

Octet baryon magnetic moments in the chiral quark model with configuration mixing

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The Coleman–Glashow sum-rule for magnetic moments is always fulfilled in the chiral quark model, independently of SU(3) symmetry breaking. This is due to the structure of the wave functions, coming from the non-relativistic quark model. Experimentally, the Coleman–Glashow sum-rule is violated by about ten standard deviations. To overcome this problem, two models of wave functions with configuration mixing are studied. One of these models violates the Coleman–Glashow sum-rule to the right degree and also reproduces the octet baryon magnetic moments rather accurately. [S0556-2821(98)00101-5]

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I. INTRODUCTION

The quark structure of baryons at low energies are probed by parameters such as magnetic moments, axial-vector form factors and decay rates of various kinds. Any refinement of the non-relativistic quark model (NQM) should improve on the experimental agreement of these parameters, if the refinement is significant. Much work has been done to effectuate such refinements and improve the agreement with the magnetic moments, the spin polarization of the nucleon, etc. Among these refinements, the chiral quark model (χ QM) suggested by Manohar and Georgi [1] has attracted some attention recently [2–8]. Other models are one with quark-gluon configuration mixing by Lipkin [9], and one with quark-diquark configuration mixing by Noda *et al.* [10].

One crucial test for quark model refinements is the Coleman–Glashow sum-rule [11]

$$\mu(p) - \mu(n) + \mu(\Sigma^-) - \mu(\Sigma^+) + \mu(\Xi^0) - \mu(\Xi^-) = 0$$

for the magnetic moments of the octet baryons, that can be derived under very general assumptions on the magnetic moment operator. Experimentally, this sum-rule is violated by ten standard deviations, the left hand side being equal to $(0.49 \pm 0.05)\mu_N$.

Franklin [12,13] and Karl [14] have shown that the Coleman–Glashow sum-rule is valid beyond the NQM. Franklin noted the validity of this sum-rule under the assumption of “baryon independence” of a given quark moment contribution. Karl considered the case of general quark spin polarizations and showed that the sum-rule is valid assuming SU(3) symmetry for the wave functions of the baryon octet states.

As we will show below, the Coleman–Glashow sum-rule turns out to hold also in the χ QM with arbitrary SU(3) symmetry breaking, as long as the wave functions for baryons with xy quarks ($x, y = u, d, s$, $x \neq y$) have the same (mirror)

symmetry. This indicates a certain over-simplification in the description of the baryons in this model and in several other models.

One possible way to remedy this is to allow the quark magnetic moments to vary between the isomultiplets. The alleged symmetry is then not relevant. This approach has the disadvantage of complicating the quark model, by making the quarks vary with environment. In fact, we know that the mass spectrum can be well accounted for using the same quark masses in all isomultiplets. It is therefore desirable to instead modify the wave functions, keeping the quark properties the same throughout.

A natural modification of the mirror symmetry occurs when the quarks are allowed to have an orbital angular momentum in the wave function. The reason is that the mass of the s quark breaks the symmetry. An example of such a model has been suggested by Casu and Sehgal [15]. Using their formulas, the Coleman–Glashow sum-rule is indeed violated and the left hand side is approximately given by $0.06\langle L_z \rangle \mu_N$, where $\langle L_z \rangle$ is the angular momentum. To reach the experimental value of $0.49\mu_N$, this requires $\langle L_z \rangle$ to be about 8, a value which is unfortunately quite unrealistic.

Another model, which also breaks the Coleman–Glashow magnetic moment sum-rule, is given by SU(3) breaking terms in a purely phenomenological SU(3) parametrization [16,17]. This model satisfies the experimental value for the left hand side. On the other hand, this model does not have any polarization of the vacuum, and therefore the violation of the Gottfried sum-rule, giving $\bar{u} - \bar{d} \approx -0.15$, cannot be explained.

Buck and Perez [18] have discussed a model in which they add a configuration term to the usual SU(6) spin function. This term involves a total angular momentum of the quarks with $L=1$. Their model violates the Coleman–Glashow sum-rule and gives $0.40\mu_N$ for the left hand side, but neither this model includes any vacuum polarization.

