Instantons and polarized structure functions

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The contribution of the quark-quark and quark-gluon interactions induced by instantons to the valence quark and proton spin-dependent structure functions $g_1(x,Q^2)$ is estimated within the instanton liquid model for a QCD vacuum. It is shown that this interaction leads to a rather large violation of the Ellis-Jaffe sum rule. [S0556-2821(98)04409-9]

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I. INTRODUCTION

One of the solutions of the famous ''spin crisis'' (see the recent review [1]), is the assumption of a large positive gluon polarization inside a nucleon [2].

Indeed, one of the next leading order (NLO) analyses of the polarized deep inelastic scattering (DIS) world data on $g_1(x,Q^2)$, which was performed to extract the polarized parton densities in a nucleon, has shown some indication for a positive value of gluon polarization [3]. However, another NLO fit leads to the conclusion that this result may be sensitive to the input shapes of the polarized parton distribution at a low normalization point [4].

The different calculations of the gluon polarization inside a nucleon have been performed (see [5]) and a contradiction between different approaches has been found, not only in the absolute value of the gluon polarization but even in its sign. For example it has been shown recently [8] that the instanton model for nonperturbative effects in QCD [9,10] rules out a positive value of gluon polarization. Therefore, it is possible that the gluon solution of the ''spin crisis'' is not viable.

Another way to resolve the proton spin problem is to take into account the quark depolarization induced by nonperturbative vacuum fluctuations of the gluon fields, so-called instantons [6,7]. The instantons describe the sub-barrier transitions between different classical QCD vacuum states that have different values of quark helicities [11]. Therefore, taking into account the quark interaction with instantons gives a direct way to obtain the magnitude of quark helicity nonconservation in QCD.

In this article we estimate the contribution from quark-quark [12] and quark-gluon interaction [13] induced by instantons to the valence quark and proton $g_1(x,Q^2)$ structure functions.

II. QUARK-QUARK AND QUARK-GLUON INTERACTION INDUCED BY INSTANTONS

The instanton model for QCD vacuum is widely used now in the description of the nonperturbative effects in strong interaction (see reviews [9,10]). The existence of instantons leads to a nonperturbative quark-quark and quark-gluon interaction through the QCD vacuum, which has the following form [12,14]:

$$\mathcal{L}_{\text{eff}}=\int\prod_q\left[m_q\rho-2\pi^2\rho^3\overline{q}_R\left(1+\frac{i}{4}\tau^aU_{aa'}\overline{\eta}_{a'\mu\nu}\sigma_{\mu\nu}\right)q_L\right]$$
$$\times\exp^{-(2\pi^2)/g\rho^2U_{bb'}\overline{\eta}_{b'\gamma\delta}G^b_{\gamma\delta}}\frac{d\rho}{\rho^5}d_0(\rho)d\hat{o}+R\leftrightarrow L,\quad(1)$$

where ρ is the instanton size, τ^a are the matrices of the $SU(2)_c$ subgroup of the $SU(3)_c$ color group, $d_0(\rho)$ is the density of the instantons, $d\hat{o}$ stands for integration over the instanton orientation in color space, $\int d\hat{o}=1$, U is the orientation matrix of the instanton, $\overline{\eta}_{a\mu\nu}$ is the numerical 't Hooft symbol and $\sigma_{\mu\nu}=[\gamma_\mu,\gamma_\nu]/2$.

From Eq. (1) one can obtain an effective quark-quark 't Hooft interaction [12] which in the framework of instanton liquid model reads [9,15]

$$\mathcal{L}^{(2)}_{\text{eff}}=F(k_1^2,k_2^2,k_3^2,k_4^2,\rho_c)\frac{4\pi^2\rho_c^2}{3}$$
$$\times\sum_{i\neq j}\{\overline{q}_{iR}(k_1)q_{iL}(k_2)\overline{q}_{jR}(k_3)q_{jL}(k_4)$$
$$\times[1+\tfrac{3}{8}(1-\tfrac{3}{4}\sigma^i_{\mu\nu}\sigma^j_{\mu\nu})t^a_ut^a_d+(R\leftrightarrow L)]\},\quad(2)$$

where $F(k_1^2,k_2^2,k_3^2,k_4^2,\rho_c)$ is the form factor, which is related to the Fourier transformation of the quark zero modes in the instanton field, and $\rho_c\approx1.6\ \text{GeV}^2$ is an average instanton size in the vacuum, $i,j=u,d,s$.

