Different low-x behavior of the spin structure functions $g_1(x)$ and $h_1(x)$

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We consider the low-x behavior of the spin structure functions $g_1(x)$ and $h_1(x)$ in the unitarized chiral quark model that combines the ideas on constituent quark structure of hadrons with a geometrical scattering picture and unitarity. A nondiffractive singular low-x dependence of $g_1^p(x)$ and $g_1^n(x)$ is obtained and a diffractive type smooth behavior of $h_1(x)$ is predicted at small x. A comparison with the experimental data is given. The estimations for the double-spin asymmetries in the low-mass Drell-Yan production at BNL RHIC in the central region are presented also. [S0556-2821(98)01711-1]

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INTRODUCTION

Low-x behavior of the spin structure functions $g_1(x)$ and $h_1(x)$ is an important issue under studies of the nucleon spin structure. Experimental evaluation of the first moments of g_1 and h_1 (and the total nucleon helicity carried by quarks and tensor charge, respectively) is sensitive to the particular extrapolation of the structure functions $g_1(x)$ and $h_1(x)$ to $x=0$. This extrapolation is a nontrivial matter in the view of the CERN Spin Muon Collaboration (SMC) [1], SLAC E154 [2], and HERMES [3] data indicating a possible rising behavior of $g_1(x)$ at small x. The most recent SMC [4] data cast doubts on such behavior but do not exclude it due to both rather large error bars and the limited kinematical region which covers the values of $x>10^{-3}$ only.

The experimental data for $\Delta\sigma_L(s)$ and $\Delta\sigma_T(s)$ could also be a useful source of information on the low-x behavior of the spin structure functions. Unfortunately, only the low-energy data are available at the moment [5]. Were the experimentally observed decreasing behavior of $\Delta\sigma_L(s)$ and $\Delta\sigma_T(s)$ also valid at high energies, it would be possible to conclude that

$$xg_1(x)\to 0 \quad \text{and} \quad xh_1(x)\to 0. \tag{1}$$

The essential point in the study of low-x dynamics is that the space-time structure of the scattering at small values of x involves large distances $l\sim 1/Mx$ on the light cone [6] and the region $x\sim 0$ is therefore sensitive to nonperturbative dynamics.

The standard part of the experimental analysis of the polarized deep-inelastic scattering is the use of a smooth Regge behavior (corresponding to the contribution of the Regge pole of unnatural parity to the difference of the helicity amplitudes) $g_1\sim x^{-\alpha_{a_1}}$ $(-0.5<\alpha_{a_1}<0)$ for extrapolation of the data to $x=0$ [7].

Meanwhile the non-Regge behavior of $g_1(x)$ at small x and its connection with a diffractive contribution were considered in [8] and continue to be under active discussion [9] since the first Europan Muon Collaboration (EMC) data have been published [10].

The general principles such as unitarity and analyticity are useful tools and provide some constraints [11,12] on the behavior of the structure functions. In particular, unitarity provides the following upper bounds for $g_1(x)$ and $h_1(x)$ [12] at $x\to 0$:

$$g_1(x)\leqslant \frac{1}{x}\ln(1/x) \quad \text{and} \quad h_1(x)\leqslant \frac{1}{x}\ln(1/x). \tag{2}$$

The experimental data are in agreement with these bounds.

In perturbative QCD the functional dependencies based either on the standard Dokshitzir-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution [13] or resummation of the leading $1/x$ logarithm contributions [14–16] provide the increasing behavior of the structure function $g_1(x)$ at small x. The rising behavior was also predicted in the nonperturbative approaches [11,17].

On the other hand the spin structure function $h_1(x)$, approximately constant at small x, was obtained in perturbative QCD [18]. $h_1(x)$ was calculated in the MIT bag model [19], chiral chromoelectric model [20], and within the QCD sum rules method [21] as well.

