Three-body amplitude analysis of the process $a_2(1320)\to\pi^+\pi^+\pi^-$

Jie-Jie Zhu and Tu-Nan Ruan*
CCAST (World Lab), P.O. Box 8730, Beijing, 100080, People's Republic of China
and Department of Modern Physics, USTC, P.O. Box 4, Hefei, Anhui, 230027, People's Republic of China
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We analyze the decay amplitudes of the process $a_2(1320)\to\pi^+\pi^+\pi^-$ using the tensor analysis technique and show how to obtain three-body amplitudes. The amplitudes can describe both the resonance and the background contribution of the process. [S0556-2821(98)00605-5]

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I. INTRODUCTION

Analytic structures of differential cross sections of collision processes are needed to fit data in high energy experimental physics. Usually, a technique called tensor analysis [1–7] is used. Comparing with the early methods of helicity or partial wave amplitude analysis [8–12], tensor analysis can give more details of the amplitudes' dependence on energies.

In the previous references, reactions with three-body final states are looked upon as sequential two-body decays. We analyze the amplitude of the process $a_2(1320)\to\pi^+\pi^+\pi^-$ in this paper. From this example one can see how to construct general three-body amplitudes without the assumption of sequential two-body decays. The derived amplitudes are general for this process. They can describe both the resonance and background terms.

In Sec. II, we will consider the covariance and boson symmetry of the amplitude. The general amplitudes for $a_2(1320)\to\pi^+\pi^+\pi^-$ are obtained. Section III will be devoted to the constraint of parity conservation on the amplitudes. The resonance structure and background terms of the process are discussed in Sec. IV.

II. CONSTRUCTION OF THREE-BODY AMPLITUDES

Let p, p_1, p_2, and p_3 be the four-momenta of $a_2(1320)$, π^+, π^+, and π^-. The helicity amplitude of this process can be formally written as

$$F_\lambda=e_{\mu\nu}(p,2\lambda)\Gamma^{\mu\nu},$$

$$\lambda=0,\ \pm1,\ \pm2, \tag{1}$$

where λ is the helicity of $a_2(1320)$. Here $e_{\mu\nu}(p,2\lambda)$ is the polarization tensor of $a_2(1320)$, and $\Gamma^{\mu\nu}$ is the effective four-leg vertex with $a_2(1320)$, π^+, π^+, and π^- as its outer legs.

$e_{\mu\nu}(p,2\lambda)$ satisfies Rarita-Schwinger conditions [13]

$$e_{\mu\nu}(p,2\lambda)=e_{\nu\mu}(p,2\lambda), \tag{2a}$$

$$g^{\mu\nu}e_{\mu\nu}(p,2\lambda)=0, \tag{2b}$$

$$p^\mu e_{\mu\nu}(p,2\lambda)=0. \tag{2c}$$

Let us analyze the Lorentz indices of the vertex function $\Gamma^{\mu\nu}$. Here $\Gamma^{\mu\nu}$ can be constructed from

$$p_1^\mu,\ p_2^\mu,\ p_3^\mu,\ g^{\mu\nu},\ \varepsilon^{\mu\nu\alpha\beta}; \tag{3}$$

here, $g^{\mu\nu}$ is the space-time metric, which is taken as diag$\{1,-1,-1,-1\}$ and $\varepsilon^{\mu\nu\alpha\beta}$ is the antisymmetric tensor. Since $e_{\mu\nu}(p,2\lambda)$ is traceless, one can safely exclude $g^{\mu\nu}$, and the antisymmetric tensor should appear as

$$Q^\mu\overset{\text{def}}{=}p_1^\alpha p_2^\beta p_3^\gamma\varepsilon^{\alpha\beta\gamma\mu}. \tag{4}$$

After consideration of the symmetry of indices μ, ν, all possible constructions are, for tensors,

$$p_1^\mu p_2^\nu+p_1^\nu p_2^\mu,\ p_1^\mu p_3^\nu+p_1^\nu p_3^\mu,\ p_2^\mu p_3^\nu+p_2^\nu p_3^\mu,$$

$$p_1^\mu p_1^\nu,\ p_2^\mu p_2^\nu,\ p_3^\mu p_3^\nu,\ Q^\mu Q^\nu, \tag{5a}$$

and for pseudotensors,

$$Q^\mu p_1^\nu+Q^\nu p_1^\mu,\ Q^\mu p_2^\nu+Q^\nu p_2^\mu,\ Q^\mu p_3^\nu+Q^\nu p_3^\mu. \tag{5b}$$