In this paper we will therefore concentrate our further discussion to the χ QM and study two models of configuration mixing in the wave functions of the octet baryons.

In the first model, this is done in the form of a gluon coupled to the three quarks in a way suggested by Lipkin [9]. The full wave function, being a superposition of the one with zero gluons and the one with one gluon, there is a natural

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room for varying the relative importance of these two components for the different isomultiplets. This creates a breaking of the mirror symmetry that generates the breaking of the Coleman–Glashow sum-rule.

In the other model, we use instead of a quark-gluon a quark-diquark configuration mixing, that is allowed to vary between the isomultiplets.

Both these models have been used originally without the Goldstone bosons that play an essential role in the χ QM. Their performance is then not satisfactory in other respects, like the \bar{u} - \bar{d} asymmetry. In our paper, we use the mechanisms of these two models to generate the configuration mixing needed to break the Coleman–Glashow sum-rule. This configuration mixing can be viewed as a correction to the SU(6) quark model baryonic wave functions. At the end of this article, we will give an example, in the form of a toy model, how such a configuration mixing could come about.

Our paper is organized as follows. In Sec. II we first review the χ QM, and then we show that the χ QM with arbitrary SU(3) symmetry breaking generates octet baryon magnetic moments that satisfy the Coleman–Glashow sum-rule. In Sec. III we then introduce two different models for configuration mixing in the octet baryon wave functions, one with quark-gluon mixing and one with quark-diquark mixing, and we show that the Coleman–Glashow sum-rule can

be violated in these models provided that the mixings are allowed to vary between the isomultiplets. At the end of this section, we discuss a toy model for configuration mixing. Finally, in Sec. IV, we present a summary of our analyses and also the main conclusions.

II. THE COLEMAN–GLASHOW SUM-RULE FOR MAGNETIC MOMENTS

A. The chiral quark model

The Goldstone bosons (GBs) of the χ QM are pseudoscalars and will be denoted by the 0^- meson names π, K, η, η' , as is usually done. For convenience, we will closely follow the notation of Ref. [3]. The Lagrangian of interaction, ignoring the space-time structure, is to lowest order

$$\mathcal{L}_I = g_8 \bar{\mathbf{q}} \Phi \mathbf{q}, \quad (1)$$

where g_8 is a coupling constant,

$$\mathbf{q} = \begin{pmatrix} u \\ d \\ s \end{pmatrix},$$

and

$$\Phi = \begin{pmatrix} c_{\pi^0} \frac{\pi^0}{\sqrt{2}} + c_{\eta} \frac{\eta}{\sqrt{6}} + c_{\eta'} \frac{\eta'}{\sqrt{3}} & c_{\pi^+ \pi^+} & c_{K^+ K^+} \\ c_{\pi^- \pi^-} & -c_{\pi^0} \frac{\pi^0}{\sqrt{2}} + c_{\eta} \frac{\eta}{\sqrt{6}} + c_{\eta'} \frac{\eta'}{\sqrt{3}} & c_{K^0 K^0} \\ c_{K^- K^-} & c_{\bar{K}^0 \bar{K}^0} & -c_{\eta} \frac{2\eta}{\sqrt{6}} + c_{\eta'} \frac{\eta'}{\sqrt{3}} \end{pmatrix},$$

where all c_i are parameters.

The effect of this coupling is that the emission of the GBs will create quark-antiquark pairs from the vacuum with quantum numbers of the pseudoscalar mesons. Goldstone boson (GB) emission will therefore in general flip the spin of the quarks. The interaction of the GBs is weak enough to be treated by perturbation theory. This means that on long enough time scales for the low energy parameters to develop we have

$$u^\uparrow \rightleftharpoons (d^\downarrow + \pi^+) + (s^\downarrow + K^+) + (u^\downarrow + \pi^0, \eta, \eta'), \quad (2a)$$

$$d^\uparrow \rightleftharpoons (u^\downarrow + \pi^-) + (s^\downarrow + K^0) + (d^\downarrow + \pi^0, \eta, \eta'), \quad (2b)$$

$$s^\uparrow \rightleftharpoons (u^\downarrow + K^-) + (d^\downarrow + \bar{K}^0) + (s^\downarrow + \eta, \eta'). \quad (2c)$$