As has been mentioned, a number of models associate the increase of $g_1(x)$ with the diffractive contribution at small values of x. Such a contribution, being dominant at the smallest values of x, would lead to the ‘‘equal’’ structure functions $g_1^p(x)$ and $g_1^n(x)$ in this kinematical region: i.e.,

$$g_1^p(x)/g_1^n(x)\to 1$$

at $x\to 0$. Such behavior has not been confirmed in the recent experiments. In particular, the SMC data demonstrate the following approximate relation in the region of $0.003\leqslant x\leqslant 0.1$:

$$g_1^p(x)\simeq -g_1^n(x). \tag{3}$$

In this connection it might happen that the diffractive contribution would not be able to serve as a single explanation of the observed experimental regularities at $x\to 0$. In the light of the available experimental data one could assume importance of a nondiffractive contribution to $g_1(x)$ at small x.

In this paper we consider a nonperturbative model and show that the non-Regge, nondiffractive behavior of the spin structure function $g_1(x)$ can be described in the unitarized chiral quark model [22] which combines ideas on the constituent quark structure of hadrons with the geometrical scattering picture and unitarity. As a consequence different functional dependence is predicted for the structure function $h_1(x)$ at small x which contrary to g_1 has a diffractive origin and obey the relation $h_1^p(x)/h_1^n(x)\to 1$ at $x\to 0$. We discuss also some predictions which might be interesting for the forthcoming RHIC spin experiments. For that purpose we consider the corresponding quark spin densities $\Delta q(x)$ and $\delta q(x)$ at small x which determine the spin asymmetries in hadron-hadron interactions, in particular, double-spin asymmetries A_{LL} and A_{TT} which are to be measured at RHIC in Drell-Yan processes with low-mass lepton pairs.

I. OUTLINE OF THE MODEL

To obtain the explicit forms for the quark spin densities $\Delta q(x)$ and $\delta q(x)$ it is convenient to use the relations between these functions and discontinuities of the helicity amplitudes of the antiquark-hadron forward scattering [23]. These relations are based on the dominance of the ''handbag'' diagrams in deep-inelastic processes and have the following form:

$$q(x)=\frac{1}{2}\,\mathrm{Im}[F_1(s,t)+F_3(s,t)]|_{t=0},$$

$$\Delta q(x)=\frac{1}{2}\,\mathrm{Im}[F_3(s,t)-F_1(s,t)]|_{t=0},$$

$$\delta q(x)=\frac{1}{2}\,\mathrm{Im}\,F_2(s,t)|_{t=0}, \tag{4}$$

where $s\simeq Q^2/x$ and F_i are the helicity amplitudes for the elastic quark-hadron scattering in the notation used for the nucleon-nucleon scattering, i.e.,

$$F_1\equiv F_{1/2,1/2,1/2,1/2},\quad F_2\equiv F_{1/2,1/2,-1/2,-1/2},$$

$$F_3\equiv F_{1/2,-1/2,1/2,-1/2},\quad F_4\equiv F_{1/2,-1/2,-1/2,1/2}$$

and

$$F_5\equiv F_{1/2,1/2,1/2,-1/2}.$$

The unitarity is explicitly taken into account using the unitary representations for the helicity amplitudes, which follow from their relations to the U matrix [24]. In the impact parameter representation,

$$F_{\Lambda_1,\lambda_1,\Lambda_2,\lambda_2}(s,b)=U_{\Lambda_1,\lambda_1,\Lambda_2,\lambda_2}(s,b)+i\rho(s)\sum_{\mu,\nu}U_{\Lambda_1,\lambda_1,\mu,\nu}(s,b)\times F_{\mu,\nu,\Lambda_2,\lambda_2}(s,b), \tag{5}$$

where λ_i and Λ_i are the quark and hadron helicities, respectively, and b is the impact parameter of quark-hadron scattering. The kinematical factor $\rho(s)$ is close to unity at high energies. Explicit solution of Eqs. (5) has a rather complicated form, however, in the approximation when the helicity-flip functions are less than the helicity nonflip ones we get simplified expressions

$$F_{1,3}(s,b)=U_{1,3}(s,b)/[1-iU_{1,3}(s,b)], \tag{6}$$

$$F_2(s,b)=U_2(s,b)/[1-iU_1(s,b)]^2. \tag{7}$$

Unitarity requires Im $U_{1,3}(s,b)\geqslant 0$. The amplitudes $F_i(s,t)$ are the corresponding Fourier-Bessel transforms of the functions $F_i(s,b)$:

$$F_i(s,t)=\frac{s}{\pi^2}\int_0^\infty bdbF_i(s,b)J_0(b\sqrt{-t}), \tag{8}$$

where $i=1,2,3$.