From condition (2c) and conservation of energy and momentum $p_1+p_2+p_3=p$, one knows that those terms in Eq. (5) that contain p_3^μ can be expressed in linear combinations of the others. Now the independent terms are, for tensors,

$$p_1^\mu p_2^\nu+p_1^\nu p_2^\mu,\ p_1^\mu p_1^\nu,\ p_2^\mu p_2^\nu,\ Q^\mu Q^\nu, \tag{6a}$$

and for pseudotensors,

$$Q^\mu p_1^\nu+Q^\nu p_1^\mu,\ Q^\mu p_2^\nu+Q^\nu p_2^\mu. \tag{6b}$$

Following the usual assumption of maximal analyticity [14–17], the vertex functions got by linear combination of covariants in Eqs. (6) are free of kinematic singularities.

According to Eqs. (6), we write the vertex function as

$$\Gamma^{\mu\nu}=c_1[p_1^\mu p_2^\nu+p_1^\nu p_2^\mu]+c_2[p_1^\mu p_1^\nu+p_2^\mu p_2^\nu]+c_3Q^\mu Q^\nu$$
$$+c_4(p_1^\mu p_1^\nu-p_2^\mu p_2^\nu)+c_5[Q^\mu(p_1^\nu-p_2^\nu)+Q^\nu(p_1^\mu-p_2^\mu)]$$
$$+c_6(Q^\mu p_3^\nu+Q^\nu p_3^\mu). \tag{7}$$

*Electronic address: tnruan@lx04.mphy.ustc.edu.cn

The coefficients in Eq. (7) are analytic functions of the dot products of four-momenta. Because of conservation of energy and momentum and mass shell conditions, there are only two independent dot products, $p_1 \cdot p_3$ and $p_2 \cdot p_3$, so that

$$c_i = c_i((p_1+p_2)\cdot p_3,\ \ (p_1-p_2)\cdot p_3), \quad i=1,2,\ldots,6, \tag{8}$$

and can be expanded as a Taylor series of $(p_1+p_2)\cdot p_3$ and $(p_1-p_2)\cdot p_3$, because c_i's are analytic functions of these two dot products.

In the process $a_2(1320) \to \pi^+\pi^+\pi^-$, the two π^+'s, particles 1 and 2, are identical particles. $\Gamma^{\mu\nu}$ should satisfy boson symmetry. Imposing boson symmetry on Eq. (7) leads to

$$\begin{aligned} &c_i((p_1+p_2)\cdot p_3,\ \ -(p_1-p_2)\cdot p_3) \\ &\quad = c_i((p_1+p_2)\cdot p_3,\ \ (p_1-p_2)\cdot p_3), \end{aligned} \tag{9a}$$

for $i=1,2,3,5$, and

$$\begin{aligned} &c_i((p_1+p_2)\cdot p_3,\ \ -(p_1-p_2)\cdot p_3) \\ &\quad = -c_i((p_1+p_2)\cdot p_3,\ \ (p_1-p_2)\cdot p_3), \end{aligned} \tag{9b}$$

for $i=4,6$. We see that there are kinematic zeros in c_4 and c_6: The even orders of $(p_1-p_2)\cdot p_3$ in their Taylor series vanish. To get a vertex function free of kinematic zeros, we must factor out $(p_1-p_2)\cdot p_3$ in c_4 and c_6:

$$\begin{aligned} \Gamma^{\mu\nu} = {} & c_1[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu] + c_2[p_1^\mu p_1^\nu + p_2^\mu p_2^\nu] + c_3 Q^\mu Q^\nu \\ & + c_4[(p_1-p_2)\cdot p_3(p_1^\mu p_1^\nu - p_2^\mu p_2^\nu)] + c_5[Q^\mu(p_1^\nu - p_2^\nu) \\ & + Q^\nu(p_1^\mu - p_2^\mu)] + c_6[(p_1-p_2)\cdot p_3(Q^\mu p_3^\nu + Q^\nu p_3^\mu)]. \end{aligned} \tag{10}$$

$c_1, c_2, \ldots, c_6$ are analytic functions of $(p_1+p_2)\cdot p_3$ and $[(p_1-p_2)\cdot p_3]^2$.

