	5.778	0.998	5.878
$2^{1/2}S$	0.940	2.490	1.047	2.597	0.955	5.835	1.063	5.943

B. $1/m_Q$ corrections

The heavy quark symmetry degeneracy of states is broken by $1/m_Q$ corrections. The corrections of order $1/m_Q$ to the potential (14) arise from the lower components of heavy quark Dirac spinors in (5). The matrix elements of the spatial components of Dirac matrices are not zero now. Thus the contributions at first order in $1/m_Q$ come from the one-gluon-exchange potential and the vector confining potential, while the scalar potential gives no contribution at first order. The resulting $1/m_Q$ correction to the heavy quark potential (14) is given by the following expression:

$$\begin{aligned}\delta V_{1/m_Q}(r) = \frac{1}{E_q m_Q}\Bigg\{ & \mathbf{p}[V_{\rm Coul}(r)+V^V_{\rm conf}(r)]\mathbf{p} \\ & + V'_{\rm Coul}(r)\frac{\mathbf{L}^2}{2r} - \frac{1}{4}\Delta V^V_{\rm conf}(r) \\ & + \left[\frac{1}{r}V'_{\rm Coul}(r) + \frac{(1+\kappa)}{r}V'^V_{\rm conf}(r)\right]\mathbf{LS} \\ & + \frac{1}{3}\Bigg(\frac{1}{r}V'_{\rm Coul}(r) - V''_{\rm Coul}(r) + (1+\kappa)^2 \\ & \times\left[\frac{1}{r}V'^V_{\rm conf}(r) - V''^V_{\rm conf}(r)\right]\Bigg) \\ & \times\left[-\mathbf{S}_q\mathbf{S}_Q + \frac{3}{r^2}(\mathbf{S}_q\mathbf{r})(\mathbf{S}_Q\mathbf{r})\right] + \frac{2}{3}[\Delta V_{\rm Coul}(r) \\ & + (1+\kappa)^2\Delta V^V_{\rm conf}(r)]\mathbf{S}_Q\mathbf{S}_q\Bigg\}, \end{aligned} \tag{16}$$

where $\mathbf{S}=\mathbf{S}_q+\mathbf{S}_Q$ is the total spin. The first three terms in (16) represent spin-independent corrections, the fourth term is responsible for the spin-orbit interaction, the fifth one is the tensor interaction, and the last one is the spin-spin interaction. It is necessary to note that the confining vector interaction gives a contribution to the spin-dependent part which is proportional to $(1+\kappa)$ or $(1+\kappa)^2$. Thus it vanishes for the chosen value of $\kappa=-1$, while the confining vector contribution to the spin-independent part is nonzero.

The quasipotential at $1/m_Q$ order is given by the sum of $V_{m_Q\to\infty}(r)$ from (14) and $\delta V_{1/m_Q}(r)$ from (16). By substituting it in the quasipotential equation (1) and treating the $1/m_Q$ correction term $\delta V_{1/m_Q}(r)$ using perturbation theory, we are now able to calculate the mass spectrum of D, D_s, B, and B_s mesons with the account of $1/m_Q$ corrections. The results of our calculations are presented in Tables II–V.

TABLE II. Mass spectrum of D mesons with the account of $1/m_Q$ corrections in comparison with other quark model predictions and experimental data. All masses are given in GeV. We use the notation (nL_J) for meson states, where J is the total angular momentum of the meson.

State	Our	[6]	[8]	Experiment [28,1,2]
$1S_0$	1.875	1.88		1.8645(5)
$1S_1$	2.009	2.04		2.0067(5)
$1P_2$	2.459	2.50	2.460	2.4589(20)
$1P_1$	2.414	2.47	2.415	2.4222(18)
$1P_1$	2.501	2.46	2.585	
$1P_0$	2.438	2.40	2.565	
$2S_0$	2.579	2.58		
$2S_1$	2.629	2.64		2.637(9) ?

IV. RESULTS AND DISCUSSION

Let us compare the obtained results with model independent predictions of HQET. In HQET the heavy-light meson mass M of ground-state pseudoscalar and vector mesons is given by [5]

$$M = \mu_Q + \overline{\Lambda} - \frac{\lambda_1 - 2[J(J+1)-\frac{3}{2}]\lambda_2}{2\mu_Q} + O(1/\mu_Q^2), \tag{17}$$

where the parameter $\overline{\Lambda}$ represents contributions arising from terms in the HQET Lagrangian which are independent of the heavy quark mass. The terms λ_1 and λ_2 parametrize contributions from the kinetic energy and chromomagnetic interaction.

