Viable range of the mass scale of the standard model

V. Agrawal,[1] S. M. Barr,[1] John F. Donoghue,[2] and D. Seckel[1]
[1]*Bartol Research Institute, University of Delaware, Newark, Delaware 19716*
[2]*Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003*
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In theories in which different regions of the universe can have different values of certain physical parameters, we would naturally find ourselves in a region where they take values favorable for life. We explore the range of such viable values of the mass parameter in the Higgs potential, μ^2. For $\mu^2<0$, the requirement that complex elements be formed suggests that the Higgs vacuum expectation value v must have a magnitude less than 5 times its observed value. For $\mu^2>0$, baryon stability requires that $|\mu|\ll M_P$, the Planck mass. Smaller values of $|\mu^2|$ may or may not be allowed depending on issues of element synthesis and stellar evolution. We conclude that the observed value of μ^2 appears reasonably typical of the viable range, and a multiple-domain scenario may provide a plausible explanation for the closeness of the QCD scale and the weak scale.
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I. INTRODUCTION

Some of the major puzzles of particle physics and cosmology concern parameters which are much smaller than their expected ''natural'' size [1]. For example, the Yukawa coupling constant of the electron, λ_e, is about 2×10^{-6}. The QCD ''vacuum angle,'' $\bar{\theta}$, is experimentally bounded to be less than about 10^{-9}. Perhaps the mother of all such small-number puzzles is the cosmological constant problem [2]. The cosmological constant, Λ, in natural gravitational units (i.e. in units of M_P^4, where $M_P\sim10^{19}$ GeV is the ''Planck mass'') is known to be less than about 10^{-120}. All of these dimensionless ratios would naturally be expected to be of order unity in the absence of some dynamical mechanism or symmetry principle that determined them to be small.

For some of these small numbers fairly simple conventional particle physics explanations are available. In particular, viable models exist for explaining the smallness of $\bar{\theta}$ and λ_e. Other small numbers are harder to explain conventionally. For example, although one can find ways (involving supersymmetry) to explain why the cosmological constant is as small as $(1\ \mathrm{TeV})^4$ (or 10^{-64} in natural units), that is still 56 orders of magnitude larger than the observational bound.

These puzzles are difficult to solve in terms of known physics and the literature contains various hypothesized remedies involving new physics. In particular, for the Higgs vacuum expectation value there is a strong suspicion that new particles and interactions such as supersymmetry or technicolor must be present at the 1 TeV scale. However, at present we have no evidence that such new physics exists, as all searches have thus far been negative. In addition, the procedure of postulating new physics just beyond the mass scale in question does not work for the cosmological constant, so that one must look for dynamical mechanisms whereby new physics at high energy forces the cosmological constant to a small or vanishing value.

In this paper we explore the consequence of a new cosmological possibility that may put some of these puzzles in a different light. In the context of chaotic inflation [3], it has been realized that there could be different domains in the universe in which the physical parameters take on different values. Because of the growth of space-time during inflation, we would live entirely within one such domain. The possibility of a multiple-domain universe may be more general than this particular known theory. For example in string theory, the moduli fields that determine many of the physical parameters of the low energy theory may not be fixed at the string scale, and perhaps the domains of the universe sample different values of these fields. Even more generally, we could argue that we need to take this possibility seriously independent of any known mechanism. The history of science has repeatedly demonstrated that our place in the universe is not unique. Our planet, solar system and galaxy are only one of many. It is then required that we raise the possibility that the same situation may hold for the small region of the universe that we can observe — is our domain equivalent to the whole universe or are there other different domains? This may be a difficult question to answer, but multiple domain theories may need different forms of analysis than do theories in which there are only a single unique set of parameters. This paper is an attempt to analyze some of the consequences of such theories on the Higgs mass parameter.

In multiple domain theories, at least some of the parameters will vary in different domains. Many of these domains will likely have extreme conditions — for example if the cosmological constant had a more natural value the universe would have lasted only a few times the fundamental Planck time. However, in the ensemble of all domains, it is an obvious requirement that we could only find ourselves in those domains that have properties that allow life. We will label domains that satisfy this bias as ''viable'' domains. Viable domains will only exist for a restricted range of parameters.

In recent work, Weinberg has addressed the question of whether this reasoning can explain the smallness of the cosmological constant [4]. Weinberg examines the requirement that the evolution of the universe be such that matter clumps into galaxies, and shows that this only occurs for a range of values of the cosmological constant. He then calculates the

mean value of viable Λ's. If the actual value of Λ in our universe turns out to be very much smaller than this mean value, one would conclude that the multiple domain hypothesis cannot provide any understanding of the cosmological constant and that other mechanisms would likely have to be invoked to explain the smallness of this parameter. Alternatively, if Λ is typical of the viable range, we may consider the multiple domain hypothesis as a possible solution to this problem. Unfortunately, the results are not yet conclusive, as neither the mean viable value nor the experimental value of Λ is yet well determined.

This type of reasoning is one version of what is often called an ''anthropic'' argument [5]. Some anthropic arguments invoke one or another ''anthropic principle'' — for example that the universe must have conscious observers. Such ''principles'' appear, for the most part, to be rather arbitrary metaphysical assertions. However, in a multiple-domain cosmology anthropic reasoning has a sounder physical basis. In such a case, one is not appealing to *a priori* assumptions about what the universe must be like, but rather analyzing the observable consequences of a definite physical hypothesis.

An analogy may make clear what is involved. Since there are many planets in the universe, having varying conditions, it would make no sense to treat the surface temperature of the earth as a fundamental parameter — like the fine-structure constant, say — to be derived from a fundamental microphysical theory. Rather, the fact that the surface temperature of our planet ''happens'' to be near 300 K (a temperature at which biochemical reactions are possible) is best explained by noting that on much hotter or colder planets the evolution of life would be difficult or impossible. We do not live, therefore, on a planet that is typical of *all* planets, but one that is presumably typical of biologically viable planets. Such considerations only become relevant if one has multiple ''domains'' (in this case planets), and if the parameter whose observed value one is trying to explain does indeed vary among those domains, rather than being truly a ''constant of nature.''

Whether the cosmological constant (or in our case the mass parameter of the Higgs field) is a fundamental parameter with a unique value derivable from microphysics, or varies among different domains of the universe, is not something that can yet be answered. An answer would require more knowledge of the details of the fundamental theory and of the cosmology to which it leads. At the least, however, it is now clear that multiple domains do occur in at least some physical theories, and the consequences of such theories need to be explored as best they can be. Weinberg's calculation gives an example of the way that such theories can lead to physical hypotheses that potentially could solve outstanding problems in physics.

The greater part of this paper will be devoted to exploring a similar hypothesis for explaining the magnitude of a single parameter of the standard model of particle physics, namely μ^2, the mass parameter of the Higgs potential. Like the cosmological constant, this parameter is many orders of magnitude smaller than its ''natural scale'' and has so far no completely satisfactory conventional particle physics explanation. Its smallness (about 10^{-34} in Planck units) is considered one of the major puzzles in particle physics, and is often called the ''fine tuning problem'' or (in the context of grand unified theories) the ''gauge hierarchy problem'' [6]. We will assume that the standard model is the correct theory of particle interactions in the limit that effects from the Planck scale or the unification scale are neglected. In the next several sections of the paper we will examine some of the consequences of varying a single parameter, μ^2, of that model [7], and their implications for the probability of life emerging. Of course, given the particular value of μ^2 in some domain, computing the probability that life evolves in that domain, or how often it is likely to do so, is an extremely hard problem that we do not know how to do. As a proxy for such a calculation we make certain plausible assumptions about necessary conditions for life to emerge in a domain. One of the conditions that we take to be necessary is that complex chemistry be possible. For us this means chemistry that involves more than just hydrogen and helium. In other words, elements other than just hydrogen and helium must (1) be allowed to exist by the theory, and (2) be actually formed in the evolution of the universe. (For further discussion of the assumptions made in our analysis, and comparison with other related work, see Appendix A.)

Without more information on the nature of the underlying multiple domain theory, we cannot be sure which parameters vary nor over what range. There are some plausible arguments for focusing particularly on μ^2 and these will be explained more fully below. Moreover, the viable range for μ^2 will turn out to be so small compared to its natural range that it might be argued that the constraint on its range is the most important anthropic bias, and that other parameters have only a secondary effect. However, many of our considerations also apply to more general situations. In particular, it is also interesting to look at the ratios of the quark masses from this perspective, and we do this in Sec. VI.

Note that we do not attempt to modify the standard model by adding new physics at low energies. To do so would make the space of possibilities too large to make meaningful arguments. There is no intrinsic conflict between the multiple domain hypothesis and other physics beyond the standard model. However, the hypothesis would certainly be superfluous if all of the parameters of the standard model could be explained without it.

