Equivalence of renormalized covariant and light-front perturbation theory. I. Longitudinal divergences in the Yukawa model

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Light-front perturbation theory (LFPT) has been proposed as an alternative to covariant perturbation theory. LFPT is only acceptable if it produces invariant S-matrix elements. Doubts have been raised concerning the equivalence of LFPT and covariant perturbation theory. The main obstacles to a rigorous proof of equivalence are algebraic complexity in the case of arbitrarily high orders in perturbation theory and the occurrence of longitudinal divergences not present in covariant perturbation theory. We show in the case of the Yukawa model of fermions interacting with scalar bosons at the one-loop level how to deal with the longitudinal divergences. Invariant S-matrix elements are obtained using our method. [S0556-2821(98)04608-6]

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I. INTRODUCTION

Covariant field theory has been very successful in describing scattering processes. However, in this framework it is difficult to describe bound states of elementary particles. Hamiltonian field theories are promising candidates to determine the properties of bound states. In a Hamiltonian framework the initial conditions are specified on some plane of quantization. The Hamiltonian then gives the evolution of the system in time. Already in 1949, Dirac [1] pointed out that there are several possible choices for the surface of quantization. Most commonly used is the equal-time plane.

For applications in, e.g., deep inelastic scattering, the light-front (LF) is favored. For the LF coordinates we use the convention of [2]

$$x^{\pm}=\frac{x^0 \pm x^3}{\sqrt{2}}, \quad x^{\perp}=(x^1,x^2). \tag{1}$$

Quantization takes place on the light-like plane $x^+=0$. This choice implies that the minus component of the momentum will play the role of energy. The advantages of light-front perturbation theory (LFPT) over quantization on the equal-time plane are given in many articles: see, e.g., Refs. [3,4]. In LFPT there can be no creation of massive particles from the vacuum or annihilation into the vacuum. This reduces the number of time-ordered diagrams and is related to the spectrum condition.

For a number of reasons, quantization on the LF is nontrivial. Subtleties arise that have no counterpart in ordinary time-ordered theories. We will encounter some of them in the present work and show how to deal with them in such a way that covariance of the perturbation series is maintained.

In naive light-cone quantization (NLCQ) some problems are not satisfactorily solved. Still, along this line rules have been proposed for LF time-ordered diagrams [2,5]. Until now, one has not succeeded in finding a better method.

In LFPT, or any other Hamiltonian theory, covariance is not manifest. Burkardt and Langnau [6] claim that, even for scattering amplitudes, rotational invariance is broken in NLCQ. In the case they studied, two types of infinities occur: longitudinal and transverse divergences. They regulate the longitudinal divergences by introducing noncovariant counterterms. In doing so, they restore at the same time rotational invariance. The transverse divergences are dealt with by dimensional regularization.

We would like to maintain the covariant structure of the Lagrangian and take the path of Ligterink and Bakker [7]. Following Kogut and Soper [2] they derive rules for LFPT by integrating covariant Feynman diagrams over the LF energy k^-. For covariant diagrams where the k^--integration is well-defined this procedure is straightforward and the rules constructed are, in essence, equal to the ones of NLCQ. However, when the k^--integration diverges the integral over k^- must be regulated first. It is our opinion that it is important to do this in such a way that covariance is maintained.

We will show that the occurrence of longitudinal divergences is related to the so-called forced instantaneous loops (FILs). If these diagrams are included and renormalized in a proper way, we can give an analytic proof of covariance. FILs were discussed before by Mustaki *et al.* [8], in the context of QED. They refer to them as *seagulls*. There are, however, some subtle differences between their treatment of longitudinal divergences and ours, which are explained in Sec. III.

Transverse divergences have a very different origin. However, they can be treated with the same renormalization method as longitudinal divergences. We found an analytic proof of the equivalence of the renormalized covariant amplitude and the sum of renormalized LF time-ordered amplitudes in two cases, the fermion and the boson self energy. In the other cases we have to use numerical techniques. They will be dealt with in forthcoming work.

A. Instantaneous terms and blinks

In the case of fermions the demonstration of equivalence is complicated because of the occurrence of instantaneous terms.

The covariant propagator for an off-shell spin-1/2 particle can be written as follows:

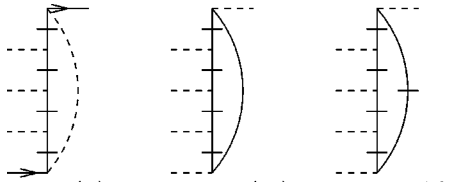
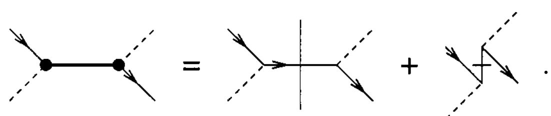


FIG. 1. Examples of FILs. In (a) a boson in the loop is forced to be instantaneous. In (b1) a fermion is obstructed in its propagation. In (b2) all fermions are instantaneous.

$$\frac{i(\not{k}+m)}{k^2-m^2+i\epsilon}=\frac{i(\not{k}_{\rm on}+m)}{k^2-m^2+i\epsilon}+\frac{i\gamma^+}{2k^+}. \tag{2}$$

The first term on the right-hand side is called the propagating part. The second one is called the instantaneous part. The splitting of the covariant propagator corresponds to a similar splitting of LF time-ordered diagrams. For any fermion line in a covariant diagram two LF time-ordered diagrams occur, one containing the propagating part of the covariant propagator, the other containing the instantaneous part. For obvious reasons we call the corresponding lines in the LF time-ordered diagrams propagating and instantaneous respectively. For a general covariant diagram the $1/k^+$-singularity in the propagating part cancels a similar singularity in the instantaneous part. Therefore the LF time-ordered diagrams with instantaneous lines are necessary; they are usually well-defined.

If the $1/k^+$-singularities are inside the area of integration, we may find it necessary to combine the propagating and instantaneous contribution again into the so-called blink:



(3)

In the LF time-ordered diagrams time increases from left to right. The dashed lines denote scalar bosons, the straight lines fermions. The thick straight line is a blink. The bar in the internal line of the third diagram denotes an instantaneous fermion. When a LF time-ordered diagram looks like the covariant diagram, we draw a cut as in the second diagram of Eq. (3) to avoid any confusion.

The difference between Eqs. (2) and (3) lies in the fact that the first uses covariant propagators, and the second has energy denominators. An example of a blink is given in Sec. II on the one-boson exchange correction.

B. Instantaneous terms and FILs

When a diagram contains a loop where all particles but one are instantaneous, a conceptual problem occurs. Should the remaining boson or fermion be interpreted as propagating or as instantaneous? Loops with this property are said to be forced instantaneous loops. Loops where all fermions are instantaneous are also considered as FILs. However, they do not occur in the Yukawa model. Examples of these three types of FILs are given in Fig. 1.