Note that the HQET heavy quark mass μ_Q is in principle different from our mass m_Q for the simple reason that HQET uses the pole heavy quark mass, while quark models use the constituent quark mass. Obviously, in the heavy quark symmetry limit this difference between constituent and pole heavy quark masses, however, disappears. In particular, we

TABLE III. Mass spectrum of D_s mesons with the account of $1/m_Q$ corrections in comparison with other quark model predictions and experimental data. All masses are given in GeV. The same notation as in Table II is used for meson states.

State	Our	[6]	[7]	Experiment [28,1,2]
$1S_0$	1.981	1.98		1.9685(6)
$1S_1$	2.111	2.13		2.1124(7)
$1P_2$	2.560	2.59	2.561	2.5735(17)
$1P_1$	2.515	2.56	2.526	2.535(4)
$1P_1$	2.569	2.55		
$1P_0$	2.508	2.48		
$2S_0$	2.670	2.67		
$2S_1$	2.716	2.73		

TABLE IV. Mass spectrum of B mesons with the account of $1/m_Q$ corrections in comparison with other quark model predictions and experimental data. All masses are given in GeV. The same notation as in Table II is used for meson states.

State	Our	[6]	[8]	[7]	Experiment [28,1,2]
$1S_0$	5.285	5.31			5.2792(18)
$1S_1$	5.324	5.37			5.3248(18)
$1P_2$	5.733	5.80	5.715	5.771	5.730(9)
$1P_1$	5.719	5.78	5.700	5.759	
$1P_1$	5.757	5.78	5.875		
$1P_0$	5.738	5.76	5.870		
$2S_0$	5.883	5.90			
$2S_1$	5.898	5.93			5.90(2) ?

TABLE V. Mass spectrum of B_s mesons with the account of $1/m_Q$ corrections in comparison with other quark model predictions and experimental data. All masses are given in GeV. The same notation as in Table II is used for meson states.

State	Our	[6]	[7]	Experiment [28,1,2]
$1S_0$	5.375	5.39		5.3693(20)
$1S_1$	5.412	5.45		5.416(4) ?
$1P_2$	5.844	5.88	5.861	5.853(15)?
$1P_1$	5.831	5.86	5.849	
$1P_1$	5.859	5.86		
$1P_0$	5.841	5.83		
$2S_0$	5.971	6.27		
$2S_1$	5.984	6.34		

find the relation $E_q=\overline{\Lambda}+O(1/m_Q)$. Thus, the values of E_q listed in Table I are just the values of the HQET parameters $\overline{\Lambda}$.

The structure of $1/m_Q$ corrections in our model is consistent with (17). However, we find that the parameter corresponding to λ_1 in our model contains not only the kinetic energy part but also other terms originating from spin-independent corrections in Eq. (16).[3] A similar result has been found in the relativistic quark model in Ref. [27]. The value of this parameter turns out to be very sensitive to the value of the heavy quark mass. Indeed, assuming that the heavy quark pole mass μ_Q and constituent quark mass m_Q are related by [27]

$$m_Q=\mu_Q-\frac{\lambda^Q}{2\mu_Q}+O(1/\mu_Q^2), \tag{18}$$

we see that only λ_1 is influenced, and it is connected to the quark model value $\lambda_1^{\rm QM}$ by

$$\lambda_1=\lambda_1^{\rm QM}+\lambda^Q. \tag{19}$$

In our model, for constituent quark masses $m_b=4.88$ GeV and $m_c=1.55$ GeV, we find $\lambda_1^{\rm QM}\approx 0.85$ GeV^2 for B mesons and $\lambda_1^{\rm QM}\approx 0.26$ GeV^2 for D mesons. If we require the parameter λ_1 to be equal to the mean value of $-\langle p^2\rangle$ (in our model $\langle p^2\rangle\approx 0.25$ GeV^2 for B and D mesons), then from (18) and (19) we get the heavy quark masses $\mu_b=4.75$ GeV and $\mu_c=1.40$ GeV. These values agree with values of the heavy quark pole masses used in HQET.

