Trace anomaly induced effective action and 2D black holes for dilaton coupled supersymmetric theories

Shin'ichi Nojiri*
Department of Mathematics and Physics, National Defence Academy, Hashirimizu Yokosuka 239, Japan

Sergei D. Odintsov†
Tomsk Pedagogical University, 634041 Tomsk, Russia
and Department de Fisica, Universidad del Valle, AA25360, Cali, Colombia

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The action for two-dimensional (2D) dilatonic supergravity with dilaton coupled matter and dilaton multiplets is constructed. A trace anomaly and anomaly induced effective action (in components as well as in supersymmetric form) for a matter supermultiplet on a bosonic background are found. The one-loop effective action and large-N effective action for quantum dilatonic supergravity are also calculated. Using an induced effective action one can estimate the back reaction of dilaton coupled matter to the classical black hole solutions of dilatonic supergravity. That is done on the example of a supersymmetric CGHS model with dilaton coupled quantum matter where Hawking radiation which turns out to be zero is calculated. A similar 2D analysis may be used to study spherically symmetric collapse for other models of 4D supergravity. [S0556-2821(98)04508-1]

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I. INTRODUCTION

There are various motivations to study two-dimensional (2D) gravitational theories and their black hole solutions (see [1–5], and references therein). First of all, it is often easier to study 2D models at least as useful laboratories. Second, starting from 4D Einstein-Maxwell-scalar theory and applying a spherically symmetric reduction ansatz [6], one is left with a specific dilatonic gravity with dilaton coupled matter. Hence, in such a case 2D gravity with dilaton coupled matter may describe the radial modes of the extremal dilatonic black holes in four dimensions [7]. Similarly, the dilaton coupled matter action appears in string inspired models. Recently, a dilaton dependent trace anomaly and induced effective action for dilaton coupled scalars have been studied in Refs. [8–10]. That opens new possibilities in the study of black holes with the back reaction of quantum matter [9].

It could be extremely interesting to present a supersymmetric generalization of the results [8–10]. The above motivations are still valid in this case. Indeed, let us consider the spherical reduction of $N=1$, $d=4$ supergravity theory to $d=2$ theory. In order to realize the spherical reduction, we assume that the metric has the following form:

$$ds^2=\sum_{\mu\nu=0,1,2,3} g_{\mu\nu}dx^\mu dx^\nu=\sum_{m,n=t,r} g_{mn}(t,r)dx^m dx^n + e^{2\phi(t,r)}(d\theta^2+\sin^2\theta d\varphi^2). \tag{1}$$

The metric (1) can be realized by choosing the vierbein fields e_μ^a as follows:

$$e^0_{\theta,\varphi}=e^3_{\theta,\varphi}=e^1_{t,r}=e^2_{t,r}=0,$$
$$e^1_\theta=e^\phi,\quad e^2_\varphi=\sin\theta e^\phi,\quad e^1_\varphi=e^2_\theta=0. \tag{2}$$

The expression (2) is unique up to a local Lorentz transformation. The local supersymmetry transformation for the vierbein field with the parameter ζ and $\overline{\zeta}$ is given by

$$\delta e^a_\mu=i(\psi_\mu\sigma^a\overline{\zeta}-\zeta\sigma^a\overline{\psi_\mu}). \tag{3}$$

Here ψ_μ is the Rarita-Schwinger field (gravitino) and we follow the standard notations of Ref. [11] (see also, [12]). If we require that the metric has the form of Eq. (1) after the local supersymmetry transformation, i.e.,

$$\delta g_{t\theta}=\delta g_{r\theta}=\delta g_{t\varphi}=\delta g_{r\varphi}=\delta g_{\theta\varphi}=0,$$
$$\delta g_{\varphi\varphi}=\sin\theta\delta g_{\theta\theta}, \tag{4}$$

we obtain, up to local Lorentz transformation,

$$\zeta_1=\overline{\zeta^1},\quad \zeta_2=\overline{\zeta^2}, \tag{5}$$

$$\psi_{\varphi 1}=\sin\theta\psi_{\theta 1},\quad \overline{\psi^1_\varphi}=\sin\theta\overline{\psi^1_\theta},$$
$$\psi_{\varphi 2}=-\sin\theta\psi_{\theta 2},\quad \overline{\psi^2_\varphi}=-\sin\theta\overline{\psi^2_\theta},$$
$$\overline{\psi^1_\theta}=-\psi_{\theta 1},\quad \overline{\psi^2_\theta}=-\psi_{\theta 2},$$
$$\psi_{r1}-\overline{\psi^1_r}=-2e^{-\phi}e^3_r\psi_{\theta 2},\quad \psi_{t1}-\overline{\psi^1_t}=-2e^{-\phi}e^3_t\psi_{\theta 2},$$
$$\psi_{r2}-\overline{\psi^2_r}=-2e^{-\phi}e^0_r\psi_{\theta 1},\quad \psi_{t2}-\overline{\psi^2_t}=-2e^{-\phi}e^0_r\psi_{\theta 1}. \tag{6}$$

*Email address: nojiri@cc.nda.ac.jp
†Email address: odintsov@quantum.univalle.edu.co, odintsov@kakuri2-pc.phys.sci.hiroshima-u.ac.jp

Equation (5) tells that the local supersymmetry of the spherically reduced theory is $N=1$, which should be compared with the torus compactified case, where the supersymmetry becomes $N=2$. Let the independent degrees of freedom of the Rarita-Schwinger fields be

$$2\psi_r^1 \equiv \psi_{r1} + \overline{\psi_r^1}, \quad 2\psi_r^2 \equiv \psi_{r2} + \overline{\psi_r^2},$$

$$2\psi_t^1 \equiv \psi_{t1} + \overline{\psi_t^1}, \quad 2\psi_t^2 \equiv \psi_{t2} + \overline{\psi_t^2}, \tag{7}$$

$$\chi_1 \equiv \psi_{\theta 1}, \quad \chi_2 \equiv \psi_{\theta 2}, \tag{8}$$

then we can regard $\psi_{t,r}$ and χ to be the gravitino and the dilatino in the spherically reduced theory.

If we coupled the massless matter multiplet, the action of the spherically reduced theory is given by

$$\begin{aligned} S &= -\frac{1}{4}\int d^4x\, d\theta \mathcal{E}(\overline{\mathcal{D}\mathcal{D}} - 8R)\Phi_i^\dagger \Phi_i \\ &\sim 4\pi \int dr\, dt \sqrt{g} e^{2\phi}\Bigg(-\partial_\mu A_i \partial^\mu A_i \\ &\quad - \frac{i}{2}[\chi_i \sigma^\mu \mathcal{D}_\mu \overline{\chi_i} + \overline{\chi_i}\overline{\sigma^\mu}\mathcal{D}_\mu \chi_i] + \cdots \Bigg). \end{aligned} \tag{9}$$

Here the ellipsis denotes the terms containing the Rarita-Schwinger fields and dilatino. Note that in the spherically reduced theory, the dilaton field ϕ couples with the matter fields. Therefore if we want to investigate 2D dilaton supergravity as the spherically reduced theory, we need to couple the dilaton field to the matter multiplet.

