Exclusive nonleptonic decays of bottom and charm baryons in a relativistic three-quark model: Evaluation of nonfactorizing diagrams

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Exclusive nonleptonic decays of bottom and charm baryons are studied within a relativistic three-quark model with a Gaussian shape for the momentum dependence of the baryon–three-quark vertex. We include factorizing as well as nonfactorizing contributions to the decay amplitudes. For heavy-to-light transitions $Q \rightarrow qud$ the total contribution of the nonfactorizing diagrams amount up to ~60% of the factorizing contributions in amplitude, and up to ~30% for $b \rightarrow c\bar{u}d$ transitions. We calculate the rates and the polarization asymmetry parameters for various nonleptonic decays and compare them to existing data and to the results of other model calculations. [S0556-2821(98)03807-7]

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I. INTRODUCTION

During the past several years there has been significant progress in the experimental study of nonleptonic decays of heavy baryons [1]. New results on the mass spectrum, lifetimes, branching ratios, and asymmetry parameters in the decays of the heavy baryons Λ_c^+, Σ_c, Ξ_c, Λ_b^0, ... were reported by various experiments by the ALEPH, ARGUS, ACCMOR, CLEO, OPAL Collaborations, etc. The heavy baryon mass spectrum has been determined with good precision (within an accuracy of a few percent). As to nonleptonic branching ratios, the accuracy of the measurements does not exceed 25–30 % even for the better studied Cabibbo-favored decay modes $\Lambda_c^+ \rightarrow \Lambda^0 + \pi^+$ and $\Lambda_c^+ \rightarrow p + \overline{K^0}$. For the decay $\Lambda_b^0 \rightarrow J/\psi\Lambda$ and the Cabibbo-suppressed decay $\Lambda_c^+ \rightarrow p\phi$ the experimental errors are even larger. The first observation of the $\Lambda_c^+ \rightarrow p\phi$ decay was reported by the NA32 Collaboration [2]. They quoted a branching ratio of $B(\Lambda_c^+ \rightarrow p\phi)/B(\Lambda_c^+ \rightarrow pK^-\pi^+) = 0.040 \pm 0.027$. A more recent measurement of the $\Lambda_c^+ \rightarrow p\phi$ decay rate by the CLEO Collaboration resulted in a ratio of branching ratios $B(\Lambda_c^+ \rightarrow p\phi)/B(\Lambda_c^+ \rightarrow pK^-\pi^+) = 0.024 \pm 0.006 \pm 0.003$ [3]. The baryonic decay $\Lambda_b^0 \rightarrow J/\psi\Lambda$ was first observed by the UA1 Collaboration [4]. The measured branching ratio was found to be $B(\Lambda_b^0 \rightarrow J/\psi\Lambda) = (1.4 \pm 0.9)\%$ [1]. The OPAL Collaboration obtained an upper limit for the branching ratio of $B(\Lambda_b^0 \rightarrow J/\psi\Lambda) < 1.1\%$ [5]. Recently the Collider Detector at Fermilab (CDF) Collaboration has reported a much smaller value for the same quantity $B(\Lambda_b^0 \rightarrow J/\psi\Lambda) = (0.037 \pm 0.017 \pm 0.004)\%$ from a larger data sample [6]. From a theoretical point of view the $\Lambda_c^+ \rightarrow p\phi$ and $\Lambda_b^0 \rightarrow J/\psi\Lambda$ decays are simple inasmuch as they are described by factorizing quark diagrams alone. Their study can shed light on the nature of the nonleptonic interactions and may serve as an additional source for determining the Cabibbo-Kobayashi-Maskawa (CKM) elements and the values of the short-distance Wilson coefficients in the effective nonleptonic Lagrangian [7–11]. In the near future one can expect large quantities of new data on exclusive charm and bottom baryon nonleptonic decays which calls for a comprehensive theoretical analysis of these decays.

There exist a number of theoretical analyses of exclusive nonleptonic heavy baryon decays in the literature (see, e.g., Refs. [12–27]) including predictions for their angular decay distributions. The analysis of nonleptonic baryon decays is complicated by the necessity of having to include nonfactorizing contributions. One thus has to go beyond the factorization approximation which had proved quite useful in the analysis of the exclusive nonleptonic decays of heavy mesons. There have been some theoretical attempts to analyze nonleptonic heavy baryon decays using factorizing contributions alone [26], the argument being that W-exchange contributions can be neglected in analogy with the power suppressed W-exchange contributions in the inclusive nonleptonic decays of heavy baryons. One might even be tempted to drop the nonfactorizing contributions on account of the fact that they are superficially proportional to $1/N_c$. However, since N_c baryons contain N_c quarks an extra combinatorial factor proportional to N_c appears in the amplitudes which cancels the explicit diagrammatic $1/N_c$ factor [14,17]. There is now ample empirical evidence in the $c \rightarrow s$ sectors that nonfactorizing diagrams cannot be neglected. For example, in the charm sector the two observed decays $\Lambda_c^+ \to \Xi^0 K^+$ and $\Lambda_c^+ \to \Sigma\pi$ can only proceed via nonfactorizing diagrams. Their sizeable observed branching ratios may thus serve to obtain a measure of the size of the nonfactorizing contributions.

In the present paper both factorizing and nonfactorizing contributions to exclusive nonleptonic decays of bottom and charm baryons are taken into account. The decay amplitudes are studied within a relativistic three-quark model with a Gaussian shape for the momentum dependence of the baryon-three-quark vertex. It is shown that the total contribution of the nonfactorizing diagrams can amount to up to $\sim$60% of the factorizing contribution for heavy-to-light transitions and up to $\sim$30% for $b \to c$ transition in amplitude. We calculate branching ratios and asymmetry parameters for bottom and charm baryon nonleptonic decays within the *Lagrangian spectator model* approach which generalizes the spectator quark model approach [28,29]. We compare our results with existing data and other theoretical approaches.

The layout of the paper is as follows. In Sec. II we present details of our *Lagrangian spectator model* approach. In Sec. III we discuss the calculation of the matrix elements of nonleptonic decays of bottom and charm baryons. In Sec. IV we present the results of our calculations. Section V contains our conclusions.

II. MODEL

A systematic and comprehensive analysis of weak semileptonic and nonleptonic decays of heavy baryons has been carried out within the spectator quark model [14,15,28,29] which is based on the ''equal-velocity'' approximation [28,29]. Namely, it is assumed that all quarks inside a hadron have equal velocities coinciding with the velocity of the hadron. In other words, the internal relative motion of quarks inside the hadrons is neglected.

The quark-hadron Bethe-Salpeter (BS) wave function satisfies the free-quark Dirac equation in each quark index, i.e., the quarks are assumed to be noninteracting. With the use of the ''equal-velocity'' assumption the equations of motion for the wave functions of the individual constituent quarks in the baryon can be rewritten in terms of the hadron velocity, thus imposing restriction on the possible form of the hadronic BS wave function [14,15,28,29]. The explicit form of the BS wave functions for hadron in the initial state is given by

$$J^P = \frac{1^+}{2}: \quad \mathcal{B}_{ABC} = \frac{1}{M}\{[(\not{P}+M)\gamma_5 C]_{\beta\gamma} u_\alpha(P)\mathcal{B}_{a[bc]} + \text{cycl.}(\alpha,a;\beta,b;\gamma,c)\},$$

$$J^P = \frac{3^+}{2}: \quad \mathcal{B}_{ABC} = \frac{1}{M}\{[(\not{P}+M)\gamma_\nu C]_{\beta\gamma} u^\nu_\alpha(P)\mathcal{B}_{\{abc\}} + \text{cycl.}(\alpha,a;\beta,b;\gamma,c)\},$$

$$J^P = 0^{-+}: \quad \mathcal{M}^B_A = [(\not{P}+M)\gamma_5]^\beta_\alpha \mathcal{M}^b_a,$$

$$J^P = 1^{--}: \quad \mathcal{M}^B_A = [(\not{P}+M)\not{\epsilon}]^\beta_\alpha \mathcal{M}^b_a. \tag{1}$$

We have suppressed color indices in (1). M is the mass of hadron, P is its total four-momentum, and $\mathcal{B}_{a[bc]}$, $\mathcal{B}_{\{abc\}}$, $\mathcal{M}^b_a$ denote the flavor part of the hadronic wave function. Analogous formulas for the final state hadronic wave functions can easily be derived from Eq. (1). They can be found in Ref. [14]. Note that the BS spin wave functions (1) contain an additional ''projector'' factor

$$V_+ = \frac{1}{2M}(\not{P}+M) = \frac{\not{v}+1}{2}, \tag{2}$$

where v is the ''on-shell'' four-velocity of hadron, i.e., $v^2 = 1$. The factor V_+ ensures that in the c.m. frame only the positive-energy components of the full BS wave function survive, as it should indeed be, when the quarks are noninteracting.

Once the explicit form of the hadron wave functions is given, the transition matrix elements for weak decays are parametrized by a few overlap integrals in terms of the spin-independent spatial part of the hadron wave functions. Previously, the overlap integrals have been treated as phenomenological parameters to be determined from a fit to experimental data [14].

In order to go beyond the approach [14] one has to develop a microscopic approach to the overlap integrals appearing in the expressions for the decay amplitudes or, equivalently, one has to specify the form of the hadron-quark transition vertex (hadronic BS wave function) including the explicit momentum dependence of the Lorentz scalar part of this vertex. In the Lagrangian model considered in this paper this dependence is given by the baryon form factor which appears in the nonlocal interaction vertex coupling the baryons to the three quarks. The Lagrangian model has been successfully applied to the description of a wide class of the low and intermediate energy hadron phenomena both in the light [30–33] and heavy [34] quark sectors.

In its present form, this model is not immediately applicable to the study of the heavy baryon nonleptonic decays since it does not reproduce the results of the spectator model analysis [14]. The purpose of our present investigation will consist in embedding, step by step, the spectator model spin structure in our Lagrangian approach. Put differently, we attempt to reformulate the spectator model using the Lagrangian language in order to be able to calculate all quantities appearing in the description of the nonleptonic decays of heavy baryons with the use of the Feynman diagram technique.

Let us begin with the formulation of the basic notions of the Lagrangian model taking into account at every step the spin structure imposed by the spectator picture.

The problem of the choice of baryonic currents was discussed in Ref. [34] (see also Refs. [35–39] and [40–42]). Let us briefly review the basic notions. Suppose that a baryon is a bound state of three quarks. Let y_i (i=1,2,3) be the position space four-coordinate of quark i with mass m_i. They are expressed through the center of mass coordinate (x) and the relative Jacobi coordinates ($\xi_1, \ldots$) as

$$y_1 = x - 3\xi_1 \frac{m_2+m_3}{\sum_i m_i},$$

$$y_2 = x + 3\xi_1 \frac{m_1}{\sum_i m_i} - 2\xi_2\sqrt{3}\frac{m_3}{m_2+m_3},$$

$$y_3 = x + 3\xi_1 \frac{m_1}{\sum_i m_i} + 2\xi_2\sqrt{3}\frac{m_2}{m_2+m_3},$$

where $x = \frac{\sum_i m_i y_i}{\sum_i m_i}, \quad \xi_1 = \frac{1}{3}\left(\frac{m_2 y_2 + m_3 y_3}{m_2+m_3} - y_1\right),$

$$\xi_2 = \frac{y_3 - y_2}{2\sqrt{3}}. \tag{3}$$

In the case of light baryons we shall work in the limit of SU(3) invariance by assuming that the masses of u,d, and s quarks are equal to each other in Eq. (3). The breaking of SU(3) symmetry through the position space variables y_i (via a difference of strange m_s and nonstrange m quark masses: $m_s - m \neq 0$) was found to be insignificant [34]. Thus, for light baryons composed of u, d, or s quarks the coordinates of the quarks may be written as

$$y_1 = x - 2\xi_1, \quad y_2 = x + \xi_1 - \xi_2\sqrt{3}, \quad y_3 = x + \xi_1 + \xi_2\sqrt{3}.$$

For a heavy-light baryon with $m_1 \gg m_2$, m_3 one has instead

$$y_1 = y_Q = x, \quad y_2 = y_{q_1} = x + 3\xi_1 - \xi_2\sqrt{3},$$

$$y_3 = y_{q_2} = x + 3\xi_1 + \xi_2\sqrt{3}.$$

We assume that the momentum distribution of the constituents inside a baryon is modelled by an effective relativistic vertex function given by

$$F\left(\frac{\Lambda_B^2}{18}\sum_{i<j}(y_i - y_j)^2\right),$$

TABLE I. Quantum numbers of heavy-light baryons.

Baryon	Quark content	J^P	(S_{qq}, I_{qq})	Mass (GeV)
Λ_c^+	c[ud]	$\frac{1}{2}^+$	(0,0)	2.285
Ξ_c^+	c[us]	$\frac{1}{2}^+$	(0,1/2)	2.470
Ξ_c^0	c[ds]	$\frac{1}{2}^+$	(0,1/2)	2.466
$\Xi_c'^+$	c{us}	$\frac{1}{2}^+$	(1,1/2)	2.470
$\Xi_c'^0$	c{ds}	$\frac{1}{2}^+$	(1,1/2)	2.466
Σ_c^0	c{dd}	$\frac{1}{2}^+$	(1,1)	2.453
Ω_c^0	c{ss}	$\frac{1}{2}^+$	(1,0)	2.704
Λ_b^0	b[ud]	$\frac{1}{2}^+$	(0,0)	5.640
Ξ_b^0	b[us]	$\frac{1}{2}^+$	(0,1/2)	5.800
Ω_b^-	b{ss}	$\frac{1}{2}^+$	(1,0)	6.040

which depends only on the sum of the relative coordinates squared in the coordinate space and on a cutoff parameter Λ_B. Generally speaking, the shape of this function should be determined from the bound state equation and may depend on the flavors of the quarks involved. In order to reduce the number of free parameters we will use a common Gaussian function for all flavors but we allow for flavor dependent values of the cutoff parameter Λ_B. The Gaussian shape guarantees ultraviolet convergence of the matrix elements. The vertex function models the long distance QCD interactions between quarks. For the present application, there are at least three different values for Λ_B corresponding to the (s,d,u), (c,d,u), and (b,d,u) sectors. However, in order to recover the Isgur-Wise symmetry in the heavy quark limit ($m_Q \to \infty$) the cutoff parameter Λ_B has to be the same for charm and bottom baryons.

The Lagrangian describing the interaction of baryons with the three-quark current is written as

$$\mathcal{L}_B^{\rm int}(x) = g_B \overline{B}(x) \int dy_1 \int dy_2 \int dy_3$$
$$\times \delta\left(x - \frac{\sum_i m_i y_i}{\sum_i m_i}\right) F\left(\frac{\Lambda_B^2}{18}\sum_{i<j}(y_i - y_j)^2\right)$$
$$\times J_B(y_1,y_2,y_3) + \text{H.c.}, \tag{4}$$

where $J_B(y_1,y_2,y_3)$ is the three-quark current with quantum numbers of a baryon B:

$$J_B(y_1,y_2,y_3) = \Gamma_1 q^{a_1}(y_1) q^{a_2}(y_2) C \Gamma_2 q^{a_3}(y_3) \varepsilon^{a_1 a_2 a_3}. \tag{5}$$

Here $\Gamma_{1,2}$ are strings of Dirac matrices, $C = \gamma^0\gamma^2$ is the charge conjugation matrix and a_i are the color indices. The strong coupling constant g_B in (4) can be calculated from *the compositeness condition* (see Refs. [34,37–39]), i.e., the renormalization constant of the hadron wave function is set equal to zero, $Z_H=1-g_H^2\Sigma_B'(M_H)=0$, with Σ_H being the hadron mass operator and M_H denotes a hadron mass. Note that the latter condition is equivalent to the well-known relativistic normalization condition for the hadronic Bethe-Salpeter (BS) wave function. However, for technical reasons it is more convenient to use the normalization condition for the elastic vector form factor at zero recoil which, of course, is completely equivalent to the compositeness condition (see discussion in Ref. [34]).