II. THE HIGGS MASS PARAMETER μ^2

Of all the parameters of the standard model, μ^2 stands out in a number of ways. First, it is the only one which is dimensionful, and because of that it sets the scale for the masses of all the known elementary particles [8]. All the elementary particles of the standard model which have mass — quarks, leptons, $W^{\pm}$, and Z^0 — derive these masses from coupling to the expectation value of the Higgs. This expectation value, which is denoted $v/\sqrt{2}$, is determined by the minimization of the effective potential of the Higgs field, $V(\phi)=\lambda(\phi^{\dagger}\phi)^2+\mu^2\phi^{\dagger}\phi$. Since $\mu^2<0$ one has that $v=\sqrt{|\mu^2|/\lambda}$. The observed value of v is 246 GeV. We will call this value v_0, and henceforth the subscript ''0'' will denote the value a parameter takes in our domain. The masses of the weak-interaction gauge bosons, $W^{\pm}$ and Z^0 are given by v times the gauge coupling constants, which are of order $\frac{1}{2}$. The masses of the quarks and leptons are given by v times Yukawa couplings

which range from about 2×10^{-6} for the electron to about 1 for the top quark. The mass of the Higgs particle is $\sqrt{2\lambda}v$, where λ is as yet unknown, but is roughly of order unity [9].

The second way in which μ^2 stands out among the parameters of the standard model is its extreme smallness. As noted above, it is of order $(100\ \mathrm{GeV})^2$, or about 10^{-34} in natural Planckian units. This is to be compared to the next smallest parameters, the electron Yukawa coupling, $\lambda_e\cong2\times10^{-6}$, and the QCD vacuum angle, $\bar{\theta}<10^{-9}$. In the simplest grand unified models μ^2 receives contributions of order $M_{GUT}^2\sim10^{32}\ \mathrm{GeV}^2$ from each of several terms, which must therefore cancel to fantastic accuracy. In such ''fine-tuned'' models, a very small change in the other parameters would disturb this cancellation and cause μ^2 to vary over an enormous range. (This may be another justification for our approach here of only considering variations of μ^2 and keeping the other parameters essentially fixed.)

Finally, μ^2 stands in contrast to the other parameters of the standard model in that fairly plausible explanations in terms of symmetry principles or other conventional particle physics considerations are available to account for their magnitudes. For example, it is generally regarded as likely that the relative values of the gauge couplings are the result of unification of the gauge groups at or below the Planck scale [10]. Likewise, there are many ideas for explaining the ratios of quark and lepton masses in terms of grand unification, family symmetries, horizontal interactions, radiative hierarchies, or a combination of these [11]. The smallness of the QCD vacuum angle can be explained by the axion mechanism [12] or by approximate CP invariance [13]. By comparison, the smallness of μ^2 is very hard to account for in conventional ways. There have been two main approaches to doing this, ''technicolor'' [14] and supersymmetry [15]. The technicolor approach is fraught with difficulties and has fallen into disfavor. The supersymmetry approach is more promising, but a completely satisfactory and simple explanation of the smallness of μ^2 does not yet exist.

It will be assumed, then, that μ^2 can be of either sign and can vary between $+M_P^2$ and $-M_P^2$. To understand the behavior of v as μ^2 is varied over this range, one must look at the potential for ϕ including the effect of its coupling to the quark-antiquark condensates (which can ordinarily be neglected):

$$V(\phi)=\lambda(\phi^\dagger\phi)^2+\mu^2\phi^\dagger\phi+\left(\sum_i \lambda_i\langle\bar{q}_iq_i\rangle\phi+\mathrm{H.c.}\right). \tag{1}$$

The $\bar{q}q$ condensates for light quarks have a value of order f_π^3, where f_π is the strong-interaction chiral-symmetry-breaking scale [8]. ($f_\pi\simeq100$ MeV.) For μ^2 negative and much larger in absolute value than f_π^2, as in our domain, one can ignore the last term in Eq. (1) and obtain $v\cong\sqrt{|\mu^2|/\lambda}\sim|\mu|$. As μ^2 becomes smaller in absolute value than f_π^2, and of either sign, one can neglect the μ^2 term and obtain $v\sim(\lambda_t/\lambda)^{1/3}f_\pi\sim f_\pi$. Finally, when μ^2 is positive and larger than f_π^2 one can neglect the quartic term and obtain $v\sim\lambda_t(f_\pi^3/\mu^2)\sim(f_\pi^3/\mu^2)$. Note that in a $\mu^2>0$ domain, the longitudinal components of the Weak interaction gauge bosons come from the ''pions'' not from the Higgs field, and $M_W\sim gf_\pi$.

III. THE $\mu^2<0$ DOMAINS

In domains with $\mu^2<0$ and greater in absolute value than in our domain, one has $v>v_0$. One is dealing, then, with ''large v'' domains. Our basic assumption that ''complex chemistry'' be possible obviously implies that a variety of nuclei be (a) stable and (b) actually formed in primordial or stellar nucleosynthesis. Specifically, since we have defined complex chemistry to be chemistry involving more than just hydrogen and helium (see Appendix A), we must have the synthesis of stable nuclei with $Z>2$.

One of the things that can go wrong when v becomes larger than v_0 is that nuclei, or even the protons and neutrons, can become unstable. For example, in our domain neutrons can be stable within nuclei; but, for sufficiently large v, as we shall see, the reaction energy for neutron decay, $Q=m_n-m_p-m_e$, becomes larger than the binding energy per nucleon in nuclei of about 8 MeV. At that point neutrons even in nuclei will decay and the only stable nuclei will be protons. Our assumptions imply that such a ''proton domain'' would be sterile.

To analyze the stability of nucleons and nuclei in large v domains, we must understand how masses and binding energies depend on v. We turn first to this question.

The quark and lepton masses simply scale with v (ignoring the relatively small effect of the logarithmic running of the Yukawa couplings). Thus, we take $m_e=0.5(v/v_0)$ MeV, $m_u=4(v/v_0)$ MeV, and $m_d=7(v/v_0)$ MeV.

We model baryon masses by $m_B=m_q+m_c+m_{em}$, where m_q is the sum of the quark masses in the baryon, m_c is the color energy, and m_{em} is the electromagnetic energy. Since for the neutron and proton the color energy is the same, the neutron-proton mass splitting is given by $m_n-m_p=m_d-m_u+m_{em,n}-m_{em,p}$. As long as the quark masses are small compared to the QCD scale (which will be true for v/v_0 less than a few hundred) the size of nucleons, and therefore the electromagnetic energy, will be relatively insensitive to v. (The size of a nucleon will scale as Λ_{QCD}^{-1}. For the dependence of this on v see below.) Thus we can take $m_{em,n}-m_{em,p}$ to have the same value which it has in our domain, namely about -1.7 MeV. Thus we have that $m_n-m_p=(3(v/v_0)-1.7)$ MeV, and the Q value for neutron beta decay, $Q\equiv m_n-m_p-m_e$ is $(2.5(v/v_0)-1.7)$ MeV.

For $v=v_0$ most of the mass of the p, n, and Δ baryons is due to color energy. The splitting between the $I=1/2$ baryons (n and p) and the $I=3/2$ baryons (Δ) is in our domain about 300 MeV. [Since the lightest baryons will be made purely of u and d quarks, we need be concerned only with isospin and not with flavor SU(3).] We will assume that both $m_{1/2}$ and $m_{3/2}$ are proportional to the QCD scale, Λ_{QCD}. Λ_{QCD} depends only indirectly and fairly weakly upon v. (This dependence arises because the renormalization group running of the strong coupling ''constant,'' α_3, depends on quark thresholds, which in turn depend on the quark masses.) We find that $\Lambda_{QCD}\sim v^\zeta$, where $0.25<\zeta<0.3$ for $-2<\log(v/v_0)<4$. Thus we take $m_{3/2}-m_{1/2}\approx300(v/v_0)^{0.3}$ MeV.

Of course, for very large v (larger than a few hundred v_0) the quark masses will become larger than the color energy, and the proton, neutron, and Δ will become non-relativistic bound states in which the color energy will go as $\alpha_3^2 m_q$ and thus be proportional to v.

The dependence of the nuclear force on v is a much more complicated matter. The long-range part of the nucleon-nucleon potential is due primarily to one-pion exchange, and therefore has a range of m_π^{-1}. As long as the u and d masses are small compared to the QCD scale, the mass of the pion is well approximated by $m_\pi \propto ((m_u+m_d) f_\pi)^{1/2}$. Assuming that $f_\pi \propto \Lambda_{QCD}$, one has that $m_\pi \sim v^{(1+\zeta)/2}$. We will take m_π to go as $v^{1/2}$, which is adequate for our discussions. The shorter-range part of the nucleon-nucleon potential comes from multi-pion exchange, the exchange of heavier states, and more complicated effects. It is difficult to estimate how these will depend on v. However, our qualitative conclusions do not depend upon this issue.

With these general considerations we map out the nature of baryons and nuclei in universes where $v > v_0$.

$v/v_0 = 1$. In our world, the splitting between isospin multiplets is large, $m_{3/2} - m_{1/2} \approx 300$ MeV $\gg m_q, m_{em}$. The lightest baryons are thus the proton and neutron. Of these the proton is lighter because the quark mass splitting $[(m_n - m_p)_{quark\ mass} = 3$ MeV$]$ wins out in competition with the electromagnetic energy splitting $[(m_n - m_p)_{em} = -1.7$ MeV$]$.

