Oscillating D-strings from type IIB matrix theory

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We present a class of BPS solutions of the type IIB matrix theory which preserves 1/4 supersymmetry. The solutions describe D-string configurations with left-moving oscillations. We demonstrate that the one-loop quantum effective action of matrix theory vanishes for this solution, confirming its BPS nature. We also study the world-volume gauge theory of oscillating strings and show its connection with static D-strings. [S0556-2821(98)03508-5]

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I. INTRODUCTION

One of the most challenging problems in string theory has been to understand its strong coupling aspects [1,2], including its moduli space structure in full quantum theory at the nonperturbative level. One also hopes that such an investigation will lead to an understanding of supersymmetry breaking in these theories and will give the correct string theory vacuum describing the real world. As is well known, the unravelling of nonperturbative aspects includes an analysis of the soliton spectrum [3] and their moduli dependence. Investigations along these lines have also led to a better understanding of the confinement mechanism in supersymmetric gauge theories from the string theory point of view [4].

A useful mechanism in studying the strong coupling aspects has been the D-brane constructions of string solitons [5]. As a result, one can obtain the soliton spectrum and their interactions using open string conformal field theory. Since the D-branes preserve a certain amount of supersymmetry, they are stable solitonic superstring vacua, around which a quantum field theory of the world-volume degrees of freedom can be formulated [6]. Many such D-brane configurations have been obtained [7] and the corresponding effective world-volume actions have been analyzed. Among them, of particular interest have been some of the six- [8] and two- [9] dimensional supersymmetric gauge theories.

In the matrix theory [10,6,11,12] proposal by Banks, Fischler, Shenker, and Susskind (BFSS), the $SU(N)$ $(N\to\infty)$ world-volume gauge theory of N D-branes has been conjectured to be a fundamental theory describing both the perturbative and nonperturbative aspects of string theory. In the simplest case, this is the dimensional reduction of the $D=10$, $N=1$ Yang-Mills theory to the relevant world-volume dimension. In this context, it has been shown that various brane solutions of string theory [11,6], including their charges, can be obtained from classical solutions in such gauge theories. We will concentrate on type IIB matrix theory [13] which proposes that the ten-dimensional type IIB string theory is described by the dimensional reduction of the $D=10$, $N=1$ $SU(N)$ guage theory to zero dimension. This possesses a manifest Lorentz invariance. The emergence of a D-string from such a matrix theory has also been shown through an analysis of its interactions. A duality among matrix theories proposed earlier for describing M theory and the one for type IIB theory has also been argued.

In this paper, we generalize some of the results in [13] and write down an infinite set of classical solutions of type IIB matrix theory [14] by solving the field equations. These are classical gauge field configurations which correspond to D-strings with chiral (left-moving) oscillations. The existence of these solutions follow from oscillating fundamental string solutions [15] in type IIB string theory and its $SL(2,Z)$ S duality in ten dimensions [3]. As in the case of fundamental strings, we show that the matrix theory solutions preserve 1/4 supersymmetry.

The Bogomol'nyi-Prasad-Sommerfield (BPS) mass formula for type IIB string theory, when compactified to nine dimensions, has been written down earlier. They are parametrized by integers (m,n), namely, internal momenta and winding in the compactified direction, as well as by the gauge charges (p,q) corresponding to the Neveu-Schwarz–Neveu-Schwarz (NS-NS) and Ramond-Ramond (RR) antisymmetric tensor fields in ten dimensions. It is also known that this BPS formula is invariant under the $SL(2,Z)$ U duality in nine dimensions, which follows from the S duality of the ten-dimensional type IIB strings. In this paper we mainly concentrate on the BPS formula for the $(p=0,q=1)$ case which corresponds to a single D-string. An explicit form for the BPS formula for this case can be derived by using the $SL(2,Z)$ duality on the mass formula of the fundamental type IIB string in nine dimensions and by restricting ourselves to the supersymmetric ground states. The mass formula for the fundamental string has the form [3]

$$M^2=\left(\frac{m}{R_B}\right)^2+(2\pi R_B n T_q)^2+4\pi T_q(N_L+N_R), \quad (1.1)$$

with

$$N_R-N_L=mn, \quad (1.2)$$

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and T_q is the string tension of the fundamental string. A general formula for the (p,q) string involves a generalization of the definition of T_q, written in terms of the ten-dimensional axion-dilaton moduli as

$$T_q^2=[p^2+e^{2\phi_0}(p\chi_0+q)^2]e^{-\phi_0}T^2. \tag{1.3}$$

For a $(0,1)$ string we then have $T_q=e^{\phi_0/2}T$.

The BPS states which preserve 1/2 supersymmetry and their interactions have already been analyzed in the matrix theory context [13]. They correspond to the supersymmetric ground states $N_L=N_R=0$. We will examine the BPS configurations of the matrix theory preserving 1/4 supersymmetry. They are the supersymmetric ground states with either $N_L=0$ or $N_R=0$ and provide a rich spectra parametrized by the integers (m,n). The BPS mass then satisfies the relation

$$M_{BPS}=(2\pi R_B n T_q+m/R_B). \tag{1.4}$$

The mass formula (1.4) is an exact expression which does not receive quantum corrections. In matrix theory we verify this by showing that the one-loop quantum effective action for our solution vanishes.