Possible choices of light and heavy-light baryonic currents have been studied in Refs. [35–39] and [40–42]. For the octet of light baryons, for the Λ-type heavy-light baryons (Λ_Q, Ξ_Q) with a light spin zero diquark system, and for the Ω-type heavy-light baryons (Ω_Q, Σ_Q) with a light spin one diquark system the currents are written as follows [34]:

Light baryon currents

$$\begin{aligned}
&\text{vector variant} && J_B^V(y_1,y_2,y_3)=\gamma^\mu\gamma^5 q^{a_1}(y_1)q^{a_2}(y_2)C\gamma_\mu q^{a_3}(y_3)\varepsilon^{a_1a_2a_3},\\
&\text{tensor variant} && J_B^T(y_1,y_2,y_3)=\sigma^{\mu\nu}\gamma^5 q^{a_1}(y_1)q^{a_2}(y_2)C\sigma_{\mu\nu}q^{a_3}(y_3)\varepsilon^{a_1a_2a_3}.
\end{aligned}\tag{6}$$

Heavy-light baryon currents

$$\begin{aligned}
&\text{pseudoscalar variant} && J^P_{\Lambda_Q}=\varepsilon^{abc}Q^a u^b C\gamma^5 d^c,\\
&\text{axial variant} && J^A_{\Lambda_Q}=\varepsilon^{abc}\gamma_\mu Q^a u^b C\gamma^\mu\gamma^5 d^c,\\
&\text{vector variant} && J^V_{\Omega_Q}=\varepsilon^{abc}\gamma_\mu\gamma^5 Q^a s^b C\gamma^\mu s^c,\quad J^{V;\mu}_{\Omega_Q^\star}=\varepsilon^{abc}Q^a s^b C\gamma^\mu s^c,\\
&\text{tensor variant} && J^T_{\Omega_Q}=\varepsilon^{abc}\sigma_{\mu\nu}\gamma_5 Q^a s^b C\sigma^{\mu\nu}s^c,\quad J^{T;\mu}_{\Omega_Q^\star}=-i\varepsilon^{abc}\gamma_\nu Q^a s^b C\sigma^{\mu\nu}s^c.
\end{aligned}\tag{7}$$

In Table I we give the quark content, the quantum numbers (spin-parity J^P, spin S_{qq}, and isospin I_{qq} of light diquark), and the experimental (when available) and theoretical mass spectrum of the heavy baryons [1,15] analyzed in this paper. Square brackets [. . .] and curly brackets { . . . } denote antisymmetric and symmetric flavor and spin combinations of the light degrees of freedom. The masses of the light baryons are taken from the Particle Data Group (PDG) book [1].

Next we write down the Lagrangian which describes the interaction of Λ_Q baryon with quarks in the heavy quark limit ($m_Q\to\infty$), i.e., to leading order in the $1/m_Q$ expansion:

$$\begin{aligned}
\mathcal{L}^{\rm int}_{\Lambda_Q}(x)=&\,g_{\Lambda_Q}\overline{\Lambda}_Q(x)\Gamma_1 Q^a(x)\int d\xi_1\int d\xi_2\\
&\times F(\Lambda^2_{B_Q}\cdot[\xi_1^2+\xi_2^2])u^b(x+3\xi_1-\xi_2\sqrt{3})C\Gamma_2\\
&\times d^c(x+3\xi_1+\xi_2\sqrt{3})\varepsilon^{abc}+\text{H.c.},
\end{aligned}\tag{8}$$

where

$$\Gamma_1\otimes C\Gamma_2=\begin{cases} I\otimes C\gamma^5 & \text{pseudoscalar current,}\\ \gamma_\mu\otimes C\gamma^\mu\gamma^5 & \text{axial vector current.}\end{cases}$$

One can see that the heavy quark is factorized from the light degrees of freedom in this limit. The vertex form factor F characterizes the distribution of u and d quarks inside the Λ_Q baryon. It is readily seen that the Lagrangian (8) exhibits the heavy quark flavor symmetry (symmetry under exchange b with c) if the parameter Λ_{B_Q} is the same for charm and bottom baryons.

In what follows we shall work with the momentum space representation of the interaction Lagrangians. Performing the requisite Fourier transformation, e.g., for the case of the Λ_Q baryon we obtain

$$\begin{aligned}
\mathcal{L}^{\rm int}_{\Lambda_Q}(p)=&\,g_{\Lambda_{B_Q}}\overline{\Lambda}_Q(p)\int dp_1\int dp_2\int dp_3\int dk_1\int dk_2\\
&\times\delta(k_1-3(p_2+p_3))\delta(k_2-\sqrt{3}(p_3-p_2))\\
&\times\delta\left(p-\sum_i p_i\right)F\left(\frac{k_1^2+k_2^2}{\Lambda^2_{B_Q}}\right)\Gamma_1 Q^a(p_1)u^b(p_2)\\
&\times C\Gamma_2 d^c(p_3)\varepsilon^{abc}+\text{H.c.},
\end{aligned}\tag{9}$$

where p and p_1,p_2,p_3 are the momenta of the baryon and the constituent quarks, respectively. The relative momenta k_1 and k_2 may be expressed in terms of the quark momenta p_i in a standard manner [34].

For our purposes we also need the effective Lagrangians that describe the coupling of pions, kaons, and the vector mesons ρ, ϕ, and J/ψ to their quark constituents. In this paper we also assume that the mesons are pointlike objects, i.e., their interaction with the constituent quarks are described by a local nonderivative Lagrangian

$$\begin{aligned}
L_M(p)=&\,g_M M(p)\int dp_1\int dp_2\,\delta(p-p_1-p_2)\\
&\times\overline{q}(p_1)\Gamma_M\lambda_M q(p_2)+\text{H.c.},
\end{aligned}\tag{10}$$

where Γ_M and λ_M are spin and flavor matrices. In other words, we choose the effective meson vertex functions to be constants in momentum space. This is a reliable approximation for the light mesons. For heavy mesons we expect that form factor effects in the meson vertex become important. This prevents us from extending the present approach to cases with heavy mesons in the final states, such as $\Lambda_b^0 \to \Lambda_c^+ + D_s^-$. In general the form factor effects in the decays involving heavy mesons in the final state are expected to suppress their rates relative to those obtained from a pointlike vertex. Exclusive nonleptonic bottom baryon decays involving heavy mesons form the subject of a separate piece of work.

To reproduce the spin amplitude structure of the spectator (or static quark) model analysis [14,15] we assign the projector $V_+=(\not{v}+1)/2$ to each light quark field in the baryon-quark vertex, where v is the ''on-shell'' four-velocity of hadron as in Ref. [14]. The conjugate antiquark fields in the mesons are multiplied by the projector $V_-=(-\not{v}+1)/2$. We shall also use the static approximation for u, d, and s quark propagators

$$\langle 0|T\{q(x)\overline{q(y)}\}|0\rangle=\frac{1}{\Lambda_q}\delta^{(4)}(x-y), \tag{11}$$

where Λ_q is the free parameter having the dimension of mass. We choose this parameter to have the same value Λ for u and d quarks and a different value Λ_s for the strange quark. The model obtained with the use of the above prescriptions will be referred to as the *Lagrangian spectator model* in what follows.

An important property of the Lagrangian spectator model is that the structure of the interaction Lagrangians of light and heavy-light baryons with quarks is simplified. Namely, the different options for the choice of baryon currents all become equivalent. For example, the vector and tensor forms of the interaction Lagrangians of $J^P=1/2^+$ light baryons are completely equivalent. For the proton the interaction Lagrangian takes the form

$$\begin{aligned}
\mathcal{L}_P^{\rm int}(p)&=4g_p\overline{p(p)}\int dk_1\int dk_2 F\left(12\frac{k_1^2+k_2^2+k_1k_2}{\Lambda_{B_q}^2}\right)\\
&\quad\times V_+u^{a_1}(k_1+p)u^{a_2}(k_2)C\gamma_5\\
&\quad\times V_+d^{a_3}(-k_1-k_2)\varepsilon^{a_1a_2a_3}+\text{H.c.}\\
&\equiv 2g_p\overline{p(p)}\int dk_1\int dk_2 F\left(12\frac{k_1^2+k_2^2+k_1k_2}{\Lambda_{B_q}^2}\right)\\
&\quad\times V_+\gamma^\mu\gamma^5 d^{a_1}(k_1+p)u^{a_2}(k_2)C\gamma_\mu\\
&\quad\times V_+u^{a_3}(-k_1-k_2)\varepsilon^{a_1a_2a_3}+\text{H.c.}
\end{aligned} \tag{12}$$

In Appendix A we provide a full list of the effective interaction Lagrangians for light baryons in the Lagrangian spectator model.

In the Lagrangian spectator model the leptonic coupling constants f_π and f_K are determined by the integrals

$$f_\pi=\frac{N_c g_\pi}{4\pi^2}\frac{1}{M_\pi\Lambda^2}\int_{reg}\frac{d^4k}{\pi^2},\qquad f_K=\frac{N_c g_K}{4\pi^2}\frac{1}{M_K\Lambda\Lambda_s}\int_{reg}\frac{d^4k}{\pi^2}. \tag{13}$$

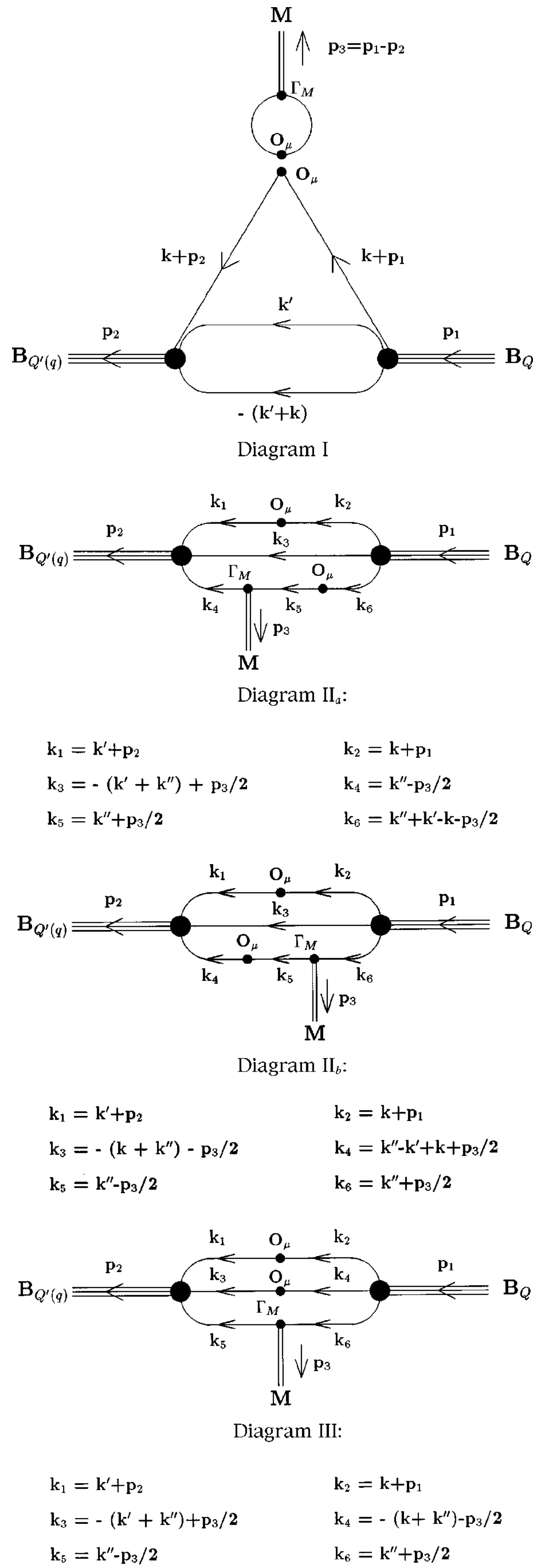

FIG. 1. Diagrams contributing to the matrix element of heavy baryon nonleptonic decay: factorizing diagram (I), nonfactorizing diagrams (II$_a$), (II$_b$), and (III).

The meson coupling constants g_π and g_K Eq. (13) are determined from *the compositeness condition* [34] which reads

$$1=\frac{N_c g_\pi^2}{4\pi^2}\frac{1}{M_\pi^2\Lambda^2}\int_{reg}\frac{d^4k}{\pi^2},\qquad 1=\frac{N_c g_K^2}{4\pi^2}\frac{1}{M_K^2\Lambda\Lambda_s}\int_{reg}\frac{d^4k}{\pi^2}. \tag{14}$$

Equations (13) and (14) contain the ultraviolet divergence since the mesons in our scheme are pointlike objects. To regularize these quantities we introduce an ultraviolet cutoff parameter Λ_{cut}. In order to reduce the number of free parameters in the model we relate the cutoff parameter in Eqs. (13) and (14) to the parameters Λ and Λ_s appearing in static light quark propagator (11) via $\Lambda_{cut}=\Lambda_{q_1}\Lambda_{q_2}/(\Lambda_{q_1}+\Lambda_{q_2})$. Here q_i corresponds to the flavor of the light quark being the constituent. After that we get

$$f_\pi=\frac{\sqrt{N_c}}{8\pi}\Lambda,\qquad f_K=\frac{\sqrt{N_c}}{2\pi}\frac{(\Lambda\Lambda_s)^{3/2}}{(\Lambda+\Lambda_s)^2}. \tag{15}$$

Substituting experimental values for f_π = 131 MeV and f_K = 160 MeV in Eqs. (15) we obtain $\Lambda=1.90$ GeV and $\Lambda_s=3.29$ GeV.

For the heavy quark propagator S_Q we will use the leading term in the inverse mass expansion. Suppose $p=M_{B_Q}v$ is the heavy baryon momentum. We introduce the parameter $\bar\Lambda_{\{q_1q_2\}}=M_{\{Qq_1q_2\}}-m_Q$ which is the difference between the heavy baryon mass $M_{\{Qq_1q_2\}}\equiv M_{B_Q}$ and the heavy quark mass. Keeping in mind that the vertex function falls off sufficiently fast such that the condition $|k|\ll m_Q$ holds (k is the virtual momentum of light quarks) one has

$$S_Q(p+k)=\frac{1}{m_Q-(\not p+\not k)}=\frac{m_Q+M_{B_Q}\not v+\not k}{m_Q^2-M_{B_Q}^2-2M_{B_Q}vk-k^2}$$
$$=S_v(k,\bar\Lambda_{\{q_1q_2\}})+O\left(\frac{1}{m_Q}\right),$$
$$S_v(k,\bar\Lambda_{\{q_1q_2\}})=-\frac{(1+\not v)}{2(v\cdot k+\bar\Lambda_{\{q_1q_2\}})}. \tag{16}$$

In what follows we will assume that $\bar\Lambda\equiv\bar\Lambda_{uu}=\bar\Lambda_{dd}=\bar\Lambda_{du}$, $\bar\Lambda_s\equiv\bar\Lambda_{us}=\bar\Lambda_{ds}$. Thus there are altogether three independent parameters: $\bar\Lambda$, $\bar\Lambda_s$, and $\bar\Lambda_{ss}$.