The present work is organized as follows. The next section is devoted to the construction of the Lagrangian for 2D dilatonic supergravity with dilaton coupled matter and dilaton supermultiplets. A dilaton dependent trace anomaly and induced effective action (as well as a large-N effective action for quantum dilatonic supergravity) for the matter multiplet are found in Sec. III. Black hole solutions and their properties are discussed for some specific models in Sec. IV. Some conclusions are presented in the final section.

II. THE ACTION OF 2D DILATONIC SUPERGRAVITY WITH MATTER

In the present section we are going to construct the action of 2D dilatonic supergravity with a dilaton supermultiplet and a matter supermultiplet. The final result is given in superfields as well as in components.

In order to construct the Lagrangian of two-dimensional dilatonic supergravity, we use the component formulation of Ref. [13]. The convention and notations are given as follows:

Signature:

$$\eta^{ab} = \delta^{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{10}$$

γ matrices:

$$\gamma^a \gamma^b = \delta^{ab} + i\epsilon^{ab}\gamma_5,$$

$$\sigma_{ab} \equiv \frac{1}{4}[\gamma_a, \gamma_b] = \frac{i}{2}\epsilon_{ab}\gamma_5. \tag{11}$$

Charge conjugation matrix C:

$$C\gamma_a C^{-1} = -\gamma_a^T,$$

$$C = C^{-1} = -C^T,$$

$$\bar{\psi} = -\psi^T C. \tag{12}$$

Here the index T means transverse.

Majorana spinor:

$$\psi = \psi^c \equiv C\overline{\psi}^T. \tag{13}$$

Levi-Civita tensor:

$$\epsilon^{12} = \epsilon_{12} = 1, \quad \epsilon^{ab} = -\epsilon^{ba}, \quad \epsilon_{ab} = -\epsilon_{ba},$$

$$\epsilon^{\mu\nu} = e e_a^\mu e_b^\nu \epsilon_{ab} \quad \epsilon_{\mu\nu} = e^{-1} e_\mu^a e_\nu^b \epsilon^{ab}. \tag{14}$$

In this paper, all the scalar fields are real and all the spinor fields are Majorana spinors.

We introduce dilaton multiplet $\Phi = (\phi, \chi, F)$ and matter multiplet $\Sigma_i = (a_i, \chi_i, G_i)$, which has the conformal weight $\lambda = 0$, and the curvature multiplet[1] W

$$W = \left(S, \eta, -S^2 - \frac{1}{2}R - \frac{1}{2}\overline{\psi^\mu}\gamma^\nu \psi_{\mu\nu} + \frac{1}{4}\overline{\psi^\mu}\psi_\mu \right). \tag{15}$$

Here R is the scalar curvature and

$$\eta = -\frac{1}{2}S\gamma^\mu \psi_\mu + \frac{i}{2}e^{-1}\epsilon^{\mu\nu}\gamma_5 \psi_{\mu\nu}, \tag{16}$$

$$\psi_{\mu\nu} = D_\mu \psi_\nu - D_\nu \psi_\mu,$$

$$D_\mu \psi_\nu = \left(\partial_\mu - \frac{1}{2}\omega_\mu \gamma_5 \right)\psi_\nu,$$

$$\omega_\mu = -ie^{-1}e_{a\mu}\epsilon^{\lambda\nu}\partial_\lambda e_\nu^a - \frac{1}{2}\overline{\psi}_\mu \gamma_5 \gamma^\lambda \psi_\lambda. \tag{17}$$

The curvature multiplet has the conformal weight $\lambda = 1$.

Then the general action of 2D dilatonic supergravity is given in terms of general functions of the dilatons $C(\phi)$, $Z(\phi)$, $f(\phi)$, and[2] $V(\phi)$ as follows:

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[1]The definition of the scalar curvature R is different than that of Ref. [13] R^{HUY} by sign:

$$R = -R^{\mathrm{HUY}}.$$

[2]The multiplet containing $C(\phi)$, for example, is given by

$$\left[C(\phi), C'(\phi)\chi, C'(\phi)F - \frac{1}{2}C''(\phi)\overline{\chi}\chi \right].$$

$$\mathcal{L}=-[C(\Phi)\otimes W]_{\rm inv}+\frac{1}{2}[\Phi\otimes\Phi\otimes T_P(Z(\Phi))]_{\rm inv}$$
$$-[Z(\Phi)\otimes\Phi\otimes T_P(\Phi)]_{\rm inv}$$
$$+\sum_{i=1}^{N}\left\{\frac{1}{2}[\Sigma_i\otimes\Sigma_i\otimes T_P(f(\Phi))]_{\rm inv}-[f(\Phi)\otimes\Sigma_i\right.$$
$$\left.\otimes T_P(\Sigma_i)]_{\rm inv}\right\}+[V(\Phi)]_{\rm inv},$$

$$[C(\Phi)\otimes W]_{\rm inv}$$
$$=e\bigg[C(\phi)\left(-S^2-\frac{1}{2}R-\frac{1}{2}\bar{\psi}^{\mu}\gamma^{\nu}\psi_{\mu\nu}\right)$$
$$+C'(\phi)(FS-\bar{\chi}\eta)-\frac{1}{2}C''(\phi)\bar{\chi}\chi S$$
$$+\frac{1}{2}\bar{\psi}_{\mu}\gamma^{\mu}[\eta C(\phi)+\chi SC'(\phi)]+\frac{1}{2}C(\phi)S\bar{\psi}_{\mu}\sigma^{\mu\nu}\psi_{\nu}\bigg],$$

$$[\Phi\otimes\Phi\otimes T_P(Z(\Phi))]_{\rm inv}$$
$$=e\bigg[\left(Z'(\phi)F-\frac{1}{2}Z''(\phi)\bar{\chi}\chi\right)(2\phi F-\bar{\chi}\chi)$$
$$+\phi^2\Box[Z(\phi)]-2\phi\bar{\chi}\not{D}[Z'(\phi)\chi]$$
$$+\frac{1}{2}\bar{\psi}_{\mu}\gamma^{\mu}\bigg\{2\chi\left(Z'(\phi)F-\frac{1}{2}Z''(\phi)\bar{\chi}\chi\right)\phi$$
$$+\not{D}[Z'(\phi)\chi]\phi^2\bigg\}$$
$$+\frac{1}{2}\left(Z'(\phi)F-\frac{1}{2}Z''(\phi)\bar{\chi}\chi\right)\phi^2\bar{\psi}_{\mu}\sigma^{\mu\nu}\psi_{\nu}\bigg],$$