Mathematically this problem also shows up. The FILs correspond to the part of the covariant amplitude where the k^--integration is ill-defined. The problem is solved in the following way. First we do not count FILs as LF time-ordered diagrams. Second we find that this special type of diagram disappears upon regularization if we use the method of Ref. [9]: minus regularization.

C. Minus regularization

The minus-regularization scheme was developed by Ligterink and Bakker [9] with the purpose to maintain the symmetries of the theory such that the amplitude is covariant order by order. It can be applied to Feynman diagrams as well as to ordinary time-ordered or LF time-ordered diagrams. Owing to the fact that minus regularization is a linear operation, minus regularization commutes with the splitting of Feynman diagrams into LF time-ordered diagrams.

Very briefly the method works as follows. Consider a diagram defined by a divergent integral. Then the integrand is differentiated with respect to the external energy, say q^-, until the integral is well defined. Next the integration over the internal momenta is performed. Finally the result is integrated over q^- as many times as it was differentiated before. This operation is the same as removing the lowest orders in the Taylor expansion in q^-. For example, if the two lowest orders of the Taylor expansion with respect to the external momentum q of a LF time-ordered diagram $\int d^3k\mathcal{F}(q,k)$ are divergent, minus regularization is the following operation:

$$\int_{q_\perp^2/2q^+}^{q^-}dq'\int_{q_\perp^2/2q^+}^{q'^-}dq''\int d^2k^\perp dk^+\left(\frac{\partial}{\partial q''^-}\right)^2\mathcal{F}(k,q''). \tag{4}$$

The point $q^2=0$ is chosen in this example as the renormalization point. This regularization method of subtracting the lowest order terms in the Taylor expansion is similar to what is known in covariant perturbation theory as the Bogoliubov-Parasiuk-Hepp-Zimmermann (BPHZ) regularization [10]. Some advantages of the minus regularization scheme are preservation of covariance and local counterterms. Another advantage is that longitudinal as well as transverse divergences are treated in the same way. A more thorough discussion on minus regularization can be found in Ref. [9].

D. Proof of equivalence for the Yukawa model

The proof of equivalence will not only hold order by order in the perturbation series, but also for every covariant diagram separately. In order to allow for a meaningful comparison with the method of Burkardt and Langnau we apply our method to the same model as they discuss. The Lagrangian of this model is

$$\mathcal{L}=\overline{\psi}(i\partial_\mu\gamma^\mu-m)\psi+\phi(\Box+\mu^2)\phi+g\overline{\psi}\psi\phi. \tag{5}$$

In the Yukawa model we have to distinguish four types of diagrams, according to their longitudinal and transverse degrees of divergence. These divergences are classified in Appendix A. The proof of equivalence is illustrated in Fig. 2.

We integrate an arbitrary covariant diagram over LF energy. For longitudinally divergent diagrams this integration is ill-defined and results in FILs. A regulator α is introduced which formally restores equivalence. Upon minus regularization the α-dependence is lost and the transverse divergences are removed. We can distinguish

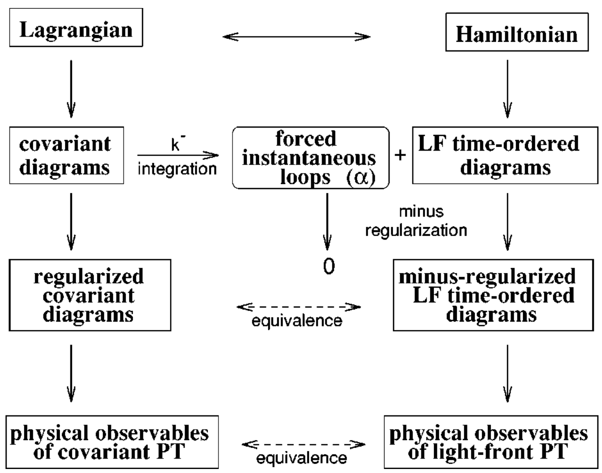


FIG. 2. Outline of the proof of equivalence.

(1) Longitudinally and transversely convergent diagrams ($D^-<0,\ D^\perp<0$).

No FILs will be generated. No regularization is needed. The LF time-ordered diagrams may contain $1/k^+$-poles, but these can be removed using blinks. A rigorous proof of equivalence for this class of diagrams is given in Ref. [7].

(2) Longitudinally convergent diagrams ($D^-<0$) with a transverse divergence ($D^\perp \geqslant 0$).

In the Yukawa model there are three such diagrams: the four fermion box, the fermion triangle and the one-boson exchange correction. Again, no FILs occur. Their transverse divergences and therefore the proof of equivalence will be postponed until a future publication. However, because the one-boson exchange correction illustrates the concept of k^--integration, the occurrence of instantaneous fermions and the construction of blinks, it will be discussed as an example in Sec. II.

(3) Longitudinally divergent diagrams ($D^-=0$) with a logarithmic transverse divergence ($D^\perp=0$).

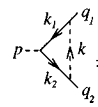
In the Yukawa model with a scalar coupling there is one such diagram: the fermion self-energy. Upon splitting the fermion propagator two diagrams are found. The troublesome one is the diagram containing the instantaneous part of the fermion propagator. It is a FIL, according to our definition, and needs a regulator. In Sec. III we show how to determine the regulator α that restores covariance formally. Since α can be chosen such that it does not depend on the LF energy, the FIL will vanish upon minus regularization.

(4) Longitudinally divergent diagrams with a quadratic transverse divergence ($D^\perp=2$). In the Yukawa model only the boson self energy is in this class. We are not able to give an explicit expression for α. However, in Sec. IV it is shown that the renormalized boson self energy is equal to the corresponding series of renormalized LF time-ordered diagrams. This implies that the contribution of FILs has again disappeared after minus regularization.

II. EXAMPLE: THE ONE-BOSON EXCHANGE CORRECTION

We will give an example of the construction of the LF time-ordered diagrams, the occurrence of instantaneous fermions and the construction of blinks. It concerns the correction to the boson–fermion–anti-fermion vertex due to the exchange of a boson by the two outgoing fermions. Here, and in the sequel, we drop the dependence on the coupling constant and numerical factors related to the symmetry of the Feynman diagrams.

