

Cosmic strings, zero modes, and supersymmetry breaking in non-Abelian $N=1$ gauge theories

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We investigate the microphysics of cosmic strings in non-Abelian gauge theories with $N=1$ supersymmetry. We give the vortex solutions in a specific example and demonstrate that fermionic superconductivity arises because of the couplings and interactions dictated by supersymmetry. We then use supersymmetry transformations to obtain the relevant fermionic zero modes and investigate the role of soft supersymmetry breaking on the existence and properties of the superconducting strings. [S0556-2821(98)03208-1]

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I. INTRODUCTION

In recent years, supersymmetry (SUSY) has become increasingly favored as the theoretical structure underlying fundamental particle interactions. In this light it is natural to investigate possible cosmological implications of SUSY.

In a recent paper [1] we discussed the effect of SUSY on the microphysics of simple cosmic string solutions of Abelian field theories. In particular we developed and applied the technique of SUSY transformations to investigate the form of fermionic zero modes, required by SUSY, which lead to cosmic string superconductivity. In the present work we extend our original ideas to a more general class of field theories, namely those with a non-Abelian gauge group. Since non-Abelian gauge theories underlie modern particle physics and, in particular, unified field theories, this class of theories is a realistic toy model for grand unified theories (GUTs). The particular example we examine, $SU(2) \times U(1) \rightarrow U(1) \times Z_2$, admits two types of string solution, one Abelian and the other non-Abelian. This model has a very similar structure to $SO(10)$ and should provide insight into cosmic strings in SUSY GUTs. Most of the features exhibited by this theory will also appear in larger non-Abelian theories. We apply the technique of SUSY transformations to the non-Abelian case and extract the behavior of the zero modes as functions of the background string fields. We then compare the results to those obtained in Ref. [1] for abelian strings.

Since SUSY is clearly broken in the universe today, it is important to know how the SUSY zero modes behave when soft-SUSY breaking occurs in the universe. We investigate this for both the Abelian and non-Abelian strings by explicitly introducing soft-SUSY breaking terms into the theory. The result is that all the zero modes are destroyed in almost all the theories, the exception being when a Fayet-Iliopoulos term is used to break the gauge symmetry in an Abelian model. We briefly comment on the physical reasons for this

and show how the effect may be seen through the breakdown of an appropriate index theorem.

These results have a cosmological significance since fermion zero modes on the string can be excited, causing a current to flow along the string. The string then behaves as a perfect conductor. The existence of charge carriers changes the cosmology of cosmic strings. In particular, they can stabilize string loops, resulting in the production of so-called vortons [2]. Such vortons can dominate the energy density of the Universe, and have been used to constrain GUT models with current-carrying strings [3]. However, if the zero modes are destroyed at the SUSY breaking energy scale, then the current, and hence vortons, will dissipate. Thus, the underlying theory may be cosmologically viable.

The plan of this paper is as follows. In Sec. II we construct a simple supersymmetric model based on the group $SU(2) \times U(1)$ and display the Abelian and non-Abelian string solutions. In Sec. III, we briefly review a powerful index theorem for finding fermion zero modes in a general theory. We then use SUSY transformations to obtain the zero modes in terms of the background string fields. Soft SUSY breaking terms are introduced in Sec. IV and their effect on the zero modes is analyzed using the results of the index theorem. In this section we consider both string solutions for the $SU(2) \times U(1)$ model and also for the $U(1)$ theory discussed in our previous paper [1]. For the non-Abelian string the SUSY breaking terms destroy the zero modes, while for the other string solutions the situation is more complicated.

II. SIMPLE MODEL: $SU(2) \times U(1)$

There exist many non-Abelian theories with breaking schemes giving rise to cosmic strings. In general both Abelian and non-Abelian strings can be produced in such a process, depending on which part of the vacuum manifold is involved in the winding.