As an application of type IIB matrix theory, we then obtain the world-volume gauge theory in the classical background of an oscillating D-string solution. It is known that the world-volume theory for a static D-string configuration of type IIB matrix theory is a two-dimensional gauge theory with $(8,8)$ supersymmetry [6,16]. In this paper, we obtain an explicit expression for the supersymmetric world-volume gauge theory action with $(8,0)$ supersymmetry from the matrix theory action. We show its Lorentz, gauge, and supersymmetry invariance. The gauge and supersymmetry invariance are the residual symmetries of the orginal type IIB theory. The supersymmetry is a global symmetry in this case, as it originates from the global supersymmetry of the Green-Schwarz superstring action, in the Schild gauge, or from the supersymmetry of $N=1$ Yang-Mills theory in ten dimensions. The gauge invariance of the $(8,0)$ world-volume action also follows from that of the gauge invariance of the ten-dimensional super Yang-Mills theory. Although the final model does not possess an explicit left-right symmetry, we will argue in Sec. IV, from a matrix theory point of view, that the particle spectrum is anomaly free. We also argue that the world-sheet actions for the static and oscillating strings define equivalent quantum field theories. This is demonstrated through a mapping of operators in the two cases. Physically this also implies that the static string is a quantum state of the world-volume theory in the classical background we have studied.

This work has been partly motivated by an analysis of BPS states in compactified M theory using the BFSS model [17]. We have carried out this analysis in the framework of S^1 compactified type IIB matrix theory [13]. The rest of the paper is organized as follows. In Sec. II, we review the oscillating fundamental string solutions from a supergravity point of view and mention how the corresponding D-strings can be obtained using the S duality of ten-dimensional type IIB string theory. In Sec. III, we obtain these solutions from the IIB matrix theory. We also show that the matrix theory solution preserves 1/4 supersymmetry. In this section, we also point out that the one-loop quantum effective action of matrix theory, for this solution, vanishes. In Sec. IV, we present the $(8,0)$ supersymmetric gauge theory of oscillating strings and show its connection with static strings. Conclusions and discussions are presented in Sec. V.

II. OSCILLATING STRING SOLUTION

We now start with a review of oscillating string solutions in string theory [15]. They were obtained as a generalization of the static fundamental strings found earlier [18] and are the solutions of the supergravity equations of motion. The singularity of the field configuration represents the position of the string. However, unlike the static case, they correspond to the states preserving only 1/4 supersymmetry.

It is also known that the static fundamental string solutions can be identified with charged extremal black holes in one lower dimension. Similarly, the oscillating string solutions, after compactification along its length, can asymptotically be identified with the supersymmetric, stationary, rotating, charged black holes. In the context of our discussion in the last section, the static string is a supersymmetric ground state and the oscillator numbers are fixed to their minimum values $N_L=N_R=0$. On the other hand, in the oscillating string configuration only $N_R=0$ and N_L is an arbitrary oscillator number. The oscillating string solutions require, from the space-time point of view, the presence of a (large) compactified direction on which the string is wrapped, as otherwise the only BPS configurations are those preserving 1/2 supersymmetry in ten noncompact dimensions. We take x^1 as the compactified coordinate of radius R.

The supergravity solution corresponding to the oscillating fundamental string is given as

$$\begin{aligned}
ds^2&=-e^{2\phi}du\,dv+[e^{2\phi}p(v)r^{-D+4}-(e^{2\phi}-1)\dot{F}(v)^2]dv^2\\
&\quad+2(e^{2\phi}-1)\dot{\mathbf{F}}(v)\cdot d\mathbf{x}\,dv+d\mathbf{x}\cdot d\mathbf{x},\\
B_{uv}&=\frac{1}{2}(e^{2\phi}-1),\\
B_{vi}&=\dot{F}_i(v)(e^{2\phi}-1),\\
e^{-2\phi}&=1+\frac{Q}{|x-F|^{D-4}},
\end{aligned} \tag{2.1}$$

where, for a fundamental string solution, $B_{\mu\nu}$ is the NS-NS antisymmetric tensor field and $F_i(v)$ are functions of the light-cone coordinate $v=x^0+x^1$ only. $u=x^0-x^1$ is the other light-cone coordinate. Overdots denote the derivative with respect to the argument v and boldfaced letters denote a vector in the transverse directions labeled by indices i's. To match properly with a string source, one also requires $p(v)=0$. The field configurations in Eqs. (2.1) define an asymptotically flat space. As a result, one can properly define the Arnowitt-Deser-Misner (ADM) mass and charge for the supergravity background. It has also been pointed out that the supergravity solution as well as the ADM energy properly matches with a string source, written in terms of the world-sheet coordinates τ and σ as

$$V(\tau,\sigma)=2Rn\sigma^{+},$$

$$U(\tau,\sigma)=(2Rn+a)\sigma^{-}+\int^{V}\dot{F}^{2}, \tag{2.2}$$

$$\mathbf{X}(\tau,\sigma)=\mathbf{F}(V),$$

where $\sigma^{\pm}=\tau\pm\sigma$ and V,U are the space-time light-cone string coordinates: $U=X^0-X^1$, $V=X^0+X^1$, and X^i are once again the string coordinates along the transverse directions. The constant a is the zero mode of $\dot{\mathbf{F}}^2$,

$$a=\frac{1}{\pi}\int_0^{2\pi Rn}\dot{\mathbf{F}}^2, \tag{2.3}$$

and F_i's have no zero modes. The oscillating string is specified by the left-moving wave profile $F_i(v)$. In [15] some specific wave profiles have been used to show the connection of the oscillating string solution with the charged rotating black holes. For our purposes, however, we do not need their specific form.

The world-sheet configuration (2.2) has been identified with a string source of momenta and winding

$$p^{\mu}=(2\alpha')^{-1}(2Rn+a,-a,\mathbf{0}), \quad n^{\mu}=(0,n,\mathbf{0}), \tag{2.4}$$

along the directions (X^0,X^1,X^i). The internal momenta m/R in the compact direction is then specified by integers

$$m=-\frac{Ra}{2\alpha'}, \tag{2.5}$$

and the oscillator number, obtained by the level-matching condition, is

$$N_L=\frac{nRa}{2\alpha'}. \tag{2.6}$$

An oscillating D-string in the supergravity context can be obtained by applying an $SL(2,Z)$ duality transformation on the fundamental string solution presented above. The general procedure, as well as the specific $SL(2,Z)$ transformation matrix (λ), is similar to the generation of a static $(p=0, q=1)$ string solution from the (1,0) solution as described in [3]. We do not elaborate on them further, except to note that the fundamental string tension will be replaced appropriately by the one for a D-string.