$$[Z(\Phi)\otimes\Phi\otimes T_P(\Phi)]_{\rm inv}$$
$$=e\bigg[Z'(\phi)(F^2\phi-\bar{\chi}\chi F-\phi\bar{\chi}\not{D}\chi)-\frac{1}{2}Z''(\phi)\bar{\chi}\chi\phi F$$
$$+Z(\phi)(\phi\Box\phi-\bar{\chi}\not{D}\chi+F^2)$$
$$+\frac{1}{2}\bar{\psi}_{\mu}\gamma^{\mu}\{\not{D}\chi Z(\phi)\phi+\chi[Z(\phi)+Z'(\phi)\phi]F\}$$
$$+\frac{1}{2}Z(\phi)\phi F\bar{\psi}_{\mu}\sigma^{\mu\nu}\psi_{\nu}\bigg],$$

$$[f(\Phi)\otimes\Sigma_i\otimes T_P(\Sigma_i)]_{\rm inv}$$
$$=e\bigg[f'(\phi)(Fa_iG_i-\bar{\chi}\xi_iG_i-a_i\bar{\chi}\not{D}\xi_i)-\frac{1}{2}f''(\phi)\bar{\chi}\chi a_iG_i$$
$$+f(\phi)(a_i\Box a_i-\bar{\xi}_i\not{D}\xi_i+G_i^2)$$
$$+\frac{1}{2}\bar{\psi}_{\mu}\gamma^{\mu}\{\not{D}\xi_if(\phi)a_i+[\xi_if(\phi)+\chi f'(\phi)a_i]G_i\}$$
$$+\frac{1}{2}Z(\phi)a_iG_i\bar{\psi}_{\mu}\sigma^{\mu\nu}\psi_{\nu}\bigg],$$

$$[\Sigma_i\otimes\Sigma_i\otimes T_P(f(\Phi))]_{\rm inv}$$
$$=e\bigg[\left(f'(\phi)F-\frac{1}{2}f''(\phi)\bar{\chi}\chi\right)(2a_iG_i-\bar{\xi}_i\xi_i)$$
$$+a_i^2\Box[f(\phi)]-2a_i\bar{\xi}_i\not{D}[f'(\phi)\chi]$$
$$+\frac{1}{2}\bar{\psi}_{\mu}\gamma^{\mu}\bigg\{2\xi_i\left(f'(\phi)F-\frac{1}{2}f''(\phi)\bar{\chi}\chi\right)a_i$$
$$+\not{D}(f'(\phi)\chi)a_i^2\bigg\}$$
$$+\frac{1}{2}\left(f'(\phi)F-\frac{1}{2}f''(\phi)\bar{\chi}\chi\right)a_i^2\bar{\psi}_{\mu}\sigma^{\mu\nu}\psi_{\nu}\bigg],$$

$$[V(\Phi)]_{\rm inv}=e\bigg[V'(\phi)F-\frac{1}{2}V''(\phi)\bar{\chi}\chi+\frac{1}{2}\bar{\psi}_{\mu}\gamma^{\mu}\chi V'(\phi)$$
$$+\left(\frac{1}{2}\bar{\psi}_{\mu}\sigma^{\mu\nu}\psi_{\nu}+S\right)V(\phi)\bigg]. \tag{18}$$

The covariant derivatives for the multiplet $Z=(\varphi,\zeta,H)$ with $\lambda=0$ are defined as

$$D_{\mu}\varphi=\partial_{\mu}\varphi-\frac{1}{2}\bar{\psi}_{\mu}\zeta,$$
$$D_{\mu}\zeta=\left(\partial_{\mu}+\frac{1}{2}\omega_{\mu}\gamma_5\right)\zeta-\frac{1}{2}D_{\nu}\varphi\gamma^{\nu}\psi_{\mu}-\frac{1}{2}H\psi_{\mu}, \tag{19}$$

$$\Box\varphi=e^{-1}\bigg\{\partial_{\nu}(eg^{\mu\nu}D_{\mu}\varphi)+\frac{i}{4}\bar{\zeta}\gamma_5\psi_{\mu\nu}\epsilon^{\mu\nu}-\frac{1}{2}\bar{\psi}^{\mu}D_{\mu}\zeta$$
$$-\frac{1}{2}\bar{\psi}^{\mu}\gamma^{\nu}\psi_{\nu}D_{\mu}\varphi\bigg\}.$$

$T_P(Z)$ is called the kinetic multiplet for the multiplet $Z=(\varphi,\zeta,H)$ and when the multiplet Z has the comformal weight $\lambda=0$, $T_P(Z)$ has the following form:

$$T_P(Z)=(H,\not{D}\zeta,\Box\varphi). \tag{20}$$

The kinetic multiplet $T_P(Z)$ has conformal weight $\lambda=1$. The product of two multiplets $Z_k=(\varphi_k,\zeta_k,H_k)$ $(k=1,2)$ with the conformal weight λ_k is defined by

$$Z_1\otimes Z_2=(\varphi_1\varphi_2,\varphi_1\zeta_2+\varphi_2\zeta_1,\varphi_1H_2+\varphi_2H_1-\bar{\zeta}_1\zeta_2). \tag{21}$$

The invariant Lagrangian $[Z]_{\rm inv}$ for multiplet Z is defined by

$$[Z]_{\rm inv}=e\bigg[H+\frac{1}{2}\bar{\psi}_{\mu}\gamma^{\mu}\zeta+\frac{1}{2}\varphi\bar{\psi}_{\mu}\sigma^{\mu\nu}\psi_{\nu}+S\varphi\bigg]. \tag{22}$$

In superconformal gauge

$$e^a_{\mu}=e^{\rho}\delta^a_{\mu}(e=e^{\rho}),\qquad \psi_{\mu}=\gamma_{\mu}\psi(\bar{\psi}_{\mu}=-\bar{\psi}\gamma_{\mu}), \tag{23}$$

we find

$$\omega_{\mu}=-i\epsilon^{\lambda}{}_{\mu}\partial_{\lambda}\rho,$$

$$eR=-2\partial_\mu\partial^\mu\rho,$$

$$\epsilon^{\mu\nu}\psi_{\mu\nu}=-2ie\gamma_5\gamma^\mu\left(\partial_\mu-\frac{1}{2}\partial_\mu\rho\right)\psi,$$

$$\eta=-S\psi+\gamma^\mu\left(\partial_\mu-\frac{1}{2}\partial_\mu\rho\right)\psi,$$

$$\overline{\psi^\mu}\gamma^\nu\psi_{\mu\nu}=-2\overline{\psi}\left(\partial_\mu-\frac{1}{2}\partial_\mu\rho\right)\psi,$$

$$\overline{\psi}_\mu\sigma^{\mu\nu}\psi_\nu=-\overline{\psi}\psi. \tag{24}$$

Hence, we constructed the classical action for 2D dilatonic supergravity with dilaton and matter supermultiplets.