$5 \gtrsim v/v_0 > 1$. As v increases the neutron becomes more unstable because $m_n - m_p$ increases, and the nuclear potential between nucleons gets weaker (since m_π is getting larger). The combined effect is to render nuclei less stable. We estimate that for $v/v_0 \gtrsim 5$ there will be no stable nuclei, as the mass excess of the neutron is greater than the nuclear binding energy. For $v/v_0 \lesssim 5$ a variety of nuclei will continue to exist, with fewer and fewer stable isotopes surviving as v/v_0 increases.

Even if nuclei are stable there is the question of whether or not they may form through nucleosynthesis. Relevant to this question is the fact that one of the first nuclei to become unstable as v/v_0 increases above 1 will be the deuteron, which even in our domain is very weakly bound. This is a particularly important case as all primordial and stellar nucleosynthesis ultimately begins with deuterium.

The critical reaction for decay of the deuteron is $d \to p + p + e^- + \bar{\nu}$ which occurs whenever $B_d < m_n - m_p - m_e \cong [2.5(v/v_0) - 1.7]$ MeV. For the binding energy of the deuteron as a function of v we consider two models, both based on the knowledge that the deuteron is a weakly bound and rather extended nuclear state, sensitive to the long-range pion-exchange component of the nuclear potential. In the simpler model we treat the nuclear potential as a square well with a hard core. The hard core mocks up the short range repulsion, whereas the square well of depth 35 MeV and width 2 fm represents the one pion exchange potential. To model the effects of changing v we decrease the width as $(v/v_0)^{-1/2}$ to account for the increase in m_π. The deuteron binding energy can be solved for analytically (see Appendix B), yielding an approximate relation

$$B_d \cong \left[2.2 - a\left(\frac{v - v_0}{v_0} \right) \right] \text{ MeV}, \tag{2}$$

where $a \approx 5.5$, for small $v - v_0$.

The shortcoming of this model is that all the deuteron binding is attributed to one-pion-exchange, which probably overstates its importance. We therefore tried a more sophisticated approximation [16], using a one-boson-exchange potential (OBEP) based on deuteron binding and scattering phase shifts. This model includes 6 bosons, and also includes s and d wave mixing for the deuteron. We varied m_π proportionally to $v^{1/2}$ (we neglect the scaling of Λ_{QCD}), but it is not clear how the other meson parameters should vary. The problem is not well defined, as several of the ''mesons'' do not correspond to physical particles but are mockups of the short range exchange of two or more pions in channels with the same quantum numbers. The masses for these mesons reflect the momentum distribution of the multiple pions as much as the mass of the pion itself. Further, the mass and coupling parameters are arrived at only after fitting, thus to change their relative values in an *ad hoc* way can destroy some sensitive cancellation, and is of questionable value. Faced with this problem we chose to vary the pion mass, but kept all other parameters of the OBEP unchanged. Solving for the deuteron bound state we find a nearly linear relation between B_d and v of the same form as Eq. (2), with $a \approx 1.3$ MeV [16].

For both our models, the deuteron binding energy is explicitly a function of m_π and implicitly a function of v/v_0. The deuteron becomes unstable to weak decay for

$$v/v_0 \approx \frac{3.9 + a}{2.5 + a} \tag{3}$$

with a in MeV. For the square-well model and the OBEP model, the deuteron is unstable at $v/v_0 = 1.2$ and 1.4 respectively.

If the deuteron lifetime against weak decay is long enough, then a chain of nuclear reactions involving intermediate unstable deuterons may be possible. In this case the critical reaction is the strong decay $d \to p + n$, which becomes possible if $B_d < 0$. The corresponding values of v/v_0 are 1.4 and 2.7 for the square-well and OBEP models, respectively.

The anthropic argument in the case where deuterons are unstable is not airtight — there exists the possibility that nuclei may form in neutron-rich regions following stellar collapse. Such a scenario would require significant rates for three-body processes, or a long-lived deuteron as may exist for the regime of $1.4 < v/v_0 < 2.7$ for the OBEP model.

$10^3 > v/v_0 > 5$. For this range of v/v_0 nuclei are unstable to decay of constituent neutrons, $(A,Z) \to (A-1,Z) + p + e^- + \bar{\nu}$. Hypothetically, there is the possibility of stable proton-rich ($Z \gg N$) nuclei, but this seems unlikely. In our world the depth of the nuclear potential is of order 50 MeV, but the binding energy per nucleon in nuclei is only about 10 MeV. The difference is primarily due to kinetic energy in the form of nucleon degeneracy energy and Coulomb energy due to the protons. In a nucleus with $Z \gg N$, with the same value of A and the same nuclear density, the fermi energy will be greater by a factor of $2^{2/3} \cong 1.6$, and the coulomb energy will be greater by roughly a factor of 4. Given, also, an expected decrease in nuclear binding due to an increase in m_π and stable proton-rich nuclei seem unlikely even in the absence of inverse β decay.

Even if stable proton-rich nuclei do exist, it will be difficult to form them. The most stable nuclei will occur for intermediate A, but there will be significant gaps in the sequence of stable nuclei necessary for nucleosynthesis. For example, either from direct experiment or by comparison to other unstable isotopes we may conclude that He^2, Li^3, and Be^4, are all strongly unstable in our domain, and therefore also in domains where $v > v_0$. It is unclear, then, how compound nuclei would form if $v/v_0 \gtrsim 5$.

Thus, the only stable nucleus will be the proton, and the only element hydrogen. We expect such ''proton domains'' to be sterile. It is interesting that the existence of neutrons close enough in mass to the proton to be stable in nuclei plays an important role in making life in our domain possible.

$v/v_0 \gtrsim 10^3$. For large enough v/v_0, the mass difference between u and d quarks is greater than the penalty in color energy, $m_{3/2} - m_{1/2}$, that must be paid to have three identical quarks in a baryon. Exactly where this occurs is not too important, but at some point $m_p = m_{\Delta^{++}}$. Comparing $m_d - m_u \approx 3(v/v_0)$ MeV to $m_{3/2} - m_{1/2} \approx 300(v/v_0)^{0.3}$ MeV suggests that equality takes place at around $v/v_0 \approx$ 500–1000.

On either side of the critical value there is a range of order 20% in v/v_0 where both p and Δ^{++} are stablized by the electron mass. This leads to what we shall call ''proton plus Δ^{++} domains.''

As v is increased above the narrow range of values where p and Δ^{++} can coexist, the proton becomes unstable to the decay $p \to \Delta^{++} + e^- + \bar{\nu}$, and at this point the only stable baryon is the Δ^{++}. We refer to such a domain as a ''Δ^{++} domain.''

It is fairly clear that the Δ^{++} domains are sterile, as in them it seems quite unlikely that two Δ^{++}'s would bind. Since v/v_0 is large enough that $m_{u,d} > f_\pi$, the pion is not a goldstone-boson-like particle, and the nuclear force has only a short range part. Here, in addition, there is a substantial Coulomb repulsion between the Δ^{++}'s.

If the Δ^{++}'s cannot fuse to form heavier nuclei, then there is only a single kind of element, which will have a single Δ^{++} as its nucleus. But this element is chemically equivalent to helium, and is therefore chemically inert as well. Therefore, in the Δ^{++} domains one expects neither nuclear nor chemical reactions to occur. It is hard to conceive, then, what kind of reactions could form the basis of life. The ''proton plus Δ^{++} universes'' are only marginally more interesting. From what we know of their molecular states, it seems obvious that hydrogen and helium alone could not form the basis of biochemistry. (For further discussion of this point and of the assumptions that we are making about ''viability'' see Appendix A.) We conclude, therefore, that the whole range from $v/v_0 \gtrsim 10^3$ to $v/v_0 \sim M_P/v_0 \sim 10^{17}$ can be excluded.

$1 > v/v_0$. Finally, although it is not strictly an issue for the hierarchy problem, we examine the nuclear consequences of $v < v_0$ in domains where $\mu^2 < 0$. As v decreases from v_0, $m_n - m_p$ falls below m_e, so that the neutron becomes stable but protons in hydrogen become unstable to electron capture. At a somewhat lower value of v, the neutron and proton masses become equal, followed eventually by a region where the proton is unstable to the decay $p \to n + e^+ + \nu_e$. The stability criteria are determined using a v dependent electron mass $m_e = \lambda_e v$.

One result of increasing neutron stability is to increase the primordial nucleosynthesis yield of He^4. Once the neutron is lighter than the proton we can no longer reliably estimate the results of nucleosynthesis. With no Coulomb barrier to suppress reactions, we anticipate that all single nucleons are bound into compound nuclei, but we cannot calculate the distribution of heavy elements. The value of $m_p - m_n$ never gets larger than 1.7 MeV, so it is never large enough to destablize nuclei in a manner similar to neutron decay in the $v/v_0 > 5$ regime. These compound nuclei are therefore stable and we see no reason why values of $v/v_0 < 1$ could not support life.