The values of the parameter λ_2, which determines the hyperfine splitting, coincide in HQET and quark models. We find $\lambda_2\approx 0.112$ GeV^2 for B mesons and $\lambda_2\approx 0.125$ GeV^2 for D mesons.

Heavy quark symmetry provides relations between excited states of B and D mesons, such as

$$\overline{M}_{B_1}-\overline{M}_{D_1}=\overline{M}_{B_{s1}}-\overline{M}_{D_{s1}}=\overline{M}_B-\overline{M}_D=\overline{M}_{B_s}-\overline{M}_{D_s}$$
$$=m_b-m_c, \tag{20}$$

where $\overline{M}_{B_1}=(3M_{B_1}+5M_{B_2})/8$, $\overline{M}_B=(M_B+3M_{B*})/4$ are appropriate spin averaged P- and S-wave states. We get from Tables II–V the following values of mass splittings:

$$\overline{M}_{B_1}-\overline{M}_{D_1}=3.29\ \text{GeV},$$
$$\overline{M}_{B_{s1}}-\overline{M}_{D_{s1}}=3.30\ \text{GeV},$$
$$\overline{M}_B-\overline{M}_D=3.34\ \text{GeV},$$
$$\overline{M}_{B_s}-\overline{M}_{D_s}=3.33\ \text{GeV}, \tag{21}$$

in agreement with (20). There also arise the following relations between hyperfine splittings of levels:

$$\Delta M_B\equiv M_{B_2}-M_{B_1}=\frac{m_c}{m_b}\Delta M_D\equiv\frac{m_c}{m_b}(M_{D_2}-M_{D_1}), \tag{22}$$

and the same for B_s and D_s mesons as well as for P_1-P_0 states. Our model predictions for these splittings are displayed in Table VI.

In Tables II–V we compare our relativistic quark model results for heavy-light meson masses with the predictions of other quark models of Godfrey and Isgur [6], Isgur [8], Eichten, Hill, and Quigg [7] and experimental data [28,1,2]. All these quark models use the expansion in inverse powers both of the heavy m_Q and light m_q quark masses for the $Q\overline{q}$ interaction potential. In Ref. [6] some relativization of the potential has been put in by hand, such as relativistic smearing of coordinates and replacing the factors $1/m_q$ by $1/\epsilon_q(p)$. However, the resulting potential in this approach accounts only for some of the relativistic effects, while the others, which are of the same order of magnitude, are missing. The considerations of Refs. [8,7] are closely related. The heavy

TABLE VI. Hyperfine splittings of P levels. All values are given in MeV.

States	ΔM_D	$\frac{m_c}{m_b}\Delta M_D$	ΔM_B	ΔM_{D_s}	$\frac{m_c}{m_b}\Delta M_{D_s}$	ΔM_{B_s}
$1P_2-1P_1$	45	14	14	45	14	13
$1P_1-1P_0$	63	20	19	61	19	18

[3]Note that, if we had also used an expansion in inverse powers of the light quark mass, then, in the static limit, we would get $\lambda_1=-\langle p^2\rangle$.

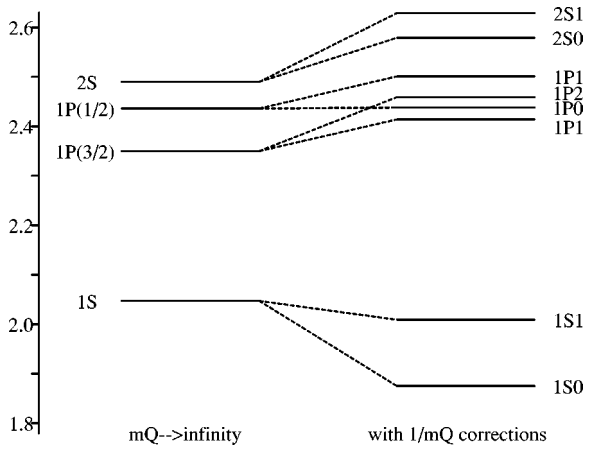

FIG. 1. The ordering pattern of D meson states. The mass scale is in GeV.