The string source (2.2) will play a crucial role in obtaining a matrix theory solution as they, with appropriate modifications of string tension, will specify the gauge field configurations, which are the solution of the matrix theory field equations. So far we have only discussed a single fundamental (1,0) and D-string (0,1) solution. The existence of multiple supersymmetric parallel string configurations has also been shown in [15]. They correspond to $(p,0)$ and $(0,q)$ type BPS states, preserving once again 1/4 supersymmetry. It may also be possible to obtain higher dimensional oscillating branes [19] and to obtain their parallel and orthogonal supersymmetric configurations.

In the next section we obtain the oscillating string as a solution to the field equation in matrix theory and examine its properties. We also show the BPS nature of $(0,q)$ or multi-D-string solutions of matrix theory from the results of a one-loop effective action.

III. TYPE IIB MATRIX THEORY

We now obtain an infinite set of solutions of type IIB matrix theory and show that they correspond to the oscillating D-strings discussed in the last section from the supergravity point of view. The type IIB matrix theory action is obtained by the dimensional reduction of the $D=10$, $N=1$ $SU(N)$ super-Yang-Mills theory to zero dimension and is written as [13]

$$S=\alpha\left(-\frac{1}{4}\mathrm{Tr}[A_{\mu},A_{\nu}]^2-\frac{1}{2}\mathrm{Tr}(\bar{\psi}\Gamma^{\mu}[A_{\mu},\psi])\right)+\beta\ \mathrm{Tr}1, \tag{3.1}$$

where the last term in the action is a "chemical potential." A similar term in the Schild-type string action is necessary to show its equivalence with the Nambu-Goto action. α and β are constants with $\sqrt{\alpha\beta}$ defining the D-string tension. Equation (3.1) without the chemical potential term is also referred to as the D-instanton matrix action [14]. The constants α and β can be determined by comparing the string interaction in matrix theory with those from open strings. The final results are

$$\alpha=\frac{8\pi^{5/2}}{\sqrt{3}\gamma}\frac{1}{\alpha'^2 g_s}, \quad \beta=\frac{24\pi^{9/2}}{\sqrt{3}\gamma}\frac{1}{g_s},$$

with γ being a numerical constant.

In [13], the target space metric, represented by the indices μ, has been chosen as Euclidean, whereas the oscillating string solutions of [15] presented in the last section are in the Minkowski metric. We take care of this discrepancy by putting appropriate factors of i in the solutions of Sec. II while computing the one-loop effective action. For the moment, however, we continue to work with the Minkowski metric.

The field equations of matrix theory are

$$[A^{\mu},[A_{\mu},A_{\nu}]]=0,$$

$$[A_{\mu},(\Gamma^{\mu}\psi)_{\alpha}]=0. \tag{3.2}$$

As fermions do not have a classical background, only the first equation of Eqs. (3.2) is considered for analyzing the classical solutions.

The action (3.1) is invariant under supersymmetry transformations

$$\delta^{(1)}\psi=\frac{i}{2}[A_{\mu},A_{\nu}]\Gamma^{\mu\nu}\epsilon, \quad \delta^{(1)}A_{\mu}=i\bar{\epsilon}\Gamma_{\mu}\psi \tag{3.3}$$

and

$$\delta^{(2)}\psi=\xi, \quad \delta^{(2)}A_{\mu}=0. \tag{3.4}$$

These are also referred to as the "dynamical" and "kinematic" supersymmetry transformations [6] and follow from the dimensional reduction of the world-sheet Green-Schwarz superstring action in the Schild gauge to zero dimension. In addition, the action (3.1) is invariant under a gauge transformation:

$$\delta_{gauge}A_{\mu}=i[A_{\mu},\alpha], \quad \delta_{gauge}\psi=i[\psi,\alpha]. \tag{3.5}$$

The field equations (3.2) are now solved by infinite dimensional Hermitian matrices A_{μ}'s. In turn, using the familiarity with quantum mechanics, these matrices are represented by the canonically conjugate variables q_i's and p_i's. The relationship of these solutions with those in string theory is established through an identification of the commutators with the Poisson brackets for the Schild action [13]:

$$\{X,Y\}=\frac{1}{\sqrt{g}}\epsilon^{ab}\partial_a X\partial_b Y, \tag{3.6}$$

where a, b denote the world-sheet coordinates τ, σ. Moreover, one also identifies

$$-i[\ ,\ ]\rightarrow\{\ ,\ \}, \quad \mathrm{Tr}\rightarrow\int d^2\sigma\sqrt{g},$$

$$\tau\rightarrow\frac{q}{\sqrt{2\pi N}}, \quad \sigma\rightarrow\frac{p}{\sqrt{2\pi N}}, \tag{3.7}$$

with the commutator $[q,p]=2\pi i$. The static D-string

$$X^0=T\tau, \quad X^1=\frac{L}{2\pi}\sigma, \quad X^i=0 \tag{3.8}$$

can then be represented by the gauge field configuration

$$A^0=\frac{T}{\sqrt{2\pi N}}q, \quad A^1=\frac{L}{\sqrt{2\pi N}}p, \quad A^i=0 \tag{3.9}$$

and satisfies the fields equations (3.2). Similarly the oscillating string can be represented by a gauge field configuration which is obtained through the identifications in Eqs. (3.7). Continuing to work in light-cone coordinates, the components A^{μ}'s are given as

$$A^V=2Rn\hat{\sigma}^+,$$

$$A^U=(2Rn+a)\hat{\sigma}^-+\int^{\hat{V}}\dot{F}^2, \tag{3.10}$$

$$A^i=F^i(\hat{V}),$$

where $\hat{\sigma}^{\pm}=q\pm p/\sqrt{2\pi N}$ and $\hat{V}$ denotes an operator replacement in the function V: $\hat{V}(\tau,\sigma)\rightarrow V(q/\sqrt{2\pi N},p/\sqrt{2\pi N})$. Once again, the gauge field configuration for a static string (3.8) corresponds to $F^i=0$ and $T=L/2\pi=2Rn$.