The vertex function F in the baryon-quark interaction Lagrangians is an arbitrary function except that it should render the Feynman diagrams ultraviolet finite as was mentioned before. In [30–34] it was found that the basic physical observables of pion and nucleon low-energy physics depend only weakly on the choice of the vertex functions. In the present paper we choose a Gaussian vertex function for simplicity. In Minkowski space we write

$$F\left(\frac{k_1^2+k_2^2}{\Lambda_B^2}\right)=\exp\left(\frac{k_1^2+k_2^2}{\Lambda_B^2}\right),$$

where Λ_B is the Gaussian range parameter which is related to the size of a baryon. Note that all calculations are done in the Euclidean region ($k_i^2=-k_{iE}^2$) where the above vertex function decreases very rapidly. We consider two different values of the Λ_B cutoff parameter: Λ_{B_q} for light baryons composed from light (u,d,s) quarks and Λ_{B_Q} for baryons containing a single heavy quark (b or c). The requirement of the unit normalization of the baryonic IW functions $\zeta(\omega)$ and $\xi_1(\omega)$ at zero recoil $\omega=1$ ($\zeta(1)=1$, $\xi_1(1)=1$) imposes the restriction $\Lambda_{B_b}=\Lambda_{B_c}$. This can be seen by expressing the baryonic IW functions for arbitrary values of Λ_{B_Q} as

$$\zeta(\omega)=\frac{\Phi(\sqrt{2\Lambda_{B_b}^2\Lambda_{B_c}^2/(\Lambda_{B_b}^2+\Lambda_{B_c}^2)},\omega)}{\sqrt{\Phi(\Lambda_{B_b},1)}\sqrt{\Phi(\Lambda_{B_c},1)}}, \tag{17}$$

$$\Phi(\Lambda_{B_Q},w)=\Lambda_{B_Q}^6(\omega+1)\int_0^\infty duu\int_0^1 dx\exp\left[-18u^2\right.$$
$$\left.-36u^2x(1-x)(\omega-1)+36u\frac{\bar\Lambda}{\Lambda_{B_Q}}\right].$$

Equation (17) shows that one recovers $\zeta(1)=1$ only when $\Lambda_{B_b}=\Lambda_{B_c}$. As was mentioned above, the parameter $\Lambda_{B_Q}=\Lambda_{B_b}=\Lambda_{B_c}$ is one of the adjustable parameters in our calculation.

Thus there is the following set of adjustable parameters in our model: the cutoff parameters Λ_B (Λ_{B_q} and Λ_{B_Q}), and a set of $\bar\Lambda_{\{q_1q_2\}}$ binding energy parameters: $\bar\Lambda$, $\bar\Lambda_s$, and $\bar\Lambda_{\{ss\}}$.

III. MATRIX ELEMENTS OF WEAK DECAYS OF HEAVY BARYONS

The weak nonleptonic decays of bottom and charm baryons are described by the diagrams I, $\mathrm{II_a}$, $\mathrm{II_b}$, and III in Fig. 1.[1]

Diagram I corresponds to the so-called factorizing contribution. Diagrams $\mathrm{II_a}$, $\mathrm{II_b}$ and III correspond to the nonfactorizing contributions. The vertices O_μ●●O_μ correspond to the nonleptonic interaction described by a standard effective four-fermion Lagrangian, [7–11] For $b\to c\bar u d$ and $c\to s\bar u d$ transitions the effective four-fermion vertices read[2]

[1]In the terminology of [26] diagram I corresponds to factorizable external and internal W emission, $\mathrm{II_a}$ to nonfactorizable internal W emission, and $\mathrm{II_b}$ and III to nonfactorizable W exchange.

[2]We employ the notation

$$\gamma_5=\begin{pmatrix}0 & -I\\ -I & 0\end{pmatrix}.$$

$$\mathcal{L}_{eff}=\frac{G_F}{\sqrt{2}}V_{cb}V^{\dagger}_{ud}[c_1(\overline{c^{a_1}}O_\mu b^{a_1})(\overline{d^{a_2}}O_\mu u^{a_2})$$
$$+c_2(\overline{c^{a_1}}O_\mu b^{a_2})(\overline{d^{a_2}}O_\mu u^{a_1})]$$
$$+\frac{G_F}{\sqrt{2}}V_{cs}V^{\dagger}_{ud}[c_1^\star(\overline{s^{a_1}}O_\mu c^{a_1})(\overline{u^{a_2}}O_\mu d^{a_2})$$
$$+c_2^\star(\overline{s^{a_1}}O_\mu c^{a_2})(\overline{u^{a_2}}O_\mu d^{a_1})]+\text{H.c.},$$
$$O_\mu=\gamma_\mu(1+\gamma_5). \tag{18}$$

Here c_1, c_2 are short distance Wilson coefficients for $b\to c\bar{u}d$ transitions and $c_1^\star, c_2^\star$ are the Wilson coefficients for $c\to s\bar{u}d$ transitions. It is well known that the factorizing contributions are proportional to the following two linear combinations:

$$a_1=c_1+\frac{c_2}{N_c}=c_1+\xi c_2, \tag{19}$$

$$a_2=c_2+\frac{c_1}{N_c}=c_2+\xi c_1 \tag{20}$$

and the same for $a_1^\star$ and $a_2^\star$. Here N_c is the number of colors and $\xi=1/N_c$ is the color singlet projection factor. Phenomenological considerations of the nonleptonic decays of D and B mesons give the following values for the Wilson coefficients:

$$a_1^\star\approx 1.2\pm 0.10\approx c_1,\quad a_2^\star\approx -0.5\pm 0.10\approx c_2\ \ [11],$$
$$a_1\approx 1.05\pm 0.10,\qquad a_2\approx 0.25\pm 0.05$$

(see, e.g., references in [10]).

The phenomenological results for the coefficients $a_{1,2}^\star$ can be seen to correspond to a suppression of the $1/N_c$ term in Eq. (19). A straightforward calculation of these coefficients in the leading logarithmic approximation has been performed in Refs. [7,8,10]. For D-meson decays it was shown that the coefficient $a_1^\star$ is weakly dependent on the choice of the renormalization scheme for fixed values of the renormalization scale and the QCD cutoff parameter: $a_1^\star=1.31\pm 0.19$ (in accordance with phenomenology). In contrast to this the value of $a_2^\star$ strongly depends on the renormalization scheme, ranging from -0.47 ± 0.15 to -0.60 ± 0.22. A detailed discussion can be found in Ref. [10]. A first calculation of the Wilson coefficients a_i for bottom hadron decays was done in Refs. [7,8]. A more refined analysis of the renormalization coefficients within various renormalization schemes can be found in Ref. [10] where it was shown that the value of the coefficient a_1 depends weakly on details of calculations: $a_1=1.01\pm 0.02$ (in accordance with phenomenological analysis). The coefficient a_2 is more sensitive to the choice of the renormalization scheme and ranges from 0.15 ± 0.05 to 0.20 ± 0.05.

The matrix elements describing heavy-to-heavy ($b\to c$) and heavy-to-light ($Q\to q$) transitions can be written as follows.

Heavy-to-heavy transition. *Factorizing contribution*, diagram I:

$$T^{fac}_{B_b\to B_c+M}=\frac{G_F}{\sqrt{2}}V_{cb}V^{\dagger}_{q_1q_2}\chi_\pm\langle B_c|J_\mu^{V+A}|B_b\rangle\cdot\langle M|J_\mu^{V+A}|0\rangle,$$

$$\langle B_c|J_\mu^{V+A}|B_b\rangle=\frac{N_c!g^2_{B_Q}}{(4\pi)^4\Lambda_{q_1}\Lambda_{q_2}}\int\frac{d^4k}{\pi^2 i}\int\frac{d^4k'}{\pi^2 i}$$
$$\times\exp\left[\frac{18k^2+6(2k'+k)^2}{\Lambda^2_{B_Q}}\right]$$
$$\times\overline{u}(v_2)\Gamma_2 S_{v_2}(k,\overline{\Lambda})O^\mu S_{v_1}(k,\overline{\Lambda})\Gamma_1 u(v_1)$$
$$\times\text{Tr}[\Gamma_2'(1+\not{v}_2)(1+\not{v}_1)\Gamma_1']. \tag{21}$$

For the matrix elements of the current operator J_μ^{V+A} sandwiched between one-meson state $\langle M|$ and the vacuum $|0\rangle$ we use the standard definitions:

$$\langle M^P(P_3)|A^\mu|0\rangle=f_P P_3^\mu \qquad \text{for the pseudoscalar mesons,}$$

$$\langle M^V(P_3)|V^\mu|0\rangle=f_V M_3\varepsilon^\mu \qquad \text{for the vector mesons.}$$

Here $\chi_+=a_1$ for transition with a charged meson in the final state and $\chi_-=a_2$ for transition with a neutral meson in the final state. P_3 and M_3 are the four-momentum and the mass of the meson, respectively, f_P is the leptonic decay constant of pseudoscalar meson, and f_V is the decay constant of vector meson into e^+e^- pair. For f_P and f_V we use the experimental values [1]: f_π = 131 MeV, f_K = 160 MeV, f_ϕ = 237 MeV, $f_{J/\psi}$ = 405 MeV.

Nonfactorizing contributions, diagram II_a:

$$T^{II_a}_{B_b\to B_c+M}=\frac{G_F}{\sqrt{2}}V_{cb}V^{\dagger}_{q_1q_2}\frac{N_c!g^2_{B_Q}g_M}{(4\pi)^6\Lambda_{q_1}\cdots\Lambda_{q_4}}\int\frac{d^4k}{\pi^2 i}\int\frac{d^4k'}{\pi^2 i}\int\frac{d^4k''}{\pi^2 i}$$
$$\times\exp\left[\frac{9k^2+9k'^2+3(2k''+2k'-k-p_3)^2+3(2k''+k'-p_3)^2}{\Lambda^2_{B_Q}}\right]$$
$$\times\overline{u}(v_2)\Gamma_2 S_{v_2}(k,\overline{\Lambda})O^\mu S_{v_1}(k,\overline{\Lambda})\Gamma_1 u(v_1)\text{Tr}[\Gamma_2'(1+\not{v}_2)\Gamma_M(1+\not{v}_3)O_\mu(1+\not{v}_1)\Gamma_1']. \tag{22}$$

Here g_M is the meson-quark coupling constant which is calculated with the use of the compositeness condition. The Dirac structure Γ_M specifies the mesonic final state, i.e., $\Gamma_M = i\gamma_5$ for pseudoscalar mesons and $\Gamma_M = \gamma_\mu$ for vector mesons.

Diagram $\mathrm{II_b}$:

$$T^{II_b}_{B_b\to B_c+M} = \frac{G_F}{\sqrt{2}} V_{cb} V^{\dagger}_{q_1q_2} \frac{N_c! g^2_{B_Q} g_M}{(4\pi)^6 \Lambda_{q_1}\cdots\Lambda_{q_4}} \int \frac{d^4k}{\pi^2 i} \int \frac{d^4k'}{\pi^2 i} \int \frac{d^4k''}{\pi^2 i} \times \exp\left[\frac{9k^2+9k'^2+3(2k''+k+p_3)^2+3(2k''+2k-k'+p_3)^2}{\Lambda^2_{B_Q}}\right] \times \overline{u}(v_2)\Gamma_2 S_{v_2}(k,\overline{\Lambda}) O^\mu S_{v_1}(k,\overline{\Lambda})\Gamma_1 u(v_1) \mathrm{Tr}[\Gamma'_2(1+\not{v}_2) O_\mu \Gamma_M (1+\not{v}_3)(1+\not{v}_1)\Gamma'_1]. \tag{23}$$

Diagram III:

$$T^{III}_{B_b\to B_c+M} = \frac{G_F}{\sqrt{2}} V_{cb} V^{\dagger}_{q_1q_2} \frac{N_c! g^2_{B_Q} g_M}{(4\pi)^6 \Lambda_{q_1}\cdots\Lambda_{q_4}} \int \frac{d^4k}{\pi^2 i} \int \frac{d^4k'}{\pi^2 i} \int \frac{d^4k''}{\pi^2 i} \exp\left[\frac{9k^2+9k'^2+3(2k''-k-p_3)^2+3(2k''-k'+p_3)^2}{\Lambda^2_{B_Q}}\right] \times \overline{u}(v_2)\Gamma_2 S_{v_2}(k,\overline{\Lambda}) O^\mu S_{v_1}(k,\overline{\Lambda})\Gamma_1 u(v_1) \mathrm{Tr}[\Gamma'_2(1+\not{v}_2) O_\mu (1+\not{v}_1)\Gamma'_1 \Gamma_M (1+\not{v}_3)]. \tag{24}$$

Heavy-to-light transition. *Factorizing contribution*, diagram I:

$$T^{fac}_{B_Q\to B_q+M} = \frac{G_F}{\sqrt{2}} V_{Qq} V^{\dagger}_{q_1q_2} \chi_\pm \langle B_q | J^{V+A}_\mu | B_Q\rangle \cdot \langle M | J^{V+A}_\mu | 0\rangle,$$

$$\langle B_q | J^{V+A}_\mu | B_Q\rangle = \frac{N_c! g_{B_Q} g_{B_q}}{(4\pi)^4 \Lambda_q \Lambda_{q_1}\Lambda_{q_2}} \int \frac{d^4k}{\pi^2 i} \int \frac{d^4k'}{\pi^2 i} \exp\left[\frac{9k^2+3(2k'+k)^2}{\Lambda^2_{B_Q}}\right] \exp\left[\frac{(3k+2p_2)^2+3(2k'+k)^2}{\Lambda^2_{B_q}}\right] \times \overline{u}(p_2)\Gamma_2 O^\mu S_{v_1}(k,\overline{\Lambda})\Gamma_1 u(v_1) \mathrm{Tr}[\Gamma'_2(1+\not{v}_2)(1+\not{v}_1)\Gamma'_1]. \tag{25}$$

Nonfactorizing contributions, diagram $\mathrm{II_a}$:

$$T^{II_a}_{B_Q\to B_q+M} = \frac{G_F}{\sqrt{2}} V_{Qq} V^{\dagger}_{q_1q_2} \frac{N_c! g_{B_Q} g_{B_q} g_M}{(4\pi)^6 \Lambda_q \Lambda_{q_1}\cdots\Lambda_{q_4}} \int \frac{d^4k}{\pi^2 i} \int \frac{d^4k'}{\pi^2 i} \int \frac{d^4k''}{\pi^2 i} \exp\left[\frac{9k^2+3(2k''+2k'-k-p_3)^2}{\Lambda^2_{B_Q}} + \frac{(3k'+2p_2)^2+3(2k''+k'-p_3)^2}{\Lambda^2_{B_q}}\right] \overline{u}(p_2)\Gamma_2 O^\mu S_{v_1}(k,\overline{\Lambda})\Gamma_1 u(v_1) \mathrm{Tr}[\Gamma'_2(1+\not{v}_2)\Gamma_M(1+\not{v}_3) O_\mu (1+\not{v}_1)\Gamma'_1]. \tag{26}$$

Diagram $\mathrm{II_b}$:

$$T^{II_b}_{B_Q\to B_q+M} = \frac{G_F}{\sqrt{2}} V_{Qq} V^{\dagger}_{q_1q_2} \frac{N_c! g_{B_Q} g_{B_q} g_M}{(4\pi)^6 \Lambda_q \Lambda_{q_1}\cdots\Lambda_{q_4}} \int \frac{d^4k}{\pi^2 i} \int \frac{d^4k'}{\pi^2 i} \int \frac{d^4k''}{\pi^2 i} \exp\left[\frac{9k^2+3(2k''+k+p_3)^2}{\Lambda^2_{B_Q}} + \frac{(3k'+2p_2)^2+3(2k''+2k-k'+p_3)^2}{\Lambda^2_{B_q}}\right] \overline{u}(p_2)\Gamma_2 O^\mu S_{v_1}(k,\overline{\Lambda})\Gamma_1 u(v_1) \times \mathrm{Tr}[\Gamma'_2(1+\not{v}_2) O_\mu \Gamma_M (1+\not{v}_3)(1+\not{v}_1)\Gamma'_1]. \tag{27}$$

Diagram III:

$$T^{III}_{B_Q\to B_q+M}=\frac{G_F}{\sqrt{2}}V_{Qq}V^{\dagger}_{q_1q_2}\frac{N_c!g_{B_Q}g_{B_q}g_M}{(4\pi)^6\Lambda_q\Lambda_{q_1}\cdots\Lambda_{q_4}}\int\frac{d^4k}{\pi^2 i}\int\frac{d^4k'}{\pi^2 i}\int\frac{d^4k''}{\pi^2 i}\exp\left[\frac{9k^2+3(2k''-kp_3)^2}{\Lambda^2_{B_Q}}+\frac{(3k'+2p_2)^2+3(2k''-k'+p_3)^2}{\Lambda^2_{B_q}}\right]\overline{u(p_2)}\Gamma_2O^{\mu}S_{v_1}(k,\overline{\Lambda})\Gamma_1u(v_1)\times\mathrm{Tr}[\Gamma_2'(1+\not v_2)O_{\mu}(1+\not v_1)\Gamma_1'\Gamma_M(1+\not v_3)]. \tag{28}$$

Details of the calculation of the matrix elements (21)–(28) can be found in Appendix B.