In this section we consider a simple example in which the gauge group $SU(2) \times U(1)$ is spontaneously broken down to the group $U(1) \times Z_2$ via the superpotential

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$$W = \mu_1 S_0 (\Phi \cdot \tilde{\Phi} - \eta^2) + \mu_2 (S \Phi^T \Lambda \Phi + \tilde{S} \tilde{\Phi}^T \Lambda \tilde{\Phi}). \quad (2.1)$$

The chiral superfields $\Phi_i(\phi_i, \psi_i, F_i)$ and $\tilde{\Phi}_i(\tilde{\phi}_i, \tilde{\psi}_i, \tilde{F}_i)$ are $SU(2)$ triplets with $U(1)$ charges ± 1 respectively. The other chiral superfields, $S_0(s_0, \omega_0, F_{S_0})$, $S(s, \omega, F_S)$ and $\tilde{S}(\tilde{s}, \tilde{\omega}, F_{\tilde{S}})$, are $SU(2)$ scalars with $U(1)$ charges 0, -2

and $+2$ respectively. Finally, defining $T^4 = \sqrt{2/3}I$, the vector supermultiplets are $V^a(A_\mu^a, \lambda^a, D^a)$, $a = 1, \dots, 4$. Since the constant matrix Λ satisfies $\Lambda T^i = -(T^i)^* \Lambda$ ($i = 1, \dots, 3$) and the $SU(2)$ gauge transformations are $\delta \Phi = iT^a n^a \Phi$ and $\delta \tilde{\Phi} = -iT^{a*} n^a \tilde{\Phi}$, the superpotential is gauge invariant.

The scalar potential, derived in the standard manner [4], is then

$$U = \mu_1^2 |\phi \cdot \tilde{\phi} - \eta^2|^2 + \mu_2^2 |2\phi_1\phi_3 - \phi_2^2|^2 + \mu_2^2 |2\tilde{\phi}_1\tilde{\phi}_3 - \tilde{\phi}_2^2|^2 + |\mu_1 s_0 \tilde{\phi} + 2\mu_2 s \Lambda \phi|^2 + |\mu_1 s_0 \phi + 2\mu_2 \tilde{s} \Lambda \tilde{\phi}|^2 + e^2 |(\phi_1 + \phi_3)\phi_2^* - (\tilde{\phi}_1 + \tilde{\phi}_3)\tilde{\phi}_2^*|^2 + \frac{e^2}{2} (|\phi_1|^2 - |\phi_3|^2 - |\tilde{\phi}_1|^2 + |\tilde{\phi}_3|^2)^2 + \frac{e^2}{3} (|\phi|^2 - |\tilde{\phi}|^2 - 2|s|^2 + 2|\tilde{s}|^2)^2. \quad (2.2)$$

This is minimized when all fields are zero except $\phi_1 = \tilde{\phi}_1 = \eta$ or at any (broken) gauge transformation of this. We note also that the theory has a local minimum with $\phi = \tilde{\phi} = 0$ and that this structure can give rise to hybrid inflation [5]. This is true even for the Abelian theory described in [1]. In both cases inflation ends with defect formation.

As we mentioned above, there are Abelian and non-Abelian string solutions to this theory. The Abelian solution is obtained from the ansatz

$$\phi_1 = \tilde{\phi}_1^* = \eta f(r) e^{i\varphi}, \quad (2.3)$$

$$A_\varphi = \frac{a(r)}{er} \sqrt{\frac{3}{5}} T^G, \quad (2.4)$$

$$F_{S_0} = \mu_1 \eta^2 [1 - f(r)^2], \quad (2.5)$$

where $T^G = \sqrt{\frac{3}{5}} T^3 + \sqrt{\frac{2}{5}} T^4$. All other fields are zero and the profile functions $a(r)$ and $f(r)$ obey the boundary conditions $f(0) = a(0) = 0$ and $\lim_{r \rightarrow \infty} f(r) = \lim_{r \rightarrow \infty} a(r) = 1$.

The non-Abelian solution is obtained from the ansatz

$$\phi = \tilde{\phi}^* = \eta \left\{ \frac{1}{2} (e^{i\varphi} \mathbf{e}_+ + e^{-i\varphi} \mathbf{e}_-) f(r) + \frac{1}{\sqrt{2}} \mathbf{e}_0 g(r) \right\}, \quad (2.6)$$

$$A_\varphi = \frac{a(r)}{er} T^1, \quad (2.7)$$

$$F_{S_0} = \frac{1}{2} \mu_1 \eta^2 [2 - f(r)^2 - g(r)^2], \quad (2.8)$$

$$F_S = F_{\tilde{S}} = \frac{1}{2} \mu_2 \eta^2 [f(r)^2 - g(r)^2], \quad (2.9)$$

where \mathbf{e}_k are unit vectors obeying $T^1 \mathbf{e}_k = k \mathbf{e}_k$. In this case $g(0)$ is finite, $\lim_{r \rightarrow \infty} g(r) = 1$ and $f(r)$ and $a(r)$ obey the same boundary conditions as in the Abelian case.