The matrix Φ in the Lagrangian (1) is the most general parameterization of the pseudoscalar GB matrix in the χ QM. In a realistic model, one should of course not use all these parameters. The reason for introducing this large set of pa-

rameters is to make the following discussion general. The parameter $c_{\eta'}$ describes U(3) symmetry breaking and the other parameters describe SU(3) symmetry breaking.

Cheng and Li have used the SU(3) symmetric model with a broken U(3) symmetry [3] and showed that it can successfully be used to calculate the quark spin polarizations in the nucleon. In a later paper [6], they have extended this model by introducing SU(3) symmetry breaking in the Lagrangian via two parameters $c_K = \alpha$ and $c_{\eta} = \beta$. Song *et al.* [7] and Weber *et al.* [8] have also studied models with SU(3) symmetry breaking, similar to the one discussed by Cheng and Li. All these extended models have lead to significantly better results for several physical quantities.

B. The Coleman–Glashow sum-rule

There is, however, one important set of data which the χ QM cannot successfully predict regardless how many symmetry breaking parameters one introduces in the Lagrangian (1): the octet baryon magnetic moments. This is the case at

least as long as one uses SU(6) symmetric wave functions for the octet baryons. This is most easily illustrated by the function

$$\Sigma_\mu \equiv \mu(p) - \mu(n) + \mu(\Sigma^-) - \mu(\Sigma^+) + \mu(\Xi^0) - \mu(\Xi^-). \quad (3)$$

Experimentally, $\Sigma_\mu = (0.49 \pm 0.05)\mu_N$, but, as we will show, in the χ QM $\Sigma_\mu = 0$ (the Coleman–Glashow sum-rule).

Writing out the explicit valence quark content of the baryons in Eq. (3) we have

$$\begin{aligned} \Sigma_\mu = & \mu(B uud) - \mu(B ddu) + \mu(B dds) - \mu(B uus) \\ & + \mu(B ssu) - \mu(B ssd). \end{aligned} \quad (4)$$

To obtain $\Sigma_\mu = 0$ we need a mirror symmetry, such that the contribution to the magnetic moment generated by GB emission from the two u quarks in $B(uud)$ cancels the corresponding contribution generated by GB emission from the two u quarks in $B(uus)$, the contribution generated by the d quark in $B(uud)$ cancels the one generated by the d quark in $B(ssd)$, etc., provided that the quark magnetic moments are constant. This is trivially true in the NQM. As mentioned in the Introduction, there is a large class of models beyond the NQM, where the Coleman–Glashow sum-rule is fulfilled [12–14]. We will now make a schematic calculation to show that the above condition is fulfilled in the χ QM with arbitrary SU(3) symmetry breaking.

First, we introduce a function \hat{B} to describe the spin structure of a baryon B

$$\hat{B} = n_{x^\uparrow} \hat{x}^\uparrow + n_{x^\downarrow} \hat{x}^\downarrow + n_{y^\uparrow} \hat{y}^\uparrow + n_{y^\downarrow} \hat{y}^\downarrow + n_{z^\uparrow} \hat{z}^\uparrow + n_{z^\downarrow} \hat{z}^\downarrow. \quad (5)$$

The coefficient $n_{q^\uparrow\downarrow}$ of each symbol $\hat{q}^{\uparrow\downarrow}$ should be interpreted as the number of $q^{\uparrow\downarrow}$ quarks. See Appendix A for a complete discussion of the function \hat{B} . Then, $\Delta q^B = n_{q^\uparrow}(B) - n_{q^\downarrow}(B)$ is the q quark spin polarization in the baryon B . Normally, there is also a contribution from the antiquarks to the spin polarization, but in the χ QM this is zero. The baryon magnetic moments can be parametrized as

$$\mu(B) = \Delta u^B \mu_u + \Delta d^B \mu_d + \Delta s^B \mu_s, \quad (6)$$

where μ_q is the quark magnetic moment of the q quark. Here the quark spin polarization, Δq^B , may vary from baryon to baryon, but the quark magnetic moment, μ_q , is the same for all baryons.

