PHYSICAL REVIEW D VOLUME 57, NUMBER 8 15 APRIL 1998

Destructive interference of dualities

Clovis Wotzasek
Instituto de Física, Universidade Federal do Rio de Janeiro, 21945, Rio de Janeiro, Brazil
and Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627

(Received 10 November 1997; published 17 March 1998)

The soldering mechanism has been shown to represent the quantum interference effect between self- and anti-self-dual aspects of a given symmetry. This mechanism was used to show that the massive mode of the 2D Schwinger model results from the constructive interference between the right and the left massless modes of chiral Schwinger models. Similarly, the topologically massive modes resulting from the bosonization of 3D massive Thirring models of opposite mass signatures are fused into the two massive modes of the 3D Proca model, thanks to the interference of dualities characteristic of the soldering mechanism. In this work, we show that the field theoretical analogue of destructive quantum mechanical interference may also be represented by the soldering mechanism. This phenomenon is illustrated by the fusion of two (diffeomorphism) invariant self-dual scalars described by right and left chiral-WZW actions, producing a Hull nonmover field. After fusion, right and left moving modes disappear from the spectrum, displaying the claimed (destructive) interference of dualities. [S0556-2821(98)02408-4]

PACS number(s): 11.10.Kk, 11.10.Ef, 11.30.Rd

The investigation of chiral bosonization began many years ago in the seminal work of Siegel [1]. Later on Gates and Siegel showed how to construct general interacting actions for chiral bosons, including the supersymmetric and the non-Abelian cases [2]. They used this construction to obtain the righton-lefton interaction by carrying out the path integral quantization in a generalized Thirring model. Alternatively, Stone [3] has shown that the method of a coadjoint orbit when applied to a representation of a group associated with a single affine Kac-Moody algebra provides an action for the chiral Wess-Zumino-Witten (WZW) model [4], a non-Abelian generalization of the Floreanini-Jackiw model [5]. This method gives a useful bosonization scheme for Weyl fermions, since a level one representation of LU(N) has an interpretation as the Hilbert space for a free chiral fermion [6]. The drawback is that only Weyl fermions can be dealt with in this way, since a 2D conformally invariant quantum field theory (QFT) has separate right and left current algebras. In order to overcome this difficulty, Stone [3] introduced the idea of soldering the two chiral scalars by introducing a nondynamical gauge field to remove the degree of freedom that obstructs the vector gauge invariance. This is connected to the observation that one needs more than the direct sum of two fermionic representations of the Kac-Moody algebra to describe a Dirac fermion. Stated differently, the equality for the weights in the two representations is physically connected with the need to abandon one of the two separate chiral symmetries, and accept that only vector gauge symmetry should be maintained. This is the main motivation for the introduction of the soldering field which pervades for the fusion of dualities in all space-time dimensions. Moreover, being just an auxiliary field, it may posteriorly be eliminated in favor of the physically relevant quantities. This restriction will force the two independent chiral representations to belong to the same multiplet, *effectively* soldering them together.

On the other hand, the role of duality as a qualitative tool in the investigation of physical systems is being gradually disclosed in a different context and dimensions [7]. In two space-time dimensions in particular, we face the intriguing situation where chirality also plays the role of duality. This enables the investigation of the former to be performed using the techniques developed for the latter. Recently, this author and collaborators [8–10] extended the techniques of fusion or soldering, introduced by Stone [3], to investigate some new aspects of dualities at different space-time dimensions, and studied the physical consequences of their combination by the soldering process. In particular, we have shown [8] that the constructive interference between the left and the right moving massless modes of two chiral Schwinger models [11] of opposite chiralities is soldered into the gauge invariant massive mode of the vector Schwinger model. In fact, by equipping the soldering technique with gauge and Bose symmetry [12] it automatically selects the massless sector of the chiral models displaying the Jackiw-Rajaraman parameter that reflects the bosonization ambiguity by $a=1$. In the 3D case, the soldering mechanism was used to show the result of fusing together two topologically massive modes generated by the bosonization of two massive Thirring models with opposite mass signatures in the long wavelength limit. The bosonized modes, which are described by self- and anti-self-dual Chern-Simons models [13,14], were then soldered onto the two massive modes of the 3D Proca model [9]. In the 4D case, the soldering mechanism produced an explicitly dual and covariant action as the result of the interference between two Schwarz-Sen [15] actions displaying opposite aspects of the electromagnetic duality [10]. It is our intention in this work to study the physical consequences of combining actions possessing truncate diffeomorphism invariance and opposite chiralities using the fusion of dualities technique.