III. EFFECTIVE ACTION IN THE LARGE-*N* APPROACH ON A BOSONIC BACKGROUND

Our purpose in this section will be the study of the trace anomaly and effective action in the large-N approximation for the 2D dilatonic supergravity discussed in the previous section. We consider only the bosonic background below as it will be sufficient for our purposes (the study of black hole type solutions).

On the bosonic background where dilatino χ and the Rarita-Schwinger fields vanish, one can show that the gravity and dilaton part of the Lagrangian have the following form:

$$[C(\Phi)\otimes W]_{\rm inv} =e\left[-C(\phi)\left(S^2+\frac{1}{2}R\right)-C'(\phi)FS\right],$$

$$[\Phi\otimes\Phi\otimes T_P(\mathbf{Z(\Phi)})]_{\rm inv} =e\{\phi^2\tilde{\Box}[Z(\phi)]+2Z'(\phi)\phi F^2\},$$

$$[Z(\Phi)\otimes\Phi\otimes T_P(\Phi)]_{\rm inv} =e[Z(\phi)\phi\tilde{\Box}\phi+Z'(\phi)\phi F^2+Z(\phi)F^2],$$

$$[V(\Phi)]_{\rm inv}=e[V'(\phi)F+SV(\phi)]. \tag{25}$$

For matter part we obtain

$$[f(\Phi)\otimes\Sigma_i\otimes T_P(\Sigma_i)]_{\rm inv}=e[f(\phi)(a_i\tilde{\Box}a_i-\overline{\xi_i}\tilde{\not{D}}\xi_i) +f'(\phi)Fa_iG_i+f(\phi)G_i^2],$$

$$[\Sigma_i\otimes\Sigma_i\otimes T_P(f(\Phi))]_{\rm inv}=e\{a_i^2\tilde{\Box}[f(\phi)]+2f'(\phi)Fa_iG_i\}. \tag{26}$$

Here the covariant derivatives for the multiplet (φ,ζ,H) with $\lambda=0$ are reduced to

$$\tilde{D}_\mu\varphi=\partial_\mu\varphi,$$

$$\tilde{D}_\mu\zeta=\left(\partial_\mu+\frac{1}{2}\omega_\mu\gamma_5\right)\zeta,$$

$$\tilde{\Box}\varphi=e^{-1}\partial_\nu(eg^{\mu\nu}\partial_\mu\varphi). \tag{27}$$

Using equations of motion with respect to the auxilliary fields S, F, G_i, on the bosonic background one can show that

$$S=\frac{C'(\phi)V'(\phi)-2V(\phi)Z(\phi)}{C'^2(\phi)+4C(\phi)Z(\phi)},$$

$$F=\frac{C'(\phi)V(\phi)+2C(\phi)V'(\phi)}{C'^2(\phi)+4C(\phi)Z(\phi)},$$

$$G_i=0. \tag{28}$$

We will be interested in the supersymmetric extension [14] of the Callan-Giddings-Harvey-Strominger (CGHS) model [1] as a specific example for the study of black holes and Hawking radiation. For such a model

$$C(\phi)=2e^{-2\phi},\quad Z(\phi)=4e^{-2\phi},\quad V(\phi)=4e^{-2\phi}, \tag{29}$$

we find

$$S=0,\quad F=-\lambda,\quad G_i=0. \tag{30}$$

Using Eq. (26), we can show that

$$\sum_{i=1}^N\left\{\frac{1}{2}[\Sigma_i\otimes\Sigma_i\otimes T_P(f(\Phi))]_{\rm inv}-[f(\Phi)\otimes\Sigma_i\otimes T_P(\Sigma_i)]_{\rm inv}\right\} =ef(\phi)\sum_{i=1}^N[g^{\mu\nu}\partial_\mu a_i\partial_\nu a_i+\overline{\xi_i}\gamma^\mu\partial_\mu\xi_i-f(\phi)G_i^2] +\text{total divergence terms}. \tag{31}$$

Here we have used the fact that

$$\overline{\xi_i}\gamma_5\xi_i=0 \tag{32}$$

for the Majorana spinor ξ_i.

Let us start now the investigation of effective action in above theory. It is clearly seen that theory (31) is conformally invariant on the the gravitational background under discussion. Then using standard methods, we can prove that theory with matter multiplet Σ_i is superconformally invariant theory. First of all, one can find trace anomaly T for the theory (31) on the gravitational background using the following relation:

$$\Gamma_{\rm div}=\frac{1}{n-2}\int d^2x\sqrt{g}b_2,\quad T=b_2, \tag{33}$$

where b_2 is the b_2 coefficient of the Schwinger–de Witt expansion and $\Gamma_{\rm div}$ is the one-loop effective action. The calculation of $\Gamma_{\rm div}$ (33) for quantum theory with Lagrangian (31) was done some time ago in Ref. [8]. Using results of this work, we find

$$T=\frac{1}{24\pi}\left\{\frac{3}{2}NR-3N\left(\frac{f''}{f}-\frac{f'^2}{2f^2}\right)(\nabla^\lambda\phi)(\nabla_\lambda\phi) -3N\frac{f'}{f}\Delta\phi\right\}. \tag{34}$$

It is remarkable that Majorana spinors do not give the contribution to the dilaton dependent terms in the trace anomaly, as was shown in [8]. They only alter the coefficient of the curvature term in T (34). Hence, except the coefficient of the curvature term in T (34), the trace anomaly (34) coincides with the correspondent expression for the dilaton coupled scalar [9]. Note also that for particular case $f(\phi)=\mathrm{e}^{-2\phi}$ the trace anomaly for the dilaton coupled scalar was recently calculated in Ref. [10].