A boson of mass μ with momentum p decays into a fermion anti-fermion pair with momenta q_1 and q_2 respectively. The covariant amplitude for the boson exchange correction can be written as

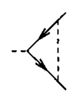


$$= \int_{\mathrm{M}} \frac{d^4k\ (\not k_1+m)(\not k_2+m)}{(k_1^2-m^2-i\epsilon)(k_2^2-m^2-i\epsilon)(k^2-\mu^2-i\epsilon)}. \tag{6}$$

The subscript M denotes that the integration is over Minkowski space. The momenta k_1 and k_2 indicated in the diagram are given by

$$k_1=k-q_1, \quad k_2=k+q_2. \tag{7}$$

We can rewrite Eq. (6) in terms of LF coordinates



$$= \int \frac{d^2k^\perp dk^+ dk^- (\not k_1+m)(\not k_2+m)}{8k_1^+k_2^+k^+(k^- - H_1^-)(k^- - H_2^-)(k^- - H^-)}, \tag{8}$$

where the poles in the complex k^--plane are given by

$$H^- = \frac{{k^\perp}^2+\mu^2-i\epsilon}{2k^+}, \tag{9}$$

$$H_1^- = q_1^- - \frac{{k_1^\perp}^2+m^2-i\epsilon}{2k_1^+}, \tag{10}$$

$$H_2^- = -q_2^- + \frac{{k_2^\perp}^2+m^2-i\epsilon}{2k_2^+}. \tag{11}$$

We will now show how the LF time-ordered diagrams, including those containing instantaneous terms, can be constructed. The LF time-ordered diagrams contain on-shell spin projections in the numerator. They are

$$\not k_{\mathrm{on}} = k^-_{i\,\mathrm{on}}\gamma^+ + k_i^+\gamma^- - k_i^\perp\gamma^\perp. \tag{12}$$

We will also use the following relation:

$$k^- - H_i^- = k_i^- - k^-_{i\,\mathrm{on}}. \tag{13}$$

We rewrite the numerator

$$(\not k_1+m)(\not k_2+m) = [(k^- - H_1^-)\gamma^+ + (\not k_{1\,\mathrm{on}}+m)] \times[(k^- - H_2^-)\gamma^+ + (\not k_{2\,\mathrm{on}}+m)]. \tag{14}$$

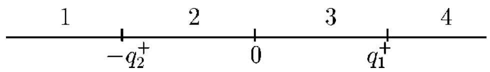
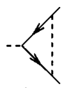


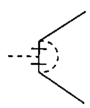
FIG. 3. Regions for the k^+-integration. At the boundaries a pole crosses the real k^--axis.

This separation allows us to write Eq. (8) as



$$= \int \frac{\mathrm{d}^2k^\perp \mathrm{d}k^+ \mathrm{d}k^-}{8k_1^+k_2^+k^+} \left\{ \frac{\gamma^+\gamma^+}{(k^- - H^-)} + \frac{(\not{k}_{1\mathrm{on}}+m)(\not{k}_{2\mathrm{on}}+m)}{(k^- - H_1^-)(k^- - H_2^-)(k^- - H^-)} + \frac{\gamma^+(\not{k}_{2\mathrm{on}}+m)}{(k^- - H_2^-)(k^- - H^-)} + \frac{(\not{k}_{1\mathrm{on}}+m)\gamma^+}{(k^- - H_1^-)(k^- - H^-)} \right\}. \tag{15}$$

The splitting corresponds to the splitting of the covariant amplitude into LF time-ordered diagrams. The numerators are written in such a form that Cauchy's theorem can be applied easily to the k^--integration. Only for the first term of Eq. (15) can k^- contour integration not be applied because the semi-circle at infinity gives a nonvanishing contribution. Such a singularity corresponds to a pole at infinity. However, we are saved by the fact that $\gamma^+\gamma^+=0$. Therefore we obtain for the first term of Eq. (15)

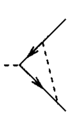


$$= 0. \tag{16}$$

Here the bars in the two internal fermion lines again denote instantaneous terms. This forces the boson line to be instantaneous too. We see that this diagram is a FIL according to the definition we gave in the previous section. The longitudinal divergences which occur due to such diagrams are discussed in the next sections. Since FILs are not LF time-ordered diagrams, rules as given by NLCQ do not apply.

The second term of Eq. (15) contains only propagating parts. It has three poles (9)–(11). We are free to close the contour either in the lower or in the upper half plane. The poles do not always lie on the same side of the real k^--axis. For example, the pole given in Eq. (9) is in the upper half plane for $k^+<0$. At $k^+=0$ it changes side. In Fig. 3 we show the four intervals that can be distinguished.

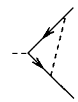
In region 1 all poles lie above the real k^--axis. By closing the contour in the lower half plane we see that the integral vanishes. At $k^+=-q^+$ the pole (11) crosses the real axis. In interval 2 the integral is proportional to its residue:



$$= -2\pi i \int \mathrm{d}^2k^\perp \int_{-q_2^+}^{0} \frac{\mathrm{d}k^+}{8k_1^+k_2^+k^+} \times \frac{(\not{k}_{1\mathrm{on}}+m)(\not{k}_{2\mathrm{on}}+m)}{(H_1^- - H_2^-)(H^- - H_2^-)}. \tag{17}$$

No cuts are drawn since this is clearly a LF time-ordered diagram. The factor $(H_1^- - H_2^-)^{-1}$ is the energy denominator corresponding to the fermion–anti-fermion state between the moment in LF time that the boson decays and the moment that the exchanged boson is emitted. $(H^- - H_2^-)^{-1}$ is the energy denominator corresponding to the state in the period that the exchanged boson exists.

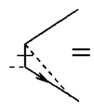
At $k^+=0$ a second pole crosses the real axis. For positive k^+ we close the contour in the upper half plane. Here only one pole (10) is present. The result is



$$= 2\pi i \int \mathrm{d}^2k^\perp \int_0^{q_1^+} \frac{\mathrm{d}k^+}{8k_1^+k_2^+k^+} \times \frac{(\not{k}_{1\mathrm{on}}+m)(\not{k}_{2\mathrm{on}}+m)}{(H_1^- - H_2^-)(H_1^- - H^-)}. \tag{18}$$

Only the second energy denominator differs from the one in Eq. (17).

The terms of Eq. (15) with one instantaneous term are easier to determine. There are two poles and a contribution only occurs if the poles are on different sides of the real k^--axis. The third term of Eq. (15) is

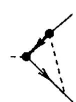


$$= -2\pi i \int \mathrm{d}^2k^\perp \int_{-q_2^+}^{0} \frac{\mathrm{d}k^+}{8k_1^+k_2^+k^+} \frac{\gamma^+(\not{k}_{2\mathrm{on}}+m)}{H^- - H_2^-}. \tag{19}$$

For the fourth and last term of Eq. (15) we have

$$= 2\pi i \int \mathrm{d}^2k^\perp \int_0^{q_1^+} \frac{\mathrm{d}k^+}{8k_1^+k_2^+k^+} \frac{(\not{k}_{1\mathrm{on}}+m)\gamma^+}{H_1^- - H^-}. \tag{20}$$

The possible $1/k^+$ poles inside the integration area can be removed using the blinks [7]: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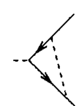
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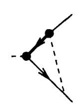
(21)

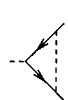
Using Eqs. (17) and (19) we get



$$= -2\pi i \int \mathrm{d}^2k^\perp \int_{-q_2^+}^{0} \frac{\mathrm{d}k^+}{8k_1^+k_2^+k^+} \times \frac{(\not{k}_{2\mathrm{on}} - \not{p} + m)(\not{k}_{2\mathrm{on}}+m)}{(H_1^- - H_2^-)(H^- - H_2^-)}. \tag{22}$$

The other blink is constructed in the same way.