Now, to verify that A_{μ}'s in Eqs. (3.10) are solutions of Eqs. (3.2), we evaluate their commutators. The nonzero ones are

$$[A^V,A^U]=-\frac{2i}{N}(2Rn)(2Rn+a),$$

$$[A^U,A^i]=\frac{2i}{N}(2Rn)(2Rn+a)\dot{F}^i. \tag{3.11}$$

These imply that the field equations are once again satisfied. We have therefore found a class of solutions of the matrix theory field equations specified by the wave profile $\mathbf{F}(V)$.

We now examine the BPS and supersymmetry properties of the solution (3.10). In the background of a static string configuration, the dynamical supersymmetry transformation is given as

$$\delta^{(1)}\psi=-\frac{TL}{2\pi N}\Gamma^{01}\epsilon, \quad \delta^{(1)}A_{\mu}=0. \tag{3.12}$$

As a result, the only way to preserve some amount of supersymmetry is to cancel the dynamical supersymmetry transformation with the kinematic one by defining $\xi=\pm(TL/2\pi N)\Gamma^{01}\epsilon$. We then have $(\delta^1\pm\delta^2)\psi=0$ and $(\delta^1\pm\delta^2)A_{\mu}=0$, which implies that the solution preserves 1/2 supersymmetry.

Now, for the oscillating string background, the dynamical supersymmetry transformation can be written as

$$\delta^{(1)}\psi=\frac{1}{2N}(2Rn)(2Rn+a)[\Gamma^{UV}\epsilon+\dot{F}^i\rangle\Gamma^{Vi}\epsilon],$$

$$\delta^{(1)}A_{\mu}=0. \tag{3.13}$$

Since the transformation $\delta^{(2)}$ is still given by Eq. (3.4), and hence to make sure that a certain amount of supersymmetry, namely, $\delta^{(1)}\pm\delta^{(2)}$, is preserved, one also has to impose the condition

$$\dot{F}_i\Gamma^{Vi}\epsilon=0. \tag{3.14}$$

Before solving this equation explicitly, we notice that Eq. (3.14) is a chirality condition on ϵ in the light-cone directions, namely, $(1+\Gamma^0\Gamma^1)\epsilon=0$. Since the string world sheet is identified with light-cone coordinates, Eq. (3.14) implies a chirality condition in the world-volume directions. More explicitly, by choosing ten-dimensional Γ matrices in the Majorana representation as

$$\Gamma^0=i\begin{pmatrix}0 & -I_8\\ I_8 & 0\end{pmatrix}, \quad \Gamma^1=-i\begin{pmatrix}0 & I_8\\ I_8 & 0\end{pmatrix}, \quad \Gamma^i=\begin{pmatrix}\gamma^i & 0\\ 0 & -\gamma^i\end{pmatrix}, \tag{3.15}$$

and by decomposing the ten-dimensional spinor ϵ in terms of the eight-dimensional ones as $\epsilon=\binom{\epsilon_L}{\epsilon_R}$, the condition (3.14) implies $\epsilon_R=0$. To summarize this part of the discussion, we have shown that a cancellation between the "dynamical" and "kinematic" supersymmetry transformations can occur in the matrix background (3.10) provided half the components of the dynamical supersymmetry transformations are zero. This, in turn, implies that our solution preserves only 1/4 supersymmetry, as expected of an oscillating string.

To further identify the solution of matrix theory (3.10) with the oscillating string solution we evaluate the classical action for this configuration. We have

$$S_B = \frac{\alpha}{2}\left(\frac{(2Rn)(2Rn+a)}{N}\right)^2 N + \beta N. \tag{3.16}$$

An extremization with respect to N and the identification $\sqrt{\alpha\beta} = 2\pi\rho$, with ρ being the string tension, now gives

$$S_B = 2\pi\rho(2Rn)(2Rn+a). \tag{3.17}$$

To verify that the action (3.17) is proportional to the area of the world sheet, we have directly evaluated the Polyakov action, acting as the source for the supergravity background and for the oscillating string solution and shown that it again gives the same value as in Eq. (3.17). Since the solutions in [15] also satisfy the Virasoro condition, the evaluation of the Nambu-Goto action in this background also gives the same value. These results once again confirm that the Yang-Mills field configurations do indeed represent the oscillating strings and, in turn, the infinite hierarchy of BPS states. In this context, we notice that the BPS mass formula (1.4) also follows from the time component of the target space momentum for these strings written in Eqs. (2.4). It is also interesting to note that the matrix solution represents a string with well-defined string tension for generic oscillations $F_i(v)$. The change in the value of the action with respect to the static string is by an amount

$$\Delta S_B = 2\pi\rho(2Rn)a = \frac{2}{\pi}N_L, \tag{3.18}$$

where the last equality follows from the relation $a = -(2\pi^2\rho)^{-1}m/R$ for a D-string which is analogous to Eq. (2.5) for a fundamental string through a replacement $1/2\pi\alpha' \to 2\pi\rho$. Solutions (3.10) can then be interpreted as an excitation over the static string state by an amount N_L from this point of view as well.