Below we list the Lorentz-spinor parts $\overline{u(p_2)}\ldots u(p_1)\mathrm{Tr}[\ldots]$ of the individual diagrams where one has to differentiate between the various possible light diquark transitions. (M_1, M_2, and M_3 denote the masses of the initial and final baryons, and the meson, respectively.)

Scalar-to-scalar diquark transitions

Factorizing diagram (I):

$$(M_1M_2M_3)v_3^{\mu}[\overline{u(}\not v_2+1)O_{\mu}(\not v_1+1)u]\mathrm{Tr}[\gamma_5(\not v_2+1)\times(\not v_1+1)\gamma_5]=8Q_+\overline{u(}M_- -M_+\gamma_5)u|_{M_2/M_1\to 0}\Rightarrow 8M_1^3\overline{u(}1-\gamma_5)u. \tag{29}$$

Diagram II$_\mathrm{a}$:

$$(M_1M_2M_3)[\overline{u(}\not v_2+1)O_{\mu}(\not v_1+1)u]\mathrm{Tr}[\gamma_5(\not v_2+1)\times\gamma_5(\not v_3+1)O_{\mu}(\not v_1+1)\gamma_5]=16M_1\overline{u}[-P_+-\gamma_5Q_+]|_{M_2/M_1\to 0}\Rightarrow 16M_1^3\overline{u(}1-\gamma_5)u. \tag{30}$$

Diagram II$_\mathrm{b}$:

$$-(M_1M_2M_3)[\overline{u(}\not v_2+1)O_{\mu}(\not v_1+1)u]\times\mathrm{Tr}[\gamma_5(\not v_2+1)O_{\mu}\gamma_5(\not v_3+1)(\not v_1+1)\gamma_5]=16M_2\overline{u}[D_+-\gamma_5Q_+]u|_{M_2/M_1\to 0}\Rightarrow 16M_1^2M_2\overline{u(}1-\gamma_5)u. \tag{31}$$

Diagram III:

$$(M_1M_2M_3)[\overline{u(}\not v_2+1)O_{\mu}(\not v_1+1)u]\times\mathrm{Tr}[\gamma_5(\not v_2+1)O_{\mu}(\not v_1+1)\gamma_5\gamma_5(\not v_3+1)]=32(M_1M_2)\sum_{i=1}^{3}M_i\overline{u}\gamma_5u|_{M_2/M_1\to 0}\Rightarrow 32M_1^2M_2\overline{u}\gamma_5u. \tag{32}$$

Vector-to-scalar diquark transitions

Diagram II$_\mathrm{a}$:

$$(M_1M_2M_3)[\overline{u}\gamma^{\beta}\gamma^5(\not v_2+1)O_{\mu}(\not v_1+1)u]\times\mathrm{Tr}[\gamma_{\beta}(\not v_2+1)\gamma_5(\not v_3+1)O_{\mu}(\not v_1+1)\gamma_5]=16M_1\overline{u}[3P_+-\gamma_5Q_+]u|_{M_2/M_1\to 0}\Rightarrow -16M_1^3\overline{u(}3+\gamma_5)u. \tag{33}$$

Diagram II$_\mathrm{b}$:

$$(M_1M_2M_3)[\overline{u}\gamma^{\beta}\gamma^5(\not v_2+1)O_{\mu}(\not v_1+1)u]\times\mathrm{Tr}[\gamma_{\beta}(\not v_2+1)O_{\mu}\gamma_5(\not v_3+1)(\not v_1+1)\gamma_5]=-48M_2\overline{u}[D_+-\gamma_5Q_+]u|_{M_2/M_1\to 0}\Rightarrow -48M_1^2M_2\overline{u(}1-\gamma_5)u. \tag{34}$$

Diagram III:

$$(M_1M_2M_3)[\overline{u}\gamma^{\beta}\gamma^5(\not v_2+1)O_{\mu}(\not v_1+1)u]\times\mathrm{Tr}[\gamma_{\beta}(\not v_2+1)O_{\mu}(\not v_1+1)\gamma_5\gamma_5(\not v_3+1)]=-96(M_1M_2)\sum_{i=1}^{3}M_i\overline{u}\gamma_5u|_{M_2/M_1\to 0}\Rightarrow -96M_1^2M_2\overline{u}\gamma_5u. \tag{35}$$

Vector-to-vector diquark transitions

Factorizing diagram (I):

$$(M_1M_2M_3)v_3^{\mu}[\overline{u}\gamma^{\alpha}\gamma^5(\not v_2+1)O_{\mu}(\not v_1+1)\gamma^{\beta}\gamma^5u]\times\mathrm{Tr}[\gamma_{\alpha}(\not v_2+1)(\not v_1+1)\gamma_{\beta}]=-8Q_+\overline{u(}3M_-+M_+\gamma_5)u|_{M_2/M_1\to 0}\Rightarrow -8M_1^3\overline{u(}3+\gamma_5)u. \tag{36}$$

Diagram $\mathrm{II_a}$:

$$(M_1M_2M_3)[\overline{u}\gamma^\alpha\gamma^5(\not{v}_2+1)O_\mu(\not{v}_1+1)\gamma^\beta\gamma^5u]$$
$$\times\mathrm{Tr}[\gamma_\alpha(\not{v}_2+1)\gamma^5(\not{v}_3+1)O_\mu(\not{v}_1+1)\gamma_\beta]$$
$$=48M_1\overline{u}[3P_+-\gamma_5Q_+]u|_{M_2/M_1\to 0}$$
$$\Rightarrow -48M_1^3\overline{u}(3+\gamma_5)u. \tag{37}$$

Diagram $\mathrm{II_b}$:

$$(M_1M_2M_3)[\overline{u}\gamma^\alpha\gamma^5(\not{v}_2+1)O_\mu(\not{v}_1+1)\gamma^\beta\gamma^5u]$$
$$\times\mathrm{Tr}[\gamma_\alpha(\not{v}_2+1)O_\mu\gamma_5(\not{v}_3+1)(\not{v}_1+1)\gamma_\beta]$$
$$=-48M_2\overline{u}[3D_++\gamma_5Q_+]u|_{M_2/M_1\to 0}$$
$$\Rightarrow -48M_1^2M_2\overline{u}(3+\gamma_5)u. \tag{38}$$

Diagram III:

$$(M_1M_2M_3)[\overline{u}\gamma^\alpha\gamma^5(\not{v}_2+1)O_\mu(\not{v}_1+1)\gamma^\beta\gamma^5u]$$
$$\times\mathrm{Tr}[\gamma_\alpha(\not{v}_2+1)O_\mu(\not{v}_1+1)\gamma_\beta\gamma^5(\not{v}_3+1)]$$
$$=-288(M_1M_2)\sum_{i=1}^{3}M_i\overline{u}\gamma_5u|_{M_2/M_1\to 0}$$
$$\Rightarrow -288M_1^2M_2\overline{u}\gamma_5u, \tag{39}$$

where

$$Q_+=(M_1+M_2)^2-M_3^2,\quad P_+=(M_2+M_3)^2-M_1^2,$$
$$D_+=(M_1+M_3)^2-M_2^2.$$

The relations Eqs. (29)–(39) are in a complete agreement with the result of spectator model analysis [14]. Note also that the contributions arising from the diagrams $\mathrm{II_b}$ and III can be seen to be down by the helicity flip factor (M_2/M_1) in agreement with the result of [14].

The general invariant matrix element describing exclusive weak nonleptonic decays of heavy baryons $1/2^+\to 1/2^+ +0^-$ is given by one

$$M=M_{\mathrm{I}}+M_{\mathrm{II_a}}+M_{\mathrm{II_b}}+M_{\mathrm{III}}\equiv A-\gamma_5B, \tag{40}$$

where the amplitudes M_{I}, $M_{\mathrm{II_a}}$, $M_{\mathrm{II_b}}$, and M_{III} are determined from the diagrams I, $\mathrm{II_a}$, $\mathrm{II_b}$, and III, respectively. Our results are given in the following form.

Factorizing contribution:

Diagram I: $$M_{\mathrm{I}}=c_W\chi_\pm f_P\frac{Q_+}{4M_1M_2}(M_-\ell_{FD}^- - M_+\ell_{FD}^+\cdot\gamma^5)f(M_1,M_2,M_3). \tag{41}$$

Nonfactorizing contributions:

Diagram $\mathrm{II_a}$: $$M_{\mathrm{II_a}}=c_Wc_-\frac{H_2(M_1,M_2,M_3)}{4M_1M_2}(P_+\ell_{II_a}^{P^+} - Q_+\ell_{II_a}^{Q^+}\cdot\gamma^5)M_1, \tag{42}$$

Diagram $\mathrm{II_b}$: $$M_{\mathrm{II_b}}=c_Wc_-\frac{H_2(M_1,M_2,M_3)}{4M_1M_2}(D_+\ell_{II_b}^{D^+} - Q_+\ell_{II_b}^{Q^+}\cdot\gamma^5)M_2, \tag{43}$$

Diagram III: $$M_{\mathrm{III}}=c_Wc_-\frac{H_3(M_1,M_2,M_3)}{4M_1M_2}\times\sum_{i=1}^{3}M_i(M_1M_2)\ell_{III}\cdot\gamma^5. \tag{44}$$

Here, $c_W=G_F/\sqrt{2}V_{QQ'(q)}V_{q_1q_2}^\dagger$, f_P $(P=\pi,\,K)$ are meson leptonic decay constants; $c_-=c_1-c_2$ and $\ell_{FD}^\pm$, $\ell_{II_a}^{P^+}$, $\ell_{II_a}^{Q^+}$, $\ell_{II_b}^{D^+}$, $\ell_{II_b}^{Q^+}$, ℓ_{III} are flavor coefficients whose values are listed in Tables II(a) and II(b). The full list of expressions for the form factors $f(M_1,M_2,M_3)$, $H_2(M_1,M_2,M_3)$, and $H_3(M_1,M_2,M_3)$ appearing in Eqs. (41)–(44) is given below. At the present stage we only give a complete analysis of the Cabibbo-favored nonleptonic decays only for $1/2^+\to 1/2^+ +0^-$ transitions. In addition to these decays we shall also consider the factorizing processes with vector mesons $\Lambda_c^+\to p\phi$ and $\Lambda_b^0\to J/\psi\Lambda$ which were recently measured by the CLEO [3] and CDF [6] Collaborations.

$b\to c$ transitions:

$$f(\omega)=\frac{R(\omega,\overline{\Lambda})}{R(1,\overline{\Lambda})},\quad \omega=\frac{M_1^2+M_2^2-M_3^2}{2M_1M_2},$$

$$H_i(\omega)=d_it_i(r)\frac{R_H(\omega,\overline{\Lambda^i},\overline{\Lambda^f})}{\sqrt{R(1,\overline{\Lambda^i})R(1,\overline{\Lambda^f})}}\frac{8}{9\pi\sqrt{3}}\frac{\Lambda_{B_Q}^4}{\Lambda^3}\quad (i=2,3), \tag{45}$$

where

$$R(\omega,\overline{\Lambda})=\int_0^\infty du\,u\int_0^1 d\alpha\exp\{-18u^2[1+2\alpha(1-\alpha)(\omega-1)]+36u\overline{\Lambda}/\Lambda_{B_Q}\},$$

$$R_H(\omega,\overline{\Lambda^i},\overline{\Lambda^f})=\int_0^\infty du\,u\int_0^1 d\alpha\exp\{-72u^2[1+2\alpha(1-\alpha)(\omega-1)]\}$$
$$\times\exp\{144u(\overline{\Lambda^i}\alpha+\overline{\Lambda^f}(1-\alpha))/\Lambda_{B_Q}-432u^2(\alpha^2+(1-\alpha)^2)\}.$$

TABLE II. (a) Flavor coefficients for heavy-heavy decays ($C\equiv\cos\delta_P$, $S\equiv\sin\delta_P$, $\delta_P=\theta_P-\theta_I$, where $\theta_P=-11°$ is the $\eta-\eta'$ mixing angle). (b) Flavor coefficients for heavy-light decays ($C\equiv\cos\delta_P$, $S\equiv\sin\delta_P$, $\tan_\pm=1\pm\tan\delta_P\cdot r\sqrt{2}$, $\cot_\pm=1\pm\cot\delta_P\cdot r\sqrt{2}$, $\delta_P=\theta_P-\theta_I$, where $\theta_P=-11°$ is the $\eta-\eta'$ mixing angle, $\theta_I=35°$).