We have now succeeded in doing the k^--integration and have rewritten the covariant expression for the one-boson exchange correction (6) in terms of LF time-ordered diagrams. The result is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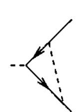
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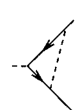
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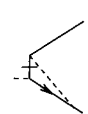
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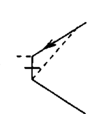
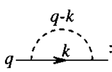

(23)

Diagrams with instantaneous parts are typical for LFPT. There is another difference with equal-time PT. Of the six possible time-orderings of the triangle diagram two have survived, which give rise to two diagrams each, upon splitting the fermion propagators into instantaneous and propagating parts. This reduction of the number of LF time-ordered diagrams compared to ordinary time-ordered ones is well known in LFPT, and explained in detail in Ref. [7].

All the calculations in this section were purely algebraic. The formulas for the LF time-ordered diagram we derived are the same as those given by NLCQ. The integrals that remain are logarithmically divergent in the transverse direction and must be regularized. This calculation will be done in a forthcoming publication in which we discuss transverse divergences.

III. EQUIVALENCE OF THE FERMION SELF ENERGY

There are two longitudinally divergent diagrams in the Yukawa model. We first discuss the fermion self energy. For our discussion the location of the poles is not relevant and therefore we ignore the $i\epsilon$ term. For a fermion momentum q we have the following self energy amplitude:



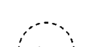
$$= \int_{\mathrm{M}} \frac{\mathrm{d}^4 k \quad (\not{k} + m)}{(k^2 - m^2)((q-k)^2 - \mu^2)}. \tag{24}$$

A. Covariant calculation

We introduce a Feynman parameter x and change the integration variable to k' given by $k = k' + xq$ in order to complete the square in the denominator. This gives

$$= \int_0^1 \mathrm{d}x \int_{\mathrm{M}} \frac{\mathrm{d}^4 k' \quad (\not{k}' + x\not{q} + m)}{(k'^2 - (1-x)m^2 - x\mu^2 + x(1-x)q^2)^2}. \tag{25}$$

The integral (25) is ill-defined. The appearance of $\not{k}$ in the numerator causes the integral to be divergent in the minus direction and obstructs the Wick rotation. However, this term is odd and is removed in accordance with common practice [10]. Wick rotation gives then



$$= i \int_0^1 \mathrm{d}x \int_{\mathrm{E}} \frac{\mathrm{d}^4 k' \quad (x\not{q} + m)}{(k'^2 + (1-x)m^2 + x\mu^2 - x(1-x)q^2)^2}. \tag{26}$$

The subscript E denotes that the integration is over Euclidean space. From Eq. (26) we can immediately infer that the fermion self energy has the covariant structure

$$= \not{q}\ F_1(q^2) + m\ F_2(q^2). \tag{27}$$

B. Residue calculation

To obtain the LF time-ordered diagram and the FIL corresponding to the fermion self energy we perform the k^--integration by doing the contour integration,

$$= \int \frac{\mathrm{d}^2 k^\perp \mathrm{d}k^+ \mathrm{d}k^-}{4k^+(q^+ - k^+)} \frac{k^-\gamma^+ + k^+\gamma^- - k^\perp\gamma^\perp + m}{(k^- - H_1^-)(k^- - H_2^-)}, \tag{28}$$

with the following poles:

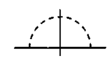
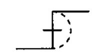
$$H_1^- = \frac{k^{\perp 2} + m^2}{2k^+}, \tag{29}$$

$$H_2^- = q^- - \frac{(q^\perp - k^\perp)^2 + \mu^2}{2(q^+ - k^+)}. \tag{30}$$

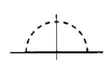
We rewrite Eq. (28) as

$$= \int \frac{\mathrm{d}^2 k^\perp \mathrm{d}k^+ \mathrm{d}k^-}{4k^+(q^+ - k^+)} \frac{H_1^-\gamma^+ + k^+\gamma^- - k^\perp\gamma^\perp + m}{(k^- - H_1^-)(k^- - H_2^-)} + \int \frac{\mathrm{d}^2 k^\perp \mathrm{d}k^+ \mathrm{d}k^-}{4k^+(q^+ - k^+)} \frac{\gamma^+(k^- - H_1^-)}{(k^- - H_1^-)(k^- - H_2^-)}. \tag{31}$$

The first term of Eq. (31) is the part that gives a convergent k^--integration. The second term contains the divergent part. This separation can also be written in terms of diagrams:

 =  +  (32)

The propagating diagram is

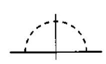


$$= 2\pi i \int \mathrm{d}^2 k^\perp \int_0^{q^+} \frac{\mathrm{d}k^+}{4k^+(q^+ - k^+)} \times \frac{\frac{m^2 + k^{\perp 2}}{2k^+}\gamma^+ + k^+\gamma^- - k^\perp\gamma^\perp + m}{H_2^- - H_1^-}. \tag{33}$$

It has the usual form for a LF time-ordered diagram. It is divergent because of the $1/k^+$ singularity in the numerator. To shed more light on the structure of this formula we introduce internal variables x and $k'^\perp$:

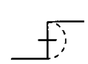
$$x = \frac{k^+}{q^+}, \quad k'^\perp = k^\perp - xq^\perp. \tag{34}$$

The denominator is now a complete square and we drop as usual the odd terms in $k'^\perp$ in the numerator. Then we find



$$= \pi i \int \mathrm{d}^2 k'^\perp \int_0^1 \mathrm{d}x \times \frac{\frac{m^2 + k'^{\perp 2} - x^2 q^2}{2xq^+}\gamma^+ + x\not{q} + m}{k'^{\perp 2} + (1-x)m^2 + x\mu^2 - x(1-x)q^2}. \tag{35}$$

The FIL is



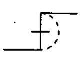
$$= \int \frac{\mathrm{d}^2 k^\perp \mathrm{d}k^+ \mathrm{d}k^-}{4k^+(q^+ - k^+)} \frac{\gamma^+}{k^- - H_2^-}. \tag{36}$$