We now analyze the one-loop effective action of matrix theory for the classical background (3.10) and show that the effective action vanishes. The effective action in a general background $A_\mu = p_\mu$ has the form [13]

$$\begin{aligned}\mathrm{Re}W = {} & \frac{1}{2}\mathrm{Tr}\,\log(P_\lambda^2\delta_{\mu\nu} - 2iF_{\mu\nu}) \\ & - \frac{1}{4}\mathrm{Tr}\,\log\left[\left(P_\lambda^2 + \frac{i}{2}F_{\mu\nu}\Gamma^{\mu\nu}\right)\left(\frac{1+\Gamma_{11}}{2}\right)\right] \\ & - \mathrm{Tr}\,\log(P_\lambda^2),\end{aligned} \tag{3.19}$$

where P_μ and $F_{\mu\nu}$ are operators acting on the space of matrices as

$$P_\mu X = [p_\mu, X], \quad F_{\mu\nu}X = [f_{\mu\nu}, X], \tag{3.20}$$

with $f_{\mu\nu} = i[p_\mu, p_\nu]$, p_μ being the operator replacement for variables A_μ. The terms in Eq. (3.19) correspond to the contributions from the bosons A_μ, the fermions ψ, and the Faddeev-Popov ghosts, repectively. It has also been noticed in [13] that the imaginary part of W vanishes when P_i is zero along at least one of the transverse directions i and implies the absence of an anomaly in the world-volume action. This holds in our case, provided $F_i = 0$ for this index i. However, it is likely that $\mathrm{Im}W = 0$ in generic cases as well.

To evaluate the effective action in our case, we rewrite the gauge field commutators in Eq. (3.11) in the Euclidean metric and notice that only nonzero components of $F_{\mu\nu}$, namely, F_{0i} and F_{1i}, satisfy the relation $F_{0i} = -iF_{1i}$. The form of the matrix $P^2\delta_{\mu\nu} - 2iF_{\mu\nu}$,

$$P^2\delta_{\mu\nu} - 2iF_{\mu\nu} = \begin{pmatrix} P^2 & 0 & -2iF_{02} & \cdot \\ 0 & P^2 & -2iF_{12} & \cdot \\ 2iF_{02} & 2iF_{12} & P^2 & 0 \\ \cdot & \cdot & 0 & \cdot \end{pmatrix}, \tag{3.21}$$

and the property of the operators P_λ^2, $F_{\mu\nu}$, in our case $[P_\lambda^2, F_{\mu\nu}] = 0$, then imply that $F_{\mu\nu}$'s cancel out in the expression of the determinant of the matrix. To show this in another way, we expand

$$\begin{aligned}\mathrm{Tr}\,\log(P_\lambda^2\delta_{\mu\nu} - 2iF_{\mu\nu}) = {} & \mathrm{Tr}\,\log P_\lambda^2\delta_{\mu\nu} + \mathrm{Tr}(2iF_{\mu\nu}/P_\lambda^2) \\ & + \frac{1}{2}(2i)^2\mathrm{Tr}[F_{\mu\alpha}F_\nu^\alpha/(P_\lambda^2)^2] + \cdots\end{aligned} \tag{3.22}$$

and use the fact that the only nonvanishing components of $\delta_{\mu\nu}, F_{\mu\nu}$ in the u,v coordinates are $\delta^{uv} = 1$ and $F^{ui} = -2F_{vi}$. It can then be shown that all the higher-order terms vanish, as one cannot form invariants out of the above nonvanishing components. A similar property of certain classical field configurations, namely, chiral-null models, has been used to show that they are an exact solution of first quantized string theory [21]. We interestingly observe the appearance of this property in the context of matrix theory.

The terms in the trace of the matrix $[P_\lambda^2 + (i/2)F_{\mu\nu}\Gamma^{\mu\nu}]$ cancel out similarly. Various other terms in the effective action (3.19) then cancel out as in the static case and imply that the one-loop contribution to the effective action for the oscillating case vanishes as well. This confirms the exactness of the BPS formula (1.4) argued on the basis of supersymmetric grounds earlier.

One can also examine the status of the multistring solution. The parallel configuration of oscillating strings from matrix theory can be obtained as block-diagonal matrices. Then the cancellations in W occur within each block in an identical fashion and they once again vanish, showing that they are BPS configurations as well.

IV. WORLD-VOLUME ACTION

In this section we obtain the world-volume gauge theory from type IIB matrix theory for the classical configuration corresponding to an oscillating string. We also analyze this world-volume gauge theory action in some detail and show its connection with static strings upon quantization.

It is known that the zero modes of a static D-string give rise to an $N=8$ $U(1)$ vector multiplet in two dimensions. We will now see that the zero modes of an oscillating string are the (8,0) $U(1)$ vector multiplets together with eight scalar multiplets containing the world-sheet fermions of opposite chirality. Similarly the zero modes of N coinciding D-branes [6] are now expected to give rise to an (8,0) $SU(N)$ gauge theory. Two-dimensional world-sheet actions with (8,0) and (4,0) supersymmetry have been written in other contexts [20,12] earlier and it may be interesting to show the exact connection among these actions.

The world-volume action describing the dynamics of these fields [6] can also be obtained by adding the quantum fluctuations to the classical backgrounds and then by expanding the matrix theory action. The one-loop effective action of matrix theory (3.19) is in fact the quantum effective action of these gauge theories. Thus in the static case we have [16]

$$A_0=-\sigma+\alpha'\widetilde{A}_0(\tau,\sigma), \quad A_1=\tau+\alpha'\widetilde{A}_1(\tau,\sigma), \tag{4.1}$$

$$A_i=\alpha'\phi_i(\tau,\sigma), \quad \psi=\alpha'\psi(\tau,\sigma), \tag{4.2}$$

where $\widetilde{A}_\alpha$ $(\alpha=0,1)$ now are the gauge fields on the world volume whereas the transverse components (ϕ_i's) are the scalar fluctuations. $\psi\equiv\binom{\psi_L}{\psi_R}$ are the world-sheet fermions which also transform as a spinor under an internal $SO(8)$ symmetry. These are, as expected, the degrees of freedom for an $N=8$ vector multiplet in two dimensions and are identified as the bosonic and fermionic zero modes of a static string.