Recently, it was shown that from Eq. (1) a new type of nonperturbative quark-gluon interaction can be obtained. This interaction has the form of anomalous chromomagnetic quark-gluon interaction [13]

$$\Delta\mathcal{L}_A=-i\mu_a\sum_q\frac{g}{2m_q^*}\overline{q}\sigma_{\mu\nu}t^aqG^a_{\mu\nu},\quad(3)$$

where $m_q^*=2\pi^2\rho_c^2\langle0|\overline{q}q|0\rangle/3$ is a quark mass in the instanton vacuum.

The value of the quark anomalous chromomagnetic moment in the liquid instanton model is

$$\mu_a=-\frac{f\pi}{2\alpha_s},\quad(4)$$

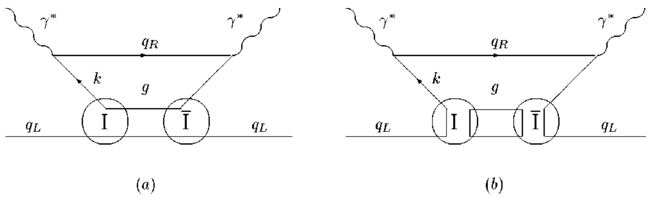

FIG. 1. The instanton contribution to the quark distribution functions: (a) the contribution to the sea quark distribution function; (b) the contribution to the valence quark distribution function. The label I(Ī) denotes instanton (antiinstanton).

where $f=n_c\pi^2\rho_c^4$ is the so-called packing fraction of instantons in vacuum. The value of n_c is connected with the value of the gluon condensate by the formula

$$n_c=\langle 0|\alpha_s G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle/16\pi\approx 7.5\ \ 10^{-4}\ \ \mathrm{GeV}^4. \qquad (5)$$

The following estimate for the value of the anomalous quark chromomagnetic moment has been obtained for $\rho_c=1.6$ GeV^{-1} in [13]:

$$\mu_a=-0.2. \qquad (6)$$

The principal difference between the instanton induced interaction Eq. (1), Eq. (2) and the perturbative quark-gluon vertex is the large quark helicity flip at instanton vertex, namely $\Delta\Sigma=-2N_f$. Therefore, this interaction can be responsible for a rather strong violation of Ellis-Jaffe sum rule [16] for the first moment of the spin-dependent structure function $g_1^p(x,Q^2)$.

III. INSTANTON CONTRIBUTION TO VALENCE QUARK AND PROTON STRUCTURE FUNCTIONS

The graphs which give the contribution to the structure functions from the quark-quark interaction (2) are presented in Fig. 1.

We calculate the contribution of this interaction to the valence quark spin-dependent structure function $g_1^q(x,Q^2)$ by using the projection of the imaginary part of the forward Compton scattering amplitude $T_{\mu\nu}$:

$$g_1^q(x,Q^2)=-\frac{ie_{\mu\nu\rho\sigma}p^\rho q^\sigma ImT_{\mu\nu}}{p\cdot q}, \qquad (7)$$

where p is the momentum of the valence quark in nucleon. The straightforward calculation of the contribution of the diagram given by Fig. 1(b)[1] leads to the result

$$g_1^{q,q}(x,Q^2)=\frac{e_q^2\rho_c^4}{128}\int_0^{Q^2(1-x)/4x}dk_\perp^2\int_{-k_\perp^2/(1-x)}^{-\infty}\times dk^2\frac{[k_\perp^4+k_\perp^2k^2(1-x)]}{x^2k^4}F^2\left(\frac{|k|\rho_c}{2}\right), \qquad (8)$$

[1]The calculation has shown that the contribution of the diagram in Fig. 1(a) is very small and therefore can be neglected. The terms which are proportional to the product of the color matrix t^a in Eq. (2) have $1/N_c$ suppression factor and give a very small contribution as well.

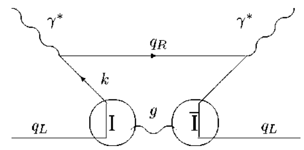

FIG. 2. The instanton contributions to proton structure function from the anomalous chromomagnetic interaction.

where

$$F(z)=z\frac{d}{dz}[I_0(z)K_0(z)-I_1(z)K_1(z)], \qquad (9)$$

and $x=Q^2/2p\cdot q$.

The graph which is responsible for the contribution from the quark-gluon chromomagnetic interaction (3) to structure functions is presented in Fig. 2.

The calculation of the contribution of the diagram in Fig. 2 leads to the result

$$g_1^{q,g}(x,Q^2)=-\frac{e_q^2}{8}\frac{|\mu_a|\rho_c^2}{(1-x)}\int_0^{Q^2(1-x)/4x}\times dk_\perp^2\frac{F^2(k\rho_c/2)}{\sqrt{1-\dfrac{4xk_\perp^2}{(1-x)Q^2}}}, \qquad (10)$$

where $k^2=k_\perp^2/(1-x)$, and the relation $\alpha_s\mu_a^2/m_q^{*2}\rho_c^2=3\pi|\mu_a|/8$ [8] has been used.