The starting point in the χ QM is the spin structure in the NQM. The NQM spin structure of an octet baryon $B(xxy)$ is

$$\hat{B}(xxy) = \frac{5}{3} \hat{x}^\uparrow + \frac{1}{3} \hat{x}^\downarrow + \frac{1}{3} \hat{y}^\uparrow + \frac{2}{3} \hat{y}^\downarrow, \quad (7)$$

so the spin polarizations are $\Delta x^B = \frac{4}{3}$, $\Delta y^B = -\frac{1}{3}$, and $\Delta z^B = 0$, where z is the non-valence quark. Using this it is easy to see that the Coleman–Glashow sum-rule is fulfilled in the NQM. With help of Eq. (7) we can express the spin structure after one iteration in the χ QM by

$$\begin{aligned} \hat{B}(xxy) = & P_x \left(\frac{5}{3} \hat{x}^\uparrow + \frac{1}{3} \hat{x}^\downarrow \right) + P_y \left(\frac{1}{3} \hat{y}^\uparrow + \frac{2}{3} \hat{y}^\downarrow \right) + \frac{5}{3} |\psi(x^\uparrow)|^2 \\ & + \frac{1}{3} |\psi(x^\downarrow)|^2 + \frac{1}{3} |\psi(y^\uparrow)|^2 + \frac{2}{3} |\psi(y^\downarrow)|^2, \end{aligned} \quad (8)$$

where P_q is the probability of no GB emission from the q quark and $|\psi(q^{\uparrow\downarrow})|^2$ are the probabilities of GB emission from the $q^{\uparrow\downarrow}$ quarks. The functions P_q and $|\psi(q^{\uparrow\downarrow})|^2$ are discussed in detail in Appendix A.

For example, the probability function $|\psi(x^\uparrow)|^2$ is of the form

$$|\psi(x^\uparrow)|^2 = b_{x^\uparrow} \hat{x}^\uparrow + b_{y^\downarrow} \hat{y}^\downarrow + b_{z^\downarrow} \hat{z}^\downarrow, \quad (9)$$

where b_{x^\uparrow} , b_{y^\downarrow} , and b_{z^\downarrow} are some constants depending on the choice of the parameters c_i in the Lagrangian. We have here omitted the quark-antiquark pair created by the GB as it will not contribute to the spin polarizations.

It is now easy to see that the sum-rule is fulfilled. For example, the two valence u quarks in $B(uud)$ give a contribution to the spin structure after GB emission, which is

$$P_u \left(\frac{5}{3} \hat{u}^\uparrow + \frac{1}{3} \hat{u}^\downarrow \right) + \frac{5}{3} |\psi(u^\uparrow)|^2 + \frac{1}{3} |\psi(u^\downarrow)|^2. \quad (10)$$

This is canceled by an identical contribution from the u quarks in $B(uus)$. Similarly, the contribution from the d quark in $B(uud)$

$$P_d \left(\frac{1}{3} \hat{d}^\uparrow + \frac{2}{3} \hat{d}^\downarrow \right) + \frac{1}{3} |\psi(d^\uparrow)|^2 + \frac{2}{3} |\psi(d^\downarrow)|^2 \quad (11)$$

will cancel the contribution from the d quark in $B(ssd)$, etc. This shows that the Coleman–Glashow sum-rule $\Sigma_\mu = 0$ is satisfied in the χ QM with arbitrary symmetry breaking in the Lagrangian (1). One can also easily show that the Coleman–Glashow sum-rule is fulfilled for arbitrary number of iterations of GB emission in the χ QM.