The commutators of matrix variables, including the fluctuations (in the static case), have the form [16]:

$$[A_0,A_1]=i\alpha'(1+\alpha'F_{01}), \quad [A_\alpha,A_i]=i\alpha'D_\alpha\phi_i,$$

$$[A_\alpha,\psi]=i\alpha'D_\alpha\psi, \tag{4.3}$$

where we have used the identification (3.7) to replace the commutators with Poisson brackets and $F_{01}=\partial_0\widetilde{A}_1-\partial_1\widetilde{A}_0+\alpha'\{\widetilde{A}_0,\widetilde{A}_1\}$ and $D_\alpha\phi^i=\partial_\alpha\phi^i+\alpha'\{\widetilde{A}_\alpha,\phi^i\}$. Then, for a single D-string, the action (3.1) reduces to a $U(1)$ gauge theory in two dimensions with $N=8$ supersymmetry. The bosonic part of the gauge theory action for the $U(1)$ case has the form

$$S_B=\frac{1}{2\pi\alpha' g_s}\int d^2\sigma(1+\alpha'^2F_{01}^2-\alpha'^2D_\alpha\phi^iD_\alpha\phi^i). \tag{4.4}$$

The first (constant) term in Eq. (4.4) is the contribution of the classical background. In the Born-Infeld action, they correspond to the term involving the induced world-volume metric. The forms of F_{01} and $D_\alpha\phi_i$ also imply the existence of higher (than two) derivative terms in the action. These have been identified with the higher-order terms in the expansion of the Born-Infeld action [16]. The two derivative terms are the standard gauge theory action of the bosonic part of an $N=8$ Abelian gauge theory.

We now obtain the world-volume gauge theory action for the oscillating configuration from matrix theory and show that they correspond to an (8,0) supersymmetric gauge theory in two dimensions. The fact that the solution preserves 1/4 supersymmetry has already been pointed out. However, this leaves us with two possibilities for the world-volume supersymmetry. One can either have a (4,4) or an (8,0) supersymmetric gauge theory in two dimensions. The latter possibility is more natural in our case, as the oscillating string solution discussed above is left-right asymmetric. We have, however, already shown the breaking of $N=8$ or (8,8) supersymmetric gauge theory in two dimensions to an (8,0) theory explicitly in Eq. (3.14).

Once again, for writing down the action in two dimensions, we expand the matrix theory fields around the classical background mentioned above in Eqs. (3.10). We now have

$$A^V=2Rn\hat{\sigma}^++\alpha'\widetilde{A}^V,$$

$$A^U=(2Rn+a)\hat{\sigma}^-+\int^V\dot{F}^2+\alpha'\widetilde{A}^U, \tag{4.5}$$

$$A^i=F^i(\hat{V})+\alpha'\phi^i.$$

The supersymmetry breaking from $N=8$ or (8,8) gauge theory to an (8,0) gauge theory can now also be seen from the background configuration in Eqs. (4.5). It is known that the R symmetry for an $N=8$ supersymmetric theory is an $SO(8)_L\times SO(8)_R$ global symmetry group which transforms the supercharges as $(8_v,1)+(1,8_v)$. Then, as a result of the background configuration for the scalars in Eqs. (4.5), the left-moving part of the world-volume scalars acquires a vacuum expectation value. This breaks the $SO(8)_L\times SO(8)_R$ R symmetry to $SO(8)_L$ and the final world-volume theory has an (8,0) supersymmetry only.

We now derive this world-volume action and show its invariance under gauge and supersymmetry transformations. To write down the world-volume action, we once again compute the commutators appearing in the action (3.1) and make the identifications (3.7). The nonzero ones are

$$\begin{aligned}[A^U,A^V]\rightarrow{}&2(2Rn)(2Rn+a)+2\alpha'(2Rn)\partial_-\widetilde{A}^U\\&+2\alpha'[(2Rn+a)\partial_+\widetilde{A}^V-(2Rn)\dot{F}^2\partial_-\widetilde{A}^V]\\&+\alpha'^2\{\widetilde{A}^U,\widetilde{A}^V\}\\\equiv{}&2(2Rn)(2Rn+a)+\alpha'F^{UV},\end{aligned} \tag{4.6}$$

$$\begin{aligned}[A^U,A^i]\rightarrow{}&2(2Rn)(2Rn+a)\dot{F}^i+2\alpha'[(2Rn+a)\partial_+\phi^i\\&-(2Rn)\dot{F}^2\partial_-\phi^i+(2Rn)\dot{F}^i\partial_-\widetilde{A}^U]+\alpha'^2\{\widetilde{A}^U,\phi^i\}\\\equiv{}&2(2Rn)(2Rn+a)\dot{F}^i+\alpha'D_+\phi^i,\end{aligned} \tag{4.7}$$

$$\begin{aligned}[A^V,A^i]\rightarrow{}&\alpha'[-2(2Rn)\partial_-\phi^i+2(2Rn)\dot{F}^i\partial_-\widetilde{A}^V]\\&+\alpha'^2\{\widetilde{A}^V,\phi^i\}\\\equiv{}&\alpha'D_-\phi^i,\end{aligned} \tag{4.8}$$

$$\begin{aligned}[A^i,A^j]\rightarrow{}&2\alpha'(2Rn)(\dot{F}^j\partial_-\phi^i-\dot{F}^i\partial_-\phi^j)+\alpha'^2\{\phi^i,\phi^j\}\\\equiv{}&\alpha'\Phi^{ij}.\end{aligned} \tag{4.9}$$

The bosonic part of the world-volume gauge theory action is then obtained by substituting the above commutators into the bosonic part of the matrix theory action (3.1) and by the identifications in Eqs. (4.6)–(4.9). For example, in the variables A_U, A_V, and A_i, the first term in Eq. (3.1) has the form

$$S_B=-\frac{\alpha}{4}\mathrm{Tr}\left(-\frac{1}{2}[A^U,A^V]^2+2[A^U,A^i][A^V,A_i]+[A^i,A^j]^2\right). \tag{4.10}$$