The very interesting feature of instanton contributions (8) and (10) is their specific Q^2 dependence. At small $Q^2\ll 1/\rho_c^2$ they are proportional to Q^2 and for $Q^2\gg 1/\rho_c^2$ they are constant. Therefore Q^2 dependence of the instanton contribution to polarized structure functions *should be different* from $\log(Q^2/\Lambda^2)$ evolution of the perturbative gluon corrections to DIS structure functions. The fundamental reason for this feature is the quark spin flip induced by instanton vertex, which gives the extra powers of $k_\perp$ to the matrix element for the forward Compton scattering amplitude. As a result, at $Q^2=0$ the instanton contribution to $g_1(x,Q^2)$ is *zero*.

In the large Q^2 limit and small x the contribution to $g_1^q(x)$ from the quark-quark interaction [Fig. 1(b)] has the anomalous $1/x^2$ behavior. This behavior comes from the pointlike instanton vertex (1),(2) which leads to very fast growth of the $\overline{q}q$ cross section induced by instantons by increasing the energy $S=(p-k)^2$. At high S the contribution of instantons should be small because the perturbative QCD should work in this region. The imaginary part of the lower part of the diagram Fig. 1(b) is proportional to a sum of the imaginary parts of the correlators of the pseudoscalar $j=\overline{q}\gamma_5 q$ and scalar $j=\overline{q}q$ currents ($s=\sqrt{S}$) [see the structure of Lagrangian (2)],

$$\Pi(s)=i\int dxe^{isx}\langle 0\|T\{j(x),j(0)\}\|0\rangle, \qquad (11)$$

that have been analyzed in QCD sum rule approach, taking into account the direct instanton contribution [17,9]. Therefore, the threshold value for the energy S_0 for the cutoff of the instanton contribution should approximately equal the value of the duality interval in the QCD sum rule for the nonet of the pseudoscalar and scalar mesons, which is $S_0 \approx 2\ \mathrm{GeV}^2$ [17].

The final formula for the contribution from quark-quark interaction $g_1^q(x)$ in the Bjorken limit $Q^2 \to \infty$ can be written in the following form:

$$g_1^q(x,Q^2)=\frac{e_q^2\rho_c^4}{128}\int_0^{S_0}dS\int_{Sx/1-x}^{\infty}\times dk^2\frac{S[xS-k^2(1-x)]}{k^4}F^2\left(\frac{k\rho_c}{2}\right), \quad (12)$$

Because of a cutoff in S, the $1/x$ divergence is absent now and the instanton contribution to the first moment of $g_1(x)$ is finite.

In the same limit the contribution to $g_1^q(x)$, due to quark-gluon chromomagnetic interaction, reads

$$g_1^q(x)=-\frac{e_q^2}{4}|\mu_a|. \quad (13)$$

The sign of both contributions is *negative* and comes from the *negative* quark polarization induced by instantons inside the proton.

To calculate the contribution to proton structure function, the simple convolution model for structure function has been used:

$$g_1^p(x)=\sum_q \int_x^1\frac{dy}{y}g_1^q\left(\frac{x}{y}\right)\Delta q_V(y), \quad (14)$$

where $\Delta q_V(y)$ are the initial valence quark polarizations, which were taken in the form

$$\Delta u_V(x)=3.7(1-x)^3, \quad \Delta d_V(x)=-1.3(1-x)^3, \quad (15)$$

and normalized to the experimental data on the weak decay coupling constants of hyperons

$$g_A^3=\Delta u_V-\Delta d_V=1.25, \quad g_A^8=\Delta u_V+\Delta d_V=0.6. \quad (16)$$

In Fig. 3 the result of the calculation of the contribution from the quark-quark interaction induced by the instanton to $g_1^p(x)$ in the region of the Bjorken variable $x>0.0001$ is presented.

It should be mentioned that this contribution is negative and rather large, especially in the low x region. The contribution to the first moment of g_1^p is

$$\delta I_{\mathrm{inst}}^{p,q}=\int_0^1 dx g_1^{p,q}(x)=-0.007. \quad (17)$$

This value is approximately one fourth of the observed violation of the Ellis-Jaffe sum rule [18]. The remaining part of the violation can be related to the contribution from the anomalous quark-gluon interaction induced by instantons.

In Fig. 4 the result of the calculation of the contribution to the $g_1^p(x)$ structure function, which comes from the quark-gluon chromomagnetic interaction induced by instantons, is shown.