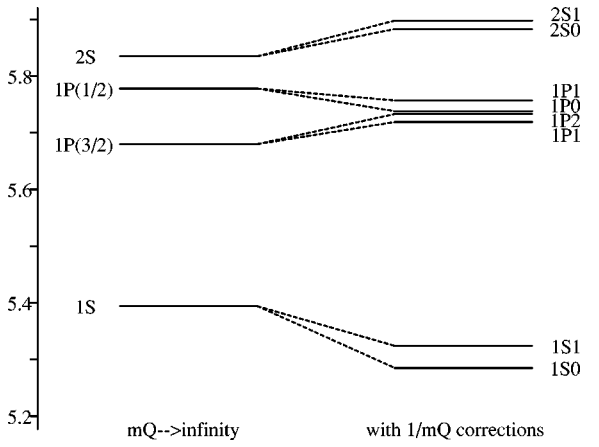

FIG. 3. The ordering pattern of B meson states. The mass scale is in GeV.

quark expansion is extended to light (u,d,s) quarks and the experimental data on P wave masses of K mesons are used to obtain predictions for B and D mesons.

In Ref. [8] it is argued that the heavy quark spin P-wave multiplets with $j=1/2$ $(0^+,1^+)$ and $j=3/2$ $(1^+,2^+)$ in B and D mesons are inverted [29]. The 2^+ and 1^+ states lie about 150 MeV below the 1^+ and 0^+ states. In the limit $m_Q \to \infty$, we find the same inversion of these multiplets in our model, but the gap between $j=1/2$ and $j=3/2$ states is smaller ($\sim$90 MeV for B and D mesons and $\sim$70 MeV for B_s and D_s mesons), and $1/m_Q$ corrections reduce this gap further. However, the hyperfine splittings among the states in these multiplets turn out to be larger than in [8]. As a result, the states from the multiplets for D, D_s, and B_s mesons overlap in our model; however, the heavy quark spin averaged centers are still inverted (see Figs. 1–4). We obtain the following ordering of P states (with masses increasing from left to right): B meson—$1P_1(\frac{3}{2})$, $1P_2$, $1P_0$, $1P_1(\frac{1}{2})$; D_s meson— $1P_0$, $1P_1(\frac{3}{2})$, $1P_2$, $1P_1(\frac{1}{2})$; D and B_s mesons— $1P_1(\frac{3}{2})$, $1P_0$, $1P_2$, $1P_1(\frac{1}{2})$. Thus, only for B meson, we get the purely inverted pattern. Note that the model [6] predicts the ordinary ordering of levels.

The results of our model agree well with available experimental data. The experimental values in Tables II–V for ground state and P-wave masses are taken from Refs. [28,1]. For the radially excited states we use the preliminary data from DELPHI [2].

V. CONCLUSIONS

In this paper we have presented the calculation of the mass spectra of orbitally and radially excited states of heavy-light mesons in the framework of the relativistic quark model. The main advantage of the proposed approach consists of the relativistic treatment of the light quarks (u,d,s). We apply only the expansion in inverse powers of the heavy quark (b,c) mass, which considerably simplifies calculations. The infinitely heavy quark limit as well as the first order $1/m_Q$ corrections are considered. Our model respects the constraints imposed by heavy quark symmetry on the number of levels and different splittings.

We find that the heavy quark spin multiplets with $j=1/2$ $(0^+,1^+)$ and $j=3/2$ $(1^+,2^+)$ are inverted in the $m_Q \to \infty$ limit. This inversion is caused by the following reason: The confining potential contribution to the spin-orbit term in (14) exceeds the one-gluon exchange contribution. Thus the sign before the spin-orbit term is negative, and the level inversion emerges. However, $1/m_Q$ corrections, which produce the hyperfine splittings of these multiplets, are substantial. As a

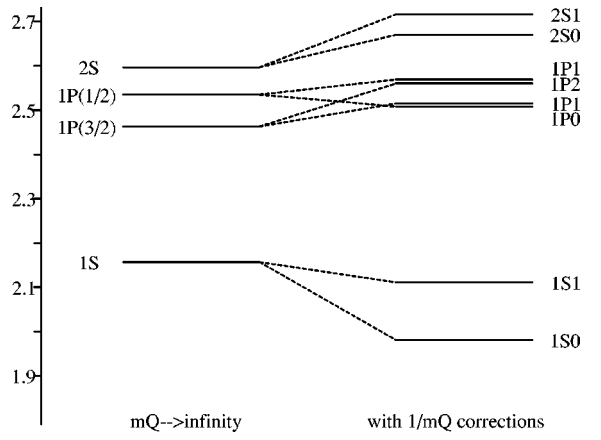

FIG. 2. The ordering pattern of D_s meson states. The mass scale is in GeV.