By ignoring the constant and total derivative terms, the bosonic world-volume action is then written as

$$-\frac{\alpha}{4}\int d^2\sigma\frac{\alpha'^3}{2\pi}\Bigg[-\frac{1}{2}F^{UV^2}-2D_+\phi^iD_-\phi^i+\Phi^{ij^2} -4(2Rn)(2Rn+a)\dot{F}^i\{\tilde{A}^V,\phi^i\}\Bigg]. \tag{4.11}$$

The last term in the above action comes from the expression $4(2Rn)(2Rn+a)\dot{F}^iD_-\phi^i$, by dropping the total derivative terms. The gauge transformations, derived from Eqs. (3.5), in two-dimensional gauge theory have the form

$$\delta_g\tilde{A}^U=2i(2Rn+a)\partial_+\epsilon-2i(2Rn)\dot{F}^2\partial_-\epsilon+i\alpha'\{\tilde{A}^U,\epsilon\},$$

$$\delta_g\tilde{A}^V=-2i(2Rn)\partial_-\epsilon+i\alpha'\{\tilde{A}^V,\epsilon\}, \tag{4.12}$$

$$\delta_g\phi^i=-2i(2Rn)\dot{F}^i\partial_-\epsilon+i\alpha'\{\phi^i,\epsilon\},$$

and imply the following transformations for the quantities F^{UV}, $D_+\phi^i$, and $D_-\phi^i$:

$$\delta_gF^{UV}=i\alpha'\{F^{UV},\epsilon\},$$

$$\delta_gD_+\phi^i=-4i(2Rn)(2Rn+a)\partial_+\dot{F}^i\partial_-\epsilon+i\alpha'\{D_+\phi_i,\epsilon\}, \tag{4.13}$$

$$\delta_gD_-\phi^i=i\alpha'\{D_-\phi^i,\epsilon\}.$$

The gauge invariance of the action S_B then follows from these transformation rules. The fermionic part of the gauge theory action can also be written as

$$S_F=-\frac{\alpha}{2}\mathrm{Tr}(\bar{\psi}\Gamma^U[A_U,\psi]+\bar{\psi}\Gamma^V[A_V,\psi]+\bar{\psi}\Gamma^i[A_i,\psi]). \tag{4.14}$$

By using the commutators

$$[A^U,\psi]=2(2Rn+a)\partial_+\psi-2(2Rn)\dot{F}^2\partial_-\psi+\alpha'\{\tilde{A}^U,\psi\},$$

$$[A^V,\psi]=-2(2Rn)\partial_-\psi+\alpha'\{\tilde{A}^V,\psi\}, \tag{4.15}$$

$$[A^i,\psi]=-2(2Rn)\dot{F}^i\partial_-\psi+\alpha'\{\phi^i,\psi\},$$

and expanding in terms of the left- and right-moving world-sheet fermions, we have the explicit form

$$\begin{aligned}S_F=-\frac{\alpha}{2}\int d^2\sigma\frac{\alpha'^2}{2\pi}[&2(2Rn+a)\psi_R^T\partial_+\psi_R\\ &-2(2Rn)\dot{F}^2\psi_R^T\partial_-\psi_R+\alpha'\psi_R^T\{\tilde{A}^U,\psi_R\}\\ &-2(2Rn)\psi_L^T\partial_-\psi_L+\alpha'\psi_L^T\{\tilde{A}^V,\psi_L\}\\ &+2i(2Rn)\dot{F}^i(\psi_R^T\gamma^i\partial_-\psi_L+\psi_L^T\gamma^i\partial_-\psi_R)\\ &-i\alpha'(\psi_R^T\gamma^i\{\phi_i,\psi_L\}+\psi_L^T\gamma^i\{\phi^i,\psi_R\})].\end{aligned} \tag{4.16}$$

To obtain the supersymmetry transformations for the two-dimensional gauge theory action, given by $S=S_B+S_F$, from matrix theory, we use the condition $\epsilon_R=0$ which follows from Eq. (3.14). For the supersymmetry transformation $\delta=\delta^1-\delta^2$ we have

$$\delta\tilde{A}^U=2i\epsilon_L^T\psi_L,\quad \delta\tilde{A}^V=0,\quad \delta\phi^i=-\epsilon_L^T\gamma^i\psi_R \tag{4.17}$$

and

$$\delta\psi_L=\frac{i}{2}(F^{UV}+\Phi^{ij}\gamma^{ij})\epsilon_L,\quad \delta\psi_R=D_-\phi_i\gamma^i\epsilon_L. \tag{4.18}$$

The supersymmetry invariance of the action can then be verified explicitly. In a compact (covariant) form, the supersymmetry transformations have the explicit form

$$\delta S_B=-\delta S_F=-i\ \mathrm{Tr}[A_\alpha,A_\beta][A^\alpha,\bar{\epsilon}\Gamma^\beta\psi]. \tag{4.19}$$

A more explicit form of these transformations in terms of the field variables $\tilde{A}^U$, $\tilde{A}^V$, ϕ^i, and ψ can be written down by using Eqs. (4.6)–(4.9) and identifications (3.7). Finally, Lorentz invariance of the world-volume action can be seen from the scaling transformations

$$\sigma^+\to\lambda\sigma^+,\quad \sigma^-\to\lambda^{-1}\sigma^-, \tag{4.20}$$

together with the transformation of the bosonic fields,

$$\tilde{A}^U\to\lambda^{-1}\tilde{A}^U,\quad \tilde{A}^V\to\lambda\tilde{A}^V,\quad \phi^i\to\phi^i,\quad \dot{F}^i\to\lambda^{-1}\dot{F}^i, \tag{4.21}$$

and those of fermions,

$$\psi_R\to\lambda^{1/2}\psi_R,\quad \psi_L\to\lambda^{-1/2}\psi_L. \tag{4.22}$$

We have already pointed out in Sec. III that the world-volume action is anomaly free. This is essentially due to the fact that the spectrum contains an equal number of left- and right-moving fermions. Moreover, as in the case of gauge theory with (8,8) supersymmetry and the corresponding Abelian Born-Infeld action, all the fermions as well as matter scalars are neutral under gauge symmetry on the world volume for the oscillating D-string as well. As a result they do not contribute to the anomaly. In Sec. III we have argued this more concretely by pointing out that $\mathrm{Im}W=0$ whenever the transverse oscillation is absent along one of the directions.