It contains the divergent part of the k^--integration and a $1/k^+$ singularity too. The single bar in Eq. (36) stands for an instantaneous part. The diagram is instantaneous because it does not depend on the external energy q^-. In order to demonstrate this we shift k^- by q^-. Then we see that the dependence on q^- disappears. However, this way of reasoning is dangerous since the integral is divergent. We make the integral well-defined by inserting a function $\mathcal{R}$ containing a regulator α:

$$\mathcal{R}=\left(\frac{\alpha(k^+)}{1-i\delta q^+k^-}+\frac{1-\alpha(k^+)}{1+i\delta q^+k^-}\right). \tag{37}$$

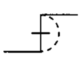
If we choose $\alpha=1$ for $k^+<0$ and $\alpha=0$ for $k^+>q^+$, the extra pole only contributes for $0<k^+<q^+$. In other words, then the spectrum condition is also satisfied for all lines in the FIL. This is convenient, but not necessary. Mustaki *et al.* do not require the spectrum condition to be fulfilled for instantaneous particles. They have as integration boundaries for the FIL $0<k^+<\infty$.

We perform the k^--integration and take the limit $\delta\to 0$. This gives



$$= 2\pi i\int \mathrm{d}^2k^\perp \int_0^{q^+} \mathrm{d}k^+ \frac{\gamma^+\alpha(k^+)}{4k^+(q^+-k^+)}. \tag{38}$$

Using internal variables (34) we obtain



$$= \pi i\frac{\gamma^+}{2q^+}\int \mathrm{d}^2k'^\perp \int_0^1 \mathrm{d}x \frac{\alpha(x)}{x(1-x)}. \tag{39}$$

C. Equivalence

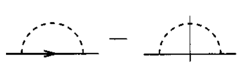
The FIL is not a LF time-ordered diagram. We think it is a remnant of the problems encountered in quantization on the light-front. We require it to satisfy two conditions:

(1) The FIL has to restore covariance and equivalence of the full series of LF time-ordered diagrams.
(2) The FIL has to be a polynomial in q^-.

The first condition will also ensure that the FIL contains a $1/k^+$ singularity that cancels a similar singularity in the propagating diagram. The second condition is that the FIL is truly instantaneous; i.e., it does not contain q^- in the denominator like a propagating diagram. To find the form of the FIL that satisfies these conditions we calculate

(40)

where we take for the covariant diagram, Eq. (26). This is a strictly formal operation. The covariant diagram is a 4-dimensional integral, whereas the propagating diagram has only 2 dimensions (not counting the x-integration). We can calculate Eq. (40) without evaluation of the integrals. In Appendix B useful relations are derived between d- and $(d-2)$-dimensional integrals. Upon using them we obtain



$$= -\pi i\frac{\gamma^+}{2q^+}\int \mathrm{d}^2k'^\perp \int_0^1 \mathrm{d}x \times\frac{m^2+k'^{\perp 2}-x^2q^2}{x\left(k'^{\perp 2}+(1-x)m^2+x\mu^2-x(1-x)q^2\right)}. \tag{41}$$

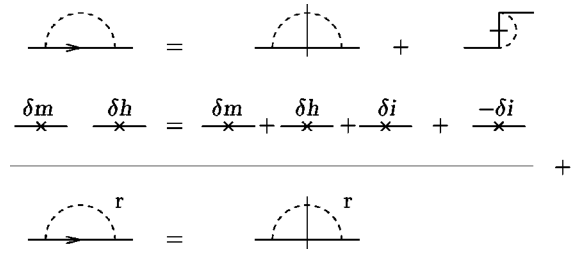
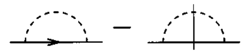


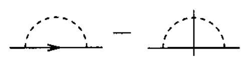
FIG. 4. Addition of the counterterms. The result is the minus-regularized fermion self energy.

This can be rewritten as



$$= -\pi i\frac{\gamma^+}{2q^+}\int \mathrm{d}^2k'^\perp \int_0^1 \mathrm{d}x \times\left(\frac{1}{x}+\frac{m^2-\mu^2+(1-2x)q^2}{k'^{\perp 2}+(1-x)m^2+x\mu^2-x(1-x)q^2}\right). \tag{42}$$

The dependence on q^2 is limited to the second term. The integral over x of the latter can be done explicitly, whence one finds that the integral is independent of q^2. Therefore we can take $q^2=0$ in Eq. (42):



$$= -\pi i\frac{\gamma^+}{2q^+}\int \mathrm{d}^2k'^\perp \int_0^1 \mathrm{d}x \times\left(\frac{1}{x}+\frac{m^2-\mu^2}{k'^{\perp 2}+(1-x)m^2+x\mu^2}\right). \tag{43}$$

This is a good moment to see if we can satisfy the two conditions we put forward in the beginning of this subsection.

The first condition is satisfied if the right-hand sides of Eqs. (43) and (39) are equal. We can verify that there is an infinite number of solutions for α to make this happen. We are free to choose α to be q^--independent. This will make formula (39) also independent of q^-. Then the second condition is trivially satisfied.

D. Conclusions

Our renormalization method is visualized in Fig. 4. There are two noncovariant counterterms (δi). One of them occurs in the LF time-ordered part; the other one is associated with a self-induced inertia. Minus regularization guarantees that they cancel provided the regulator α is chosen appropriately. The other counterterms δm and δh are covariant. After the (infinite) counterterms have been added the renormalized amplitude (denoted by the superscript r) remains. An illustration of the full procedure of minus regularization is given in the next section.

We take another look at Fig. 4. The first line contains three ill-defined objects. The covariant amplitude (24) has a Minkowskian measure and contains odd terms. Divergent odd terms are dropped as part of the regularization procedure. To calculate the LF time-ordered diagram (33) we also dropped surface terms. Can these assumptions be justified? Would another set of assumptions give different physical amplitudes? We conjecture that any set of assumptions corresponds to a certain class of choices for α. The α-dependence is only present in the FILs. In the process of

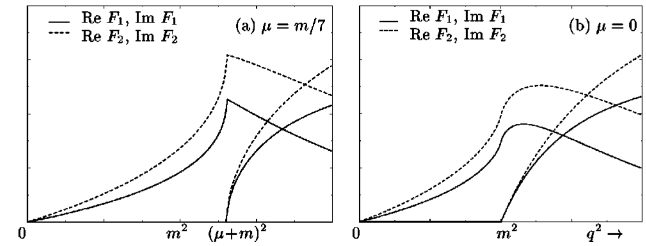
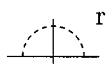


FIG. 5. The renormalized fermion self energy. The left hand panel (a) shows the case $\mu = m/7$; the right hand panel (b) is for $\mu = 0$.

minus regularization the α-dependence is lost, as we see for the fermion self energy in Fig. 4. Therefore the physical observables do not depend on the assumptions we started out with.