Making now the conformal transformation of the metric $g_{\mu\nu}\to \mathrm{e}^{2\sigma}g_{\mu\nu}$ in the trace anomaly, and using relation

$$T=\frac{1}{\sqrt{g}}\frac{\delta}{\delta\sigma}W[\sigma] \tag{35}$$

one can find the anomaly induced action $W[\sigma]$. In the covariant, nonlocal form it may be found as the following:

$$W=-\frac{1}{2}\int d^2x\sqrt{g}\left[\frac{N}{32\pi}R\frac{1}{\Delta}R-\frac{N}{16\pi}\frac{f'^2}{f^2}\nabla^\lambda\phi\nabla_\lambda\phi\frac{1}{\Delta}R\right.$$
$$\left.-\frac{N}{8\pi}\ln fR\right]. \tag{36}$$

Hence, we got the anomaly induced effective action for the dilaton coupled matter multiplet in the external dilaton-gravitational background. We should note that the same action W (36) gives the one-loop large-N effective action in the quantum theory of supergravity with matter (18) (i.e., when all fields are quantized).

We can now rewrite W in a supersymmetric way. In order to write down the effective action expressing the trace anomaly, we need the supersymmetric extention of $(1/\Delta)R$. The extension is given by using the inverse kinetic multiplet in [15], or equivalently by introducing two auxiliary fields $\Theta=(t,\theta,T)$ and $\Upsilon=(u,v,U)$. We can now construct the following action:

$$[\Theta\otimes[T_P(\Upsilon)-W]]_{\mathrm{inv}}. \tag{37}$$

The Θ equation of motion tells that, in the superconformal gauge (23),

$$u\sim\rho\sim-\frac{1}{2\Delta}R,\quad v\sim\psi. \tag{38}$$

Then we find

$$\sqrt{g}R\frac{1}{\Delta}R\sim 4[W\otimes\Upsilon]_{\mathrm{inv}},$$

$$\sqrt{g}\frac{f'^2(\phi)}{f^2(\phi)}\nabla_\lambda\phi\nabla^\lambda\phi\frac{1}{\Delta}R,$$
$$\sim-\left[\Phi\otimes\Phi\otimes T_P\left(\frac{f'^2(\Phi)}{f^2(\Phi)}\otimes\Upsilon\right)\right]_{\mathrm{inv}}$$
$$+2\left[\frac{f'^2(\Phi)}{f^2(\Phi)}\otimes\Upsilon\otimes\Phi\otimes T_P(\Phi)\right]_{\mathrm{inv}},$$

$$\sqrt{g}\ln f(\phi)R\sim 2[\ln f(\Phi)\otimes W]_{\mathrm{inv}}. \tag{39}$$

In components,

$$[\Theta\otimes[T_P(\Upsilon)-W]]_{\mathrm{inv}}=e\left[t\left(\Box u+\frac{1}{2}R+\frac{1}{2}\overline{\psi}^\mu\gamma^\nu\psi_{\mu\nu}-S\overline{\psi}^\mu\psi_\mu\right)+T(U-S)-\overline{\theta}(\not{D}v-\eta)+\frac{1}{2}\overline{\psi}_\mu\gamma^\mu\{(\not{D}v-\eta)t+\theta(U-S)\}\right.$$
$$\left.+\frac{1}{2}t(U-S)\overline{\psi}_\mu\sigma^{\mu\nu}\psi_\nu\right], \tag{40}$$

$$4[W\otimes\Upsilon]_{\mathrm{inv}}=4e\left[u\left(-\frac{1}{2}R-\frac{1}{2}\overline{\psi}^\mu\gamma^\nu\psi_{\mu\nu}+S\overline{\psi}^\mu\psi_\mu\right)+US-\overline{v}\eta+\frac{1}{2}\overline{\psi}_\mu\gamma^\mu\{\eta u+vS\}+\frac{1}{2}uS\overline{\psi}_\mu\sigma^{\mu\nu}\psi_\nu\right], \tag{41}$$

$$\left[\Phi\otimes\Phi\otimes T_P\left(\frac{f'^2(\Phi)}{f^2(\Phi)}\otimes\Upsilon\right)\right]_{\mathrm{inv}}$$
$$=e\left[\phi^2\Box[uh'^2(\phi)]+(2\phi F-\overline{\chi}\chi)\{h'^2(\phi)U+u(2h'(\phi)h''(\phi)F-[h''^2(\phi)+h'(\phi)h'''(\phi)]\overline{\chi}\chi)\}\right.$$
$$-2\phi\overline{\chi}\not{D}[h'^2(\phi)v+2uh'(\phi)h''(\phi)\chi]+\frac{1}{2}\overline{\psi}_\mu\gamma^\mu(\{\not{D}[h'^2(\phi)v+2uh'(\phi)h''(\phi)\chi]\}\phi^2$$
$$+2\chi\phi\{h'^2(\phi)U+u\{2h'(\phi)h''(\phi)F-[h''^2(\phi)+h'(\phi)h'''(\phi)]\overline{\chi}\chi\}\})$$
$$\left.+\frac{1}{2}\phi^2\{h'^2(\phi)U+u\{2h'(\phi)h''(\phi)F-[h''^2(\phi)+h'(\phi)h'''(\phi)]\overline{\chi}\chi\}\}\overline{\psi}_\mu\sigma^{\mu\nu}\psi_\nu\right], \tag{42}$$

$$\left[\frac{f'^2(\Phi)}{f^2(\Phi)}\otimes\Upsilon\otimes\Phi\otimes T_P(\Phi)\right]_{\rm inv}$$
$$=e\Big[uh'^2(\phi)\phi\Box\phi+F\{\phi[h'^2(\phi)U+u\{2h'(\phi)h''(\phi)F-(h''^2(\phi)+h'(\phi)h'''(\phi))\bar{\chi}\chi\}$$
$$-2h'(\phi)h''(\phi)\bar{\chi}v]+uh'^2(\phi)F-\bar{\chi}[vh'^2(\phi)+2\chi uh'(\phi)h''(\phi)]\}$$
$$-[uh'^2(\phi)\bar{\chi}+\phi h'^2(\phi)\bar{v}+2u\phi h'(\phi)h''(\phi)\bar{\chi}]\not{D}\chi$$
$$+\frac{1}{2}\bar{\psi}_\mu\gamma^\mu\{\chi uh'^2(\phi)\phi+[\chi uh'^2(\phi)+v\phi h'^2(\phi)+2\chi u\phi h'(\phi)h''(\phi)]F\}$$
$$+\frac{1}{2}uh'^2(\phi)\phi F\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu\Big], \tag{43}$$

$$[\ln f(\Phi)\otimes W]_{\rm inv}=e\Big[h(\phi)\left(-S^2+\frac{1}{2}R-\frac{1}{2}\bar{\psi}^\mu\gamma^\nu\psi_{\mu\nu}\right)+h'(\phi)(FS-\bar{\chi}\eta)-\frac{1}{2}h''(\phi)\bar{\chi}\chi S+\frac{1}{2}\bar{\psi}_\mu\gamma^\mu[\eta h(\phi)$$
$$+\chi Sh'(\phi)]+\frac{1}{2}h(\phi)S\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu\Big]. \tag{44}$$

Here

$$h(\phi)\equiv\ln f(\phi). \tag{45}$$

That finishes the construction of the large-N supersymmetric anomaly induced effective action for 2D dilatonic supergravity with matter.