The main points of the model used [22] which allows us to get explicit form for the U matrix are the following.

We consider quark as a structured hadronlike object since at small x the photon converts to a quark pair at long distance before it interacts with the hadron. At large distances perturbative QCD vacuum undergoes transition into a nonperturbative one with formation of the quark condensate. Appearance of the condensate means the spontaneous chiral symmetry breaking and the current quark transforms into a massive quasiparticle state—a constituent quark. The constituent quark is embedded into the nonperturbative vacuum (condensate) and therefore we can treat it in a way similar to a hadron. Arguments in favor of such a picture have been given in [6,22,25]. Thus, quark-hadron scattering at small x can be considered similar to the hadron-hadron scattering.

A hadron is consisting of the constituent quarks located at the central part embedded into a quark condensate [26,27]. We refer to effective QCD approach and use the Nambu–Jona-Lasinio (NJL) model [28] as a basis. The Lagrangian in addition to the four-fermion interaction of the original NJL model includes the six-fermion $U(1)_A$-breaking term. Transition to partonic picture in this model is described by the introduction of a momentum cutoff $\Lambda=\Lambda_\chi\simeq 1$ GeV, which corresponds to the scale of chiral symmetry spontaneous breaking.

The constituent quark masses can be expressed in terms of quark condensates [29]; e.g.,

$$m_U=m_u-2g_4\langle 0|\bar{u}u|0\rangle-2g_6\langle 0|\bar{d}d|0\rangle\langle 0|\bar{s}s|0\rangle. \tag{9}$$

In this approach constituent quarks appear as quasiparticles, i.e., as current quarks and the clouds of quark-antiquark pairs which consist of a mixture of quarks of the different flavors. It is worth to stress that in addition to u and d quarks the constituent quark (U, for example) contains pairs of strange quarks [cf. Eq. (9)]. Quantum numbers of the constituent quarks coincide with the quantum numbers of the respective current quarks due to conservation of the corresponding currents in QCD. The only exception is the flavor-singlet, axial-vector current, it has a Q^2 dependence due to axial anomaly which arises under quantization [30].

Quark radii are determined by the radii of the clouds surrounding it. We assume that the strong interaction radius of quark Q is determined by its Compton wavelength $r_Q=\xi/m_Q$, where constant ξ is universal for different flavors. The quark form factor $F_Q(q)$ is taken in the dipole form, i.e.,

$$F_Q(q)\simeq(1+\xi^2\vec{q}^2/m_Q^2)^{-2} \tag{10}$$

and the corresponding quark matter distribution $d_Q(b)$ is of the form [27]

$$d_Q(b)\propto\exp(-m_Q b/\xi). \tag{11}$$

Spin of constituent quark J_U in this approach is given by the following sum:

$$J_U=1/2=J_{u_v}+J_{\{\bar{q}q\}}+\langle L_{\{\bar{q}q\}}\rangle=1/2+J_{\{\bar{q}q\}}+\langle L_{\{\bar{q}q\}}\rangle\,. \tag{12}$$

The value of the orbital momentum contribution into the spin of constituent quark can be estimated according to the relation between contributions of current quarks into a proton spin and corresponding contributions of current quarks into a spin of the constituent quarks and that of the constituent quarks into a proton spin [31]:

$$(\Delta\Sigma)_p=(\Delta U+\Delta D)(\Delta\Sigma)_U\,. \tag{13}$$

It is also important to note the exact compensation between the spins of quark-antiquark pairs and their angular orbital momenta:

$$\langle L_{\{\bar{q}q\}}\rangle=-J_{\{\bar{q}q\}}\,. \tag{14}$$

Since we consider an effective Lagrangian approach where gluon degrees of freedom are overintegrated, we do not discuss problems of the principal separation and mixing of the quark orbital angular momentum and gluon effects in QCD (see Ref. [32]). The only effective degrees of freedom here are quasiparticles; mesons and baryons are the bound states arising due to residual interactions between the quasiparticles. In the NJL model [29] the six-quark fermion operator simulates the effect of gluon operator

$$\frac{\alpha_s}{2\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a,$$

where $G_{\mu\nu}$ is the gluon field tensor in QCD.