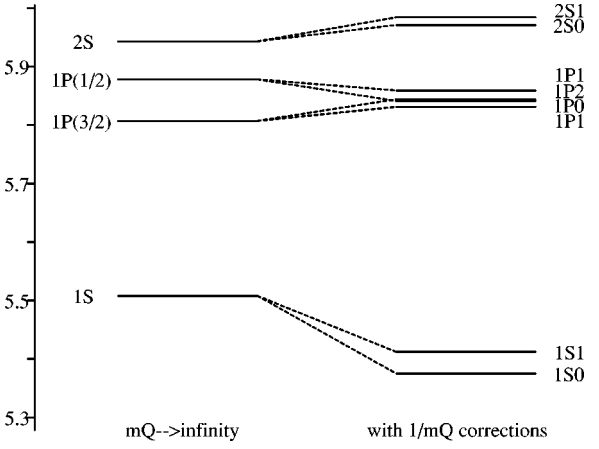

FIG. 4. The ordering pattern of B_s meson states. The mass scale is in GeV.

result the purely inverted pattern of P levels occurs only for the B meson. For D and B_s mesons the levels from these multiplets begin to overlap. This effect is more pronounced in the D_s meson, where the ordinary ordering is restoring. Thus we see that the D_s meson occupies an intermediate position between heavy-light mesons and heavy-heavy mesons (quarkonia, B_c meson). It is necessary to note that both the presence of the relativistic light quark and the light-to-heavy quark mass ratio play an important role in the formation of level ordering patterns. The light quark determines the meson radius, while the m_q/m_Q ratio indicates the validity of the application of the heavy quark symmetry limit.

The found mass values of orbitally and radially excited heavy-light mesons are in good agreement with available experimental data. At present only narrow P-wave $2^+,1^+$ ($j=3/2$) levels of D, D_s and B mesons have been measured. It will be very interesting to observe also $0^+,1^+$ ($j=1/2$) levels, which is more complicated, because these states are expected to be broad. This will allow us to determine the ordering of P levels and to test quark dynamics in a heavy-light meson. We plan to apply the found meson wave functions for the calculation of semileptonic and nonleptonic decays of B mesons into P-wave D and D_s mesons, which are important for the experimental observation of these states.

ACKNOWLEDGMENTS

We are grateful to D. V. Antonov, A. B. Kaidalov, T. Lohse, V.I. Savrin, Yu. A. Simonov, and A. S. Vshivtsev for useful discussions of the results. One of the authors (V.O.G.) gratefully acknowledges the warm hospitality of the colleagues in the particle theory group of the Humboldt University extended to him during his stay there. He was supported in part by the Deutsche Forschungsgemeinschaft under contract Eb 139/1-3 and in part by the Russian Foundation for Fundamental Research under Grant No. 96-02-17171. R.N.F. was supported in part by the Russian Foundation for Fundamental Research under Grant No. 96-02-17171 and in part by Graduiertenkolleg Elementarteilchenphysik.

[1] DELPHI Collaboration, P. Abreu *et al.*, Phys. Lett. B **345**, 598 (1995); ALEPH Collaboration, D. Buskulic *et al.*, Z. Phys. C **73**, 601 (1997); CLEO Collaboration, R. Anastassov *et al.*, hep-ex/9708035, 1997.

[2] DELPHI Collaboration, M. Feindt and O. Podobrin, Report No. DELPHI 96-93 CONF 22 (1996); DELPHI Collaboration, D. Bloch *et al.*, Report No. DELPHI 97-102 CONF 84 (1997).

[3] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990); M. B. Voloshin and M. A. Shifman, Sov. J. Nucl. Phys. **45**, 292 (1987); **47**, 511 (1988).

[4] N. Isgur and M. B. Wise, Phys. Rev. Lett. **66**, 1130 (1991).

[5] M. Neubert, Phys. Rep. **245**, 259 (1994); T. Mannel, Rep. Prog. Phys. **60**, 1113 (1997).