IV. THE $\mu^2 > 0$ DOMAINS

For $\mu^2 < 0$ the argument based on viable chemistry worked very effectively. The situation is not as simple for $\mu^2 > 0$. In this case, $v \sim f_\pi^3/\mu^2$, and therefore for $\mu^2 \gtrsim |\mu_0^2| \sim (10^2 \text{ GeV})^2$ one has $v \lesssim 10^{-9} v_0$, and all the quark and lepton masses are extremely small. For example, $m_e \sim 5 \times 10^{-4}$ eV(μ_0^2/μ^2). Also, symmetry breaking of SU(2) $\times$ U(1) is driven by quark condensates, so that $M_W \sim f_\pi$. These facts have dramatic consequences for the chemical energy scale of life, the structure of elements and stellar evolution; all of which play a role in the genesis of life.

Our critical assumption is that life requires the chemistry of complex molecules. A typical biochemical energy is $E_{chem} = \epsilon \alpha^2 m_e$, where ϵ is a numerical factor, which in our domain is of order 10^{-3}. (This gives, not coincidentally, $E_{chem} = 300$ K, the average surface temperature of the earth.) What ϵ would be in the bizarre small-v domains we shall consider is hard to say. We shall keep it as a parameter in our formulas. We think it unlikely that it is large compared to one. The crucial point is that E_{chem} is proportional to m_e, which in $\mu^2 > 0$ domains is tiny compared to its value in our domain.

It is clear that chemical life cannot emerge until at least the time, which we will call t_{chem}, when the temperature of the cosmic background radiation cools below E_{chem}. To put it picturesquely, there is the problem of life being fried by the cosmic background radiation. When this radiation has temperature $T \sim E_{chem}$, the matter density will be $\rho(t_{chem}) \sim \eta_B m_p E_{chem}^3$, where η_B is the baryon to photon ratio. We assume η_B to be set by physics that is insensitive to the value of μ^2, such as the interactions of a grand unified theory, and therefore to have a value similar to that in our universe, namely 10^{-10}. Using $t_{chem} \sim M_P/\rho(t_{chem})^{1/2}$, $E_{chem} = \epsilon \alpha^2 m_e$, and $m_e \sim m_{e0}(f_\pi^3/\mu^2 v_0)$, one derives

$$t_{chem} \sim m_p^{-1} \left[\left(\frac{M_P}{m_p} \right) \left(\frac{m_p}{m_{e0}} \right)^{3/2} \left(\frac{v_0 |\mu_0^2|}{f_\pi^3} \right)^{3/2} \alpha^{-3} \eta_B^{-1/2} \right] \times \epsilon^{-3/2} \left(\frac{\mu^2}{\mu_0^2} \right)^{3/2}, \tag{4}$$

or

$$t_{chem} \sim 10^{19} \text{ yrs } \epsilon^{-3/2} \left(\frac{\mu^2}{\mu_0^2} \right)^{3/2} . \quad (5)$$

By this time one of several disasters may occur which prevent the emergence of life. We will discuss two of these: the burning out of stars, and the decaying away of all baryons. Before we do so, however, we must reexamine one of the assumptions that went into Eq. (5), namely that chemistry is based on electrons.

It is not hard to show that if we replace m_e by either m_μ or m_τ in the formula for t_{chem} the resulting time is still short compared to the lifetimes of the μ or τ. This is because the β-decay lifetime of a lepton goes as one over the fifth power of its mass, and therefore as $v^{-5} \sim (f_\pi^3 / v_0 \mu^2)^{-5}$. (Recall that the Fermi constant, $G_F \sim f_\pi^{-2}$ in this limit rather than as v^{-2}.)

Since μ's and τ's are stable on the relevant time scales, it is possible that the valence leptons of biochemistry would be some lepton other than (or in addition to) electrons. This cannot be ruled out, and in fact whether it is so depends on the details of baryo/leptogenesis. Whatever the mechanism of baryo/leptogenesis, sphaleron processes will certainly have the effect of reshuffling baryon and lepton numbers in such a way that there will be non-zero values of all three lepton numbers. Further, there is no reason for all three lepton numbers to be negative, even though sphaleron processes will try to minimize $B+L$. Let us consider two cases to see some of what is possible. (1) If primordially only τ-number was produced, and it was negative, then sphaleron-processes, weak scattering processes, and beta decay would lead to a distribution with a net positive charge in τs, $Q_\tau > 0$. Additionally, baryon's (see below) tend to be positively charged, so $Q_B > 0$. These charges are balanced by $Q_e < 0$ and $Q_\mu < 0$. This means that baryons and τ^+'s would become clothed with e^-'s and μ^-'s. Since the μ^-'s would be much more tightly bound, chemistry in this case would be electron chemistry. (2) If, on the other hand, primordially only electron number were produced, and it were negative, then the baryons would end up being clothed exclusively with τ^-'s — as long as $N_\tau > Q_B$. In any baryonic atom, a τ could replace a μ, with a gain in binding energy. The remaining τs and μs would bind, albeit more weakly, with electrons.

If the chemistry relevant for life is dominated by lepton l^-, then one simply multiplies the formula for t_{chem} by $(m_{e0}/m_{l0})^{3/2}$. This result is to be compared with the baryon lifetime. If $\tau_B > t_{chem}$, we then consider the structure of nuclei, and stellar evolution.

(i) The decay of baryons.[1] The unification of gauge couplings suggests grand unification at a scale of order 10^{16} GeV, and therefore the existence of gauge bosons of that mass whose exchange leads to violation of baryon number. But even if there is no unification at that scale, it is plausible to suppose that at least at the Planck scale there will be states whose exchange violates baryon number. Thus we will parametrize the baryon decay rate as $\Gamma_B = m_p^5 / M^4$, where M is assumed to lie between 10^{16} GeV and 10^{19} GeV.

The time when baryons decay is determined by the condition $\Gamma_B = H \simeq \rho^{1/2} / M_P$. The temperature at this time is given implicitly by the relation $\rho_{rad} = f \rho_B$, where f is a numerical factor that we take to be about 0.3 [17]. This gives $T_{rad} \simeq (f/g_{rad})^{1/4} (\Gamma_B M_P)^{1/2} \simeq 0.3 (\Gamma_B M_P)^{1/2}$. Requiring that this be less than $E_{chem} = \epsilon \alpha^2 m_e \sim \epsilon \alpha^2 m_{e0} (f_\pi^3 / \mu^2 v_0)$, and using the parametrization of Γ_B, yields the constraint

$$\mu^2 \lesssim \epsilon \alpha^2 \left(\frac{m_{e0} f_\pi^2 M_P^{3/2}}{v_0 m_p^{5/2}} \right) \left(\frac{M}{M_P} \right)^2$$

$$\cong \epsilon (4 \times 10^5 \text{ GeV})^2 \left(\frac{M}{10^{16} \text{ GeV}} \right)^2 . \quad (6)$$

Thus if $M \sim 10^{16}$ GeV, the constraint that baryons still exist when the universe is cooled down to E_{chem} tells us that μ^2 must be at least 27 orders of magnitude smaller than the natural scale of M_P^2. The value in our domain is about 33 ± 1 orders of magnitude smaller than the natural scale. If biochemistry is controlled by μ's or τ's, then the above constraint on μ^2 would be weakened by a factor of $m_\mu / m_e \approx 200$, or $m_\tau / m_e \approx 3000$.

There are a number of considerations that may actually strengthen the bound on μ^2. In the first place, we have neglected the possibility that baryons might decay predominantly into channels that are blocked in our domain by the heaviness of the higher generation quarks and leptons. For example, in some supersymmetric theories the exchange of superheavy colored Higgsinos would produce very fast proton decay (in our domain) if it were not for the fact that the final states include flavors heavier than the proton.

Even more interesting is the possibility that baryons could decay *via* sphalerons [18]. Since in $\mu^2 > 0$ domains the weak interactions are broken not at v_0 but at f_π, non-perturbative weak baryon decay would not be as strongly suppressed as in our domain.

Finally, there is the question of baryogenesis. If it occurs through standard ''drift and decay'' mechanisms [19] involving grand unified interactions, then one would expect the asymmetry to be essentially independent of μ^2. But if baryogenesis takes place at the weak scale in our domain [20], it is probable that in the $\mu^2 > 0$ domains the asymmetry would be very different.

(ii) The structure of the elements. With the Higgs system not generating a vacuum expectation value, the breaking of the standard model occurs through the Goldstone bosons of the strong interactions. The W mass is of order 50 MeV. However, the quark masses are very small since they are sensitive only to the Higgs vacuum expectation value. With six essentially massless quarks, the lightest baryon multiplet will be a 70-plet, with 27 members which are neutral, 27 which have charge +1 and eight each with charge -1 and charge +2. The lightest meson multiplet starts out with 35 members, but three are ''eaten'' to become the longitudinal gauge bosons, leaving 32 pseudoscalar mesons, which are the Goldstone bosons of the dynamically broken chiral SU(6) symmetry.

[1] It is also possible to obtain a ''viability'' bound on μ^2 for $\mu^2 < 0$ from the requirement that baryons be sufficiently stable. However, it turns out that the resulting bound of about $(10^{11} \text{ GeV})^2$ is weaker than the bounds we obtained in Sec. IV from other considerations.