(a)

Decay	$\mathcal{C}^-_{FD}$	$\mathcal{C}^+_{FD}$	$\mathcal{C}^{P_+}_{II_a}$	$\mathcal{C}^{Q_+}_{II_a}$	$\mathcal{C}^{D_+}_{II_b}$	$\mathcal{C}^{Q_+}_{II_b}$	$\mathcal{C}_{III}$
$\Lambda_b^0\to\Lambda_c^+\pi^-$	-1	-1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-2
$\Lambda_b^0\to\Sigma_c^+\pi^-$	0	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$-2\sqrt{3}$
$\Lambda_b^0\to\Sigma_c^0\pi^0$	0	0	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$2\sqrt{3}$
$\Lambda_b^0\to\Sigma_c^0\eta$	0	0	$-\frac{\sqrt{3}}{2}S$	$-\frac{1}{2\sqrt{3}}S$	$\frac{\sqrt{3}}{2}S$	$\frac{\sqrt{3}}{2}S$	$2\sqrt{6}S$
$\Lambda_b^0\to\Sigma_c^0\eta'$	0	0	$\frac{\sqrt{3}}{2}C$	$\frac{1}{2\sqrt{3}}C$	$-\frac{\sqrt{3}}{2}C$	$-\frac{\sqrt{3}}{2}C$	$-2\sqrt{6}C$
$\Lambda_b^0\to\Xi_c^0K^0$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	-2
$\Lambda_b^0\to\Xi_c'^0K^0$	0	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2\sqrt{3}}$	0	0	$-2\sqrt{3}$
$\Xi_b^0\to\Xi_c^+\pi^-$	-1	-1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0
$\Xi_b^0\to\Xi_c'^+\pi^-$	0	0	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2\sqrt{3}}$	0	0	0
$\Xi_b^0\to\Xi_c^0\pi^0$	0	0	$\frac{1}{2\sqrt{2}}$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	0
$\Xi_b^0\to\Xi_c^0\eta$	0	0	$\frac{1}{2\sqrt{2}}S$	$-\frac{1}{2\sqrt{2}}S$	$-\frac{1}{2\sqrt{2}}S$	$-\frac{1}{2\sqrt{2}}S$	$-2C$
$\Xi_b^0\to\Xi_c^0\eta'$	0	0	$-\frac{1}{2\sqrt{2}}C$	$\frac{1}{2\sqrt{2}}C$	$\frac{1}{2\sqrt{2}}C$	$\frac{1}{2\sqrt{2}}C$	$-2S$
$\Xi_b^0\to\Xi_c'^0\pi^0$	0	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	0
$\Xi_b^0\to\Xi_c'^0\eta$	0	0	$\frac{\sqrt{3}}{2\sqrt{2}}S$	$\frac{\sqrt{3}}{2\sqrt{2}}S$	$-\frac{\sqrt{3}}{2\sqrt{2}}S$	$-\frac{\sqrt{3}}{2\sqrt{2}}S$	$-2\sqrt{3}C$
$\Xi_b^0\to\Xi_c'^0\eta'$	0	0	$-\frac{\sqrt{3}}{2\sqrt{2}}C$	$-\frac{\sqrt{3}}{2\sqrt{2}}C$	$\frac{\sqrt{3}}{2\sqrt{2}}C$	$\frac{\sqrt{3}}{2\sqrt{2}}C$	$-2\sqrt{3}S$
$\Xi_b^0\to\Lambda_c^+K^-$	0	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	2
$\Xi_b^0\to\Sigma_c^+K^-$	0	0	0	0	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$2\sqrt{3}$
$\Xi_b^0\to\Sigma_c^0\overline{K^0}$	0	0	0	0	0	0	$2\sqrt{6}$
$\Xi_b^0\to\Omega_c^0K^0$	0	0	$-\sqrt{\frac{3}{2}}$	$-\frac{1}{\sqrt{6}}$	0	0	0
$\Xi_b^-\to\Xi_c^0\pi^-$	-1	-1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0
$\Xi_b^-\to\Xi_c'^0\pi^-$	0	0	0	0	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	0
$\Xi_b^-\to\Sigma_c^0K^-$	0	0	0	0	$-\sqrt{\frac{3}{2}}$	$-\sqrt{\frac{3}{2}}$	0
$\Omega_b^-\to\Omega_c^0\pi^-$	-1	$\frac{1}{3}$	0	0	0	0	0

TABLE II. (*Continued.*)

(b)

Decay	ℓ^{-}_{FD}	ℓ^{+}_{FD}	$\ell^{P_+}_{II_a}$	$\ell^{Q_+}_{II_a}$	$\ell^{D_+}_{II_b}$	$\ell^{Q_+}_{II_b}$	ℓ_{III}
$\Lambda_c^+\to\Lambda^0\pi^+$	-1	-1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-2
$\Lambda_c^+\to\Sigma^0\pi^+$	0	0	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$2\sqrt{3}$
$\Lambda_c^+\to\Sigma^+\pi^0$	0	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$-2\sqrt{3}$
$\Lambda_c^+\to\Sigma^+\eta$	0	0	$-\frac{\sqrt{3}}{2}\cdot S\cdot\cot_+$	$-\frac{1}{2\sqrt{3}}\cdot S\cdot\cot_-$	$\frac{\sqrt{3}}{2}\cdot S$	$\frac{\sqrt{3}}{2}\cdot S$	$\sqrt{3}\cdot S$
$\Lambda_c^+\to\Sigma^+\eta'$	0	0	$\frac{\sqrt{3}}{2}\cdot C\cdot\tan_-$	$\frac{1}{2\sqrt{3}}\cdot C\cdot\tan_+$	$-\frac{\sqrt{3}}{2}\cdot C$	$-\frac{\sqrt{3}}{2}\cdot C$	$-\sqrt{3}\cdot C$
$\Lambda_c^+\to p\overline{K^0}$	$\frac{3}{\sqrt{6}}$	$\frac{3}{\sqrt{6}}$	$-\frac{3}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	0	0	0
$\Lambda_c^+\to\Xi^0K^+$	0	0	0	$\frac{2}{\sqrt{6}}$	0	0	$-2\sqrt{6}$
$\Xi_c^+\to\Sigma^+\overline{K^0}$	$\frac{3}{\sqrt{6}}$	$\frac{3}{\sqrt{6}}$	0	0	$\frac{3}{\sqrt{6}}$	$\frac{3}{\sqrt{6}}$	0
$\Xi_c^+\to\Xi^0\pi^+$	$\frac{3}{\sqrt{6}}$	$\frac{3}{\sqrt{6}}$	0	0	$-\frac{3}{\sqrt{6}}$	$-\frac{3}{\sqrt{6}}$	0
$\Xi_c^0\to\Lambda^0\overline{K^0}$	$-\frac{1}{2}$	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	-2
$\Xi_c^0\to\Sigma^0\overline{K^0}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$2\sqrt{3}$
$\Xi_c^0\to\Sigma^+K^-$	0	0	0	$\frac{2}{\sqrt{6}}$	0	0	$-2\sqrt{6}$
$\Xi_c^0\to\Xi^0\pi^0$	0	0	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	0
$\Xi_c^0\to\Xi^0\eta$	0	0	$-\frac{\sqrt{3}}{2}\cdot S\cdot\cot_+$	$\frac{1}{2\sqrt{3}}\cdot S\cdot\cot_-$	$\frac{\sqrt{3}}{2}\cdot S$	$\frac{\sqrt{3}}{2}\cdot S$	$\sqrt{6}\cdot C$
$\Xi_c^0\to\Xi^0\eta'$	0	0	$\frac{\sqrt{3}}{2}\cdot C\cdot\tan_-$	$-\frac{1}{2\sqrt{3}}\cdot C\cdot\tan_+$	$-\frac{\sqrt{3}}{2}\cdot C$	$-\frac{\sqrt{3}}{2}\cdot C$	$\sqrt{6}\cdot S$
$\Xi_c^0\to\Xi^-\pi^+$	$-\frac{3}{\sqrt{6}}$	$-\frac{3}{\sqrt{6}}$	$-\frac{3}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	0	0	0
$\Omega_c^0\to\Xi^0\overline{K^0}$	-1	$\frac{1}{3}$	0	0	-3	1	0
$\Lambda_b^0\to\Lambda^0\pi^0$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	$\sqrt{2}$
$\Lambda_b^0\to pK^-$	$\frac{3}{\sqrt{6}}$	$\frac{3}{\sqrt{6}}$	$\frac{3}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$	0	0	0

Here $d_2=1$ and $d_3=0.5\exp[9M_3^2/2\Lambda_{B_Q}^2]$. The parameters $\overline{\Lambda^i}$ and $\overline{\Lambda^f}$ correspond to initial and final baryons, respectively. The parameters $t_i(r)$, where $r=\Lambda/\Lambda_s$, are given in Table III(a).

It is well known that there are altogether three IW functions $\zeta(\omega)$, $\xi_1(\omega)$, and $\xi_2(\omega)$ describing current induced ground state to ground state transitions. Here $\zeta(\omega)$ describes Λ_Q-type baryon transitions and Ω_Q-type baryon transitions [43,44]. In our approach they are expressed via a single universal function $f(\omega)$:

$$\zeta(\omega)=\xi_1(\omega)=\xi_2(\omega)(1+\omega)=f(\omega)\frac{Q_+}{M_1M_2}$$
$$=f(\omega)\frac{\omega+1}{2},\quad f(1)=1. \tag{46}$$

TABLE III. (a) Factors $t_i(r)$ for heavy-heavy decays ($C \equiv \cos\delta_P$, $S \equiv \sin\delta_P$, $\delta_P = \theta_P - \theta_I$, where $\theta_P = -11^{\mathrm{o}}$ is the $\eta-\eta'$ mixing angle, $\theta_I = 35^{\mathrm{o}}$). (b) Factors $\chi(r)$ and $t_i(r)$ for heavy-light decays.

(a)

Decay	$t_2(r)$	$t_3(r)$
$\Lambda_b^0 \to \Lambda_c^+ \pi^-$	1	1
$\Lambda_b^0 \to \Sigma_c^+ \pi^-$	1	1
$\Lambda_b^0 \to \Sigma_c^0 \pi^0$	1	1
$\Lambda_b^0 \to \Sigma_c^0 \eta$	$r^2/\sqrt{C^2 \cdot r^2 + S^2}$	$r^2/\sqrt{C^2 \cdot r^2 + S^2}$
$\Lambda_b^0 \to \Sigma_c^0 \eta'$	$r^2/\sqrt{S^2 \cdot r^2 + C^2}$	$r^2/\sqrt{S^2 \cdot r^2 + C^2}$
$\Lambda_b^0 \to \Xi_c^0 K^0$	$(1+r)^2/4$	$(1+r)^2/4$
$\Lambda_b^0 \to \Xi_c'^0 K^0$	$(1+r)^2/4$	$(1+r)^2/4$
$\Xi_b^0 \to \Xi_c^+ \pi^-$	1	1
$\Xi_b^0 \to \Xi_c'^+ \pi^-$	1	1
$\Xi_b^0 \to \Xi_c^0 \pi^0$	1	1
$\Xi_b^0 \to \Xi_c^0 \eta$	$r^2/\sqrt{C^2 \cdot r^2 + S^2}$	$r^3/\sqrt{C^2 \cdot r^2 + S^2}$
$\Xi_b^0 \to \Xi_c^0 \eta'$	$r^2/\sqrt{S^2 \cdot r^2 + C^2}$	$r^3/\sqrt{S^2 \cdot r^2 + C^2}$
$\Xi_b^0 \to \Xi_c'^0 \pi^0$	1	1
$\Xi_b^0 \to \Xi_c'^0 \eta$	$r^2/\sqrt{C^2 \cdot r^2 + S^2}$	$r^3/\sqrt{C^2 \cdot r^2 + S^2}$
$\Xi_b^0 \to \Xi_c'^0 \eta'$	$r^2/\sqrt{S^2 \cdot r^2 + C^2}$	$r^3/\sqrt{S^2 \cdot r^2 + C^2}$
$\Xi_b^0 \to \Lambda_c^+ K^-$	$(1+r)^2/4$	$(1+r)^2/4$
$\Xi_b^0 \to \Sigma_c^+ K^-$	$(1+r)^2/4$	$(1+r)^2/4$
$\Xi_b^0 \to \Sigma_c^0 \overline{K^0}$	0	$(1+r)^2/4$
$\Xi_b^0 \to \Omega_c^0 K^0$	$(1+r)^2/4$	0
$\Xi_b^- \to \Xi_c^0 \pi^-$	1	1
$\Xi_b^- \to \Xi_c'^0 \pi^-$	1	1
$\Xi_b^- \to \Sigma_c^0 K^-$	$(1+r)^2/4$	0
$\Omega_b^- \to \Omega_c^0 \pi^-$	1	1

(b)

Decay	χ_f	$t_2(r)$	$t_3(r)$
$\Lambda_c^+ \to \Lambda^0 \pi^+$	$\frac{2}{3r}+\frac{1}{3}$	1	1
$\Lambda_c^+ \to \Sigma^0 \pi^+$	$\frac{2}{3r}+\frac{1}{3}$	1	1
$\Lambda_c^+ \to \Sigma^+ \pi^0$	$\frac{2}{3r}+\frac{1}{3}$	1	1
$\Lambda_c^+ \to \Sigma^+ \eta$	$\frac{2}{3r}+\frac{1}{3}$	1	1
$\Lambda_c^+ \to \Sigma^+ \eta'$	$\frac{2}{3r}+\frac{1}{3}$	1	1
$\Lambda_c^+ \to p \overline{K^0}$	1	$(1+r)^2\sqrt{r}/4$	$(1+r)^2\sqrt{r}/4$
$\Lambda_c^+ \to \Xi^0 K^+$	$\frac{2r}{3}+\frac{1}{3}$	$(1+r)^2\sqrt{r}/4$	$(1+r)^2\sqrt{r}/4$
$\Xi_c^+ \to \Sigma^+ \overline{K^0}$	$\frac{2}{3}+\frac{r}{3}$	$(1+r)^2\sqrt{r}/4$	$(1+r)^2\sqrt{r}/4$
$\Xi_c^+ \to \Xi^0 \pi^+$	$\frac{2}{3}+\frac{1}{3r}$	1	1
$\Xi_c^0 \to \Lambda^0 \overline{K^0}$	$\frac{2}{3}+\frac{r}{3}$	$(1+r)^2\sqrt{r}/4$	$(1+r)^2\sqrt{r}/4$

TABLE III. (*Continued.*)

Decay	$t_2(r)$	$t_3(r)$	
$\Xi_c^0 \to \Sigma^0 \overline{K^0}$	$\frac{2}{3}+\frac{r}{3}$	$(1+r)^2\sqrt{r}/4$	$(1+r)^2\sqrt{r}/4$
$\Xi_c^0 \to \Sigma^+ K^-$	$\frac{2}{3}+\frac{r}{3}$	$(1+r)^2\sqrt{r}/4$	$(1+r)^2\sqrt{r}/4$
$\Xi_c^0 \to \Xi^0 \pi^0$	$\frac{2}{3}+\frac{1}{3r}$	1	1
$\Xi_c^0 \to \Xi^0 \eta$	$\frac{2}{3}+\frac{1}{3r}$	1	r
$\Xi_c^0 \to \Xi^0 \eta'$	$\frac{2}{3}+\frac{1}{3r}$	1	r
$\Xi_c^0 \to \Xi^- \pi^+$	$\frac{2}{3}+\frac{1}{3r}$	1	1
$\Omega_c^0 \to \Xi^0 \overline{K^0}$	$\frac{2r}{3}+\frac{1}{3}$	$(1+r)^2\sqrt{r}/4$	$(1+r)^2\sqrt{r}/4$
$\Lambda_b^0 \to \Lambda^0 \pi^0$	$\frac{2}{3r}+\frac{1}{3}$	1	1
$\Lambda_b^0 \to p K^-$	1	$(1+r)^2\sqrt{r}/4$	$(1+r)^2\sqrt{r}/4$

This result coincides with the prediction of large-N$_c$ QCD [45] and reproduces the result of the spectator quark model [15].