Account for axial anomaly in the framework of chiral quark models results in compensation of the valence quark helicity by helicities of quarks from the cloud in the structure of the constituent quark. The specific nonperturbative mechanism of such compensation is different in different approaches [29,33,34], e.g., the modification of the axial U(1) charge of the constituent quark is considered to be generated by the interaction of current quarks with flavor singlet field φ^0.

On these grounds we can conclude that significant part of the spin of the constituent quark should be associated with the orbital angular momentum of quarks inside this constituent quark, i.e., the quarks from the cloud should rotate coherently inside the constituent quark.

The important point what the origin of this orbital angular momentum is. It was proposed [35] to use an analogy with an anisotropic extension of the theory of superconductivity which seems to match well with the above picture for a constituent quark. The studies [36] of that theory show that the presence of anisotropy leads to axial symmetry of pairing correlations around the anisotropy direction $\hat{\vec{l}}$ and to the particle currents induced by the pairing correlations. In other words it means that a particle of the condensed fluid is surrounded by a cloud of correlated particles (‘‘hump’’) which rotate around it with the axis of rotation $\hat{\vec{l}}$ [cf. Eq. (12)]. The calculation of the orbital momentum shows that it is proportional to the density of the correlated particles. Thus, it is clear that there is a direct analogy between this picture and that describing the constituent quark. An axis of anisotropy $\hat{\vec{l}}$ can be associated with the polarization vector of current valence quark located at the origin of the constituent quark. The orbital angular momentum $\vec{L}$ lies along $\hat{\vec{l}}$ [cf. Eq. (12)].

We argued that the existence of this orbital angular momentum, i.e., orbital motion of quark matter inside the constituent quark, is the origin of the observed asymmetries in inclusive production at moderate and high transverse momenta [35]. Thus, we assume the standard SU(6) spin structure of a nucleon consisting of the three constituent quarks (embedded into the condensate), i.e., all the nucleon spin is composed from the spins of the constituent quarks. Constituent quarks, however, have a complex internal spatial and spin structure.

Such a picture for the hadron structure implies that overlapping and interaction of peripheral condensates in hadron collision occurs at the first stage. In the overlapping region the condensates interact and as a result virtual massive quark pairs appear. Being released, a part of the hadron energy carried by the peripheral condensates goes to generation of massive quarks. In other words nonlinear field couplings transform kinetic energy into internal energy of dressed quarks (see the arguments for this mechanism in [37], and references therein, for the earlier works). Of course, the number of such quarks fluctuates. The average number of quarks is proportional to the convolution of the condensate distributions $D_c^{Q,H}$ of the colliding constituent quark and hadron:

$$N(s,b)\simeq N(s)\cdot D_c^Q\otimes D_c^H\,, \tag{15}$$

where the function $N(s)$ is determined by a transformation thermodynamics of the kinetic energy of interacting condenstates to the internal energy of massive quarks. To estimate the $N(s)$ it is feasible to assume that it is proportional to the maximal possible energy dependence

$$N(s)\simeq\kappa\frac{(1-\langle x_Q\rangle)\sqrt{s}}{\langle m_Q\rangle}, \tag{16}$$

where $\langle x_Q\rangle$ is the average fraction of energy carried by the constituent quarks and $\langle m_Q\rangle$ is the mass scale of the constituent quarks.

In the model each of the constituent valence quarks located in the central part of the hadron is supposed to scatter in a quasi-independent way by the produced virtual quark pairs at a given impact parameter and by the other valence quarks. When smeared over longitudinal momenta the scattering amplitude of constituent valence quark Q may be represented in the form

$$\langle f_Q(s,b)\rangle=[N(s,b)+N-1]\langle V_Q(b)\rangle, \tag{17}$$

where $N=N_H+1$ is the total number of quarks in the system of the colliding constituent quark and hadron and $\langle V_Q(b)\rangle$ is the averaged amplitude of single quark-quark scattering.