Electroweak interactions will split the multiplets and our estimates for these effects are based on the understanding of electromagnetic mass splittings of the observed hadrons. For the Goldstone bosons the mass splittings can be understood using effective Lagrangians with chiral symmetry, which in the world under consideration would involve a $SU(6)_L \times SU(6)_R$ chiral invariance. The purely lefthanded interactions of the charged W's will not produce masses for the Goldstone bosons, since such interactions lead to an effective chiral lagrangian which involves derivatives [21], and hence vanishes at $p^2=0$. However vectorial interactions do generate masses, in analogy to the electromagnetic mass shifts of pions and kaons. The electromagnetic and Z^0 interactions will have such vectorial effects, and display an $SU(3)_u \times SU(3)_d$ invariance for separate rotations of up-type and down-type quarks. By a generalization of Dashen's theorem [22], these interactions will leave the 16 neutral Goldstone bosons massless, while giving a common mass to all 16 charged mesons. Using the observed pion mass splitting we can estimate that the charged mesons will have a mass of about 35 MeV. The neutral mesons will develop very small masses due to chiral symmetry breaking, $m_\pi \propto (m_q f_\pi)^{1/2} \approx \lambda_q^{1/2} f_\pi^2/\mu$. Even for μ as small as 100 GeV, some meson masses will be less than a KeV. The tiny mass of the neutral mesons implies that nucleons will have a long-ranged neutral mesonic cloud, and that the nuclear force will have a correspondingly long-range potential.

The baryon mass splittings are less amenable to an analysis based only on symmetry, and we must include quark model ideas about the effects of electroweak interactions. The short-distance electroweak effects preserve the quark chirality, which implies that they will not generate quark mass shifts. The longest range effects of the vectorial interactions will be proportional to the square of the baryon's electric or weak charges. Describing these by the electric charge Q and the strong isospin t, and ignoring the masses of the W,Z, we would expect that the lightest baryons should be the 19 $(Q,t)=(0,\frac{1}{2})$ states, a group which contains the neutron. With splittings of order an MeV, the 19 $(Q,t)=(1,\frac{1}{2})$ charged baryons (containing the proton and related states) could likely be the next grouping. Hyperfine interactions and quark masses could further split the states within these groups. All of the baryon and meson states which are shifted up in mass are unstable and can decay down to the ground states via weak or electromagnetic interactions, although perhaps with very long lifetimes.

Given these building blocks we outline the nature of nuclei. In our world, consisting of just protons and neutrons, any substantial nucleus has comparable numbers of neutrons and protons, as dictated by a need to minimize the Fermi energy. There is a tendency to have fewer protons so as to simultaneously lower the Coulomb energy. In the $\mu^2>0$ domains, there are many species of neutral and charged nucleons, so the effects of degeneracy energy will only come into play for larger nuclei. At the same time, for modest size nuclei (R of a few fermi) the weak bosons are effectively long range, $m_W R \approx m_Z R < 1$, so the one must take into account weak-interaction energy as well as electromagnetic Coulomb energy.

A nucleus consisting of N baryons of type $(Q,t_3)=(0,-\frac{1}{2})$ and Z of type $(Q,t_3)=(1,\frac{1}{2})$ will have strong isospin $t_3=\frac{1}{2}(Z-N)$ and weak hypercharge $Y/2=\frac{1}{4}N+\frac{3}{4}Z$. (Since right-handed quarks have vanishing weak isospin, the effective weak isospin of a nucleon is half of its strong isospin, and $Y/2=Q-t_3/2$.) The Coulomb energy will have contributions proportional to $[g_1^2(Y/2)^2+g_2^2 t(t+1)/4]$. For large t we can approximate $t(t+1)$ by t^2. Clearly, this is minimized by making t as small as possible, namely $t=|t_3|$. This allows us to express the Coulomb energy in terms of N and Z, and minimizing with respect to Z subject to the constraint that $Z+N=A$, one finds easily that

$$Z=\frac{A}{2}\frac{g_2^2-g_1^2}{g_2^2+g_1^2}=\frac{A}{2}[1-2\sin^2(\theta)]\approx\frac{A}{4}.$$

This relation holds for intermediate A; for large A the weak force will saturate due to its finite range, and eventually we must also include degeneracy effects (filling Fermi levels) as well as other nucleon states, while for small A we must include the nucleon mass differences.

Coulomb energy may cause nuclei larger than some critical size to be unstable to spontaneous fission, as in our domain. However, above the value of A for which the weak force saturates (which we estimate to be on the order of a few hundred) the ratio Z/A is determined predominantly by minimizing electric Coulomb energy rather than weak-interaction Coulomb energy. This would lead (in these $\mu^2>0$ domains) to large nuclei having $Z \ll A$. The smallness of Z/A may well mean that there is no maximum size set to nuclei by spontaneous fission. The situation is complicated by the longer range of the strong nuclear force and the effects of degeneracy energy with many degrees of freedom. Therefore, we cannot say with certainty whether or not there is a maximum nuclear size in the $\mu^2>0$ domains.

Similarly, the spectrum of nuclei that result from primordial nucleosynthesis is not certain. Early stages of nucleosynthesis will occur with neutral baryons combining to form light nuclei, but as nuclei grow in size and charge, it is not clear to us whether or not Coulomb barriers will result in a termination of nucleosynthesis at small or modest nuclei, or whether nuclear burning will ''run away'' to give only very massive nuclei. The difficulty lies in estimating how screening and thermal contributions to meson masses will affect the long-range nuclear potential in its competition with the Coulomb barrier between light nuclei.

With the uncertainty in these issues, it seems possible that light nuclei with small charge may exist and provide a basis for chemical life. It is also possible that nucleosythesis will result in a small number of low-charge, superheavy nuclei, which does not seem conducive to the development of chemical life.

(iii) Stars burning out. Even if an interesting mix of elements develops during nucleosynthesis, the time t_{chem} is so large that it is natural to wonder if there would be any stars left by the time biochemistry could take place in a small-v domain. On the other hand, with extremely small lepton masses, stellar cooling may be so obstructed that stars never contract to the nuclear burning phase at all. We give an overview of these issues.

Stars are supported by either gas pressure, radiation pressure, or degeneracy pressure. If the support is either gas, radiation, or degeneracy of a relativistic species then as the star cools (loses energy) it contracts and heats up (T increases). If supported by degeneracy of non-relativistic fermions, then it cools but does not contract. A system of the first kind contracts until it becomes hot enough for nuclear burning to proceed against the Coulomb barriers of the nuclei. Once the fuel is gone contraction continues until either another fuel burning stage is reached, the object is supported by non-relativistic degeneracy pressure (white dwarf or neutron star), or the object contracts within its Schwarzschild radius and disappears as a black hole. Systems supported by degeneracy pressure before nuclear burning cool into planets.

Whether a cloud of gas turns into a star or a planet depends on its initial mass and (to a lesser extent) composition. Initially the cloud is non-degenerate and is supported by gas pressure. As it contracts degeneracy pressure increases as R^{-5} while gas or radiation pressure increase as R^{-4}. When the star reaches a size R_d degeneracy pressure will halt further contraction. From dimensional analysis,

$$R_d \sim \frac{N^{1/3}}{M_* m_f},$$

where $M_* = (G_N m_B^2)^{3/2} N \approx M/M_\odot$, N is the number of baryons in the star, and m_f is the mass of the degenerate fermion [23].

The temperature at this point is $T_d \sim M_*^2 m_f$, and the Fermi momentum of the degenerate fermions is $k_{f_d} \sim M_* m_f$. If T_d is greater than the temperature necessary for nuclear burning, $T_N \approx$ 1 keV [24], then a star is born before degeneracy occurs. In our domain, $m_f = m_e$ and, after including numerical factors, $T_d > T_N$ for $N_B > .08 N_\odot$. Low mass stars develop, burn, and then turn into white dwarfs. In a $\mu^2 > 0$ domain, the lepton may be e, μ, or τ, but in any case $m_f < 1$ eV. T_d will therefore be too cool to support nuclear burning. Protostars with $M_* < 1$ turn into planets.

If $M_* > 1$ the leptons become relativistic before they become degenerate, and the collapse cannot be halted until nuclear fusion takes place. After a burning phase contraction continues until either another nuclear burning phase occurs or the core contracts inside its Schwarzschild radius and a black hole results. If M_* is not too much greater than 1, a neutron star may form.

Since $M_* > 1$ for nuclear fusion, stellar burning will take place in a plasma of relativistic leptons and anti-leptons. This plasma is not degenerate: the degeneracy parameter is $k_f/T \approx 1/M_*$. As the leptons are relativistic, opacities for photons will be large, dominated by the photon-lepton cross-section α^2/T^2. On the other hand neutrino interactions are also much larger, so neutrinos dominate stellar cooling in the nuclear burning phase.