There are still kinematic zeros caused by parity conservation. This will be discussed in the next section.

III. SYMMETRY OF SPACE REFLECTION

We denote the intrinsic parities of $a_2(1320)$, π^+, π^+, and π^- as η, η_1, η_2, and η_3; their spins are J, J_1, J_2, and J_3, and the corresponding helicities are λ, λ_1, λ_2, and λ_3. Write the initial state as $|\vec{p},J,\lambda\rangle$, the final state as $|\vec{p}_1,J_1,\lambda_1;\vec{p}_2,J_2,\lambda_2;\vec{p}_3,J_3,\lambda_3\rangle$. Here $\mathbf{p}$ is the parity operator.

Using the notation of Ref. [5], one obtains

$$\mathbf{p}|\vec{p},J,\lambda\rangle \overset{\text{def}}{=} \eta e^{-i\pi J}|-\vec{p},J,-\lambda\rangle,$$

$$\mathbf{p}|\vec{p}_1,J_1,\lambda_1;\vec{p}_2,J_2,\lambda_2;\vec{p}_3,J_3,\lambda_3\rangle \equiv \eta_1\eta_2\eta_3 e^{-i\pi(J_1+J_2+J_3)}|-\vec{p}_1,J_1,-\lambda_1;-\vec{p}_2,J_2,-\lambda_2;-\vec{p}_3,J_3-\lambda_3\rangle. \tag{11}$$

Spin-1 polarization vectors are given by

$$(e^\mu(p,0)) = \begin{pmatrix} \dfrac{|\vec{p}|}{W} \\ \dfrac{E}{W}\sin\theta\cos\phi \\ \dfrac{E}{W}\sin\theta\sin\phi \\ \dfrac{E}{W}\cos\theta \end{pmatrix},$$

$$(e^\mu(p,\pm 1)) = \begin{pmatrix} 0 \\ \mp\cos\theta\cos\phi + i\sin\phi \\ \mp\cos\theta\sin\phi - i\cos\phi \\ \pm\sin\phi \end{pmatrix}, \tag{12}$$

where

$$(p^\mu) = (E,\vec{p}),$$

$$\vec{p} = (|\vec{p}|\sin\theta\cos\phi, |\vec{p}|\sin\theta\sin\phi, |\vec{p}|\cos\theta),$$

$$W = \sqrt{p^\mu p_\mu}. \tag{13}$$

We define the polarization tensor of arbitrary order recurrently by

$$\begin{aligned} e^{\mu_1\mu_2\cdots\mu_j}(p,j,\lambda) \overset{\text{def}}{=} & \sum_{\lambda_1,\lambda_2} (1,\lambda_1;j-1,\lambda_2|j,\lambda) e^{\mu_j}(p,\lambda_1) \\ & \times e^{\mu_1\mu_2\cdots\mu_{j-1}}(p,j-1,\lambda_2). \end{aligned} \tag{14}$$

The space reflection matrix is defined as

$$(P^\mu{}_\nu) = \mathrm{diag}\{1,1,1,1\}, \tag{15}$$

and one has

$$\overline{p^\mu} \equiv P^\mu{}_\nu p^\nu = (E,-\vec{p}),$$

$$\overline{p_i^{\mu}} \equiv P^{\mu}{}_{\nu} p_i^{\nu} = (E_i, -\vec{p}_i), \quad i=1,2,3,$$

$$\overline{e^{\mu_1\mu_2\cdots\mu_j}} \equiv P^{\mu_1}{}_{\nu_1} P^{\mu_2}{}_{\nu_2} P^{\mu_j}{}_{\nu_j} e^{\nu_1\nu_2\cdots\nu_j}. \tag{16}$$

From Eqs. (12) one can see that

$$e^{\mu}(\overline{p},\lambda) = \overline{e^{\mu}}(p,-\lambda). \tag{17}$$

As a consequence of the definition of Eqs. (12), we have

$$e^{\mu_1\mu_2\cdots\mu_j}(\overline{p},j,\lambda) = \overline{e^{\mu_1\mu_2\cdots\mu_j}}(p,j,-\lambda). \tag{18}$$