It should be mentioned that such a procedure has a typical quantum mechanical nature, with no classical parallel. It is completely meaningless to perform the sum of two classical actions, which, although describing opposite aspects of some (duality) symmetry, would depend on the same field. On the other hand, the direct sum of duality symmetric actions depending on different fields would not give anything new. It is the soldering process that leads to a new and nontrivial result. In 2D, this result has been interpreted [8] as the consequence of the constructive interference of chiralities, by coupling the chiral scalar fields to a dynamical gauge field. The resulting effective gauge theory, obtained after the elimination of the soldered scalar field through equations of motion, shows the presence of a mass term that is typical of the right-left quantum interference [16].

In this work we show that it is also possible to obtain the field theoretical analogue of the ''quantum destructive interference'' phenomenon, by coupling the chiral scalars to appropriately truncated metric fields, known as chiral WZW models, or non-Abelian Siegel models. By soldering the two (Siegel) invariant representations of the chiral WZW model [1] of opposite chiralities, the effective action that results from this process is shown to be invariant under the full diffeomorphism group, which is not a mere sum of two Siegel symmetries. In fact, this effective action does not contain either right or left movers, but can be identified with the non-Abelian generalization of the bosonic nonmover action proposed by Hull [18], thanks to the richer symmetry structure induced over it by the soldering mechanism.

To begin with, let us review some facts about the non-Abelian Siegel model [17]. The action for a left mover chiral scalar is given as[1]

$$S_0^{(+)}(g)=\int d^2x\ \mathrm{tr}(\partial_+ g\partial_-\tilde{g}+\lambda_{++}\partial_- g\partial_-\tilde{g})+\Gamma_{WZ}(g), \tag{1}$$

where $g\in G$ is a matrix-valued field taking values on some compact semisimple Lie group G, with an algebra $\hat{G}$. The term $\Gamma_{WZ}(g)$ is the topological Wess-Zumino functional, as defined in Ref. [19]. It is invariant under a chiral diffeomorphism known as Siegel transformation where

$$\delta\lambda_{++}=-\partial_+\epsilon^-+\lambda_{++}\partial_-\epsilon^-+\epsilon_-\partial_-\lambda_{++} \tag{2}$$

and g transforming as a scalar. This action can be seen as the WZW action, immersed in a gravitational background, with an appropriately truncated metric tensor,

$$S_0^{(+)}(g)=\frac{1}{2}\int d^2x\sqrt{-\eta_+}\,\eta_+^{\mu\nu}\mathrm{tr}(\partial_\mu g\partial_\nu\tilde{g})+\Gamma_{WZ}(g), \tag{3}$$

with $\eta^+=det(\eta^+_{\mu\nu})$ and

$$\frac{1}{2}\sqrt{-\eta_+}\,\eta_+^{\mu\nu}=\begin{pmatrix}0 & \frac{1}{2}\\ \frac{1}{2} & \lambda_{++}\end{pmatrix}. \tag{4}$$

[1]Our notation is as follows: $x^{\pm}=1/2(t\pm x)$ are the light-cone variables and $\tilde{g}=g^{-1}$ denotes the inverse matrix.