At temperatures less than M_W cross sections for neutrino pair production, scattering from leptons or baryons, absorption, etc., will be of order T^2/f_π^4. For stars of $M_* = 1$ the density of baryons and thermal leptons will be comparable, but for larger stars thermal particles will dominate, so we may estimate mean free paths and emissivities from thermal pair processes. Mean free paths for weak interactions are $\lambda \sim f_\pi^4/T^5 \approx 10^{12}/T_{\rm keV}^5$ cm; i.e., solar type stars are likely to cool by volume emission or have a very deep neutrino sphere. The energy loss rate per lepton from pair annihilation to neutrinos is $\approx 10^{-8} T_{\rm keV}^6$ GeV s^{-1}; i.e., solar mass stars cool on time scales of roughly a year, and larger stars in much less time. This is very much less than t_{chem}.

The next issue is star formation. Of particular concern are the cooling rates during collapse, when temperatures are too low for efficient neutrino emission, and cooling is regulated by the photon opacity either in the interior or at the surface. Opacities are determined by the chemistry of the lightest charged lepton. For us this is the electron mass, although in some scenarios the active species will in fact be positrons bound in either μ or τ-onium.

There are two stages in the collapse of a cloud to the point of nuclear ignition. At first, temperature gradients are not large and convection provides the energy transport. The photosphere is essentially held at a fixed temperature $T_e \approx \alpha^2 m_e$ at which material is no longer ionized and the opacity drops. The luminosity is $L \sim R^2 T_e^4$. Cooling is initially very fast but slows as the star shrinks. Eventually, temperature gradients increase to where radiative transport is effective and convection is cut off. At that point the cooling time scale is the Kelvin-Helmholtz time for photon diffusion to the surface:

$$t_{KH} \sim \frac{R^2}{\lambda} \sim \frac{N}{R}\sigma \approx \frac{10^{38}\alpha^2 T}{T^2 + m_e^2},$$

where λ is the mean free path and σ is the cross section for photon-lepton scattering. To derive the last relation: note that if $T < m_e$ then $\sigma \sim \alpha^2/m_e^2$ but for higher temperatures $\sigma \sim \alpha^2/T^2$, and that $RT \approx 10^{19} M_*$ is roughly constant during collapse. The cooling time is dominated by the epoch when $T = m_e$, or $t_{cool} \approx 10^{34}/m_e \approx 10^{17}\mu^2/\mu_0^2$ yr. This is less than t_{chem} [see Eq. (5)], but not by so much that the energy from stellar burning may not be available for life to form.

Thus, within this crude treatment of stellar evolution, stars are expected to form slowly, but burn nuclear fuel very quickly. The actual stage of nuclear burning seems too short to benefit life, but there are other possible energy sources than surface heating of planets by stars. For example, planets may be volume heated by radioactivity, residual gravitational energy, baryon decay, or even absorption of the background of neutrinos produced by stellar burning. The energy flow to the surface would in principle be usable for the evolution of life.

V. QUARK MASSES

We have been discussing theories in which the Higgs mass parameter μ^2 can vary in different regions of the universe, under the assumption that the Yukawa couplings do not change from the values that they have in our portion of the universe. It is possible that the underlying theory also allows the values of the Yukawa couplings to vary in different domains. In this section, we discuss some of the possible implications of this situation. However, we stress that without knowing the details of the fundamental theory we do not know whether these masses are in fact subject to variation or whether they occur in fixed ratios due to some other mechanism. For example, we can see no anthropic argument which

would force neutrinos to be as light as they seem to be (if indeed they have a mass). However, a ''see-saw'' mechanism [9] would make them very light automatically, and we would not need consider anthropic reasoning.

The masses of the quarks and leptons of the second and third generations have very little impact on the particles and reactions which occur naturally in our universe. Therefore anthropic arguments would not place constraints on their masses. In an ensemble of viable domains, these masses could be randomly distributed. In practice, the observed masses appear to be distributed without any obvious pattern, and randomness appears as good an ''explanation'' as anything else.

The masses of the light quarks are constrained by the physics which has been discussed above. In particular, the requirement that the deuteron exist yields an upper bound on the sum of the up and down quark masses:

$$\frac{(m_u+m_d)_{max}}{(m_u+m_d)_{real}} \leqslant 2. \tag{7}$$

If the d quark mass was lighter, or the both the masses were small, the proton would be heavier than the neutron, and could decay into it. Hydrogen would then not exist as a stable atom (at least for small values of the electron mass). However, the complex elements would still exist, and there would seem to be sufficient building blocks for some form of life. Thus, we will use the above inequality as the sole constraint on the light quark masses.

The constraints on the electron mass come from other sources. An electron mass larger that the binding energy of a nucleon, around 10 MeV, would lead to the decay of atoms, through the process $e^- + p \to n + \nu$, with the final neutron ejected from the nucleus. However, stronger constraints can be obtained in nucleosynthesis. If the electron mass is higher than the temperature at the time of nucleosynthesis, the electrons will have all disappeared, converting via $e^- + p \to n + \nu$ leaving only neutrons behind. Likewise, the reactions which would burn neutrons, such as $n + n \to d + e^- + \overline{\nu_e}$ which would be the neutron equivalent of the start of the pp cycle, use weak interaction transitions and would be shut off if the electron mass is large. The precise constraint depends on the neutron and proton masses, but is generically in the range of a few MeV.

VI. CONCLUSIONS

In a universe with domains which can have different values of some of the underlying parameters, life may only be able to develop in some of those domains. If this is the case, we would expect that the parameters of our domain should be typical of the viable range. If the viable range is so large that our values of the parameters are unnaturally small within this range, then this hypothesis fails to help us understand the observed sizes of these parameters.[2] However, we have found that within the overall structure of the standard model there is a relatively small acceptable range for the Higgs parameter μ^2 and the light quark masses. The physical values of these parameters are quite typical of this range, raising the possibility that the anthropic bias of multiple domain theories could be an ''explanation'' of these magnitudes.

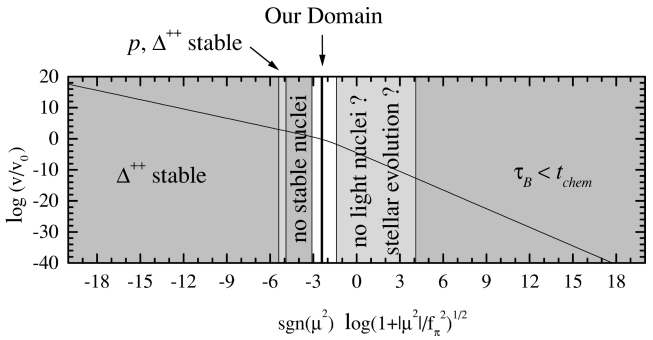

FIG. 1. A summary of arguments that $|\mu^2| \ll M_P$ is necessary for life to develop. For $\mu^2 < 0$, large values of $|\mu^2|$ imply large values of v, and hence larger masses for leptons, quarks, and baryons. The increasing difference between the light quark masses, $m_d - m_u \propto v/v_0$, implies universes with but a single species of stable nuclei, which we argue would not allow for chemistry rich enough to support life. There is a narrow band where both p and Δ^{++} are stable, but the chemical equivalent of a mix of hydrogen and helium is plausibly also sterile. For $\mu^2 > 0$, quark chiral condensates lead to $v \propto f_\pi^3/\mu^2$ and quark and lepton masses become very small. Light lepton masses imply that biochemical processes cannot occur until cosmologically late times, when baryons may have already decayed. We show a constraint for a baryon lifetime estimated from exchange of intermediate grand unified theory scale ($M_X \approx 10^{16}$ GeV) particles. Even if baryons are stable, formation of a biologically acceptable mix of elements or the nature of stellar evolution may make development of life improbable. What is left is a rather narrow range of $\mu^2 < 0$ which includes the physical values in our universe.

The arguments behind this conclusion are summarized in Fig. 1. For μ^2 negative, as in our domain, it seems that the whole range of values for the vacuum expectation value from M_P down to about 5 (or perhaps even down to 1.2) times the value in our domain can be excluded. For most of that range (down to about $10^3 v_0$) the universe would consist of sterile, helium-like atoms whose nuclei were Δ^{++}. There would be essentially no reactions either chemical or nuclear. For the lower part of the excluded range, there would be virtually no nuclei other than protons, and the pp and pn processes that are needed for nucleosynthesis would be endothermic as the deuteron would not be stable.

For positive values of μ^2 the condition that baryons still exist at the time biochemistry becomes possible forces μ^2 to be many orders of magnitude smaller than the ''natural'' Planck scale. [Cf. Eq. (4).] Our arguments for smaller values of μ^2 are less certain. It may be that long range nuclear forces cause all baryons to clump into superheavy nuclei with small charge, which does not appear to be promising for life. If these forces are screened in a mesonic plasma, then light nuclei will continue to exist, and can burn in stars, and stars may ignite at sufficiently late times to fuel life. Individual stars, however, will be extremely short lived compared to the cosmological time scales. If life is to develop in such a universe, the energy source is not likely to be the

[2]For example, the θ parameter in QCD appears to be much smaller than its viable range, and thus the multiple domain theories do not appear to provide any insight into the strong CP problem.

photoluminescence of an individual star.