Finally we give the result for the fermion self energy:



$$^{\mathrm{r}} = -\pi^2 i \int_0^1 \mathrm{d}x\, (x\not{q} + m) \times \log\left(1 - \frac{x(1-x)q^2}{(1-x)m^2 + x\mu^2}\right) \tag{44}$$

This integral can be done analytically, but the result is a rather long formula, which we give in the Appendix C. Here we display the result in pictorial form. Figure 5 shows F_1 and F_2 for values of the fermion momentum squared in the range $q^2 \in [0,2m^2]$ for the case of a massless boson and the case where $\mu = m/7$ corresponding to the self energy correction for a nucleon due to a scalar pion. The case $\mu = 0$ is included because it was calculated before by Ligterink and Bakker [9].

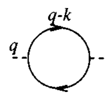
The threshold behavior in the two cases is clearly seen in this figure. Above threshold, $q^2 > (m+\mu)^2$, the self energy becomes complex.

We have verified that our result is in agreement with the result given by dimensional regularization and the result given by Bjorken and Drell [11], using Pauli-Villars regularization.

For the following reasons our analysis differs essentially from the analysis of Mustaki *et al.* [8]. First of all, we make an explicit distinction between LF time-ordered diagrams and FILs. Second, we make the integration over the longitudinal coordinates well-defined by introducing a regulator $\alpha(k^+)$. Mustaki *et al.* make the k^+-integration well-defined by using cutoffs. The form of the cutoffs depends on the regularization scheme of the divergences in the transverse directions. In our calculation the form of $\alpha(k^+)$ is determined by requiring equivalence to the covariant calculation. In our opinion, this is the most important constraint on the FIL. We do not think that the cutoffs can always be determined from an analysis of the transverse divergences. For example, in two dimensions $(D = 1+1)$ there are no transverse divergences, but longitudinal divergences are still present and $\alpha(k^+)$ has to ensure that covariance is maintained. Moreover, in $D = 1+1$ the covariant calculation of the fermion self energy gives a finite result. Our choice of $\alpha(k^+)$, independent of $k^\perp$, ensures also in this case that the LF time-ordered calculation reproduces the covariant result. The same is true for the calculation by Mustaki *et al.* if they make a particular choice for the cutoffs.

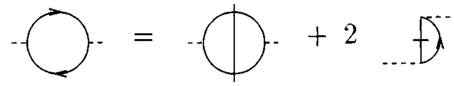
IV. EQUIVALENCE OF THE BOSON SELF ENERGY

Our analysis of the boson self energy serves two purposes. First of all it illustrates in detail the concept of minus regularization. Second it concludes our proof of equivalence for one-loop diagrams with longitudinal divergences in the scalar Yukawa model. The covariant expression for the boson self energy at one-loop level is



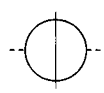
$$= \int_{\mathrm{M}} \frac{\mathrm{d}^4k \;\mathrm{Tr}\,[(\not{k} + m)(\not{k} - \not{q} + m)]}{(k^2 - m^2)((q-k)^2 - m^2)}. \tag{45}$$

The momenta are chosen in the same way as for the fermion self energy. The location of the poles is given by Eqs. (29), (30) with μ replaced by m. In order to do the k^--integration we separate the numerator into three parts. We find



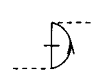
$$\tag{46}$$

The second term on the right-hand side are the two FILs, which are identical. The first term is the LF time-ordered boson self energy. It can be rewritten as



$$= 2\pi i \int \mathrm{d}^2k^\perp \int_0^{q^+} \frac{\mathrm{d}k^+}{4k^+(q^+ - k^+)} \times \frac{\mathrm{Tr}\,[(\not{k}_{\mathrm{on}} + m)\,((\not{k} - \not{q})_{\mathrm{on}} + m)]}{H_2^- - H_1^-}. \tag{47}$$

The FIL is given by



$$= \int \frac{\mathrm{d}^2k^\perp \mathrm{d}k^+ \mathrm{d}k^-}{4k^+(q^+ - k^+)} \frac{\mathrm{Tr}\,[\gamma^+\,((\not{k} - \not{q})_{\mathrm{on}} + m)]}{k^- - H_2^-}. \tag{48}$$

We have seen in our discussion of the fermion self energy that it is possible to determine the exact form of the FIL that maintains covariance. However, we have also seen that taking this step is not necessary, since upon minus regularization the FILs disappear. An analysis along lines similar to those in Sec. III C will show that the FIL is also in this case independent of q^-. Therefore we limit ourselves to the calculation and renormalization of the propagating diagram.

A. Minus regularization

We will now apply the minus regularization scheme to the LF time-ordered boson self energy. For a self energy diagram the following ten steps can be used to find the regularized diagram. Some steps are explained in more detail for the boson self energy.

(1) Write the denominator in LF coordinates.

(2) Complete the squares in the denominator by introducing internal variables ($k'^\perp$ and x).

(3) Write the numerator in terms of internal and external LF coordinates.

(4) Remove odd terms in $k'^{\perp}$ in the numerator.
These steps were also taken in our discussion of the fermion self energy. Next we diverge.

(5) Subtraction of the lowest order in the Taylor expansion is equivalent to inserting a multiplier X. Construct the multiplier.

(6) Compensate for the subtraction by adding counterterms. Verify that they are infinite. If they are not, the corresponding divergence was only apparent and we should not subtract it. We do not allow for finite renormalizations.

For the boson self energy all terms have the same denominator. For them we can write the expansion

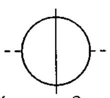
$$\frac{1}{k'^{\perp 2}+m^2-x(1-x)q^2}=\frac{1}{k'^{\perp 2}+m^2}\sum_{j=0}^{\infty}X^j, \tag{49}$$

where the multiplier X has the form

$$X=\frac{x(1-x)q^2}{k'^{\perp 2}+m^2}. \tag{50}$$

(7) Identify, term by term, the degree of divergence and insert the corresponding multiplier. To compensate for this, add a polynomial of the appropriate degree with infinite coefficients.

Steps (1)–(7) lead to the following result for the boson self energy:



$$= A+Bq^2+\pi i\int d^2k'^{\perp}\int_0^1 dxX\ \mathrm{Tr}\left[\left(\frac{Xk'^{\perp 2}+x^2q^{\perp 2}+m^2}{2xq^+}\gamma^+ + x(q^+\gamma^- - q^{\perp}\gamma^{\perp})+m\right)\right.$$
$$\left.\times\left(\frac{Xk'^{\perp 2}+(x-1)^2q^{\perp 2}+m^2}{2(x-1)q^+}\gamma^+ + (x-1)(q^+\gamma^- - q^{\perp}\gamma^{\perp})+m\right)+X(k'^{\perp}\gamma^{\perp})^2\right]\left(k'^{\perp 2}+m^2-x(1-x)q^2\right)^{-1}. \tag{51}$$

Longitudinal divergences appear as $1/x$ singularities. Transverse divergences appear as ultraviolet $k'^{\perp}$ divergences. Since every term in the boson self energy is at least logarithmically divergent, there is an overall factor X. Some of the terms are quadratically divergent in $k'^{\perp}$ and have an extra factor X. We use the fact that terms containing the factor $\gamma^+\gamma^+$ vanish. We are not interested in the exact form of the counterterms A and B. We can verify that they are infinite. They are included to allow for comparison with other regularization schemes.