Next, let us compute the Noether currents for the axial vector, vectorial, and the right and left chiral transformations. The variation of the Siegel-WZW action (1) or (3) gives

$$\delta S_0^{(+)}(g)=\begin{cases}\int d^2x\ \mathrm{tr}\{\delta g\tilde{g}2[\partial_+(\partial_- g\tilde{g})+\partial_-(\lambda_{++}\partial_- g\tilde{g})]\},\\ \int d^2x\ \mathrm{tr}\{\tilde{g}\delta g2[\partial_-(\tilde{g}\partial_+ g)+\partial_-(\lambda_{++}\tilde{g}\partial_- g)]\}.\end{cases} \tag{5}$$

From Eq. (5) and the axial transformation $(g\to kgk)$ we obtain

$$J_A^+=2g\partial_-\tilde{g},$$

$$J_A^-=-2[\tilde{g}\partial_+ g+\lambda_{++}(\tilde{g}\partial_- g+\partial_- g\tilde{g})], \tag{6}$$

where $k\in K$ take their values in some subgroup $K\subset G$. From the transformation $(g\to\tilde{k}gk)$ we obtain the vector current

$$J_V^+=2g\partial_-\tilde{g},$$

$$J_V^-=2[\tilde{g}\partial_+ g+\lambda_{++}(\tilde{g}\partial_- g-\partial_- g\tilde{g})]. \tag{7}$$

Incidentally, it should be observed that the axial vector and the vectorial currents (6) and (7) are dual to each other only if the following extended definition is adopted,

$$*T^\mu=\sqrt{-\eta_+}\,\eta_+^{\mu\nu}\epsilon_{\mu\lambda}T^\lambda, \tag{8}$$

and use of the following relations is made:

$$J_+=J^- -2\lambda_{++}J^+,$$

$$J_-=J^+, \tag{9}$$

which is valid for all currents. Similarly, the chiral currents can be obtained from the left $(g\to gk)$ and right $(g\to\tilde{k}g)$ transformation. The result is

$$J_L^{(+)}=0,$$

$$J_L^{(-)}=2(\tilde{g}\partial g+\lambda_{++}\tilde{g}\partial_- g) \tag{10}$$

and

$$J_R^{(+)}=-2g\partial_-\tilde{g},$$

$$J_R^{(-)}=-2\lambda_{++}g\partial_-\tilde{g}. \tag{11}$$

It is crucial to notice that out of the two affine invariances of the original WZW model, only one is left over due to the chiral constraint $\partial_- g\approx 0$. Indeed, the affine invariance is only present in the left sector since $J_L^{(+)}=0$ and $\partial_- J_L^{(-)}=0$, which implies $J_L^{(-)}=J_L^{(-)}(x^+)$, while $J_R^{(-)}\neq 0$ and $J_R^{(+)}\neq J_R^{(+)}(x^-)$.

Next we work out the details for the right chirality action,

$$S_0^{(-)}(h)=\int d^2x\ \mathrm{tr}(\partial_+ h\partial_-\tilde{h}+\lambda_{--}\partial_+ h\partial_+\tilde{h})-\Gamma_{WZ}(h)$$
$$=\frac{1}{2}\int d^2x\sqrt{-\eta_-}\,\eta_-^{\mu\nu}\mathrm{tr}(\partial_\mu h\partial_\nu\tilde{h})-\Gamma_{WZ}(h), \tag{12}$$

where

$$\frac{1}{2}\sqrt{-\eta_-}\,\eta_-^{\mu\nu}=\begin{pmatrix}\lambda_{--} & \frac{1}{2}\\ \frac{1}{2} & 0\end{pmatrix}. \tag{13}$$

Notice that $S_0^{(+)}(g)$ and $S_0^{(-)}(h)$ are chosen at opposite critical points; otherwise, they will not carry different chiralities, a crucial condition for the soldering to be performed. The set of axial vector, vector, and chiral Noether currents is similarly obtained:

$$J_A^{(+)}(h)=2[\tilde{h}\partial_- h+\lambda_{--}(\tilde{h}\partial_+ h+\partial_+ h\tilde{h})],$$
$$J_A^{(-)}(h)=2\partial_+ g\tilde{g}, \tag{14}$$
$$J_V^{(+)}(h)=2[\tilde{h}\partial_- h+\lambda_{--}(\tilde{h}\partial_+ h-\partial_+ h\tilde{h})],$$
$$J_V^{(-)}(h)=-2\partial_+ h\tilde{h}, \tag{15}$$
$$J_L^{(+)}(h)=2\partial_+ h\tilde{h},$$
$$J_L^{(-)}(h)=2\lambda_{--}\partial_+ h\tilde{h}, \tag{16}$$
$$J_R^{(+)}(h)=2(\tilde{h}\partial_- h+\lambda_{--}\tilde{h}\partial_+ h),$$
$$J_R^{(-)}(h)=0, \tag{17}$$

with the corresponding interpretation analogous to that following Eq. (11). Since the actions (1) and (12) do correspond to opposite aspects of a symmetry (chirality), the stage is set for the soldering.