One can thus plausibly argue that for life to exist the μ^2 parameter has to be negative and has to be close to the value it has in our universe.

One of the interesting features of our multiple-domain hypothesis is that it explains — as no other approaches do at present — the curious fact that the weak scale is near to the QCD scale. In order for protons and neutrons (rather than Δ^{++}) to be the lightest baryons, $m_d - m_u$ has to be less than the chromodynamic energy which splits the baryon decuplet from the octet. For the deuteron to have large enough binding energy to save neutrons from virtual extinction in the early universe and also to allow the pp reactions to be exothermic, the pion has to have a long Compton wavelength compared to the nucleon, and this in turn means that the u and d masses have to be not only less than but small compared to the QCD energy scale by about the amount seen in nature. This also provides a possible resolution to the ''fine-tuning problem'' — in an ensemble of different domains of the universe, the Higgs mass parameter will occasionally fall into the viable range without having to be fine-tuned in general. If the cosmological constant is confirmed to have a non-zero value close to present estimates, and no new physics is found in the TeV energy region, we may be faced with de-facto evidence for the presence of fine-tuning. In such a situation, the anthropic or multiple-universe considerations become highly attractive.

Finally, let us comment on the ability of these ideas to be tested. Many of the ideas presently being explored theoretically, such as the hidden sector of supersymmetry breaking or compactification in string theory, are difficult to probe directly and the idea of multiple domains is in general similarly difficult. Negatively, we can say that if the electroweak scale is what it is for ''anthropic'' reasons, there would be no need to invoke supersymmetry or technicolor or other structure at the electroweak scale to make the fine-tuning ''natural'' [1]. If no such structure is found, then, it would be a point in favor of an anthropic explanation; indeed, in that case there would be few if any physical alternatives. Positive evidence is harder to come by. Of course, we are not able to explore other domains in the universe. However, the multiple domain structure will ultimately be contained as part of specific physical theories, such as chaotic inflation, and the consequences of these theories can be testable by other more conventional means. For example, the community is hoping to be able to test the details of inflationary theories through cosmological measurements and perhaps these could give indications as to the validity of chaotic inflation. Likewise, direct physical experimentation, such as rare decays and the pattern of neutrino masses, has the potential to eventually sort out the correct underlying theory. The consideration of this class of theories is thus far largely unexplored, and we cannot yet predict how successful the idea will be. Ideally, through standard means we may be able to learn if the fundamental theory in fact produces multiple domains, in which case these considerations automatically become relevant. Until the time that this happens, our conclusion must be modest: the observed values of the mass parameters are reasonably typical of the viable range.

APPENDIX A

In this paper we make the following assumptions: (a) physics below the unification scale (which may be at or below the Planck scale) is described by the standard model of particle physics together with Einstein's theory of gravity and some set of dimension-6 operators (suppressed by M^{-2}, where M is between 10^{16} and 10^{19} GeV) that lead to baryon number violation; (b) the universe has a multiple domain structure; (c) the parameter μ^2 of the standard model Higgs potential varies among the domains over the whole allowed range from $+M_P^2$ to $-M_P^2$, while the other parameters of physics (as evaluated at the Planck scale) are fixed; (d) life requires complex chemistry, and in particular chemistry involving more than just hydrogen and helium; and in a portion of the work we assume that (e) life requires stars as a source of energy (or, rather, of low entropy).

It is clear that assumption (a) could be changed in various ways without affecting our arguments. For example, it would seem to matter little what one assumes about the existence of axions, or about the existence of right-handed neutrinos. Clearly, there could be many kinds of new physics that would have little effect on the ordinary sector of particles. Thus assumption (a) is unnecessarily strong. However, we are not interested at this point in finding the *weakest* set of assumptions that would allow one to explain the observed value of μ^2 by means of a multi-domain hypothesis, but only in exploring whether there is *some* reasonable set of assumptions that might allow one to do so.

Assumption (c) is undoubtedly also unnecessarily strong. One can consider instead the possibility that not only μ^2 but other parameters vary among domains. For example, if both μ^2 and α_s varied, there would be domains where the strong and weak scale differed from their values in our domain but had the same ratio. It would be interesting to examine the consequences of such possibilities. However, again, we are interested only in finding *some* reasonable multi-domain hypothesis that might explain the gauge hierarchy. Variant models may ultimately prove to be superior.

The validity of assumption (d) can certainly be called into question. For example, there are interesting speculations due to Feinberg and Shapiro [25] that complex structures might arise in liquid hydrogen from the interactions of orthohydrogen and parahydrogen molecules. If this is true, life could conceivably arise composed only of hydrogen. However, even such a radical possibility would not undercut most of our arguments. It would have no relevance to the ''Δ^{++} domains'' discussed in Sec. III, as they contain only exotic helium rather than hydrogen. Even the ''proton domains'' discussed in the same section would still be sterile if assumption (e) is valid, since they would not be able to produce hydrogen-burning stars. For the $\mu^2 > 0$ domains to allow *any* kind of biochemistry, however exotic or simple, there would still have to baryons around when the universe had cooled sufficiently to allow the biochemistry. Thus, even if life can emerge from fairly simple biochemistry, most of our discussion would carry through.

One can imagine the even wilder possibility of life based on nuclear rather than chemical reactions, though it is hard to envision how this would be realized. But even if such a possibility were realizable, most of the range of negative μ^2

would still be excluded, as in both the Δ^{++} and proton domains there is only one stable nucleus and therefore no nuclear reactions. The positive μ^2 case is obscure.

It is clear that one cannot exclude rigorously all such exotic possibilities. However, it is not clear that is necessary to do so in order to claim to have a reasonable hypothesis for explaining the value of μ^2. We would maintain that to have a reasonable hypothesis it is only necessary to find a set of plausible assumptions that would explain the data in a simple way, not to rigorously prove the validity of all of those assumptions *in advance*. It is true, however, that to the extent that exotic possibilities for life which are not based on chemistry can be made plausible — or even better, are actually found to be realized in nature — then the kind of hypothesis we are exploring would become less plausible, and perhaps could be overthrown altogether. This at least says that our hypothesis is falsifiable.

Perhaps the most dubious assumption is (e), since there are various sources of energy in our world besides stars. Geothermal energy is one example. (Though, of course, the radioactive isotopes whose decay heats the earth's interior were synthesized in stellar interiors.) The validity of assumption (e) is relevant for small positive values of μ^2. [See Sec. IV (iii), ''Stars burning out.''] One would hope that arguments which exclude small positive μ^2 could be found which do not make use of assumption (e). Perhaps, when more is understood about baryon-number violation the viability bounds coming from baryon stability [Sec. IV (i)] can be strengthened.

We should comment on some previous work that bears some resemblance to our own. Certainly ''anthropic'' arguments based on the stability of the deuteron are not new. They have been discussed by many authors, including Rozental [26], Barrow and Tipler [4], and Okun [27]. Generally these discussions consider a variation in the strength of the nucleon-nucleon potential due to varying α_s rather than μ^2 as we do. Moreover, the discussions in the literature of which we are aware contemplate only what happens for small variations of parameters away from their observed values, as opposed to considering the whole possible range of a parameter as we do. In particular, the observation that most of the range of $\mu^2<0$ gives rise to Δ^{++} domains that are sterile in both a nuclear and chemical sense is new. Moreover, as far as we know, no one has considered before the question of what happens for positive μ^2. One must, of course, do this if one is to explain the magnitude of μ^2. More generally, we are unaware of any previous discussion which frames the question explicitly in terms of solving the gauge hierarchy problem.

There exist in the literature [28] models which have ''mirror'' sectors with $SU(3)'\times SU(2)'\times U(1)'$ interactions, the same particle content as the ordinary sector, but large values of the vacuum expectation value that breaks $SU(2)'\times U(1)'$. These were constructed as models of sterile neutrinos in order to explain recent experimental results on neutrino oscillations. The mirror sector looks just as would the ordinary sector if v were simply scaled up in value. (The requirement that the weak scale of the mirror sector be large arises from observational constraints.) Mohapatra and Teplitz [28] have explored some of the phenomenology of such a mirror sector with large v_{mirror}, and, in fact, have noted that for large enough v_{mirror} they have no stable nuclei except the mirror analogue of protons — our ''proton domain'' case. They did not, however, discuss the ''Δ^{++} domain'' case, and, of course, did not consider the small v case. Moreover, they were not considering multi-domain or ''anthropic'' arguments, and were discussing only the phenomenology of the mirror sector rather than trying to explain the value of v in the ordinary sector.

Cahn [29] has discussed the implications of various parameters of the standard model having different values. He has a very brief discussion of the implications of varying v. However, he only contemplates very small variations of v, and the only effect he considers is the change in reaction rates in the sun due to changing the Fermi constant, G_F.

The general kind of multi-domain hypothesis we are proposing has been discussed by many physicists. This includes Sakharov [30], Zeldovich [31], Carter [4], Carr and Rees [5], Barrow and Tipler [5], Rozental [26], Okun [27], Linde [32], Weinberg [3], Vilenkin [4], and Shrock [33]. We would imagine that some of these authors have entertained ideas quite similar to some of those discussed here.