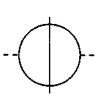
(8) Rewrite the numerator in terms of objects having either covariant or γ^+/q^+ structure.
For our integral we use the following relation:

$$\frac{x^2q^{\perp 2}+m^2}{2xq^+}\gamma^+ + x(q^+\gamma^- - q^{\perp}\gamma^{\perp})=\frac{x^2q^2+m^2}{2x}\frac{\gamma^+}{q^+}+x\not{q}. \tag{52}$$

(9) Perform the trace, if present.

(10) Do the x and $k'^{\perp}$ integrations.

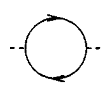
Application of the last two steps gives



$$= A+Bq^2-2i\pi^2\left(3q^2-8m^2 +2(4m^2-q^2)\sqrt{\frac{4m^2-q^2}{q^2}}\arctan\sqrt{\frac{q^2}{4m^2-q^2}}\right). \tag{53}$$

B. Equivalence

We will now compare the result of the minus regularization scheme applied to the LF time-ordered boson self energy with dimensional regularization applied to the covariant diagram. Using the standard rules of dimensional regularization [10] we obtain



$$= A'+B'q^2-4\pi^2 i(4m^2-q^2)\times\sqrt{\frac{4m^2-q^2}{q^2}}\arctan\sqrt{\frac{q^2}{4m^2-q^2}}. \tag{54}$$

A' and B' are constants containing $1/\varepsilon$. In the limit of $\varepsilon\to 0$ they diverge. Of course, A' and B' cannot be related to the infinite constants generated by minus regularization. However, this is not necessary. Both schemes are equivalent if the same physical amplitudes are generated. To calculate them we have to construct the counterterms or, equivalently, fix the amplitude and its first derivative at the renormalization point. For the unrenormalized amplitudes (53),(54) the coefficients A or A' of the constant term are used to determine the physical mass μ_{ph} of the boson. The coefficients B or B' determine the fermion wave function renormalization. Only the q^4 and higher order terms can be used to make predictions. These coefficients must be the same for the two methods. We see that Eqs. (53) and (54) only differ in the first two coefficients of the polynomial in q^2. Therefore the two methods generate the same physical amplitudes.

V. CONCLUSIONS

We discussed in this paper the problem of covariance, which includes the problem of nonmanifest rotational invariance, in LFPT.

For diagrams which are both longitudinally and transversely convergent one can give a rigorous demonstration of equivalence, without discussing renormalization explicitly. It is given by Ligterink and Bakker [7].

For longitudinally divergent diagrams such a proof is not possible because the integration over LF energy is ill-defined. Still, LF time-ordered diagrams can be constructed applying the rules of NLCQ. However, FILs have to be included to make the full series add up to the covariant diagram. These FILs contain the ambiguity related to the ill-defined integration, as can be shown by our analysis involving the regulator α.

We conjecture that the FILs are remnants of the difficulty of quantizing on the light-front. Just like NLCQ, we are not able to provide general rules to construct them. However, we can identify the conditions for their occurrence. We show that it is not necessary to find an explicit expression for the FILs. Upon minus regularization they vanish. Therefore the α-dependence drops too. The remaining series of regularized LF time-ordered diagrams is again covariant.

The main difficulty we encountered was to show that the FILs are instantaneous indeed. This can be shown by proving that the regulator α does not depend on the LF energy, as we did for the fermion self energy. Another way is to show that the regularized covariant amplitude equals the corresponding series of minus-regularized LF time-ordered diagrams. We used this technique for the boson self energy.

This concludes our proof of equivalence of renormalized covariant and LF perturbation theory for longitudinally divergent diagrams in the Yukawa model. Three diagrams with transverse divergences remain. They require a more elaborate analysis of minus regularization and numerical implementation of the method. Therefore this work is postponed until a future publication [12].

ACKNOWLEDGMENTS

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APPENDIX A: TYPES OF DIVERGENCES OF FEYNMAN AMPLITUDES

1. Transverse divergences

In a discussion on LF time-ordered diagrams we encounter divergences in the perpendicular direction. In most cases this divergence is the same as what is known in covariant PT as *the* divergence D of a diagram. There it is the divergence one finds if in the covariant amplitude odd terms are removed and Wick rotation is applied. For a one-loop Feynman diagram in d space-time dimensions with f internal fermion lines and b internal (scalar) boson lines the transverse degree of divergence is

$$D^{\perp} = d - f - 2b. \tag{A1}$$

2. Longitudinal divergences

We relate covariant PT and LFPT by integrating over LF energy k^-. In this process we can find divergences for the integration which are classified by D^-. The formula for the longitudinal degree of divergence of a diagram is

$$D^- = 1 - b. \tag{A2}$$

Longitudinally divergent diagrams contain zero or one boson in the loop. Since any loop contains at least two lines, a longitudinally divergent diagram contains at least one fermion line. For the model we discuss, the Yukawa model with a scalar coupling, the divergence is reduced. For scalar coupling g it turns out that $\gamma^+ g \gamma^+ = 0$ and therefore two instantaneous parts can not be neighbors. The longitudinal degree of divergence for the Yukawa model with scalar coupling is

$$D^-_{\mathrm{Yuk}} = 1 - b - \left[\frac{1+f-b}{2}\right]_{\mathrm{entier}} = 1 - \left[\frac{1+f+b}{2}\right]_{\mathrm{entier}}. \tag{A3}$$

3. Divergent diagrams in the Yukawa model

In Table I we list all one-loop diagrams up to order g^4 that are candidates to have either longitudinal or transverse divergences. There are five diagrams with transverse divergences $D^{\perp} \geqslant 0$, of which two also have a longitudinal divergence $D^- \geqslant 0$. These are the boson and the fermion self energies.