Note that f , g and a are solutions to simple coupled second order ordinary differential equations. Their forms can be obtained numerically and are well known [8].

III. SUSY TRANSFORMATIONS AND ZERO MODES

The string solutions obtained above have all the fermion fields set to zero. In this section we investigate what happens when these fields are excited in the background of the cosmic string.

We can find the number of zero modes with an appropriate index theorem [6]. To apply the theorem, the fermion fields must be expressed as eigenstates of the string generator. The resulting mass matrix is then split up into irreducible parts, and the theorem applied to each part separately.

For most irreducible mass matrices, the eigenvalues of the fermion fields are split into positive (q_i^+) and negative (q_j^-) eigenvalues, and then ordered. If there are n_+ positive and n_- negative eigenvalues, then $q_1^+ \geq q_2^+ \geq \dots \geq q_{n_+}^+ > 0$ and $q_1^- \leq q_2^- \leq \dots \leq q_{n_-}^- < 0$. Any zeros are ignored. The numbers of complex left and right moving zero modes are then given by

$$N_L = \sum_{j=1}^{\min(n_-, n_+)} [q_j^+ + q_j^-]_+ + \sum_{j=n_-+1}^{n_+} q_j^+, \quad (3.1)$$

$$N_R = \sum_{j=1}^{\min(n_-, n_+)} [-q_j^+ - q_j^-]_+ + \sum_{j=n_++1}^{n_-} -q_j^-. \quad (3.1)$$

where $[x]_+ = x$ if $x > 0$, and 0 otherwise.

The exception to this is when the mass matrix can be put into the form

$$\begin{pmatrix} 0 & A \\ B & 0 \end{pmatrix}, \quad (3.2)$$

where A and B are $n \times n$ matrices. If the fermion states are put in a corresponding form, then in this case the eigenvalues are split up according to whether their eigenstates occupy the first or second half of the $2n$ component fermion vector. The

two sets of eigenvalues are then ordered so that $q_1^A \geq q_2^A \geq \dots \geq q_n^A$ and $q_1^B \leq q_2^B \leq \dots \leq q_n^B$. The resulting number of zero modes is

$$N_L = \sum_{j=1}^n [q_j^A + q_j^B]_+,$$

$$N_R = \sum_{j=1}^n [-q_j^A - q_j^B]_+. \quad (3.3)$$

The derivation of this theorem assumed that none of the fermion fields involved were massless at large r . In fact the results still hold for massless fields if all the eigenvalues are whole integers.

We therefore know that there must exist fermion zero modes on the string. Rather than attempting to solve the difficult fermion equations of motion to obtain these solutions, we shall use a different technique [1] which exploits the power of SUSY to obtain some of the solutions.

A. Abelian string

The relevant part of the Lagrangian is the Yukawa sector which is entirely determined by supersymmetry. In the abelian case, the nonzero Yukawa couplings are

$$\begin{aligned} \mathcal{L}_Y = & - \left[\mu_1 (e^{i\varphi} \tilde{\psi}_1 + e^{-i\varphi} \psi_1) \omega_0 + \sqrt{\frac{5}{6}} i e (e^{-i\varphi} \psi_1 \right. \\ & - e^{i\varphi} \tilde{\psi}_1) \lambda^G + 2\mu_2 (e^{i\varphi} \psi_3 \omega + e^{-i\varphi} \tilde{\psi}_3 \tilde{\omega}) \\ & \left. + \frac{ie}{\sqrt{2}} (e^{-i\varphi} \lambda_+ \psi_2 + e^{i\varphi} \lambda_- \tilde{\psi}_2) \right] \eta f(r), \end{aligned} \quad (3.4)$$

where $\lambda_{\pm} = (\lambda^1 \mp i\lambda^2)/\sqrt{2}$. With respect to the string generator, the only fields with non-zero eigenvalues are ψ_1 and λ_+

(eigenvalue 1) and $\tilde{\psi}_1$ and λ_- (eigenvalue -1). The Yukawa Lagrangian can be split up into 5 independent parts. Applying Eq. (3.4) to these reveals that there are a total of three left moving and three right moving complex zero modes.