At the end of this section, we will find the one-loop effective action (its divergent part) for the whole quantum theory (25),(26) on the bosonic background under discussion. Using Eqs. (25),(26), one can write the complete Lagrangian as follows:

$$e^{-1}L=-\left(\tilde{V}+\tilde{C}R+\frac{1}{2}\tilde{Z}(\nabla_\mu\phi)(\nabla^\mu\phi)-f(\phi)\times\sum_{i=1}^N[(\nabla_\mu a_i)(\nabla^\mu a_i)+\bar{\xi}_i\gamma^\mu\partial_\mu\xi_i]\right), \tag{46}$$

where

$$-\tilde{V}=-CS^2-C'FS+2Z'\phi F^2+Z'\phi F^2+ZF^2+V'F+SV,$$

$$\tilde{C}=\frac{C}{2},$$

$$2\tilde{Z}=3\phi Z'+Z, \tag{47}$$

where auxilliary fields equations of motion which lead to Eq. (28) should be used.

The calculation of the one-loop effective action for the theory (46) is given in Ref. [8] in the harmonic gauge with the following result:

$$\Gamma_{\rm div}=-\frac{1}{4\pi(n-2)}\int d^2x\sqrt{g}\left\{\frac{48-3N}{12}R+\frac{2}{\tilde{C}}\tilde{V}+\frac{2}{\tilde{C}'}\tilde{V}'\right.$$
$$+\left(\frac{\tilde{C}''}{\tilde{C}}-\frac{3\tilde{C}'^2}{\tilde{C}^2}-\frac{\tilde{C}''\tilde{Z}}{\tilde{C}'^2}-\frac{Nf'^2}{4f^2}+\frac{Nf''}{2f}\right)(\nabla^\lambda\phi)(\nabla_\lambda\phi)$$
$$+\left(\frac{\tilde{C}'}{\tilde{C}}-\frac{\tilde{Z}}{\tilde{C}'}+\frac{Nf'}{2f}\right)\Delta\phi$$
$$\left.-\left(\frac{3f\tilde{Z}}{4\tilde{C}'^2}+\frac{3f}{4\tilde{C}}-\frac{f'}{\tilde{C}'}\right)\sum_{i=1}^N\bar{\xi}_i\gamma^\mu\partial_\mu\xi_i\right\}. \tag{48}$$

Thus, we found the one-loop effective action for dilatonic supergravity with matter on a bosonic background. Of course, the contribution of fermionic superpartners is missing there. However, Eq. (48) also gives the divergent one-loop effective action in the large-N approximation (one should keep only terms with multiplier N). This effective action may also be used for the construction of renormalization group improved effective Lagrangians and the study of their properties, such as BH solutions in Ref. [16].

IV. BLACK HOLES IN SUPERSYMMETRIC EXTENSION OF CGHS MODEL WITH MATTER BACK REACTION

In the present section we discuss the particular 2D dilatonic supergravity model which represents the supersymmetric extension of the CGHS model. Note that as matter, we use a dilaton coupled matter supermultiplet. We would like to estimate the back reaction of such a matter supermultiplet to black holes and Hawking radiation working in the large-N approximation. Since we are interested in the vacuum (black hole) solution, we consider a background where matter fields, the Rarita-Schwinger field, and the dilatino vanish.

In the superconformal gauge the equations of motion can be obtained by the variation over $g^{\pm\pm}$, $g^{\pm\mp}$, and ϕ:

$$\begin{aligned}0&=T_{\pm\pm}\\&=e^{-2\phi}[4\partial_\pm\rho\partial_\pm\phi-2(\partial_\pm\phi)^2]+\frac{N}{8}(\partial_\pm^2\rho-\partial_\pm\rho\partial_\pm\rho)+\frac{N}{8}\left\{[\partial_\pm h(\phi)\partial_\pm h(\phi)]\rho+\frac{1}{2}\frac{\partial_\pm}{\partial_\mp}[\partial_\pm h(\phi)\partial_\mp h(\phi)]\right\}\\&\quad+\frac{N}{8}\{-2\partial_\pm\rho\partial_\pm h(\phi)+\partial_\pm^2 h(\phi)\}+t^\pm(x^\pm)+\frac{N}{64}\frac{\partial_\pm}{\partial_\mp}[h'(\phi)^2F^2],\end{aligned}\tag{49}$$

$$\begin{aligned}0&=T_{\pm\mp}\\&=e^{-2\phi}\left(2\partial_+\partial_-\phi-4\partial_+\phi\partial_-\phi-\lambda^2e^{2\rho}\right)-\frac{N}{8}\partial_+\partial_-\rho-\frac{N}{16}\partial_+h(\phi)\partial_-h(\phi)-\frac{N}{8}\partial_+\partial_-h(\phi)-\frac{N}{64}h'(\phi)^2F^2\\&\quad+\left(\frac{N}{16}US+\frac{N}{2}[-h(\phi)S^2+h'(\phi)FS]\right)e^{2\rho},\end{aligned}\tag{50}$$

$$\begin{aligned}0&=e^{-2\phi}(-4\partial_+\partial_-\phi+4\partial_+\phi\partial_-\phi+2\partial_+\partial_-\rho+\lambda^2e^{2\rho})-\frac{Nf'}{f}\left\{\frac{1}{16}\partial_+[\rho\partial_-h(\phi)]+\frac{1}{16}\partial_-[\rho\partial_+h(\phi)]\right.\\&\quad\left.-\frac{1}{8}\partial_+\partial_-\rho\right\}.\end{aligned}\tag{51}$$

Here $t^\pm(x^\pm)$ is a function which is determined by the boundary condition. Note that there is, in general, a contribution from the auxilliary fields to $T_{\pm\mp}$ besides the contribution from the trace anomaly.