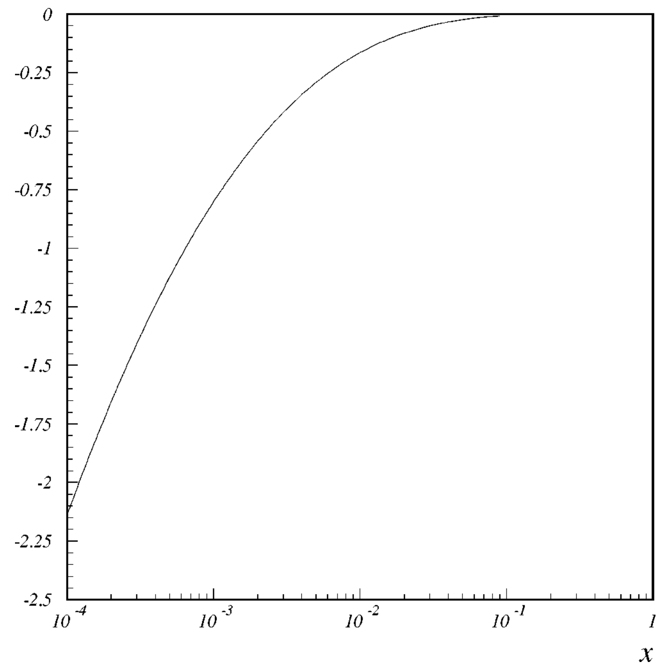

FIG. 3. The contribution to proton spin-dependent structure function $g_1^p(x)$ due to quark-quark instanton induced interaction.

This contribution is also negative and has a harder x dependence than the contribution from the quark-quark interaction. Therefore by taking into account the quark-gluon chromomagnetic interaction induced by instantons we can explain the decrease of the $g_1^p(x)$ structure function at the large Bjorken variable x.

The contribution to the first moment of g_1^p from this interaction is

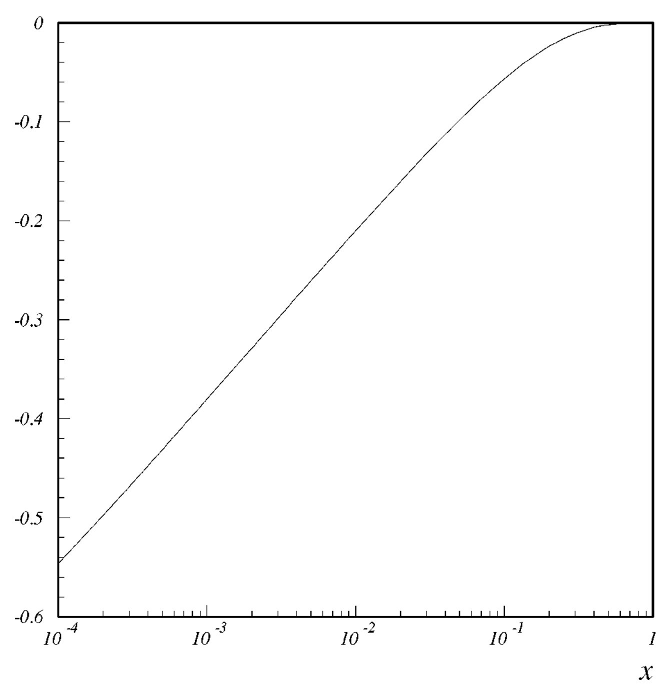

FIG. 4. The contribution to the proton spin-dependent structure function $g_1^p(x)$ due to the quark-gluon chromomagnetic interaction.

$$\delta I_{\text{inst}}^{p,g} = \int_0^1 dx g_1^{p,g}(x) = -0.019. \tag{18}$$

The total contribution from both interactions is $\delta I_{\text{inst}}^{p} = -0.026$. One can compare this number with the modern experimental data on the value of the violation of the Ellis-Jaffe sum rule $\delta I_{\text{exp}}^{p} = -0.02 - -0.04$ [18].

Thus, by taking into account the accuracy of the available experimental data and some ambiguities in the extrapolations of the $g_1(x)$ to very low x region in the current experiments, we can conclude that the instanton model gives a rather good description of the Ellis-Jaffe sum rule violation for protons. The prediction of the instanton model for the neutron g_1 structure function is very sensitive to the details of the violation of the SU(6) symmetry for the valence quark distribution function and will be discussed elsewhere.

IV. SUMMARY

In summary, the instanton induced quark-quark and quark-gluon interaction leads to a *large negative* contribution to the proton spin-dependent structure function $g_1^p(x,Q^2)$. This contribution allows us to explain the observed violation of the Ellis-Jaffe sum rule.

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