Now, following the techniques of [1], we perform an infinitesimal SUSY transformation (with Grassmann parameter ξ_α) followed by a gauge transformation to return to the Wess-Zumino gauge. The (bosonic) string fields all transform quadratically and so are unchanged to first order. However, the fermions transform linearly and, in terms of the background string fields, it is possible to find two complex (or 4 real) fermion zero mode solutions given by

$$\omega_0 = \sqrt{2} \mu_1 \eta^2 (1 - f^2) \xi, \quad (3.5)$$

$$\lambda^G = -i \sqrt{\frac{3}{5}} \frac{a}{er} \sigma^z \xi, \quad (3.6)$$

$$\psi_1 = i\sqrt{2} \eta e^{i\varphi} \left[f' \sigma^r + i \frac{(1-a)}{r} f \sigma^\varphi \right] \bar{\xi}, \quad (3.7)$$

$$\tilde{\psi}_1 = i\sqrt{2} \eta e^{-i\varphi} \left[f' \sigma^r - i \frac{(1-a)}{r} f \sigma^\varphi \right] \bar{\xi}. \quad (3.8)$$

Setting either component of ξ to zero gives one of the zero modes, one of which is left moving and the other right moving.

B. Non-Abelian string

In the non-Abelian case it is convenient to split ψ , $\tilde{\psi}$ and λ^i into eigenvectors of T^1 , and label them by their eigenvalues. Defining $\chi_i = (\psi_i + \tilde{\psi}_i)/\sqrt{2}$, $\zeta_i = (\psi_i - \tilde{\psi}_i)/\sqrt{2}$ and $\omega_{(\pm)} = (\omega \pm \tilde{\omega})/\sqrt{2}$, the nonzero Yukawa couplings are

$$\begin{aligned} \mathcal{L}_Y = & - \mu_1 \eta \left[\chi_0 g(r) + \frac{1}{\sqrt{2}} (\chi_+ e^{-i\varphi} + \chi_- e^{i\varphi}) f(r) \right] \omega_0 - \mu_2 \eta [-\sqrt{2} \chi_0 g(r) + (\chi_+ e^{-i\varphi} + \chi_- e^{i\varphi}) f(r)] \omega_{(+)} \\ & - \frac{ie\eta}{2} (\chi_+ e^{-i\varphi} - \chi_- e^{i\varphi}) f(r) \lambda^0 - \mu_2 \eta [-\sqrt{2} \zeta_0 g(r) + (\zeta_+ e^{-i\varphi} + \zeta_- e^{i\varphi}) f(r)] \omega_{(-)} + \frac{e\eta}{2} [\sqrt{2} (\zeta_- \lambda^+ - \zeta_+ \lambda^-) g(r) \\ & + \zeta_0 (\lambda^+ e^{-i\varphi} - \lambda^- e^{i\varphi}) f(r)] - \frac{ie\eta}{\sqrt{6}} [\sqrt{2} \zeta_0 g(r) + (\zeta_+ e^{-i\varphi} + \zeta_- e^{i\varphi}) f(r)] \lambda^4. \end{aligned} \quad (3.9)$$

In this case χ_{\pm} , ζ_{\pm} and λ^{\pm} have eigenvalues ± 1 . Applying Eq. (3.4) to the two irreducible parts of Eq. (3.9) shows that there are just two complex zero modes, moving in opposite directions.