APPENDIX B: RELATIONS BETWEEN EUCLIDIAN INTEGRALS

The two basic formulas are

$$\int \mathrm{d}^d k f(k^2) = \frac{2\pi^{d/2}}{\Gamma(d/2)} \int_0^{\infty} \mathrm{d}k k^{d-1} f(k^2), \tag{B1}$$

$$\int_0^{\infty} \mathrm{d}k \frac{k^{d-1}}{(k^2+C^2)^m} = \frac{\Gamma(d/2)\Gamma(m-d/2)}{2\Gamma(m)} (C^2)^{d/2-m}, \tag{B2}$$

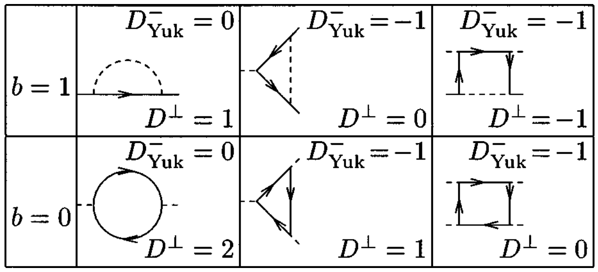
with $d \geqslant 1$ and $m > 0$. If we take $d \geqslant 2$ and $m > 1$, the following manipulations are valid. Formulas (B1) and (B2) can be combined to give

$$\int \mathrm{d}^d k \frac{1}{(k^2+C^2)^m} = \pi^{d/2} \frac{\Gamma(m-d/2)}{\Gamma(m)} (C^2)^{d/2-m}, \tag{B3}$$

$$\int \mathrm{d}^d k \frac{A+Bk^2}{(k^2+C^2)^m} = \pi^{d/2} \frac{\Gamma(m-1-d/2)}{\Gamma(m)} (C^2)^{d/2-m} \times [(m-1-d/2)A + dBC^2/2]. \tag{B4}$$

We can formulate the same equation for $d-2$ dimensions and $m-1$ powers in de denominator. We find that the right hand sides differ only slightly:

TABLE I. Transverse and longitudinal divergences in the Yukawa model for $d=4$.



$$\int d^{d-2}k\, \frac{A+Bk^2}{(k^2+C^2)^{m-1}} = \frac{\pi^{d/2}}{\pi}\frac{\Gamma(m-1-d/2)}{\Gamma(m-1)}(C^2)^{d/2-m}[(m-1-d/2)A + (d-2)BC^2/2]. \tag{B5}$$

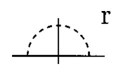
A comparison of these formulas gives

$$\int d^dk\, \frac{A+Bk^2}{(k^2+C^2)^m} = \frac{\pi}{m-1}\int d^{d-2}k\, \frac{A+B\dfrac{d}{d-2}k^2}{(k^2+C^2)^{m-1}}, \tag{B6}$$

provided we have $d>2$ and $m>1$.

APPENDIX C: THE FERMION SELF ENERGY IN CLOSED FORM

Here we give the results for the integral (44) in closed form. We write for the renormalized self energy



$$= \not{q}\, F_1(q^2) + m\, F_2(q^2). \tag{C1}$$

Then the two functions $F_{1,2}$ are found to be

$$\frac{F_1(q^2)}{\pi^2 i} = -\int_0^1 dx\, x\, \log\left(1-\frac{x(1-x)q^2}{(1-x)m^2+x\mu^2}\right) \tag{C2}$$

and

$$\frac{F_2(q^2)}{\pi^2 i} = -\int_0^1 dx\, \log\left(1-\frac{x(1-x)q^2}{(1-x)m^2+x\mu^2}\right). \tag{C3}$$

For $\mu=0$ we find the result to be in agreement with the formula given by Ligterink and Bakker [9] and by Bjorken and Drell [11]. They use the vector coupling appropriate for the photon and therefore overall numerical factors are different:

$$\frac{F_1(q^2)}{\pi^2 i} = \frac{1}{4}+\frac{m^2}{2q^2}-\left(\frac{1}{2}-\frac{m^4}{2q^4}\right)\log\frac{m^2-q^2}{m^2}, \tag{C4}$$

$$\frac{F_2(q^2)}{\pi^2 i} = 1-\left(1-\frac{m^2}{q^2}\right)\log\frac{m^2-q^2}{m^2}. \tag{C5}$$

For $\mu>0$ we have

$$\frac{F_1(q^2)}{\pi^2 i} = \frac{1}{4}+\frac{(\mu^2-m^2)^2-\mu^2q^2}{2(m^2-\mu^2)q^2}+\left(\frac{(m^2-\mu^2+q^2)^2-2m^2q^2}{4q^4}-\frac{m^4}{2(m^2-\mu^2)^2}\right)\log\frac{\mu^2}{m^2}+\left(\log\frac{D^{1/2}+m^2-\mu^2-q^2}{D^{1/2}-m^2+\mu^2+q^2} - \log\frac{D^{1/2}+m^2-\mu^2+q^2}{D^{1/2}-m^2+\mu^2-q^2}\right)\frac{D^{1/2}(m^2-\mu^2+q^2)}{4q^4} \tag{C6}$$

and

$$\frac{F_2(q^2)}{\pi^2 i} = 1+\left(\frac{m^2}{\mu^2-m^2}+\frac{m^2-\mu^2+q^2}{2q^2}\right)\log\frac{\mu^2}{m^2}+\frac{D^{1/2}}{2q^2}\left(\log\frac{D^{1/2}+m^2-\mu^2-q^2}{D^{1/2}-m^2+\mu^2+q^2}-\log\frac{D^{1/2}+m^2-\mu^2+q^2}{D^{1/2}-m^2+\mu^2-q^2}\right), \tag{C7}$$

where the variable D contains the threshold behavior

$$D=[q^2-(m+\mu)^2][q^2-(m-\mu)^2]. \tag{C8}$$

We checked that the limit $\mu\to 0$ of Eqs. (C6),(C7) exists and is equal to Eqs. (C4),(C5), respectively.

[1] P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).
[2] J. B. Kogut and D. E. Soper, Phys. Rev. D **1**, 2901 (1970).
[3] H. Leutwyler and J. Stern, Ann. Phys. (N.Y.) **112**, 94 (1978).
[4] S. J. Brodsky, H. C. Pauli and S. Pinsky, ''Quantum chromodynamics and other field theories on the light cone,'' report, 1997.
[5] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).
[6] M. Burkardt and A. Langnau, Phys. Rev. D **44**, 3857 (1991).
[7] N. E. Ligterink and B. L. G. Bakker, Phys. Rev. D **52**, 5954 (1995).
[8] D. Mustaki, S. Pinsky, J. Shigemitsu and K. Wilson, Phys. Rev. D **43**, 3411 (1991).
[9] N. E. Ligterink and B. L. G. Bakker, Phys. Rev. D **52**, 5917 (1995).
[10] J. C. Collins, *Renormalization* (Cambridge University Press, Cambridge, England, 1984).
[11] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
[12] N. C. J. Schoonderwoerd and B. L. G. Bakker, Vrije Universiteit Report No. VUTH 98-8, hep-ph/9801433, 1998.