Next, let us discuss the gauging procedure to be adopted in the soldering of the right and the left chiral-WZW actions just reviewed. The basic idea of the soldering procedure is to lift a global Noether symmetry present at each individual chiral component into a local symmetry for the composite system that, consequently, defines the soldered action. It is of vital importance to notice that the coupling with the (auxiliary) soldering gauge field is only consistent if use is made of the correspondent chiral current. Otherwise the equations of motion, after the gauging, will result being incompatible with the covariant chiral constraint, by the presence of an anomaly. Anomalies can certainly be incorporated into the theory, but not at the expense of violating the consistency between equations of motion and gauge constraints. Here we shall adopt an iterative Noether procedure to lift the global (left) chiral symmetry of Eq. (1),

$$g\to gk,$$
$$\lambda_{++}\to\lambda_{++},$$
$$A_-\to\tilde{k}A_-k+\tilde{k}\partial_- k, \tag{18}$$

into a local one. To compensate for the noninvariance of $S_0^{(+)}$, we introduce the coupling term

$$S_0^{(+)}\to S_1^{(+)}=S_0^{(+)}+A_-J_L^-(g), \tag{19}$$

along with the soldering gauge field A_-, taking values in the subalgebra $\hat{K}$ of K, whose transformation properties are defined in Eq. (18). Using such transformations, it is a simple algebra to find that

$$\delta(S_1^{(+)}-\lambda_{++}A_-^2)=2\partial_+\omega A_-, \tag{20}$$

with $\omega\in\hat{K}$ being an infinitesimal element of the algebra. One can see that

$$S_2^{(+)}=S_1^{(+)}-\lambda_{++}A_-^2 \tag{21}$$

cannot be made gauge invariant by additional Noether counterterms, but it has the virtue of being independent of the transformation properties of g while depending only on the elements of the gauge algebra $\hat{K}$. Similarly, for the right chirality we find

$$\delta S_2^{(-)}=-2A_+\partial_-\omega \tag{22}$$

for

$$S_2^{(-)}(h)=S_0^{(-)}(h)-A_+J_R^+(h)-\lambda_{--}A_+^2 \tag{23}$$

when the basic fields transform as

$$h\to hk,$$
$$A_+\to kA_+\tilde{k}+k\partial_+\tilde{k},$$
$$\lambda_{--}\to\lambda_{--}. \tag{24}$$

It is important to observe that the action for the right sector depends functionally on a different field, namely, $h\in H$. Although the gauged actions for each chirality could not be made gauge invariant separately, with the inclusion of a contact term, the combined action

$$S_{eff}=S_2^{(+)}+S_2^{(-)}+2A_+A_- \tag{25}$$

is invariant under the set of transformations (18) and (24) simultaneously.

Following Ref. [3], we eliminate the (nondynamical) gauge field A_μ. From the equations of motion one gets

$$\mathcal{J}=2\mathbf{M}\mathcal{A}, \tag{26}$$

where we have introduced the following matricial notation:

$$\mathcal{J}=\begin{pmatrix} J_L^-(g) \\ J_R^+(h) \end{pmatrix}, \tag{27}$$

$$\mathcal{A}=\begin{pmatrix} A_+ \\ A_- \end{pmatrix}, \tag{28}$$

and

$$\mathbf{M}=\begin{pmatrix} 1 & \lambda_{++} \\ \lambda_{--} & 1 \end{pmatrix}. \tag{29}$$