Once again performing an infinitesimal SUSY transformation and a (non-Abelian) gauge transformation we obtain the two complex zero modes

$$\omega_0 = \frac{1}{\sqrt{2}} \mu_1 \eta^2 (2 - g^2 - f^2) \xi, \quad (3.10)$$

$$\omega_{(+)} = \mu_2 \eta^2 (g^2 - f^2) \xi, \quad (3.11)$$

$$\lambda^0 = -i \frac{a}{er} \sigma^z \xi, \quad (3.12)$$

$$\chi_+ = i\eta e^{i\varphi} \left(f' \sigma^r + i \frac{1-a}{r} f \sigma^\varphi \right) \bar{\xi}, \quad (3.13)$$

$$\chi_- = i\eta e^{-i\varphi} \left(f' \sigma^r - i \frac{1-a}{r} f \sigma^\varphi \right) \bar{\xi}, \quad (3.14)$$

$$\chi_0 = i\sqrt{2} \eta g' \sigma^r \bar{\xi}. \quad (3.15)$$

Thus in this case there are no zero modes beyond those implied by SUSY, in contrast to the Abelian case. This is related to the fact that there are components of the Higgs fields that do not wind in the non-Abelian case.

IV. SOFT SUSY BREAKING

Perhaps the most attractive feature of SUSY arises from the nonrenormalization theorems. These ensure that quadratic divergences are absent in SUSY theories and so protect any tree-level hierarchy of scales from receiving quantum corrections.

When SUSY is broken, as it must be at a scale of around 1 TeV, it is crucial that these quadratic divergences remain absent from the theory. This is achieved by adding only *soft* SUSY breaking terms to the model. Such terms are defined as being those which are noninvariant under SUSY transformations but which do not induce quadratic divergences.

In a general model, one may obtain soft SUSY breaking terms by the following prescription.

- (1) Add arbitrary mass terms for all scalar particles to the scalar potential.
- (2) Add all trilinear scalar terms in the superpotential, plus their Hermitian conjugates, to the scalar potential with arbitrary coupling.
- (3) Add mass terms for the gauginos to the Lagrangian density.

Since the techniques we have used are strictly valid only when SUSY is exact, it is necessary to investigate the effect of these soft terms on the fermionic zero modes we have identified.

As we have already commented, the existence of the zero modes can be seen as a consequence of an index theorem [6]. The index is insensitive to the size and exact form of the Yukawa couplings, as long as they are regular for small r , and tend to a constant at large r . In fact, the existence of zero modes relies only on the existence of the appropriate Yukawa couplings and that they have the correct φ dependence. Thus there can only be a change in the number of zero modes if the soft breaking terms induce specific new Yukawa couplings in the theory and it is this that we must check for. Further, it was conjectured in [6] that the destruction of a zero mode occurs only when the relevant fermion mixes with another massless fermion.

We have examined each of our theories, both from this paper and Ref. [1], with respect to this criterion and list the results below.

A. $U(1)$ Abelian models

1. Theory F

In Ref. [1] we referred to an Abelian theory in which the gauge symmetry is broken via an F term as “theory F ”. The corresponding superpotential was

$$W = \mu \Phi_0 (\Phi_+ \Phi_- - \eta^2). \quad (4.1)$$

The trilinear and mass terms that arise from soft SUSY breaking in this theory are

$$m_0^2 |\phi_0|^2 + m_-^2 |\phi_-|^2 + m_+^2 |\phi_+|^2 + \mu M \phi_0 \phi_+ \phi_-. \quad (4.2)$$

The derivative of the scalar potential with respect to ϕ_0^* becomes

$$\phi_0 (\mu^2 |\phi_+|^2 + \mu^2 |\phi_-|^2 + m_0^2) + \mu M (\phi_+ \phi_-)^*. \quad (4.3)$$

This will be zero at a minimum, and so $\phi_0 \neq 0$ only if $M \neq 0$.

New Higgs mass terms will alter the values of ϕ_+ and ϕ_- slightly, but will not produce any new Yukawa terms. Thus these soft SUSY-breaking terms have no effect on the existence of the zero modes.

However, the presence of the trilinear term gives ϕ_0 a non-zero expectation value, which gives a Yukawa term coupling the ψ_+ and ψ_- fields. This destroys all the zero modes in the theory since the left and right moving zero modes mix.

For completeness note that a gaugino mass term also mixes the left and right zero modes, aiding in their destruction.

In terms of the index theorem, the change in the number of zero modes arises because Eq. (3.2) applies after the SUSY breaking, while Eq. (3.4) applied before it. Although the fermion eigenvalues do not change, the expression relating them to the zero modes does.