In the large-N limit, where classical part can be ignored, field equations become simpler:

$$\begin{aligned}0&=\frac{1}{N}T_{\pm\pm}\\&=\frac{1}{8}(\partial_\pm^2\rho-\partial_\pm\rho\partial_\pm\rho)+\frac{1}{8}\left\{[\partial_\pm h(\phi)\partial_\pm h(\phi)]\rho\right.\\&\quad\left.+\frac{1}{2}\frac{\partial_\pm}{\partial_\mp}[\partial_\pm h(\phi)\partial_\mp h(\phi)]\right\}\\&\quad+\frac{1}{8}\{-2\partial_\pm\rho\partial_\pm h(\phi)+\partial_\pm^2h(\phi)\}+t^\pm(x^\pm),\end{aligned}\tag{52}$$

$$\begin{aligned}0&=\frac{1}{N}T_{\pm\mp}\\&=-\frac{1}{8}\partial_+\partial_-\rho-\frac{1}{16}\partial_+h(\phi)\partial_-h(\phi)-\frac{1}{8}\partial_+\partial_-h(\phi),\end{aligned}\tag{53}$$

$$0=\frac{1}{16}\partial_+[\rho\partial_-h(\phi)]+\frac{1}{16}\partial_-[\rho\partial_+h(\phi)]-\frac{1}{8}\partial_+\partial_-\rho.\tag{54}$$

Here we used the Θ equation and the equations for the auxilliary fields S and F; i.e.,

$$U=S,\quad u=\rho=-\frac{1}{2\Delta}R,\quad S=F=0.\tag{55}$$

The function $t^\pm(x^\pm)$ in Eq. (52) can be absorbed into the choice of the coordinate and we can choose

$$t^\pm(x^\pm)=0.\tag{56}$$

Combining Eq. (52) and Eq. (53), we obtain

$$-\frac{1}{2}(\partial_\pm\rho)^2+\frac{1}{2}\rho[\partial_\pm h(\phi)]^2-\partial_\pm\rho\partial_\pm h(\phi)=0,\tag{57}$$

i.e.,

$$\partial_\pm h(\phi)=\frac{1+\sqrt{1+\rho}}{\rho}\partial_\pm\rho$$

or

$$\frac{1-\sqrt{1+\rho}}{\rho}\partial_\pm\rho.\tag{58}$$

This tells us that

$$h(\phi)=\int d\rho\frac{1\pm\sqrt{1+\rho}}{\rho}.\tag{59}$$

Substituting Eq. (59) into Eq. (54), we obtain

$$\partial_+\partial_-\{(1+\rho)^{3/2}\}=0,\tag{60}$$

i.e.,

$$\rho=-1+[\rho^+(x^+)+\rho^-(x^-)]^{2/3}.\tag{61}$$

Here $\rho^\pm$ is an arbitrary function of $x^\pm=t\pm x$. We can straightforwardly confirm that the solutions (59) and (61) satisfy Eq. (53). The scalar curvature is given by

$$R=8\mathrm{e}^{-2\rho}\partial_+\partial_-\rho$$
$$=-4\frac{\mathrm{e}^{-2\{-1+[\rho^+(x^+)+\rho^-(x^-)]^{2/3}\}}}{[\rho^+(x^+)+\rho^-(x^-)]^{4/3}}\rho^{+\prime}(x^+)\rho^{-\prime}(x^-). \tag{62}$$

Note that when $\rho^+(x^+)+\rho^-(x^-)=0$, there is a curvature singularity. Especially if we choose

$$\rho^+(x^+)=\frac{r_0}{x^+},\quad \rho^-(x^-)=-\frac{x^-}{r_0} \tag{63}$$

there are curvature singularities at $x^+x^-=r_0^2$ and horizon at $x^+=0$ or $x^-=0$. Hence we got the black hole solution in the model under discussion. The asymptotic flat regions are given by $x^+\to+\infty$ $(x^-<0)$ or $x^-\to-\infty$ $(x^+>0)$. Therefore we can regard $x^\pm$ as corresponding to the Kruskal coordinates in 4 dimensions.

We now consider the Hawking radiation. The quantum part of the energy momentum tensor for the generalized dilatonic supergravity is given by

$$T^q_{\pm\pm}=\frac{N}{8}(\partial_\pm^2\rho-\partial_\pm\rho\partial_\pm\rho)+\frac{N}{8}\left\{[\partial_\pm h(\phi)\partial_\pm h(\phi)]\rho+\frac{1}{2}\frac{\partial_\pm}{\partial_\mp}[\partial_\pm h(\phi)\partial_\mp h(\phi)]\right\}$$
$$+\frac{N}{8}\{-2\partial_\pm\rho\partial_\pm h(\phi)+\partial_\pm^2 h(\phi)\}$$
$$+\frac{N}{64}\frac{\partial_\pm}{\partial_\mp}[h'(\phi)^2F^2]+t(x^\pm), \tag{64}$$

$$T^q_{\pm\mp}=-\frac{N}{8}\partial_+\partial_-\rho-\frac{N}{16}\partial_+h(\phi)\partial_-h(\phi)$$
$$-\frac{N}{8}\partial_+\partial_-h(\phi)-\frac{N}{64}h'(\phi)^2F^2$$
$$+\left(\frac{N}{16}US+\frac{N}{2}[-h(\phi)S^2+h'(\phi)FS]\right)\mathrm{e}^{2\rho}. \tag{65}$$

Here we consider the bosonic background and put the fermionic fields to be zero. We now investigate the case that

$$f(\phi)=\mathrm{e}^{\alpha\phi}\quad [h(\phi)=\alpha\phi]. \tag{66}$$

Substituting the classical black hole solution which appeared in the original CGHS model [1],

$$\rho=-\frac{1}{2}\ln\left(1+\frac{M}{\lambda}e^{\lambda(\sigma^--\sigma^+)}\right), \tag{67}$$

$$\phi=-\frac{1}{2}\ln\left(\frac{M}{\lambda}+e^{\lambda(\sigma^+-\sigma^-)}\right) \tag{68}$$

(here M is the mass of the black hole and we used asymptotic flat coordinates) and using Eq. (30), we find

$$T^q_{+-}=\frac{N\lambda^2}{64}(4+4\alpha+\alpha^2)\frac{1}{[1+(M/\lambda)e^{\lambda(\sigma^--\sigma^+)}]^2}$$
$$-\frac{N\lambda^2}{16}(1+\alpha)\frac{1}{[1+(M/\lambda)e^{\lambda(\sigma^--\sigma^+)}]}-\frac{N\lambda^2\alpha^2}{64},$$

$$T^q_{\pm\pm}=-\frac{N\lambda^2}{32}\left\{1-\frac{1}{[1+(M/\lambda)e^{\lambda(\sigma^--\sigma^+)}]^2}\right\}$$
$$-\frac{N\lambda^2\alpha^2}{16}\frac{\ln[1+(M/\lambda)e^{\lambda(\sigma^--\sigma^+)}]-1}{[1+(M/\lambda)e^{\lambda(\sigma^--\sigma^+)}]^2}+t^\pm(\sigma^\pm). \tag{69}$$

Then when $\sigma^+\to+\infty$, the energy momentum tensor behaves as

$$T^q_{+-}\to 0,$$
$$T^q_{\pm\pm}\to\frac{N\lambda^2}{16}\alpha^2+t^\pm(\sigma^\pm). \tag{70}$$