APPENDIX B

Let us make a simple qualitative estimate of the deuteron binding energy. A well-known pedagogical model of the deuteron involves a square well of depth $V_0=35$ MeV and range $R=2$ fm, with a hard core of radius r_0. Let us neglect the D-wave component, so that the S-wave solutions are

$$u_<(r)=r\psi_<(r)=A\sin\kappa(r-r_0), \quad r_0\leqslant r\leqslant R, \tag{B1}$$

and

$$u_>(r)=r\psi_>(r)=Be^{-\gamma r}, \quad r\geqslant R, \tag{B2}$$

with $\kappa\equiv\sqrt{m(V_0-B_d)}$, and $\gamma\equiv\sqrt{mB_d}$. The boundary condition is therefore

$$\kappa\cot\kappa(R-r_0)=-\gamma. \tag{B3}$$

There will be a range, R_c, at which the binding energy goes to zero: i.e.,

$$\kappa_c\cot\kappa_c(R_c-r_0)=0, \quad \kappa_c\equiv\sqrt{mV_0}. \tag{B4}$$

If we let $\delta R=R-R_c$, we can solve for the present value of R *via* a Taylor series. To first order in $\sqrt{B}$ we have

$$\cot\kappa_c(\delta R+(R_c-r_0))\simeq-\sqrt{\frac{B}{V_0}}\Rightarrow\tan\kappa_c\delta R\simeq-\kappa_c\delta R \simeq-\sqrt{\frac{B}{V_0}}. \tag{B5}$$

Now, the outer range of the potential is determined by the pion mass, $R\propto 1/m_\pi$. If we search for the value of the pion mass at which the deuteron becomes unbound, we equate

$$\frac{\delta m_\pi}{m_\pi}=\frac{\delta R}{R}=\sqrt{\frac{B}{V_0}}\frac{1}{\kappa_c R}\approx 0.2. \tag{B6}$$

Thus, according to this calculation, only a 20% increase in

the pion mass would cause an unbound deuteron. And since the pion mass and the quark masses very nearly obey the relation

$$m_\pi^2=(m_u+m_d)\frac{\langle 0|\bar{\psi}\psi|0\rangle}{F_\pi^2}\propto v, \tag{B7}$$

a 20% increase in the pion mass corresponds to a 40% increase in v, or a factor of 2 increase in $|\mu^2|$.

We shall parametrize the vacuum expectation value as

$$B_d\cong\left[2.2-a\left(\frac{v-v_0}{v_0}\right)\right]\ \text{MeV}, \tag{B8}$$

for small $v-v_0$, where a is some positive constant. The square-well calculation gives $a\simeq 5.5$.

[1] G. 't Hooft, in *Recent Developments in Gauge Theories*, proceedings of the Cargèse Summer Institute, Cargèse, France, 1979, edited by G. 't Hooft *et al.* (NATO Advanced Study Institute, Series B: Physics, Vol. 59) (Plenum, New York, 1980); H. Georgi and S. L. Glashow, Phys. Rev. D **6**, 2977 (1972).

[2] For a comprehensive review see S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).

[3] A. Linde, Phys. Lett. **129B**, 177 (1983); Phys. Lett. B **175**, 395 (1986); **202**, 194 (1988); Phys. Scr. **15**, 169 (1987).

[4] S. Weinberg, Phys. Rev. Lett. **59**, 2607 (1987); S. Weinberg, astro-ph/9610044, to be published in the proceedings of the conference *Critical Dialogues in Cosmology* at Princeton University, 1996; H. Martel, P. Shapiro, and S. Weinberg, astro-ph/9701099.

[5] B. Carter, in *I.A.U. Symposium, Vol. 63*, edited by M. Longair (Reidel, Dordrecht, 1974); J. Barrow and F. Tipler, *The Anthropic Cosmological Principle* (Clarendon, Oxford, 1986); B. J. Carr and M. J. Rees, Nature (London) **278**, 605 (1979); A. Vilenkin, Phys. Rev. Lett. **74**, 846 (1995).

[6] E. Gildener and S. Weinberg, Phys. Rev. D **13**, 3333 (1976); E. Gildener, *ibid.* **14**, 1667 (1976).

[7] Changes in μ^2 can indirectly make small changes in the dimensionless parameters of the standard model through the renormalization group. These are generally negligible. The cases where they are not will be indicated explicitly.

[8] Even though μ^2 is the only dimensionful parameter that appears explicitly in the Lagrangian of the theory, a scale, Λ_{QCD}, is dynamically produced by the strong interactions through ''dimensional transmutation.'' This sets the scale for chiral-symmetry breaking and for the baryon masses. However, the masses of the elementary particles (quarks, leptons, Higgs particle, and weak gauge bosons) are set by the Higgs potential.

[9] The light neutrinos are generally believed to get a ''seesaw'' mass of order v^2/M_R, where M_R is the scale of lepton-number violation. But these are negligible for our purposes.

[10] H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).

[11] See, for example, K. S. Babu and S. M. Barr, Phys. Rev. Lett. **75**, 2088 (1995); G. Anderson, S. Raby, S. Dimopoulos, and L. J. Hall, Phys. Rev. D **49**, 3660 (1994); Z. Berezhiani, hep-ph/9602325; and references therein.

[12] R. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); S. Weinberg, *ibid.* **40**, 223 (1978); F. Wilczek, *ibid.* **40**, 279 (1978).

[13] S. M. Barr, in *CP Violation in Particle Physics and Astrophysics*, edited by J. Tran Thanh Van (Editions Frontières, Gif-sur-Yvette, France, 1989), and references therein.

[14] S. Weinberg, Phys. Rev. D **13**, 974 (1976); S. Dimopoulos and L. Susskind, Nucl. Phys. **B155**, 237 (1979).

[15] S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981); Phys. Lett. **117B**, 287 (1982).

[16] V. Agrawal, Ph.D. thesis, University of Delaware, 1996.

[17] R. J. Scherrer and M. S. Turner, Phys. Rev. D **31**, 681 (1985).

[18] V. Kuzmin, V. Rubakov, and M. Shaposhnikov, Phys. Lett. **155B**, 36 (1985); Phys. Lett. B **191**, 171 (1987); P. Arnold and L. McLerran, Phys. Rev. D **36**, 581 (1987); **37**, 1020 (1988).

[19] S. Dimopoulos and L. Susskind, Phys. Lett. **81B**, 416 (1979); D. Toussaint, S. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D **19**, 1036 (1979); S. Weinberg, Phys. Rev. Lett. **42**, 850 (1979).

[20] For a recent review see A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Annu. Rev. Nucl. Part. Sci. **43**, 27 (1993).

[21] J. F. Donoghue, E. Golowich, and B. R. Holstein, *Dynamics of the Standard Model* (Cambridge University Press, Cambridge, England, 1992).

[22] R. Dashen, Phys. Rev. **183**, 1245 (1969); J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982); J. F. Donoghue, in *Building Blocks of Creation - TASI 1993*, edited by S. Raby (World Scientific, Singapore, 1995), hep-ph/9403263.

[23] We have explicitly ignored questions of composition and taken the number of baryons to be equal to the number of fermions. We have also ignored numerical factors that arise from geometry (e.g. 4π) or from more realistic solutions to hydrostatic equilibrium which would result in central concentration of the material.

[24] With light mesons, and long range nuclear reactions, burning may take place at slightly cooler temperatures. This is the same issue faced in primordial nucleosynthesis. If screening of the nuclear force is effective, then light nuclei exist and stars burn with $T_N\gtrsim 1$ keV. If screening is not efficient then the whole discussion of stars must be greatly modified.

[25] G. Feinberg and R. Shapiro, *Life Beyond Earth* (Wm. Morrow, New York, 1980).

[26] I. L. Rozental, Usp. Fiz. Nauk **131**, 239 (1980) [Sov. Phys. Usp. **23**, 296 (1980)]; *Geometry, Dynamics and Universe* (Moscow, Nauka, 1987) (in Russian); *Big Bang, Big Bounce* (Springer, Berlin, 1988).

[27] L. Okun, in *Sakharov Memorial Lectures in Physics*, edited by L. V. Keldysh and V. Ya. Fainberg (Nova Science Publishers, Inc., Commack, New York, 1992), Vol. 2, pp. 819–840.
[28] Z. Berezhiani and R. N. Mohapatra, Phys. Rev. D **52**, 6607 (1995); R. N. Mohapatra and V. L. Teplitz, Astrophys. J. **478**, 29 (1997).
[29] R. Cahn, Rev. Mod. Phys. **68**, 951 (1996), cf. p. 957.
[30] A. D. Sakharov, ''Nobel lecture,'' read by Ye. Bonner on Dec. 11, 1975.
[31] Ya. B. Zeldovich, Pis'ma Astron. Zh. **7**, 579 (1981)[Sov. Astron. Lett. **7**, 322 (1980)].
[32] A. Linde, *Particle Physics and Inflationary Cosmology* (Gordon and Breach, New York, 1990).
[33] R. Shrock (private communication).