Heavy-light transitions:

$$f(M_1,M_2,M_3) = \frac{R_{FD}(M_1,M_2,M_3,\overline{\Lambda})}{\sqrt{R(1,\overline{\Lambda})}} \frac{8R^2}{(1+R)^3} \frac{1}{\sqrt{\chi(r)}}, \qquad R = \frac{\Lambda_{B_Q}^2}{\Lambda_{B_q}^2}, \tag{47}$$

$$R_{FD}(M_1,M_2,M_3,\overline{\Lambda}) = \int_0^\infty d\alpha \exp\left[-9\alpha^2(1+R) + 18\alpha\frac{\overline{\Lambda}}{\Lambda_{B_Q}}(1+R)\right] \times \exp\left[-12\alpha R\omega\frac{M_2}{\Lambda_{B_Q}} + \frac{4R}{R+1}\frac{M_2^2}{\Lambda_{B_Q}^2}\right],$$

$$H_i(M_1,M_2,M_3,\overline{\Lambda}) = \frac{t_i(r)}{\sqrt{\chi(r)}} \frac{R_{H_i}(M_1,M_2,M_3,\overline{\Lambda})}{(1+R)\sqrt{R(1,\overline{\Lambda})}} \frac{4}{9\pi\sqrt{3}} \frac{\Lambda_{B_Q}^4}{\Lambda^3} \quad (i=2,3),$$

where

TABLE IV. Branching ratios (in %) in nonleptonic decays $1/2^+ \to 1/2^+ + 0^-$ of heavy baryons (heavy-light transitions). Numerical values of CKM elements and Wilson coefficients: $|V_{cs}|=0.975$, $|V_{ud}|=0.975$, $|V_{ub}|=0.0035$, $a_1^\star=1.3$, $a_2^\star=-0.65$.

Process	Körner, Krämer [14]	Xu, Kamal [22]	Cheng, Tseng [24]	Our	Experiment [1]
$\Lambda_c^+ \to \Lambda \pi^+$	0.76	1.67	0.91	0.79	0.79± 0.18
$\Lambda_c^+ \to \Sigma^0 \pi^+$	0.33	0.35	0.74	0.88	0.88± 0.20
$\Lambda_c^+ \to \Sigma^+ \pi^0$	0.33	0.35	0.74	0.88	0.88± 0.22
$\Lambda_c^+ \to \Sigma^+ \eta$	0.16			0.11	0.48± 0.17
$\Lambda_c^+ \to \Sigma^+ \eta'$	1.28			0.12	
$\Lambda_c^+ \to p \overline{K^0}$	2.16	1.24	1.30	2.06	2.2± 0.4
$\Lambda_c^+ \to \Xi^0 K^+$	0.27	0.10		0.31	0.34± 0.09
$\Xi_c^+ \to \Sigma^+ \overline{K^0}$	5.11	0.35	0.67	3.08	
$\Xi_c^+ \to \Xi^0 \pi^+$	2.80	2.66	3.12	4.40	1.2±0.5±0.3
$\Xi_c^0 \to \Lambda \overline{K^0}$	0.11	0.32	0.24	0.42	
$\Xi_c^0 \to \Sigma^0 \overline{K^0}$	1.03	0.08	0.12	0.20	
$\Xi_c^0 \to \Sigma^+ K^-$	0.11	0.11		0.27	
$\Xi_c^0 \to \Xi^0 \pi^0$	0.03	0.49	0.25	0.04	
$\Xi_c^0 \to \Xi^0 \eta$	0.21			0.28	
$\Xi_c^0 \to \Xi^0 \eta'$	0.74			0.31	
$\Xi_c^0 \to \Xi^- \pi^+$	0.91	1.52	1.10	1.22	
$\Omega_c^0 \to \Xi^0 \overline{K^0}$	1.10		0.08	0.02	
$\Lambda_b^0 \to \Lambda \pi^0$				4.92×10^{-5}	
$\Lambda_b^0 \to p K^-$				2.11×10^{-4}	

TABLE V. Asymmetry parameters α in the nonleptonic decays $1/2^+ \to 1/2^+ + 0^-$ of heavy baryons (heavy-light transitions). Numerical values of CKM elements and Wilson coefficients: $|V_{cs}|=0.975$, $|V_{ud}|=0.975$, $|V_{ub}|=0.0035$, $a_1^\star=1.3$, $a_2^\star=-0.65$.

Process	Körner, Krämer [14]	Xu, Kamal [22]	Cheng, Tseng [24]	Our	Experiment [1]
$\Lambda_c^+ \to \Lambda \pi^+$	-0.70	-0.67	-0.95	-0.95	-0.98±0.19
$\Lambda_c^+ \to \Sigma^0 \pi^+$	0.70	0.92	0.78	0.43	
$\Lambda_c^+ \to \Sigma^+ \pi^0$	0.71	0.92	0.78	0.43	-0.45±0.31±0.06
$\Lambda_c^+ \to \Sigma^+ \eta$	0.33			0.55	
$\Lambda_c^+ \to \Sigma^+ \eta'$	-0.45			-0.05	
$\Lambda_c^+ \to p \overline{K^0}$	-1.0	0.51	-0.49	-0.97	
$\Lambda_c^+ \to \Xi^0 K^+$	0	0		0	
$\Xi_c^+ \to \Sigma^+ \overline{K^0}$	-1.0	0.24	-0.09	-0.99	
$\Xi_c^+ \to \Xi^0 \pi^+$	-0.78	-0.81	-0.77	-1.0	
$\Xi_c^0 \to \Lambda \overline{K^0}$	-0.76	1.0	-0.73	-0.75	
$\Xi_c^0 \to \Sigma^0 \overline{K^0}$	-0.96	-0.99	-0.59	-0.55	
$\Xi_c^0 \to \Sigma^+ K^-$	0	0		0	
$\Xi_c^0 \to \Xi^0 \pi^0$	0.92	0.92	-0.54	0.94	
$\Xi_c^0 \to \Xi^0 \eta$	-0.92			-1.0	
$\Xi_c^0 \to \Xi^0 \eta'$	-0.38			-0.32	
$\Xi_c^0 \to \Xi^- \pi^+$	-0.38	-0.38	-0.99	-0.84	
$\Omega_c^0 \to \Xi^0 \overline{K^0}$	0.51		-0.93	-0.81	
$\Lambda_b^0 \to \Lambda \pi^0$				-1.0	
$\Lambda_b^0 \to p K^-$				-0.88	

TABLE VI. Invariant amplitudes in the decay $\Lambda_c^+\to\Lambda\pi^+$ and $\Lambda_c^+\to\Sigma^+\pi^0$ (in units of $G_F V_{cs} V_{ud}\times 10^{-2}$ GeV^2).

Reference	$\Lambda_c^+\to\Lambda\pi^+$		$\Lambda_c^+\to\Sigma^+\pi^0$	
	A	B	A	B
CLEO II [46]	$-3.0^{+0.8}_{-1.2}$	$12.7^{+2.7}_{-2.5}$	$1.3^{+0.9}_{-1.1}$	$-17.3^{+2.3}_{-2.9}$
Xu and Kamal [22]	-2.7	20.8	-2.9	-6.0
Cheng and Tseng [23]	-3.5	13.2	-2.4	-14.6
Körner and Krämer [14]	-1.9	13.9	-1.3	-9.9
Our	-4.2	9.0	-1.2	-17.2

$$R_{H_2}=\int_0^\infty d\alpha \exp\left[36\alpha^2(1+R)(3R+4)+72\alpha\frac{\bar\Lambda}{\Lambda_{B_Q}}(1+R)\right.$$
$$\left.-12\alpha R\omega\frac{M_2}{\Lambda_{B_Q}}+\frac{R}{1+R}\frac{M_2^2}{\Lambda_{B_Q}^2}\right],$$

$$R_{H_3}=\int_0^\infty d\alpha \exp\left[36\alpha^2(1+R)(3R+4)+72\alpha\frac{\bar\Lambda}{\Lambda_{B_Q}}(1+R)\right.$$
$$\left.-12\alpha R\frac{M_2^2+M_3^2}{\Lambda_{B_Q}M_1}\right]\exp\left[\frac{R}{1+R}\frac{M_2^2+6M_3^2}{\Lambda_{B_Q}^2}\right].$$

The parameters $\chi(r)$, $t_2(r)$, and $t_3(r)$ are given in Table III(b). The terms proportional to $(M_1-M_2)/\Lambda_{B_Q}$ in the exponents in Eqs. (45) and (47) have been dropped for physical reasons.

IV. RESULTS

In this section we give our numerical results for the decay rates and the asymmetry parameters in the nonleptonic decays of Λ_Q, Ξ_Q, and Ω_Q baryons. Let us specify the model parameters. Our model contains the following set of parameters: the cutoff parameters Λ_{B_q} and Λ_{B_Q} and the binding energy parameters $(\bar\Lambda,\bar\Lambda_s$ and $\bar\Lambda_{ss})$. Three of the parameters $(\Lambda_{B_q}$, Λ_{B_Q}, and $\bar\Lambda)$ are used to fit known branching ratios of five nonleptonic decays: $\Lambda_c^+\to\Lambda^0\pi^+$, $\Lambda_c^+\to\Sigma^0\pi^+$, $\Lambda_c^+\to\Sigma^+\pi^0$, $\Lambda_c^+\to p\overline{K^0}$, and $\Lambda_c^+\to\Xi^0K^+$. Moreover, in the fit we impose the condition $\rho^2=1$ on the slope of the baryonic Isgur-Wise function. The fit yields the following values for these model parameters: Λ_{B_q}=3.037 GeV, Λ_{B_Q}=2.408 GeV, $\bar\Lambda$=0.9 GeV. One has to remark that the values Λ_{B_q} and Λ_{B_Q} are the phenomenological parameters which in principle are related to the size of a baryon. However their magnitude is not strongly constrained by the experimental values of baryon observables and allows for the variation in a rather wide range. Note that the obtained value $\Lambda_{B_Q}=2.408$ GeV is close to Λ_{B_Q}=2.5 GeV coming from analysis of semileptonic heavy baryon decays in a relativistic three-quark model which uses the constituent quark masses [34]. As to the cutoff parameter in the light-baryon vertex, in Ref. [33] it was demonstrated that the experimental data both for the dimensionless (nucleon magnetic moments) as well as dimensionful (nucleon charge radii) observables can be described successfully, using the value of the parameter Λ_{B_q} from the interval $\sim$ (1–3) GeV provided the constituent quark mass is properly fitted. In particular, for the value Λ_{B_q}=3.037 GeV, with the constituent quark mass m_q=315 MeV, we obtain for the nucleon magnetic moments and charge radii: μ_p=2.62 (experiment 2.79), $\mu_n=-1.61$ (experiment -1.91), $r_p^E=$ 0.82 fm (experiment 0.86 ± 0.01 fm), $\langle r^2\rangle_n^E=-0.188$ fm^2 (experiment -0.119 ± 0.004 fm^2), r_p^M=0.74 fm (experiment 0.86 ± 0.06 fm), r_n^M=0.76 fm (experiment 0.88 ± 0.07 fm). The parameters $\bar\Lambda_s$ and $\bar\Lambda_{ss}$ cannot be determined at present due to the lack of experimental information on the decays of heavy-light baryons containing one or two strange quarks. For the time being we fix them at the values $\bar\Lambda_s$=1 GeV and $\bar\Lambda_{ss}$=1.1 GeV. The masses of hadrons are taken from [1,15]. In what follows we will use the following values for the Cabibbo-Kobayashi-Maskawa matrix elements $V_{qq'}$ [1]:

$$|V_{cb}|=0.04,\quad |V_{ud}|\approx|V_{cs}|=0.975,$$
$$|V_{us}|\approx|V_{cd}|=0.22,\quad |V_{ub}|=0.0035. \tag{48}$$

TABLE VII. Decay rates and asymmetry parameters in heavy-heavy transitions. Numerical values of CKM elements and Wilson coefficients: $|V_{cb}|=0.04$, $|V_{ud}|=0.975$, $a_1=1.03$, $a_2=0.10$.

Process	Γ (in 10^{10} s^{-1})	α	Process	Γ (in 10^{10} s^{-1})	α
$\Lambda_b^0\to\Lambda_c^+\pi^-$	0.382	-0.99	$\Xi_b^0\to\Xi_c'^0\pi^0$	0.014	0.94
$\Lambda_b^0\to\Sigma_c^+\pi^-$	0.039	0.65	$\Xi_b^0\to\Xi_c'^0\eta$	0.015	-0.98
$\Lambda_b^0\to\Sigma_c^0\pi^0$	0.039	0.65	$\Xi_b^0\to\Xi_c'^0\eta'$	0.021	0.97
$\Lambda_b^0\to\Sigma_c^0\eta$	0.023	0.79	$\Xi_b^0\to\Lambda_c^+K^-$	0.010	-0.73
$\Lambda_b^0\to\Sigma_c^0\eta'$	0.029	0.99	$\Xi_b^0\to\Sigma_c^+K^-$	0.030	-0.74
$\Lambda_b^0\to\Xi_c^0K^0$	0.021	-0.81	$\Xi_b^0\to\Sigma_c^0\overline{K^0}$	0.021	0
$\Lambda_b^0\to\Xi_c'^0K^0$	0.032	0.98	$\Xi_b^0\to\Omega_c^0K^0$	0.023	0.65
$\Xi_b^0\to\Xi_c^+\pi^-$	0.479	-1.00	$\Xi_b^-\to\Xi_c^0\pi^-$	0.645	-0.97
$\Xi_b^0\to\Xi_c'^+\pi^-$	0.018	0.61	$\Xi_b^-\to\Xi_c'^0\pi^-$	0.007	-1.00
$\Xi_b^0\to\Xi_c^0\pi^0$	0.002	-0.99	$\Xi_b^-\to\Sigma_c^0K^-$	0.016	-0.98
$\Xi_b^0\to\Xi_c^0\eta$	0.012	-0.86	$\Omega_b^-\to\Omega_c^0\pi^-$	0.352	0.60
$\Xi_b^0\to\Xi_c^0\eta'$	0.003	0.71			

TABLE VIII. Decay $\Lambda_c^+ \to \Lambda \pi^+$: contribution of nonfactorizing diagrams (in % relative to the factorizing contribution).

Amplitude	Diagram			
	II_a	II_b	$\mathrm{II}_a + \mathrm{II}_b$	III
A	-29.8%	-18.5%	-48.3%	
B	-32.4%	-15.9%	-48.3%	-13.9%

The Wilson coefficients are taken to be $a_1 = 1.03$, $a_2 = 0.10$, $a_1^\star = 1.3$, and $a_2^\star = -0.65$.

In order to check on the consistency of our approach, we shall prove that the Isgur-Wise functions ξ_1 and ξ_2 satisfy the model-independent Bjorken-Xu inequalities [47]. As was mentioned in Sec. III the baryonic IW functions $\zeta(\omega)$, $\xi_1(\omega)$, and $\xi_2(\omega)$, corresponding to Λ_Q-type and Ω_Q-type heavy-heavy weak baryon transitions, are expressed via a single universal function $f(\omega)$ [see Eqs. (45) and (46)].

The IW-functions ξ_1 and ξ_2 must satisfy to the two model-independent Bjorken-Xu inequalities in [47]. The first inequality reads

$$1 \geqslant \frac{2+\omega^2}{3}\xi_1^2(\omega) + \frac{(\omega^2-1)^2}{3}\xi_2^2(\omega) + \frac{2}{3}(\omega-\omega^3)\xi_1(\omega)\xi_2(\omega). \tag{49}$$

The inequality (49) implies a second inequality, namely, a model-independent restriction on the slope (radius) of the form factor $\xi_1(\omega)$:

$$\rho_{\xi_1}^2 \equiv -\left.\frac{d\xi_1(\omega)}{d\omega}\right|_{\omega=1} \geqslant \frac{1}{3} - \frac{2}{3}\xi_2(1). \tag{50}$$

From the inequality (49) we find an upper limit for the universal function $f(\omega)$

$$\xi_1(\omega) \leqslant 1 \quad \text{or} \quad f(\omega) \leqslant \sqrt{\frac{2}{1+\omega}}, \tag{51}$$

which we impose as a condition.