In order to evaluate $t^\pm(\sigma^\pm)$, we impose the boundary condition that there is no incoming energy. This condition requires that T^q_{++} should vanish at the past null infinity ($\sigma^-\to-\infty$) and if we assume $t^-(\sigma^-)$ is black hole mass independent, T^q_{--} also should vanish at the past horizon ($\sigma^+\to-\infty$) after taking a $M\to 0$ limit. Then we find

$$t^-(\sigma^-)=-\frac{N\lambda^2\alpha^2}{16}, \tag{71}$$

and one obtains

$$T^q_{--}\to 0 \tag{72}$$

at the future null infinity ($\sigma^+\to+\infty$). Equations (70) and (72) might tell that there is no Hawking radiation in the dilatonic supergravity model under discussion when the quantum back reaction of the matter supermultiplet in the large-N approach is taken into account. (That indicates that the above black hole is an extremal one.) This might be the result of the positive energy theorem [17]. If Hawking radiation is positive and mass independent, the energy of the system becomes unbounded below. On the other hand, the negative radiation cannot be accepted physically. Of course, this result may be changed in the next order of the large-N approximation or in other models of dilatonic supergravity. From another side, since we work in the strong coupling regime it could be that new methods to study Hawking radiation should be developed.

V. DISCUSSION

In summary, we studied 2D dilatonic supergravity with dilaton coupled matter and dilaton supermultiplets. Some results of this work have been briefly reported in [18]. The trace anomaly and induced effective action for matter supermultiplet as well as the large-N effective action for dilatonic supergravity are calculated. Using these results one can estimate matter quantum corrections in the study of black holes and their properties, such as Hawking radiation. Such a study is presented on the example of the supersymmetric CGHS model which corresponds to a specific choice of generalized dilatonic couplings in the initial theory. Similarly, one can investigate quantum spherical collapse for different 4D or higher-dimensional supergravities using 2D models.

It is interesting to note that there are the following directions to generalize our work. First of all, one can consider extended 2D supergravities with dilaton coupled matter. The general structure of the trace anomaly and effective action will be the same. Second, one can consider other types of black hole solutions in the model under discussion with arbitrary dilaton couplings. Unfortunately, since such models are not exactly solvable, one can only apply numerical methods for the study of black holes and their properties. Third, it could be important to discuss the well-known C theorem for the dilaton dependent trace anomaly. We hope to investigate some of these questions in the near future.

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[1] C.G. Callan, S.B. Giddings, J.A. Harvey, and A. Strominger, Phys. Rev. D **45**, 1005 (1992).

[2] J.G. Russo, L. Susskind, and L. Thorlacius, Phys. Lett. B **292**, 13 (1992); Phys. Rev. D **47**, 533 (1993).

[3] S.P. de Alwis, Phys. Lett. B **289**, 278 (1992); A. Bilal and C. Callan, Nucl. Phys. **B394**, 73 (1993); S. Nojiri and I. Oda, Phys. Lett. B **294**, 317 (1992); Nucl. Phys. **B406**, 499 (1993); T. Banks, A. Dabholkar, M. Douglas, and M. O'Loughlin, Phys. Rev. D **45**, 3607 (1992); R.B. Mann, *ibid.* **47**, 4438 (1993); D. Louis-Martinez and G. Kunstatter, *ibid.* **49**, 5227 (1994): T. Klobsch and T. Strobl, Class. Quantum Grav. **13**, 965 (1996); G. Amelino-Camelia, L. Griguolo, and D. Seminara, Phys. Lett. B **371**, 41 (1996).

[4] S. Bose, L. Parker, and Y. Peleg, Phys. Rev. D **52**, 3512 (1995); M. Katanaev, W. Kummer, and H. Liebl, *ibid.* **53**, 5609 (1996).

[5] T. Banks, in *String Theory, Gauge Theory and Quantum Gravity*, Proceedings of the Spring School, Trieste, Italy, 1994, edited by R. Dijkgraaf *et al.* [Nucl. Phys. B (Proc. Suppl.) **41**, 21 (1995)], hep-th/9412131; A. Strominger, *Les Houches Lectures on Black Holes*, hep-th/9501071; S. Giddings, in *1994 Summer School in High Energy Physics and Cosmology*, Proceedings, Trieste, Italy, edited by E. Gava *et al.* (World Scientific, Singapore, 1995), hep-th/9412138.

[6] S.P. Trivedi, Phys. Rev. D **47**, 4233 (1993); A. Strominger and S.P. Trivedi, *ibid.* **48**, 5778 (1993).

[7] G.W. Gibbons, Nucl. Phys. **B207**, 337 (1982); G.W. Gibbons and K. Maeda, *ibid.* **B298**, 741 (1988); S.B. Giddings and A. Strominger, Phys. Rev. Lett. **67**, 1930 (1991): D. Garfinkle, G.T. Horowitz, and A. Strominger, Phys. Rev. D **43**, 3140 (1991).

[8] E. Elizalde, S. Naftulin, and S.D. Odintsov, Phys. Rev. D **49**, 2852 (1994).

[9] S. Nojiri and S.D. Odintsov, Mod. Phys. Lett. A **12**, 2083 (1997); Phys. Rev. D **57**, 2363 (1998).

[10] R. Bousso and S.W. Hawking, Phys. Rev. D **56**, 7788 (1997); S. Ichinose, hep-th/9707025; W. Kummer, H. Liebl, and D.V. Vassilevich, Mod. Phys. Lett. A **12**, 2683 (1997).

[11] J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton University Press, Princeton, NJ, 1991).

[12] S.J. Gates, M.T. Grisaru, M. Ricek, and W. Siegel, *Superspace or One Thousand and One Lessons in Supersymmetry*, Vol. 58 of *Frontiers in Physics* (Benjamin Cummings, New York, 1983).

[13] K. Higashijima, T. Uematsu, and Y.Z. Yu, Phys. Lett. B **139**, 161 (1994); T. Uematsu, Z. Phys. C **29**, 143 (1985): T. Uematsu, *ibid.* **32**, 33 (1986).

[14] Shin'ichi Nojiri and Ichiro Oda, Mod. Phys. Lett. A **8**, 53 (1993).

[15] Mihoko M. Nojiri and Shin'ichi Nojiri, Prog. Theor. Phys. **76**, 733 (1986).

[16] S. Nojiri and S.D. Odintsov, Mod. Phys. Lett. A **12**, 925 (1997).

[17] Y. Park and A. Strominger, Phys. Rev. D **47**, 1569 (1993).

[18] S. Nojiri and S.D. Odintsov, Phys. Lett. B **416**, 85 (1998).