Parity-conserving helicity amplitudes hold the following relation:

$$\begin{aligned}
&\langle \vec{p},J,\lambda|\mathbf{M}|\vec{p}_1,J_1,\lambda_1;\vec{p}_2,J_2,\lambda_2;\vec{p}_3,J_3,\lambda_3\rangle \\
&= \langle \vec{p},J,\lambda|\mathbf{p}^{\dagger}\mathbf{M}\mathbf{p}|\vec{p}_1,J_1,\lambda_1;\vec{p}_2,J_2,\lambda_2;\vec{p}_3,J_3,\lambda_3\rangle \\
&= \eta^{*}\eta_1\eta_2\eta_3 e^{i\pi(J-J_1-J_2-J_3)}\langle -\vec{p},J,-\lambda|\mathbf{M}|-\vec{p}_1,J_1,-\lambda_1;-\vec{p}_2,J_2,-\lambda_2;-\vec{p}_3,J_3,-\lambda_3\rangle,
\end{aligned} \tag{19}$$

with

$$\langle \vec{p},J,\lambda|\mathbf{M}|\vec{p}_1,J_1,\lambda_1;\vec{p}_2,J_2,\lambda_2;\vec{p}_3,J_3,\lambda_3\rangle \equiv e^{*}(p,J,\lambda)\Gamma(p,p_1,p_2,p_3,g^{\mu\nu},\varepsilon^{\alpha\beta\gamma\delta})e(p_1,J_1,\lambda_1)e(p_2,J_2,\lambda_2)e(p_3,J_3,\lambda_3). \tag{20}$$

Some indices are abbreviated in the above expression. Γ is a tensor contracted from the products of p^{μ}, p_1^{μ}, p_2^{μ}, p_3^{μ}, $g^{\mu\nu}$, and $\varepsilon^{\alpha\beta\gamma\delta}$. The requirement of parity conservation is equivalent to

$$\begin{aligned}
&e^{*}(\overline{p},J,-\lambda)\Gamma(\overline{p},\overline{p_1},\overline{p_2},\overline{p_3},g^{\mu\nu},\varepsilon^{\alpha\beta\gamma\delta})e(\overline{p_1},J_1,-\lambda_1)e(\overline{p_2},J_2,-\lambda_2)e(\overline{p_3},J_3,-\lambda_3) \\
&= \eta\eta_1^{*}\eta_2^{*}\eta_3^{*}(-1)^{J-J_1-J_2-J_3}e^{*}(p,J,\lambda)\Gamma(p,p_1,p_2,p_3,g^{\mu\nu},\varepsilon^{\alpha\beta\gamma\delta})e(p_1,J_1,\lambda_1)e(p_2,J_2,\lambda_2)e(p_3,J_3,\lambda_3).
\end{aligned} \tag{21}$$

The unitary conditions $\eta^{*}\eta=1$ and $\eta_i^{*}\eta_i=1$ $(i=1,2,3)$ have been used.

We should set $J=2$, $\eta=+1$, $J_i=0$, $\eta_i=-1$ $(i=1,2,3)$ [18]. The polarization tensors for final state particles become

$$e(p_1,J_1,\lambda_1)=e(p_2,J_2,\lambda_2)=e(p_3,J_3,\lambda_3)=1. \tag{22}$$

Substituting these into Eq. (21), one finds

$$\begin{aligned}
&e^{*}_{\mu\nu}(p,J,\lambda)\Gamma^{\mu\nu}(p,p_1,p_2,p_3,g^{\sigma\tau},\varepsilon^{\alpha\beta\gamma\delta}) \\
&= -e^{*}_{\mu\nu}(\overline{p},J,-\lambda)\Gamma^{\mu\nu}(\overline{p},\overline{p_1},\overline{p_2},\overline{p_3},g^{\sigma\tau},\varepsilon^{\alpha\beta\gamma\delta}),
\end{aligned} \tag{23}$$

where $\Gamma^{\mu\nu}$ is symmetric and traceless. From Eqs. (18), (23), and

$$\overline{g^{\mu\nu}} \overset{\text{def}}{=} P^{\mu}{}_{\alpha}P^{\nu}{}_{\beta}g^{\alpha\beta} \equiv g^{\mu\nu},$$