For simplicity consider for a moment the case of spinless particles. In this approach the elastic scattering amplitude satisfies the unitarity since it is constructed as a solution of the following equation [38]:

$$F=U+iUDF \tag{18}$$

which is presented here in operator form. The function $U(s,b)$ (generalized reaction matrix) [38]—the basic dynamical quantity of this approach—is then chosen as a product of the averaged quark amplitudes

$$U(s,b)=\prod_{Q=1}^{N}\langle f_Q(s,b)\rangle \tag{19}$$

in accordance with assumed quasi-independent nature of valence quark scattering.

The b dependence of the function $\langle f_Q\rangle$ related to the quark form factor $F_Q(q)$ has a simple form $\langle f_Q\rangle \propto \exp(-m_Q b/\xi)$. The smeared quark amplitudes can have a nontrivial phase, but for simplicity we suppose that they are such that the resulting generalized reaction matrix is a pure imaginary function. Thus, the generalized reaction matrix in this case can be represented in the form

$$U(s,b)=i\left[\gamma N_H+\frac{a\sqrt{s}}{\langle m_Q\rangle}\right]^N \exp(-Mb/\xi). \tag{20}$$

In Eq. (20) N_H is the number of the constituent quarks in the hadron, $M=\Sigma_{Q=1}^{N}m_Q$, and b is the impact parameter of the colliding constituent quark and hadron [22]; a and γ are the constants.

In impact parameter representation the scattering amplitude may be written in the form

$$F(s,b)=U(s,b)[1-iU(s,b)]^{-1}. \tag{21}$$

This is solution of Eq. (18) at $s\gg 4m^2$. The unitarity is satisfied provided the inequality

$$\mathrm{Im}\, U(s,b)\geqslant 0 \tag{22}$$

is fulfilled. Note that the more familiar way to provide the direct channel unitarity consists in representation of the scattering amplitude in the eikonal form

$$F(s,b)=\frac{i}{2}(1-e^{i\chi(s,b)}),$$

where $\chi(s,b)$ is the eikonal function related to the function $U(s,b)$ by the equation

$$\chi(s,b)=i\ \ln\frac{1-iU(s,b)}{1+iU(s,b)}.$$

Now we can return to our case of the two 1/2 spin particles scattering. The mechanism of quark helicity flip in this picture is associated with the constituent quark interaction with the quark generated under interaction of the condensates [24]. The quark exchange process between the valence quark and an appropriate quark with relevant orientation of its spin and the same flavor will provide the necessary helicity flip transition, i.e., $Q_+\rightarrow Q_-$.

Of course, such processes are relatively suppressed in comparison with the quark scattering preserving helicity and therefore do not contribute into to the spin-averaged observables at the leading order. However, measuring the transverse spin asymmetries is sensitive to the subleading contribution and serves therefore as a filter of the quark exchange processes $Q_+\rightarrow Q_-$. This transition occurs when the valence quark knocks out a quark with the opposite helicity and the same flavor. The quark helicity flip amplitude in the model is determined by the relation

$$\langle f_Q^f(s,b)\rangle=\langle V_Q^f(b)\rangle,$$

[cf. Eq. (17)], where $\langle V_Q^f(b)\rangle \sim \exp(-\alpha m_Q b/\xi)$ and the parameter α determines the smaller radius of quark helicity flip interaction. The value of $\alpha>1$ corresponds to the central type of the quark helicity flip mechanism accepted in the model [24].

This interaction has energy suppression by the factor $[N(s,b)+N_H]^{-1}$ and leads to the obvious relation between the functions $U_1(s,b)$ and $U_2(s,b)$

$$\frac{U_2(s,b)}{U_1(s,b)}\sim\frac{\langle m_Q\rangle^2}{s}\exp[2(\alpha-1)\langle m_Q\rangle b/\xi] \tag{23}$$

at large values of s.

Consider now the helicity nonflip functions U_1 and U_3. These differ in the helicities of the initial and the final states but both functions describe helicity nonflip scattering. The second term in the square brackets of Eq. (20) results from the quark interaction with the component of the effective field which in its turn arises from the interaction of the condensates. Since the hadron spin is composed from the spins of the constituent quarks, this part of the interaction does not depend on the quark spin orientation and should be the same for the functions U_1 and U_3 due to parity conservation. This argument does not work for the first term of Eq. (20) which follows from the quark interaction with a self-consistent field of the valence quarks. Indeed, we should assume different forms for the quark amplitudes with parallel (p) and antiparallel (a) spin to the hadron's one