From the inequality (50) for the slope of the function $\xi_1(\omega)$ we see that $\rho_{\xi_1}^2 \geqslant 0$. For the choice of model parameters corresponding to *the best fit* the universal function $f(\omega)$ and the slope of the ξ_1 satisfy to the Bjorken-Xu inequalities (49) and (50). In this case the charge radii of the ζ and ξ functions are equal to 0.84. Our form factor function $f(\omega)$ is well approximated by the formula

$$f(\omega) \approx \left[\frac{2}{1+\omega}\right]^{1+0.68/\omega}. \tag{52}$$

In Table IV we present the branching ratios of the decays $\Lambda_c^+ \to \Lambda^0\pi^+$, $\Lambda_c^+ \to \Sigma^0\pi^+$, $\Lambda_c^+ \to \Sigma^+\pi^0$, $\Lambda_c^+ \to p\overline{K^0}$, and $\Lambda_c^+ \to \Xi^0 K^+$ which are described nicely using a three-parameter fit. Our predictions for the other heavy-to-light decay modes are listed in Table IV. In Table V we give the calculated values for the asymmetry parameters in the nonleptonic decays of $1/2^+$ charm and bottom baryons into octet of light baryons and pseudoscalar mesons (pions and kaons). The relevant formulas for the decay rates and the asymmetry parameters in terms of the invariant amplitudes A and B are listed in Ref. [14]. For comparison in Tables IV and V we quote the results predicted by other phenomenological approaches. It is seen that rates of decays which proceed only via the nonfactorizing diagrams are not suppressed. In Table VI we list our predictions for the parity-violating (A) and parity-conserving (B) amplitudes in the decays $\Lambda_c^+ \to \Lambda\pi^+$ and $\Lambda_c^+ \to \Sigma^+\pi^0$ in units of $G_F V_{cs} V_{ud} \times 10^{-2}$ GeV^2.

TABLE IX. Decay $\Lambda_b^0 \to \Lambda_c^+ \pi^-$: contribution of nonfactorizing diagrams (in % relative to the factorizing contribution).

Amplitude	Diagram			
	II_a	II_b	$\mathrm{II}_a + \mathrm{II}_b$	III
A	-13.9%	-6.2%	-20.1%	
B	-14.3%	-5.8%	-20.1%	-8.5%

TABLE X. Predictions for $\Lambda_c^+ \to p\phi$ decay rate for different values of the Wilson coefficient $a_2^\star$.

Ratio of interest	$B(\mathrm{p}\phi)/B(\mathrm{pK}^-\pi^+)$ (in %)
CLEO [3]	$0.024 \pm 0.006 \pm 0.003$
NA32 [2]	0.04 ± 0.03
Körner & Krämer [14]	0.05
Cheng & Tseng [25]	0.016
Datta [27]	0.01
	0.022 ($a_2^\star$=-0.30)
	0.030 ($a_2^\star$=-0.35)
	0.040 ($a_2^\star$=-0.40)
Our	0.050 ($a_2^\star$=-0.45)
	0.062 ($a_2^\star$=-0.50)
	0.075 ($a_2^\star$=-0.55)
	0.090 ($a_2^\star$=-0.60)
	0.105 ($a_2^\star$=-0.65)

TABLE XI. Predictions for $\Lambda_b^0 \to J/\psi\Lambda$ decay rate for different values of the Wilson coefficient a_2.

Ratio of interest	$B(\Lambda_b^0 \to J/\psi\Lambda)$ (in %)
UA1 [1,4]	1.4 ± 0.9
OPAL [5]	< 1.1
CDF [6]	$0.037 \pm 0.017 \pm 0.004$
Cheng & Tseng [25]	0.011
Cheng [26]	0.016
	0.027 ($a_2 = 0.10$)
	0.061 ($a_2 = 0.15$)
Our	0.108 ($a_2 = 0.20$)
	0.169 ($a_2 = 0.25$)
	0.243 ($a_2 = 0.25$)

In Table VII we give the predictions for the rates and the asymmetry parameters in the nonleptonic decays of bottom baryons into charm baryons with the use of the same model parameters. A clear pattern emerges. The dominant rates are into channels with factorizing contributions. Rates which proceed only via nonfactorizing diagrams are small but not negligibly small. The total contribution of the nonfactorizing diagrams can be seen to be destructive. The sum of nonfactorizing contributions amounts up to 30% of the factorizing contribution in amplitude. Using $\tau(\Lambda_b)=(1.14\pm0.08)\times10^{-12}$ s [1] we predict a branching ratio of the mode $\Lambda_b\to\Lambda_c\pi$ to be $(0.44\pm0.003)\%$. If one neglects the nonfactorizing contributions for this mode as was done in [26] one would obtain an enhanced rate of $\Gamma=0.665\times10^{10}$ s^{-1}. The prediction for the asymmetry parameter remains at $\alpha\simeq-1$ and is thus not affected by such an omission.

In Tables VIII and IX we analyze the nonfactorizing contributions to the decay amplitudes for the transitions $\Lambda_c^+\to\Lambda\pi^+$ and $\Lambda_b^0\to\Lambda_c^+\pi^-$. It is seen that the total contribution of the nonfactorizing diagrams are destructive. They can amount to up to $\sim60\%$ of the factorizing contribution in amplitude of heavy-to-light transition and up to $\sim30\%$ of the factorizing contribution in amplitude of $b\to c$ transition. Also we calculate the values for overlap integrals f, H_2, and H_3 for these modes. They turn out to be equal to $f=0.51$, $H_2=43$ MeV, and $H_3=14$ MeV for $\Lambda_c^+\to\Lambda\pi^+$ and $f=0.61$, $H_2=24$ MeV, and $H_3=12$ MeV for $\Lambda_b^0\to\Lambda_c^+\pi^-$. For comparison we quote the results for overlap integrals evaluated for the decay $\Lambda_c^+\to\Lambda\pi^+$ in Ref. [14] : $f=0.34$, $H_2=40$ MeV, and $H_3=-4$ MeV.

In Tables X and XI we present the predictions for the $\Lambda_c^+\to p\phi$ and $\Lambda_b^0\to J/\psi\Lambda$ decays for various values of the a_2 and $a_2^\star$ parameters. As mentioned before these processes are described by the factorizing diagram alone. The corresponding weak hadronic matrix elements in the spectator approximation have a trivial spin structure given by the matrix O_μ. For this reason the asymmetry parameter for these transitions does not depend on the model parameters and can be expressed through the hadron masses

$$\alpha\left(\frac{1}{2}^+\to\frac{1}{2}^++1^-\right)=-\frac{M_1^2-M_2^2-2M_3^2}{\sqrt{Q_+Q_-}+\dfrac{3}{2}M_3^2(Q_++Q_-)}. \tag{53}$$

In particular, the asymmetry parameter in the decay $\Lambda_c^+\to p\phi$ is equal to -0.26 and $\alpha=0.21$ for $\Lambda_b^0\to J/\psi\Lambda$ transition. It is seen that for the accepted value of the Wilson coefficient $a_2=0.10$ our approach gives the prediction for the branching $B(\Lambda_b^0\to J/\psi\Lambda)=0.027$ which is consistent with the recent CDF data $B(\Lambda_b^0\to J/\psi\Lambda)=0.037\pm0.017\pm0.004$ [6]. For the rare decay $\Lambda_c^+\to p\phi$ our approach for the accepted value of the corresponding Wilson coefficient $a_2^\star=-0.65$ yields the branching ratio $B(p\phi)/B(pK^-\pi^+)=0.105$ which overestimates the known experimental data from CLEO [3] and NA32 [2] measurements.

V. CONCLUSION

We have studied the exclusive nonleptonic decays of heavy-light baryons into charm and light baryons. The decay rates and the asymmetry parameters have been calculated. It would be interesting to test our predictions in $b\to c$ transitions in future experiments.

We have shown that rates of decays which proceed only via the nonfactorizing diagrams are suppressed but not completely suppressed for both cases of heavy-to-light and heavy-to-heavy transitions. We have analyzed in detail the nonfactorizing contributions to the decay amplitudes for the transitions $\Lambda_c^+\to\Lambda\pi^+$ and $\Lambda_b^0\to\Lambda_c^+\pi^-$. It was shown that the total contribution of the nonfactorizing diagrams are destructive. They amount to up to $\sim60\%$ of the factorizing contribution in amplitude of heavy-to-light transition and up to $\sim30\%$ of the factorizing contribution in amplitude of $b\to c$ transition. Finally, we give the predictions for the $\Lambda_c^+\to p\phi$ and $\Lambda_b^0\to J/\psi\Lambda$ decays for various values of the a_2 and $a_2^\star$ parameters.

The generalization to the channels $\frac{1}{2}^+\to\frac{1}{2}^++1^-$, $\frac{1}{2}^+\to\frac{3}{2}^++0^-$, and $\frac{1}{2}^+\to\frac{1}{2}^++1^-$ involving the ground state partners of the mesons and baryons in the final state is straightforward and will be treated in a subsequent paper. In this paper we have only discussed the Cabibbo favored decays induced by the transitions $b\to c\bar{u}d$ with a light pseudoscalar meson in the final state. There are also a number of Cabibbo favored decays with heavy mesons in the final state which include the decays induced by the quark transitions $b\to c\bar{c}s$. The treatment of heavy mesons in the final state requires some refinements in our simple Lagrangian spectator model. Again, exclusive nonleptonic heavy baryon decays involving heavy mesons in the final state are the subject of a future publication.

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APPENDIX A: HADRON-QUARK INTERACTION LAGRANGIANS

Below we present a complete list of hadronic interaction Lagrangians used in the calculations. We start from the consideration of various possible couplings of three quarks in the light baryons. It is well known that there are five possible nonderivative forms of such coupling for octet baryons [38]

$$\begin{array}{ll}
\text{Pseudoscalar variant} & \bar{B}^{km_1} q^{a_1}_{m_1} q^{a_2}_{m_2} C\gamma^5 q^{a_3}_{m_3} \varepsilon^{a_1a_2a_3}\varepsilon^{km_2m_3} \\
\text{Scalar variant} & \bar{B}^{km_1}\gamma^5 q^{a_1}_{m_1} q^{a_2}_{m_2} C q^{a_3}_{m_3} \varepsilon^{a_1a_2a_3}\varepsilon^{km_2m_3} \\
\text{Axial vector variant} & \bar{B}^{km_1}\gamma^\mu q^{a_1}_{m_1} q^{a_2}_{m_2} C\gamma_\mu\gamma_5 q^{a_3}_{m_3} \varepsilon^{a_1a_2a_3}\varepsilon^{km_2m_3} \\
\text{Vector variant} & \bar{B}^{km}\lambda_i^{mm_1}\gamma^\mu\gamma^5 q^{a_1}_{m_1} q^{a_2}_{m_2}\lambda_i^{nm_3} C\gamma_\mu q^{a_3}_{m_3} \varepsilon^{a_1a_2a_3}\varepsilon^{km_2n} \\
\text{Tensor variant} & \bar{B}^{km}\lambda_i^{mm_1}\sigma^{\mu\nu}\gamma^5 q^{a_1}_{m_1} q^{a_2}_{m_2}\lambda_i^{nm_3} C\sigma_{\mu\nu} q^{a_3}_{m_3} \varepsilon^{a_1a_2a_3}\varepsilon^{km_2n},
\end{array} \tag{A1}$$

where $\bar{B}^{km}$ is the baryonic octet matrix

$$\bar{B}^{km} = \begin{pmatrix} \bar{\Sigma}^0/\sqrt{2}+\bar{\Lambda}^0/\sqrt{6} & \bar{\Sigma}^- & -\bar{\Xi}^- \\ \bar{\Sigma}^+ & -\bar{\Sigma}^0/\sqrt{2}+\bar{\Lambda}^0/\sqrt{6} & \bar{\Xi}^0 \\ \bar{p} & \bar{n} & -2\bar{\Lambda}^0/\sqrt{6} \end{pmatrix}. \tag{A2}$$

It is well known [37,38] that these five forms can be combined in the two linearly independent SU(3) invariant combinations called *vector variant* and *tensor variant* [see Eqs. (6)].

In order to reproduce the results of the spectator model in the Lagrangian formulation, one has to modify the baryonic currents writing them in terms of the ''projected'' quark fields, replacing $q \to V_+ q$, where $V_+ = 1/2(\not{v}+1)$ is the projector introduced is Sec. II. With the use of the ''on-shell'' conditions $\bar{B}V_+ = \bar{B}$ and $v^2=1$ it is easy to verify that there exist simple relations between various interaction Lagrangians obtained from Eq. (A1) via the substitution $q \to V_+ q$:

$$\bar{B}^{km_1} V_+ q^{a_1}_{m_1} q^{a_2}_{m_2} C\gamma^5 V_+ q^{a_3}_{m_3} \varepsilon^{a_1a_2a_3}\varepsilon^{km_2m_3}$$
$$= -\bar{B}^{km_1}\gamma^\mu V_+ q^{a_1}_{m_1} q^{a_2}_{m_2} C\gamma_\mu\gamma_5 V_+ q^{a_3}_{m_3} \varepsilon^{a_1a_2a_3}\varepsilon^{km_2m_3},$$
$$\bar{B}^{km_1}\gamma_5 V_+ q^{a_1}_{m_1} q^{a_2}_{m_2} C V_+ q^{a_3}_{m_3} \varepsilon^{a_1a_2a_3}\varepsilon^{km_2m_3} = 0,$$

$$\bar{B}^{km}\lambda_i^{mm_1}\gamma^\mu\gamma^5 V_+ q^{a_1}_{m_1} q^{a_2}_{m_2}\lambda_i^{nm_3} C\gamma_\mu V_+ q^{a_3}_{m_3} \varepsilon^{a_1a_2a_3}\varepsilon^{km_2n}$$
$$= \frac{1}{2}\bar{B}^{km}\lambda_i^{mm_1}\sigma^{\mu\nu}\gamma^5 V_+ q^{a_1}_{m_1} q^{a_2}_{m_2}\lambda_i^{nm_3} C\sigma_{\mu\nu} \times V_+ q^{a_3}_{m_3} \varepsilon^{a_1a_2a_3}\varepsilon^{km_2n}. \tag{A3}$$

Since the *vector* and *tensor* Lagrangians (6) are completely equivalent to each other on the baryon mass shell one can start with either of them. Note that the *vector* and *pseudoscalar* forms of interaction Lagrangians transform into each other under Fierz transformations (on baryon mass shell)

$$[\bar{B}^{\alpha_1} V_+^{\alpha_1\alpha_2} q^{\alpha_2}] \otimes [q^{\alpha_3}(C\gamma_5 V_+)^{\alpha_3\alpha_4} q^{\alpha_4}]$$
$$= \frac{1}{2}\{[\bar{B}^{\alpha_1} V_+^{\alpha_1\alpha_4} q^{\alpha_4}] \otimes [q^{\alpha_3}(C\gamma_5 V_+)^{\alpha_3\alpha_2} q^{\alpha_2}]$$
$$+ [\bar{B}^{\alpha_1}(\gamma^\mu\gamma^5 V_+)^{\alpha_1\alpha_4} q^{\alpha_4}] \otimes [q^{\alpha_3}(C\gamma_\mu V_+)^{\alpha_3\alpha_2} q^{\alpha_2}]\}. \tag{A4}$$

Here (α_i) denote the spinor indices.

For SU(3) octet of light baryons the interaction Lagrangians are listed in Table XII. The interaction

TABLE XII. Light baryon Lagrangians.