$$\overline{\varepsilon^{\alpha\beta\gamma\delta}} \overset{\text{def}}{=} P^{\alpha}{}_{\alpha'}P^{\beta}{}_{\beta'}P^{\gamma}{}_{\gamma'}P^{\delta}{}_{\delta'}\varepsilon^{\alpha'\beta'\gamma'\delta'} \equiv -\varepsilon^{\alpha\beta\gamma\delta}, \tag{24}$$

one finds

$$\begin{aligned}
&e^{*}_{\mu\nu}(p,J,\lambda)\Gamma^{\mu\nu}(p,p_1,p_2,p_3,g^{\sigma\tau},\varepsilon^{\alpha\beta\gamma\delta}) \\
&= -e^{*}_{\mu\nu}(p,J,\lambda)\Gamma^{\mu\nu}(p,p_1,p_2,p_3,g^{\sigma\tau},-\varepsilon^{\alpha\beta\gamma\delta})
\end{aligned}$$

or

$$\begin{aligned}
&\Gamma^{\mu\nu}(p,p_1,p_2,p_3,g^{\sigma\tau},\varepsilon^{\alpha\beta\gamma\delta}) \\
&= -\Gamma^{\mu\nu}(p,p_1,p_2,p_3,g^{\sigma\tau},-\varepsilon^{\alpha\beta\gamma\delta}).
\end{aligned} \tag{25}$$

That is, to preserve parity conservation in the decay $a_2(1320)\to\pi^{+}\pi^{+}\pi^{-}$, the antisymmetric tensor $\varepsilon^{\alpha\beta\gamma\delta}$ should appear only an odd number of times in $\Gamma^{\mu\nu}$. Applying this to Eq. (10) gives

$$\begin{aligned}
\Gamma^{\mu\nu} &= c[Q^{\mu}(p_1^{\nu}-p_2^{\nu})+Q^{\nu}(p_1^{\mu}-p_2^{\mu})] \\
&\quad + d(p_1-p_2)\cdot p_3(Q^{\mu}p_3^{\nu}+Q^{\nu}p_3^{\mu}).
\end{aligned} \tag{26}$$

The vertex function given in Eq. (26) is free of kinematic singularities and zeros.

IV. RESONANCE STRUCTURE AND BACKGROUND TERMS

The predominant resonance decay mode of the process $a_2(1320)\to\pi^{+}\pi^{+}\pi^{-}$ is $a_2(1320)\to\pi^{+}\rho_0$, $\rho_0\to\pi^{+}\pi^{-}$. The result of a tensor analysis of this sequential two-body decay [5] is

$$F_{\lambda}^{(\text{res})} = c_r e_{\mu\nu}(p,2\lambda)\Phi^{\mu\nu},$$

$$\Phi^{\mu\nu} = (D_{23}Q^{\mu}p_1^{\nu} - D_{13}Q^{\mu}p_2^{\nu}). \tag{27}$$

Here c_r is a Lorentz scalar, and D_{13}, D_{23} are Breit-Wigner factors:

$$D_{23} = \frac{1}{(p_2+p_3)^2 - M_{\rho}^2 + i\Gamma_{\rho}M_{\rho}},$$

$$D_{13} = \frac{1}{(p_1+p_3)^2 - M_{\rho}^2 + i\Gamma_{\rho}M_{\rho}}. \tag{28}$$

M_ρ, Γ_ρ are the mass and the width of ρ_0. Note that the expression (25) satisfies the boson symmetry of particles 1 and 2.

According to the results of Sec. III, the general helicity amplitude of the process $a_2(1320)\to\pi^+\pi^+\pi^-$ is

$$\begin{aligned} F_\lambda &= e_{\mu\nu}(p,2\lambda)\Gamma^{\mu\nu} \\ &= c e_{\mu\nu}(p,2\lambda)[Q^\mu(p_1^\nu - p_2^\nu) + Q^\nu(p_1^\mu - p_2^\mu)] \\ &\quad + d e_{\mu\nu}(p,2\lambda)(p_1 - p_2)\cdot p_3(Q^\mu p_3^\nu + Q^\nu p_3^\mu). \end{aligned} \tag{29}$$