$$\langle f_Q^{p,a}(s,b)\rangle=N(s,b)\langle V_Q(b)\rangle+N_H\langle V_Q^{p,a}(b)\rangle, \tag{24}$$

and assuming that

$$\langle V_Q^{p,a}(b)\rangle=\tau_{p,a}\langle V_Q(b)\rangle$$

we obtain that the constant γ, in fact, has different values in the expressions for the functions U_1 and U_3. It is flavor dependent also. Thus, instead of Eq. (20) we have

$$U_{1,3}(s,b)=i\left[\gamma_{1,3}N_H+\frac{a\sqrt{s}}{\langle m_Q\rangle}\right]^N \exp(-Mb/\xi). \tag{25}$$

Then Eqs. (4), (6), and (7) allow us to obtain the quark densities $\Delta q(x)$ and $\delta q(x)$ at small values of x.

II. RESULTS AND DISCUSSION

Now we can calculate the helicity amplitudes $F_{1,2,3}(s,t)|_{t=0}$ at high values of s and obtain then the functional dependencies for the quark densities $q(x)$, $\Delta q(x)$, and $\delta q(x)$ at small x. It can be done using explicit forms for the functions $U_{1,2,3}(s,b)$.

$U_{1,3}(s,b)$ can be represented at large values of s as follows:

$$U_{1,3}(s,b)=U_0(s,b)(1+\beta_{1,3}\langle m_Q\rangle/\sqrt{s}), \tag{26}$$

where

$$U_0(s,b)=i\left[\frac{a\sqrt{s}}{\langle m_Q\rangle}\right]^N \exp(-Mb/\xi)$$

and

$$\beta_{1,3}=\gamma_{1,3}N_H(N_H+1)/a.$$

We need to keep the subleading terms in the expressions for $U_1(s,b)$ and $U_3(s,b)$ since the $\Delta q(x)$ is determined by their difference. For $U_2(s,b)$ one can keep only leading term and the expression for this function is

$$U_2(s,b)=g_f^2\frac{\langle m_Q\rangle^2}{s}\exp[2(\alpha-1)\langle m_Q\rangle b/\xi]U_0(s,b). \tag{27}$$

For the difference of the helicity amplitudes $F_1(s,b)-F_3(s,b)$ one has

$$F_1(s,b)-F_3(s,b)=\frac{U_1(s,b)-U_3(s,b)}{[1-iU_1(s,b)][1-iU_3(s,b)]}$$

and calculating integrals of Eq. (8) at high energies we can obtain the corresponding quark densities $q(x)$, $\Delta q(x)$, and $\delta q(x)$ at small x according to Eq. (4) (where $x\simeq Q^2/s$ at large values of s):

$$q(x)\sim\frac{1}{x}\ln^2(1/x), \tag{28}$$

$$\Delta q(x)\sim\frac{1}{\sqrt{x}}\ln(1/x), \tag{29}$$

and

$$\delta q(x)\sim x^c\ \ln(1/x), \tag{30}$$

where

$$c=\frac{\alpha-1}{N_H+1}.$$

The analysis of the experimental data on elastic hadron scattering [24] provides the value of the parameter $\alpha\simeq 2$ and then the value of c is about 0.25.

The behavior of $q(x)$ [and $F_1(x)$] and $\delta q(x)$ [and $h_1(x)$] is determined by the leading terms in $U_{1,3}(s,b)$ and $U_2(s,b)$ and therefore the small-x dependence of $F_1(x)$ and $h_1(x)$ will be universal for the proton and neutron: i.e.,