[6] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985); S. Godfrey and R. Kokoski, *ibid.* **43**, 1679 (1991).

[7] E. J. Eichten, C. T. Hill, and C. Quigg, Phys. Rev. Lett. **71**, 4116 (1993); Report No. FERMILAB-CONF-94/118-T (1994).

[8] N. Isgur, Phys. Rev. D **57**, 4041 (1998).

[9] A. F. Falk and T. Mehen, Phys. Rev. D **53**, 231 (1996); S. N. Gupta and J. M. Johnson, *ibid.* **51**, 168 (1995); Y.-B. Dai, C.-S. Huang, M.-Q. Huang, and C. Liu, Phys. Lett. B **390**, 350 (1997).

[10] V. O. Galkin, A. Yu. Mishurov, and R. N. Faustov, Yad. Fiz. **55**, 2175 (1992) [Sov. J. Nucl. Phys. **55**, 1207 (1992)].

[11] R. N. Faustov, V. O. Galkin, and A. Yu. Mishurov, Phys. Lett. B **356**, 516 (1995); **367**, 391(E) (1996); Phys. Rev. D **53**, 6302 (1996); R. N. Faustov and V. O. Galkin, *ibid.* **52**, 5131 (1995).

[12] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D **56**, 312 (1997).

[13] R. N. Faustov and V. O. Galkin, Z. Phys. C **66**, 119 (1995).

[14] A. A. Logunov and A. N. Tavkhelidze, Nuovo Cimento **29**, 380 (1963).

[15] A. P. Martynenko and R. N. Faustov, Teor. Mat. Fiz. **64**, 179 (1985).

[16] H. J. Schnitzer, Phys. Rev. Lett. **35**, 1540 (1975); W. Lucha, F. F. Schöberl, and D. Gromes, Phys. Rep. **200**, 127 (1991); V. D. Mur, V. S. Popov, Yu. A. Simonov, and V. P. Yurov, Zh. Eksp. Teor. Fiz. **78**, 1 (1994) [J. Exp. Theor. Phys. **78**, 1 (1994)]; A. Yu. Dubin, A. B. Kaidalov, and Yu. A. Simonov, Phys. Lett. B **323**, 41 (1994); Yu. A. Simonov, Phys. Usp. **39**, 313 (1996).

[17] W. Buchmüller, Phys. Lett. **112B**, 479 (1982).

[18] R. McClary and N. Byers, Phys. Rev. D **28**, 1692 (1983).

[19] V. O. Galkin and R. N. Faustov, Yad. Fiz. **44**, 1575 (1986) [Sov. J. Nucl. Phys. **44**, 1023 (1986)]; V. O. Galkin, A. Yu. Mishurov, and R. N. Faustov, *ibid.* **51**, 1101 (1990) [*ibid.* **51**, 705 (1990)].

[20] N. Brambilla and A. Vairo, Phys. Lett. B **407**, 167 (1997); Phys. Rev. D **55**, 3974 (1997); M. Baker, J. S. Ball, N. Brambilla, and A. Vairo, Phys. Lett. B **389**, 577 (1996).

[21] A. P. Szczepaniak and E. S. Swanson, Phys. Rev. D **55**, 3987 (1997).

[22] E. Eichten and F. Feinberg, Phys. Rev. D **23**, 2724 (1981).

[23] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer-Verlag, Berlin, 1957).

[24] A. P. Martynenko and R. N. Faustov, Teor. Mat. Fiz. **66**, 399 (1986).

[25] I. B. Khriplovich and A. I. Milstein, Report No. BINP 96-49, hep-ph/9607374, 1996; P. Labelle and G. P. Lepage, Phys. Rev. Lett. **72**, 2006 (1994).

[26] V. O. Galkin and R. N. Faustov, Teor. Mat. Fiz. **85**, 155 (1990) [Theor. Math. Phys. **85**, 1119 (1990)]; R. N. Faustov, V. O. Galkin, A. V. Tatarintsev, and A. S. Vshivtsev, hep-ph/9705421, 1997.

[27] S. Simula, Phys. Lett. B **415**, 273 (1997).

[28] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).

[29] The idea that these multiplets can be inverted was originally proposed in H. J. Schnitzer, Phys. Lett. **76B**, 461 (1978).