Bringing these results into the effective action (25) gives

$$S_{eff}=S_0^{(+)}(g)+S_0^{(-)}(h)+\int d^2x\,\frac{1}{1-\lambda^2}\mathrm{tr}\{2[\tilde{g}\partial_+g\tilde{h}\partial_-h+\lambda^2\tilde{g}\partial_-g\tilde{h}\partial_+h+\lambda_{++}\tilde{g}\partial_-g\tilde{h}\partial_-h+\lambda_{--}\tilde{g}\partial_+g\tilde{h}\partial_-h]$$
$$+\lambda_{--}(\partial_+g\partial_+\tilde{g}+2\lambda_{++}\partial_+g\partial_-\tilde{g}+\lambda_{++}^2\partial_-g\partial_-\tilde{g})+\lambda_{++}(\partial_-h\partial_-\tilde{h}+2\lambda_{--}\partial_+h\partial_+\tilde{h}+\lambda_{--}^2\partial_+h\partial_+\tilde{h})\}. \tag{30}$$

where $\lambda^2=\lambda_{++}\lambda_{--}$. It is now a simple algebra to show that this effective action does not depend on the fields g and h individually, but only on a gauge invariant combination of them, defined below, Eq. (32). This effective action corresponds to that of a (nonchiral-)WZW model coupled minimally to an effective metric built out of the Lagrange multipliers of the original Siegel actions,

$$\frac{1}{2}\sqrt{-\eta}\eta^{\mu\nu}=\frac{1}{1-\lambda^2}\begin{pmatrix} \lambda_{--} & \frac{1+\lambda^2}{2} \\ \frac{1+\lambda^2}{2} & \lambda_{++} \end{pmatrix}, \tag{31}$$

and a new (effective) field

$$\mathcal{G}=g\tilde{h} \tag{32}$$

and reads

$$S=\frac{1}{2}\int d^2x\sqrt{-\eta}\eta^{\mu\nu}\mathrm{tr}(\partial_\mu\mathcal{G}\partial_\nu\tilde{\mathcal{G}})+\Gamma_{WZ}(\mathcal{G}). \tag{33}$$

Here we have used the well-known property of the Wess-Zumino functional $\Gamma_{WZ}(h)=-\Gamma_{WZ}(\tilde{h})$, and the Polyakov-Weigman identity [19].

It is interesting to notice that the original chiral transformations (18) and (24) are now hidden, since the effective action is composed of only the gauge invariant objects (31) and (32). To unravel the physical contents of the effective soldered action (33), it is important to study the new set of symmetries of the composite theory. We first observe that under diffeomorphism the metric transform as a symmetric tensorial density,

$$\delta\lambda_{++}=-\partial_+\epsilon^-+\lambda_{++}^2\partial_-\epsilon^++(\partial_+\epsilon^+-\partial_-\epsilon^-+\epsilon^+\partial_+ +\epsilon^-\partial_-)\lambda_{++},$$
$$\delta\lambda_{--}=-\partial_-\epsilon^++\lambda_{--}^2\partial_+\epsilon^-+(\partial_-\epsilon^--\partial_+\epsilon^++\epsilon^+\partial_+ +\epsilon^-\partial_-)\lambda_{--}, \tag{34}$$

while the $\mathcal{G}$ transforms as a scalar. It is important to observe that if we restrict the diffeomorphism to just one sector, say, by requiring $\epsilon^+=0$, we reproduce the original Siegel symmetry for the sector described by the pair $\mathcal{G},\lambda_{++}$ in the same way as it appears in the original chiral theory (1). However, under this restriction, λ_{--} transforms in a nontrivial way as

$$\delta\lambda_{--}=\lambda_{--}^2\partial_+\epsilon^-+(\partial_-\epsilon^-+\epsilon^-\partial_-)\lambda_{--}. \tag{35}$$

The original Siegel symmetry, therefore, is not a subgroup of the diffeomorphism group but it is only recovered if we also make a further truncation, by imposing that $\lambda_{--}=0$. The existence of the residual symmetry (35) seems to be related to a duality symmetry satisfied by the effective action (33) when the metric is parametrized as in Eq. (31). Under the discrete transformation

$$\lambda_{\pm\pm}\rightarrow\frac{1}{\lambda_{\mp\mp}}, \tag{36}$$

the residual transformation (35) swaps to (2) while that becomes the residual symmetry for the opposite chiral sector. Indeed we see that the classical equations of motion remain invariant under Eq. (36) while the effective action changes its signature, very much like in the original electromagnetic duality transformation. This is obviously related to the interchange symmetry between the right and the left moving sectors of the theory, and seems to be of general validity [10]. Also notice that the gauged Lagrangian in one sector, either S_2^+ or S_2^-, cannot be written in a diffeomorphism invariant manner. Therefore, gauging in one of the sectors breaks Siegel invariance. However, let us note that if we integrate out either the A_- or the A_+ field, the Siegel theory changes chirality with the identification provided by Eq. (36), that is, again related to the discrete duality symmetry.