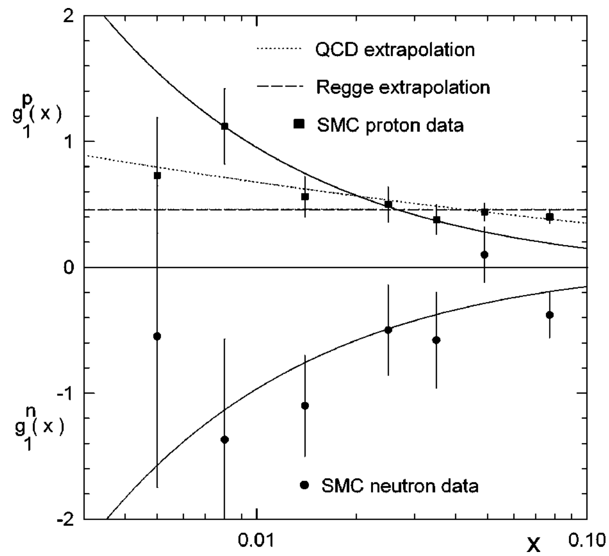

FIG. 1. The x dependence of the proton spin structure function $g_1^p(x)$ and the neutron spin structure function $g_1^n(x)$ at $x<0.1$ in the model (solid lines) and comparison with SMC experimental data evolved to common Q^2 values (proton data are taken from Ref. [4] and neutron data are taken from Ref. [41]).

$$F_1^p(x)/F_1^n(x)\rightarrow 1,$$

$$h_1^p(x)/h_1^n(x)\rightarrow 1$$

at $x\rightarrow 0$, and the explicit forms are as follows:

$$F_1^p(x)\sim\frac{1}{x}\ln^2(1/x), \tag{31}$$

$$h_1^p(x)\sim x^c\ \ln(1/x). \tag{32}$$

Equations (31) and (32) originate from the leading contributions and in that sense have a feature of a diffractive contribution. It is also seen that $h_1(x)$ has a smooth behavior at $x\rightarrow 0$, i.e., $h_1(x)\rightarrow 0$ in this limit since $c>0$.

The behavior of $\Delta q(x)$ and correspondingly $g_1(x)$ is determined by the difference $U_3(s,b)-U_1(s,b)$ or the subleading terms in $U_{1,3}$ and therefore the small-x dependence of $g_1(x)$ shows different constant factors for the proton and neutron: i.e.,

$$g_1^{p,n}(x)\simeq\frac{C^{p,n}}{\sqrt{x}}\ln(1/x). \tag{33}$$

Contrary to h_1 the spin structure function g_1 has a singular behavior at $x\rightarrow 0$.

The values of the parameters $C^{p,n}$ do not follow from the model and can be determined by the fit to the SMC experimental data available at lowest x. Comparison with the SMC data provides a satisfactory agreement of Eq. (33) at small x $(0<x<10^{-1})$ (cf. Fig. 1) and leads to the values $C^p=2.07\times 10^{-2}$ and $C^n=-2.10\times 10^{-2}$. As was mentioned the 1996 SMC data are less supportive of the singular dependence of $g_1(x)$ at $x\rightarrow 0$ but as it follows from the Fig. 1 do not exclude it. A Regge-type extrapolation of $g_1(x)$,

$$g_1(x) \sim \text{const}$$

and the perturbative QCD extrapolation

$$g_1(x) \sim \exp\{C\sqrt{\ln[\alpha_s(Q_0^2)/\alpha_s(Q^2)]\ln(1/x)}\}$$

based on the DGLAP resummation and assumption that the initial structure function is flat were also used to fit the data. Our analysis of the data has a qualitative character and its aim is to demonstrate that the model does not contradict to experiment. The same is true for the Regge and perturbative QCD extrapolations represented in Fig. 1. Comprehensive NLO QCD analysis of the world data was performed in [2]. The essential point of this analysis is the particular parametrization of the polarized parton densities at the low initial scale $Q_0^2 = 0.34$ which, in fact, has a model nature and the final result has a nontrivial dependence on the particular choice.

To evaluate the first moment

$$\Gamma_1 = \int_0^1 g_1(x)dx \tag{34}$$

we use the value of

$$I(0.1,1) = \int_{0.1}^1 g_1^p(x)dx = 0.092, \tag{35}$$

obtained with the standard parametrization of the data at medium and large values of x [11]. Equation (33) then gives

$$I(0,0.1) = \int_0^{0.1} g_1^p(x)dx = 0.057 \tag{36}$$

and for the first moment Γ_1^p we obtain

$$\Gamma_1^p = \int_0^1 g_1^p(x)dx = 0.149. \tag{37}$$

The above simple estimation of Γ_1^p alongside with the known $g_A = 1.257$ and $3F - D = 0.579$ provides the following values for the quark spin contributions:

$$\Delta\Sigma \simeq 0.25, \quad \Delta u \simeq 0.81, \quad \Delta d \simeq -0.45, \quad \Delta s \simeq -0.11, \tag{38}$$

which are in agreement with the results obtained in the comprehensive analysis with account for the QCD evolution and higher twist contributions [1,2,13,14]. Equation (38) demonstrates that the singular behavior of $g_1^p(x)$ in the form of Eq. (33) does not lead to significant deviations from the results of the experimental analysis [1] where the smooth extrapolation of the data to $x = 0$ is used. The functional dependence of the spin structure function

$$g_1^n(x) \sim \frac{1}{\sqrt{x}}\ln(1/x)$$

is in a good agreement with the new E154 [2] and HERMES data [3] as well.

It is important to note that Eqs. (29) and (30) demonstrate that the relation $\Delta q(x) \simeq \delta q(x)$ is not valid at small x contrary to the result of the nonrelativistic quark model. The explicit forms Eqs. (29) and (30) lead to the conclusion that the longitudinal and transverse double-spin asymmetries for the Drell-Yan processes at $x_F \simeq 0$ are small, i.e.,

$$A_{LL}^{l\bar{l}} \simeq 0 \quad \text{and} \quad A_{TT}^{l\bar{l}} \simeq 0 \tag{39}$$

when the invariant mass of the lepton pair is $M_{l\bar{l}}^2 \ll s$. It is just this kinematical region which is sensitive to low-x behavior of the spin densities $\Delta q(x)$ and $\delta q(x)$ since

$$A_{LL}^{l\bar{l}} = -\frac{\Sigma_i e_i^2[\Delta q_i(x_1)\Delta\bar{q}(x_2) + \Delta\bar{q}_i(x_1)\Delta q_i(x_2)]}{\Sigma_i e_i^2[q_i(x_1)\bar{q}_i(x_2) + \bar{q}_i(x_1)q_i(x_2)]} \tag{40}$$

and

$$A_{TT}^{l\bar{l}} = a_{TT}\frac{\Sigma_i e_i^2[\delta q_i(x_1)\delta\bar{q}(x_2) + \delta\bar{q}_i(x_1)\delta q_i(x_2)]}{\Sigma_i e_i^2[q_i(x_1)\bar{q}_i(x_2) + \bar{q}_i(x_1)q_i(x_2)]}, \tag{41}$$

where $x_1x_2 = M_{l\bar{l}}^2/s$ and $x_F = x_1 - x_2$.

The ratio of the asymmetries $A_{TT}^{l\bar{l}}$ and $A_{LL}^{l\bar{l}}$ is also small in the central region of low-mass Drell-Yan production:

$$A_{TT}^{l\bar{l}}/A_{LL}^{l\bar{l}} \simeq 0. \tag{42}$$

This result agrees with the predictions made in [39].

The above results were obtained in the limit $s \to \infty$ which corresponds to $x \to 0$, i.e., they have an asymptotic nature. However, it might happen that the kinematical region of the SMC experiment lies in the preasymptotic domain and the above formulas, in fact, are valid at much smaller values of x, than the range covered by the present experiments. Indeed, it was shown that the preasymptotic effects are very important to understand the experimental regularities observed in the hadron interactions and unpolarized deep-inelastic scattering [40].

Thus, the measurements of the double-spin asymmetries at the BNL Relativistic Heavy Ion Collider (RHIC) would provide important information on the x dependence of the spin quark densities $\Delta q(x)$ and $\delta q(x)$.

CONCLUSION

We have obtained the low-x behavior of the quark spin densities $\Delta q(x)$ and $\delta q(x)$ [and spin structure functions $g_1(x)$ and $h_1(x)$] in the framework of the nonperturbative approach based on the assumed structures of the hadron and the constituent quarks and their scattering picture in the effective two-component field. Thus, our considerations could be regarded as a kind of a bootstrap approach.

The dependence of $g_1(x)$ at $x \to 0$ can describe the data measured at small x in the SMC experiment. The model predicts a smooth behavior of $h_1(x)$ at $x \to 0$ which, contrary to g_1, corresponds to the leading diffractive contribution. It was also shown that the singular behavior of the structure function $g_1(x)$ does not affect much the numerical results on the partition of a nucleon spin.

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