This can be formally written as

$$F_\lambda = \begin{pmatrix} c \\ d \end{pmatrix}; \tag{30}$$

i.e., the part proportional to c and the part proportional to d in F_λ are regarded as two vectors orthogonal to each other. The helicity amplitude for the process $a_2(1320)\to\pi^+\pi^+\pi^-$ satisfying parity conservation and boson symmetry is a vector in the two-dimensional space spanned by these two vectors. Now the resonance term becomes

$$F_\lambda^{(\mathrm{res})} = c_r \begin{pmatrix} \frac{1}{4}(D_{13}+D_{23}) \\ \dfrac{D_{13}-D_{23}}{4(p_1-p_2)\cdot p_3} \end{pmatrix} = c_r \begin{pmatrix} \frac{1}{4}(D_{13}+D_{23}) \\ -\frac{1}{2}D_{13}D_{23} \end{pmatrix}. \tag{31}$$

F_λ can be expanded as the sum of a term that is parallel to $F_\lambda^{(\mathrm{res})}$ and another term perpendicular to it:[1]

$$\begin{aligned} F_\lambda &= c_\parallel \begin{pmatrix} \frac{1}{4}(D_{13}+D_{23}) \\ -\frac{1}{2}D_{13}D_{23} \end{pmatrix} + c_\perp \begin{pmatrix} \frac{1}{2}D_{13}D_{23} \\ \frac{1}{4}(D_{13}+D_{23}) \end{pmatrix} \\ &= c_r \begin{pmatrix} \frac{1}{4}(D_{13}+D_{23}) \\ -\frac{1}{2}D_{13}D_{23} \end{pmatrix} \\ &\quad + \begin{pmatrix} (c_\parallel - c_r)\,\dfrac{D_{13}+D_{23}}{4} + c_\perp\,\dfrac{D_{13}D_{23}}{2} \\ (c_\parallel - c_r)\,\dfrac{-D_{13}D_{23}}{2} + c_\perp\,\dfrac{D_{13}+D_{23}}{4} \end{pmatrix} \\ &\equiv F_\lambda^{(\mathrm{res})} + F_\lambda^{(b)}. \end{aligned} \tag{32}$$

$F_\lambda^{(b)}$ stands for background terms. In the region close to ρ_0 resonance points in the phase space, $F_\lambda^{(\mathrm{res})}$ is the leading term of F_λ, and the dependence of the background terms on energies is weak. The background terms can be treated as constants:

$$F_\lambda^{(b)} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix},$$

$$F_\lambda = F_\lambda^{(\mathrm{res})} + F_\lambda^{(b)} = \begin{pmatrix} \dfrac{c_r}{4}(D_{13}+D_{23}) + b_1 \\ \dfrac{-c_r}{2}D_{13}D_{23} + b_2 \end{pmatrix}. \tag{33}$$

The helicity amplitude after consideration of the ρ_0 resonance reads

$$\begin{aligned} F_\lambda &= \left[\frac{c_r}{4}(D_{13}+D_{23}) + b_1\right] e_{\mu\nu}(p,2\lambda) \\ &\quad \times [Q^\mu(p_1^\nu - p_2^\nu) + Q^\nu(p_1^\mu - p_2^\mu)] + \left[\frac{-c_r}{2}D_{13}D_{23} + b_2\right] \\ &\quad \times (p_1 - p_2)\cdot p_3 e_{\mu\nu}(p,2\lambda)(Q^\mu p_3^\nu + Q^\nu p_3^\mu), \end{aligned} \tag{34}$$

where c_r, b_1, and b_2 can be dealt with as constants when fitting data.

V. CONCLUSIONS

We derive the general form of the decay amplitude for the process $a_2(1320)\to\pi^+\pi^+\pi^-$ through the covariance and symmetry analysis of the four-leg vertex. The derived amplitude is more general compared with the sequential two-body decay amplitudes. It contains both the background terms and the resonance term of the process.

Consider the example in this paper. The amplitude given by two-body sequential decays properly describes the process in the ρ_0 resonance region, but without background terms it might be unreasonable in the region not so close to resonance points. Background terms might contribute a non-negligible portion to the amplitude. General three-body decay amplitudes given in this paper are needed to include the background contribution.

A similar analysis can be applied to the case in which the particles of the initial and final states are of arbitrary spins, but the problem may be much harder depending on symmetry relations and the number of Lorentz indices involved.

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[1] In order to be consistent on dimensions, we should substitute p_i by p_i/m_π in Eqs. (29)–(33). The constant factor m_π could be absorbed into the coefficients in the result of Eq. (34).

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