Now comes the crucial observation. By solving the equations of motion and setting the $\lambda_\pm$ to zero by invoking the diffeomorphism invariance discussed above, it is simple to see that the composite field (32) of the effective action (33) describes a nonmover field, as first proposed by Hull [18]. The right and the left moving modes have therefore disappeared from the spectrum. The soldering procedure has clearly produced a destructive interference between left and right movers of the original chiral components. Moreover, it can be easily seen that the coupling of chiral scalars to a dynamical gauge field before soldering, as done in [8] (see the Appendix), will decouple the gauge sector from the effective soldered action. This seems to be a natural result since a nonmover field cannot couple to either right or left components of the vector gauge field. This is a distinctive result produced by the presence of the full group of diffeomorphism resulting from the soldering process, which constrains the matter scalar field to the nonmoving sector, quite in opposition to the constructive interference result that comes from soldering the noninvariant models.

The author would like to thank the members of the Department of Physics and Astronomy of University of Rochester for the hospitality during the visit that was made possible by the bilateral agreement CNPq-NSF. The author is partially supported by CNPq, FINEP, and FUJB, Brazil.

[1] W. Siegel, Nucl. Phys. **B238**, 307 (1984).
[2] S. J. Gates, Jr. and W. Siegel, Phys. Lett. B **206**, 631 (1988).
[3] M. Stone, Phys. Rev. Lett. **63**, 731 (1989); Nucl. Phys. **B327**, 399 (1989); Illinois Report No. ILL-TH-89-23, 1989.
[4] E. Witten, Commun. Math. Phys. **92**, 455 (1984).
[5] R. Floreanini and R. Jackiw, Phys. Rev. Lett. **59**, 1873 (1987).
[6] A. Pressley and G. Segal, *Loop Groups* (Oxford University Press, Oxford, 1986).
[7] L. Alvarez-Gaume and S. F. Hassan, Fortschr. Phys. **45**, 159 (1997); also see L. Alvarez-Gaume and F. Zamora, ''Duality in Quantum Field Theory (and String Theory),'' hep-th/9709180.
[8] E. M. C. de Abreu, R. Banerjee, and C. Wotzasek, Nucl. Phys. **B509**, 519 (1998).
[9] R. Banerjee and C. Wotzasek, ''Bosonization and Soldering of Dual Symmetries in Two and Three Dimensions,'' hep-th/9709105.
[10] R. Banerjee and C. Wotzasek, ''Duality Symmetry and Soldering at Different Dimensions,'' hep-th/9710060.
[11] R. Jackiw and R. Rajaraman, Phys. Rev. Lett. **54**, 1219 (1985).
[12] N. Banerjee and R. Banerjee, Nucl. Phys. **B445**, 516 (1995).
[13] P. K. Townsend, K. Pilch, and P. van Nieuwenhuizen, Phys. Lett. **136B**, 452 (1984).
[14] S. Deser and R. Jackiw, Phys. Lett. **139B**, 371 (1984).
[15] J. Schwarz and A. Sen, Nucl. Phys. **B411**, 35 (1994).
[16] R. Jackiw, *Diverse Topics in Theoretical and Mathematical Physics* (World Scientific, Singapore, 1995).
[17] Y. Frishman and J. Sonnenschein, Nucl. Phys. **B301**, 346 (1988).
[18] C. Hull, Phys. Lett. B **206**, 234 (1988).
[19] A. Polyakov and P. B. Weigman, Phys. Lett. **131B**, 121 (1983); **141B**, 223 (1984).