Baryon	Lagrangian
p	$g_p\bar{p}\gamma^\mu\gamma^5 V_+ d^{a_1}u^{a_2} C\gamma_\mu V_+ u^{a_3}\varepsilon^{a_1a_2a_3}$ $= 2g_p\bar{p}V_+ u^{a_1}u^{a_2} C\gamma_5 V_+ d^{a_3}\varepsilon^{a_1a_2a_3}$
n	$-g_n\bar{n}\gamma^\mu\gamma^5 V_+ u^{a_1}d^{a_2} C\gamma_\mu V_+ d^{a_3}\varepsilon^{a_1a_2a_3}$ $= -2g_n\bar{n}V_+ d^{a_1}d^{a_2} C\gamma_5 V_+ u^{a_3}\varepsilon^{a_1a_2a_3}$
Σ^+	$-g_{\Sigma^+}\bar{\Sigma}^+\gamma^\mu\gamma^5 V_+ s^{a_1}u^{a_2} C\gamma_\mu V_+ u^{a_3}\varepsilon^{a_1a_2a_3}$ $= -2g_{\Sigma^+}\bar{\Sigma}^+ V_+ u^{a_1}u^{a_2} C\gamma_5 V_+ s^{a_3}\varepsilon^{a_1a_2a_3}$
Σ^-	$g_{\Sigma^-}\bar{\Sigma}^-\gamma^\mu\gamma^5 V_+ s^{a_1}d^{a_2} C\gamma_\mu V_+ d^{a_3}\varepsilon^{a_1a_2a_3}$ $= 2g_{\Sigma^-}\bar{\Sigma}^- V_+ d^{a_1}d^{a_2} C\gamma_5 V_+ s^{a_3}\varepsilon^{a_1a_2a_3}$
Σ^0	$\sqrt{\tfrac{1}{2}}g_{\Sigma^0}\bar{\Sigma}^0\gamma^\mu\gamma^5 V_+ s^{a_1}(u^{a_2} C\gamma_\mu V_+ d^{a_3} + d^{a_2} C\gamma_\mu V_+ u^{a_3})\varepsilon^{a_1a_2a_3}$ $= \sqrt{2}g_{\Sigma^0}\bar{\Sigma}^0 V_+(d^{a_1}u^{a_2} C\gamma_5 V_+ s^{a_3} + u^{a_1}d^{a_2} C\gamma_5 V_+ s^{a_3})\varepsilon^{a_1a_2a_3}$
Λ^0	$\sqrt{\tfrac{2}{3}}g_{\Lambda^0}\bar{\Lambda}^0\gamma^\mu\gamma^5 V_+(u^{a_1}d^{a_2} C\gamma_\mu V_+ s^{a_3} - d^{a_1}u^{a_2} C\gamma_\mu V_+ s^{a_3})\varepsilon^{a_1a_2a_3}$ $= -\sqrt{6}g_{\Lambda^0}\bar{\Lambda}^0 V_+ s^{a_1}u^{a_2} C\gamma_5 V_+ d^{a_3}\varepsilon^{a_1a_2a_3}$
Ξ^0	$g_{\Xi^0}\bar{\Xi}^0\gamma^\mu\gamma^5 V_+ u^{a_1}s^{a_2} C\gamma_\mu V_+ s^{a_3}\varepsilon^{a_1a_2a_3}$ $= 2g_{\Xi^0}\bar{\Xi}^0 V_+ s^{a_1}s^{a_2} C\gamma_5 V_+ u^{a_3}\varepsilon^{a_1a_2a_3}$
Ξ^-	$g_{\Xi^-}\bar{\Xi}^-\gamma^\mu\gamma^5 V_+ d^{a_1}s^{a_2} C\gamma_\mu V_+ s^{a_3}\varepsilon^{a_1a_2a_3}$ $= 2g_{\Xi^-}\bar{\Xi}^- V_+ s^{a_1}s^{a_2} C\gamma_5 V_+ d^{a_3}\varepsilon^{a_1a_2a_3}$

TABLE XIII. Heavy-light baryon Lagrangians.

Baryon	Lagrangian
Λ_Q	$-g_{\Lambda_Q}\bar{\Lambda}_Q Q^{a_1}u^{a_2}C\gamma_5 V_+ d^{a_3}\varepsilon^{a_1a_2a_3}$
Ξ_Q	$g_{\Xi_Q}\bar{\Xi}_Q Q^{a_1}u^{a_2}C\gamma_5 V_+ s^{a_3}\varepsilon^{a_1a_2a_3}$
	$g_{\Xi_Q}\bar{\Xi}_Q Q^{a_1}d^{a_2}C\gamma_5 V_+ s^{a_3}\varepsilon^{a_1a_2a_3}$
	$\frac{1}{\sqrt{6}}g_{\Sigma_Q}\bar{\Sigma}_Q\gamma^\mu\gamma^5 Q^{a_1}u^{a_2}C\gamma_\mu V_+ u^{a_3}\varepsilon^{a_1a_2a_3}$
Σ_Q	$\frac{1}{\sqrt{6}}g_{\Sigma_Q}\bar{\Sigma}_Q\gamma^\mu\gamma^5 Q^{a_1}d^{a_2}C\gamma_\mu V_+ d^{a_3}\varepsilon^{a_1a_2a_3}$
	$\frac{1}{\sqrt{3}}g_{\Sigma_Q}\bar{\Sigma}_Q\gamma^\mu\gamma^5 Q^{a_1}u^{a_2}C\gamma_\mu V_+ d^{a_3}\varepsilon^{a_1a_2a_3}$
Ξ'_Q	$g_{\Xi'_Q}\bar{\Xi}'_Q\gamma^\mu\gamma^5 Q^{a_1}u^{a_2}C\gamma_\mu V_+ s^{a_3}\varepsilon^{a_1a_2a_3}$
	$g_{\Xi'_Q}\bar{\Xi}'_Q\gamma^\mu\gamma^5 Q^{a_1}d^{a_2}C\gamma_\mu V_+ s^{a_3}\varepsilon^{a_1a_2a_3}$
Ω_Q	$\frac{1}{\sqrt{6}}g_{\Omega_Q}\bar{\Omega}_Q\gamma^\mu\gamma^5 Q^{a_1}s^{a_2}C\gamma_\mu V_+ s^{a_3}\varepsilon^{a_1a_2a_3}$

Lagrangians for heavy-light baryons are given in Table XIII. The meson-quark-antiquark interaction Lagrangians are listed in Table XIV.

The baryon-quark couplings g_B are determined from the normalization condition for vector current. For heavy-light baryons they are given by

$$g_{B_Q}^{-2}=\frac{N_c!}{(4\pi)^4}\frac{\Lambda_{B_Q}^6}{18\Lambda_{q_1}\Lambda_{q_2}}\cdot R_Q, \tag{A5}$$

where Λ_{q_1} and Λ_{q_2} are light quark cutoff parameters and R_Q is the structure integral which depends on the ratio $\bar{\Lambda}/\Lambda_{B_Q}$

$$R_Q=\int_0^\infty du\,u\exp\left[-18u^2+36u\frac{\bar{\Lambda}}{\Lambda_{B_Q}}\right].$$

In the case of light baryons the couplings are given by

$$g_{B_q}^{-2}=\frac{N_c!}{(4\pi)^4}\frac{\Lambda_{B_q}^8}{27\Lambda_q^4}\cdot\kappa,$$

where

TABLE XIV. Meson Lagrangians.

Meson	Lagrangian
π	$g_\pi\pi^+\bar{u}i\gamma^5 d+\text{H.c.}$
K	$g_K[K^+\bar{u}i\gamma^5V_+s+K^0\bar{d}i\gamma^5V_+s]+$ H.c.
ϕ	$g_\phi\phi_\mu\bar{s}(\gamma^\mu-v^\mu)V_+s$
J/ψ	$g_{J/\psi}(J/\psi)_\mu\bar{c}(\gamma^\mu-v^\mu)V_+c$

$$\kappa=\begin{cases}1 & \text{for nucleons}\\ r(2/3+r/3) & \text{for } \Lambda^0 \text{and the triplet of } \Sigma \text{ hyperons}\\ r^2(2r/3+1/3) & \text{for the doublet of } \Xi \text{ hyperons.}\end{cases} \tag{A6}$$

APPENDIX B: THE CALCULATION TECHNIQUE

To elucidate the calculation of the matrix elements (21)–(28) we consider the four relevant integrals in Euclidean space corresponding to the factorizing contributions from $b\to c$ and heavy-light transitions and the typical nonfactorizing ones coming from the diagram II_a. The calculations of the nonfactorizing contributions from diagrams II_b and III can be carried out analogously.

Factorizing contribution ($b\to c$ transition):

$$I_F^{b\to c}(\omega_E)=\int\frac{d^4k_E}{\pi^2}\int\frac{d^4k'_E}{\pi^2}\times\exp\left[-\frac{18k_E^2+6(2k'_E+k)^2}{\Lambda_{B_Q}^2}\right]\times\frac{1}{k_Ev_{1E}-\bar{\Lambda}}\frac{1}{k_Ev_{2E}-\bar{\Lambda}}. \tag{B1}$$

Factorizing contribution ($Q\to q$ transition):

$$I_F^{Q\to q}(\omega_E,M_2)=\int\frac{d^4k_E}{\pi^2}\int\frac{d^4k'_E}{\pi^2}\times\exp\left[-\frac{9k_E^2+3(2k'_E+k_E)^2}{\Lambda_{B_Q}^2}\right]\times\exp\left[-\frac{(3k_E+2p_{2E})^2+3(2k'_E+k_E)^2}{\Lambda_{B_Q}^2}\right]\times\frac{1}{k_Ev_{1E}-\bar{\Lambda}}. \tag{B2}$$

Nonfactorizing contribution ($b\to c$ transition):

$$I_{NF}^{b\to c}(\omega_E,M_2)=\int\frac{d^4k_E}{\pi^2}\int\frac{d^4k'_E}{\pi^2}\int\frac{d^4k''_E}{\pi^2}\times\exp\left[-\frac{9k_E^2+3(2k''_E+2k'_E-k_E-p_{3E})^2}{\Lambda_{B_Q}^2}\right]\times\exp\left[\frac{9k_E'^2+3(2k''_E+k'_E-p_{3E})^2}{\Lambda_{B_Q}^2}\right]\times\frac{1}{k_Ev_{1E}-\bar{\Lambda}}\frac{1}{k'_Ev_{2E}-\bar{\Lambda}}. \tag{B3}$$

Nonfactorizing contribution ($Q\to q$ transition)

$$I_{NF}^{Q\to q}(\omega_E,M_2)=\int\frac{d^4k_E}{\pi^2}\int\frac{d^4k'_E}{\pi^2}\int\frac{d^4k''_E}{\pi^2}\times\exp\left[-\frac{9k_E^2+3(2k''_E+2k'_E-k_E-p_{3E})^2}{\Lambda_{B_Q}^2}\right]\times\exp\left[-\frac{(3k'_E+2p_{2E})^2+3(2k''_E+k_E-p_{3E})^2}{\Lambda_{B_Q}^2}\right]\times\frac{1}{k_Ev_{1E}-\bar\Lambda}. \tag{B4}$$

The final light baryon state carries the Euclidean momenta p_{2E} with the mass-shell condition: $p_{2E}^2=-M_2^2$. The dimensionless variable ω_E is defined as $\omega_E=v_{1E}\cdot p_{2E}/M_2=-\omega$.

Scaling all momentum variables in the above integrals by Λ_{B_Q} and using the Feynman parametrization

$$\frac{1}{A}=\int_0^\infty d\alpha\exp(-\alpha A), \tag{B5}$$

we have

$$I_F^{b\to c}(\omega_E)=4\Lambda_{B_Q}^6\int_0^\infty d\alpha\int_0^\infty d\beta\int\frac{d^4k_E}{\pi^2}\int\frac{d^4k'_E}{\pi^2}\times\exp[-18k_E^2-24k_E'^2]\times\exp\left[-\frac{(\alpha+\beta)^2}{18}+\frac{\alpha\beta}{9}(\omega_E+1)+2(\alpha+\beta)\frac{\bar\Lambda}{\Lambda_{B_Q}}\right],$$

$$I_F^{Q\to q}(\omega_E,M_2)=2\Lambda_{B_Q}^7\int_0^\infty d\alpha\int\frac{d^4k_E}{\pi^2}\int\frac{d^4k'_E}{\pi^2}\times\exp[-9(1+R)k_E^2-12(1+R)k_E'^2]\times\exp\left[-\frac{\alpha^2-12R\alpha\omega_E-36RM_2^2}{9(1+R)}+2\alpha\frac{\bar\Lambda}{\Lambda_{B_Q}}\right],$$

$$I_{NF}^{b\to c}(\omega_E)=4\Lambda_{B_Q}^{10}\int_0^\infty d\alpha\int_0^\infty d\beta\int\frac{d^4k_E}{\pi^2}\int\frac{d^4k'_E}{\pi^2}\int\frac{d^4k''_E}{\pi^2}\times\exp[-12k_E^2-21k_E'^2]\exp\left[-\frac{72}{7}k_E''^2-\frac{(\alpha+\beta)^2}{72}+\frac{\alpha\beta}{72}(\omega_E+1)+2(\alpha+\beta)\frac{\bar\Lambda}{\Lambda_{B_Q}}-\frac{\alpha^2+\beta^2}{12}\right],$$

$$I_{NF}^{Q\to q}(\omega_E,M_2)=2\Lambda_{B_Q}^{11}\int_0^\infty d\alpha\int\frac{d^4k_E}{\pi^2}\int\frac{d^4k'_E}{\pi^2}\int\frac{d^4k''_E}{\pi^2}\times\exp[-12k_E^2-3R(3+4R)k_E'^2]\times\exp\left[-36\frac{R(1+R)}{3+4R}k_E''^2-\frac{\alpha^2-12R\alpha\omega_E-9M_2^2}{9(1+R)}+2\alpha\frac{\bar\Lambda}{\Lambda_{B_Q}}\right].$$

After integration over k_E, k'_E, and k''_E we arrive at

$$I_F^{b\to c}(-\omega)=\frac{\Lambda_{B_Q}^6}{12^2}\int_0^\infty du\,u\int_0^1 dx\exp\left[-18u^2-36u^2x(1-x)\times(\omega-1)+36u\frac{\bar\Lambda}{\Lambda_{B_Q}}\right],$$

$$I_F^{Q\to q}(-M_2,-\omega)=\frac{2\Lambda_{B_Q}^7}{36^2(1+R)^3}\int_0^\infty du\times\exp\left[-9(1+R)u^2+18(1+R)u\frac{\bar\Lambda}{\Lambda_{B_Q}}\right]\times\exp\left[-12Ru\omega\frac{M_2}{\Lambda_{B_Q}}+\frac{4R}{R+1}\frac{M_2^2}{\Lambda_{B_Q}^2}\right],$$

$$I_{NF}^{b\to c}(-\omega)=\frac{\Lambda_{B_Q}^{10}}{36^2}\int_0^\infty du\,u\int_0^1 dx\times\exp[-72u^2-144u^2x(1-x)(\omega-1)]\times\exp\left[144u\frac{\bar\Lambda}{\Lambda_{B_Q}}-432u^2[x^2+(1-x)^2]\right],$$

$$I_{NF}^{Q\to q}(-M_2^2,-\omega)=\frac{2\Lambda_{B_Q}^{11}}{216^2R^2(1+R)}\int_0^\infty du\exp[-36(1+R)\times(3R+4)u^2]\exp\left[72(1+R)u\frac{\bar\Lambda}{\Lambda_{B_Q}}-12Ru\omega\frac{M_2}{\Lambda_{B_Q}}+\frac{R}{R+1}\frac{M_2^2}{\Lambda_{B_Q}^2}\right].$$

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