2. Theory D

The $U(1)$ theory with gauge symmetry broken via a Fayet-Iliopoulos term and no superpotential is simpler to analyze [1]. New Higgs mass terms have no effect, as in the above case, and there are no trilinear terms. Further, although the gaugino mass terms affect the form of the zero mode solutions, they do not affect their existence, and so, in theory D , the zero modes remain even after SUSY breaking.

B. $SU(2) \times U(1)$ model

1. Abelian strings

The effect of soft SUSY breaking terms on the zero modes which were found analytically in Eq. (3.8) is identical to the equivalent $U(1)$ theory. Thus, all SUSY zero modes are destroyed in this case. In this larger theory, there are also other, non-SUSY, zero modes. Not all of these are destroyed by a gaugino mass term, as some do not involve the gaugino fields. However, if the trilinear terms give s_0 a non-zero vacuum expectation value, all the zero modes are destroyed. The extra Yukawa terms mean there are fewer irreducible parts to the fermion mass matrices. This results in more terms cancelling in Eq. (3.4), reducing the number of zero

modes. As in the other cases, the physical reason behind the destruction of the zero modes is that left and right movers mix.

2. Non-Abelian strings

As in the other cases above, nonzero gaugino mass or trilinear terms destroy the zero modes that were found with SUSY transformations (3.15). Similarly Eq. (3.2) is required instead of Eq. (3.4), implying that the left and right moving modes mix. For non-Abelian strings these are the only zero modes and so none remain after SUSY breaking.

V. COMMENTS AND CONCLUSIONS

We have examined the microphysics of Abelian and non-Abelian cosmic string solutions to the $SO(10)$ inspired supersymmetric $SU(2)\times U(1)$ model. By performing infinitesimal SUSY transformations on the background string fields we have obtained the form of the fermionic zero modes responsible for cosmic string conductivity. These solutions may be compared to those found in our earlier work [1] on Abelian defects.

Our results mean that fermion zero modes are always present around cosmic strings in SUSY. We conjecture that in theories with F -term gauge symmetry breaking, the zero modes given by SUSY always occur in pairs, one left and one right moving. It also seems likely that such theories always have hybrid inflation.

Further, in the Abelian case there were additional zero modes that were not a consequence of supersymmetry. We expect that similar extra zero modes will be present in a larger theory, even in the non-Abelian case.

We have also analyzed the effect of soft SUSY breaking on the existence of fermionic zero modes. The $SU(2)\times U(1)$ model and two simple Abelian models were examined. In all cases Higgs mass terms did not affect the existence of the zero modes. In the theories with F -term symmetry breaking, gaugino mass terms destroyed all zero modes which involved gauginos and trilinear terms created extra Yukawa couplings which destroyed all the zero modes present. In the Abelian theory with D -term symmetry breaking, the zero modes were unaffected by the SUSY breaking

terms. It was conjectured in [6] that zero modes would only disappear when they mixed with another massless fermion field and this is consistent with the results obtained in this section. If the remaining zero modes survive subsequent phase transitions, then stable vortons could result. Such vortons would dominate the energy density of the universe, rendering the underlying GUT cosmologically problematic.

Therefore, although SUSY breaking may alleviate the cosmological disasters faced by superconducting cosmic strings [3], there are classes of string solution for which zero modes remain even after SUSY breaking. It remains to analyze all the phase transitions undergone by specific SUSY GUT models to see whether or not fermion zero modes survive down to the present time. If the zero modes do not survive SUSY breaking, the universe could experience a period of vorton domination beforehand, and then reheat and evolve as normal afterwards.

If the zero modes do occur in pairs (one left and one right moving) in F -term gauge symmetry breaking, it is possible that they could scatter off each other [7]. This would cause the current to decay, and could stop vorton domination.

There is the possibility that even if zero modes are destroyed they become low-lying bound states. Such bound states may still be able to carry a persistent current. If this is the case, even such theories may not be safe cosmologically. Work on this is under investigation.

It may also be possible to extend our analysis of the effect of SUSY breaking on the bosonic fields. The resulting potential is very complex, even in the Abelian case. However, it may be possible to use some sort of approximation or numerical solution. The change in potential also affects the hybrid inflation which occurs in the model, although at this stage it is not clear how.

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