We now discuss the connection of our solution with static D-strings by arguing that one can obtain the particle spectrum of static strings, from that of the classical configuration discussed above, after quantization. In two dimensions, this implies the quantum equivalence of the world-volume action (4.4) for the static case with the oscillating one (4.10) and (4.16). This is true in spite of the fact that the manifest symmetries of the two actions are quite different. However, this is expected from a different angle, namely, the absence of spontaneous symmetry breaking in two dimensions. Our results of this part therefore imply that the collective modes of oscillating strings have, in their spectrum, the static string as well.

To analyze the quantum spectrum corresponding to the action (4.10) and (4.16), we set the gauge field fluctuations to zero and make the choice $F^i=0$, for $i\neq 1$, and $F^1=F$. Furthermore, we restrict the analysis to the bosonic sector only, as the fermionic part can also be analyzed in a similar manner. After these simplifying assumptions, the bosonic action for fields ϕ^i ($i=2,\dots,8$) are those of a free field. The field $\phi\equiv\phi^1$ is described by the action

$$S_B^1=\left(-\frac{\alpha}{4}\right)(2Rn)\int d^2\sigma[(2Rn+a)\partial_+\phi\partial_-\phi -(2Rn)\dot{F}^2(\partial_-\phi)^2]. \tag{4.23}$$

The equation of motion corresponding to the action (4.23) can be written as $\partial_- J^-=0$, where

$$J^-=(2Rn+a)\partial_+\phi-(2Rn)\dot{F}^2\partial_-\phi. \tag{4.24}$$

J^- is a chiral conserved current in the theory. The equation of motion for closed strings can be solved as

$$\phi=p_L\sigma^+ + p_R\left(\sigma^- + \frac{1}{(2Rn+a)}\int^V \dot{F}^2\right) + \sum_{m\neq 0}\left(\frac{\alpha_{-m}}{m}\right)e^{2im\sigma^+} + \sum_{m\neq 0}\left(\frac{\tilde{\alpha}_{-m}}{m}\right)\exp\left[-2i\frac{m}{(2Rn)}\times\left((2Rn+a)\sigma^- + \int^V \dot{F}^2\right)\right]. \tag{4.25}$$

The canonical formulation can be applied to the time- and space-dependent Lagrangian (4.23) in a standard way and leads to a Hamiltonian density of the form

$$\mathcal{H}=\left(-\frac{\alpha}{4}\right)\frac{(2Rn)}{2(2Rn+a)}\{J^{-2}+[(2Rn+a)^2-(2Rn)^2\dot{F}^4]\times(\partial_-\phi)^2\}. \tag{4.26}$$

In writing down Eq. (4.26), we have replaced the canonical momentum by the space and time derivatives, to present a simple form.

To show the mapping of the spectrum of oscillating strings, specified by the oscillators α and $\tilde{\alpha}$ with that of the static stings, specified by $\bar{\alpha}$, $\tilde{\bar{\alpha}}$, we choose a specific wave profile $F(v)$ of the classical oscillating sting solution, namely,

$$F(v)=\sqrt{a(2Rn)}\left([\sin(V/2Rn)+\cos(V/2Rn)]+\frac{2}{2\pi Rn}\right).$$

This choice satisfies the condition that $F(v)$ have no zero mode, namely, $\int^{2\pi Rn}F(v)=0$. Then, by representing the oscillators α and $\tilde{\alpha}$ as Fourier components of two commuting operators J^- and $\partial_-\phi$, respectively, and comparing them with the Fourier components of similar operators in the static case, $F=0$, it can be shown that, at $\tau=0$, the oscillators are mapped as

$$\bar{\alpha}_m=\frac{(2Rn+a)}{(2Rn)}\alpha_m, \quad \tilde{\bar{\alpha}}_m=\frac{(2Rn+a)}{(2Rn)}\sum_q J_{m-q}\left(\frac{2aq}{Rn}\right)\tilde{\alpha}_q, \tag{4.27}$$

where $J_m(x)$ are the Bessel functions. Equation (4.27) gives a mapping between operators in the spectrum of oscillating and static strings. Using the above relationship between the operators J^- and $\partial_-\phi$ in the two cases, the solutions for fields themselves can be shown to be related to each other. The mapping at $\tau\neq 0$ is given by the time evolution of these operators with respect to the corresponding Hamiltonians. A similar mapping should be possible for the fermionic oscillators as well.

V. CONCLUSIONS

We have presented a class of solutions of type IIB matrix theory and shown that the solutions preserve 1/4 supersymmetry. The supersymmetry that is preserved is chiral in nature in terms of the wave motion on the D-string. We have confirmed the BPS nature of these solutions by computing the one-loop effective action and derived world-volume gauge theory. It was also shown that the world-volume action in the classical background of oscillating strings is anomaly free for a large class of models. However, it should be possible to show this property without making any assumption about the form of the transverse oscillations.

There can be several applications and generalizations of these results. First, it will be interesting to extend the results of this paper to other extended objects with oscillations. A membrane solution of this type is already known [19] and implies that a similar analysis in BFSS matrix theory should be possible. Another interesting aspect of this analysis may be to examine gauge theories that might arise through other oscillating D-brane configurations. The BPS states of strings have been analyzed using the results in four-dimensional gauge theories [17] in the context of BFSS matrix theory. However, it should be possible to carry out our analysis in similar circumstances. One may also be able to apply the results of this paper to study black holes in the type IIB matrix theory picture. This can be done through the identification of compactified oscillating strings with extremal black holes.

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