Note that expression (10) contains a part of the spin polarization of all three quarks, u , d , and s , as can be seen from Eq. (9). Similarly, the original d quark in the proton contributes by GB emission to the spin polarization of all three quarks. The contribution to the spin polarization of the u quark generated by the original d quark in the proton is in general different from the one generated by the s quark in Σ^+ , due to the symmetry breaking in the Lagrangian. This means that in general $\Delta u^p \neq \Delta u^{\Sigma^+}$. Therefore the sum-rule is fulfilled only because of the mirror symmetry in the NQM wave functions used as input in Eq. (8). The sum-rule is not a result of baryon independent quark spin polarizations, but a result of the fact that the total contribution from all six baryons to a given flavor cancels. Thus, we have the relation

$$\Delta q^p - \Delta q^n + \Delta q^{\Sigma^-} - \Delta q^{\Sigma^+} + \Delta q^{\Xi^0} - \Delta q^{\Xi^-} = 0, \quad q = u, d, s, \quad (12)$$

rather than simple relations as *e.g.* $\Delta u^p = \Delta u^{\Sigma^+}$. This can also be seen from the explicit expressions in Appendix B (when $\theta_N = \theta_\Sigma = \theta_\Xi = 0$).

III. THE CHIRAL QUARK MODEL WITH CONFIGURATION MIXING

As we have shown above, the Coleman–Glashow sum-rule is satisfied in the χ QM. There are in principle two ways of overcoming this problem as discussed before, one is to let the quark magnetic moments vary between the isomultiplets of the octet baryons, and the other one is to introduce symmetry breaking in the wave functions. For reasons discussed in Sec. I, we will here adopt the second alternative. One way of doing this is to add configuration mixing terms in the wave functions.

In our models, the wave functions will have the general structure

$$|B^\uparrow\rangle \equiv |B; S = \frac{1}{2}, S_z = +\frac{1}{2}\rangle = \cos\theta_B |B_1^\uparrow\rangle + \sin\theta_B |B_1^{\prime\uparrow}\rangle, \quad (13)$$

where $|B_1^\uparrow\rangle$ is the usual SU(6) wave function and $|B_1^{\prime\uparrow}\rangle$ is the configuration mixing term. The angle θ_B is a measure of the amount of mixing. We will let the angles of configuration mixing be the same within each baryon isomultiplet, but let them vary between different isomultiplets. We also assume, for simplicity, that the mixing angle for Λ is equal to the one for Σ . Thus we have three mixing angles θ_N , θ_Σ , and θ_Ξ .

A. Wave functions with quark-gluon mixing

First, we will discuss a simple model with a wave function with an additional term where a color-octet baryon state is coupled to a spin one color-octet gluon state. We call this model the chiral quark model with quark-gluon mixing (χ QMg).

The wave function for the octet baryons in this model is a mixture of two different wave functions [9]

$$|B^\uparrow\rangle = \cos\theta_B |B_1^\uparrow\rangle + \sin\theta_B |(B_8G)^\uparrow\rangle. \quad (14)$$

Thus, in this case we have set $|B_1^{\prime\uparrow}\rangle = |(B_8G)^\uparrow\rangle$ in Eq. (13).

The octet baryon color-singlet wave function for the xy baryons is given by

$$|B_1^\uparrow(xy)\rangle = \frac{1}{\sqrt{6}}(2|x^\uparrow x^\uparrow y^\downarrow\rangle - |x^\uparrow x^\downarrow y^\uparrow\rangle - |x^\downarrow x^\uparrow y^\uparrow\rangle) \quad (15)$$

and for the Λ baryon by

$$|\Lambda_1^\uparrow(uds)\rangle = \frac{1}{\sqrt{2}}(|u^\uparrow d^\downarrow s^\uparrow\rangle - |u^\downarrow d^\uparrow s^\uparrow\rangle). \quad (16)$$

We have suppressed color and permutations in flavor in the above wave functions. We will do so also in the following, as this will not affect the spin structures.

The gluonic octet baryon color-singlet wave function is a coupling of an octet baryon color-octet wave function, $|B_8\rangle$, and a spin-one color-octet gluon wave function, $|G\rangle$, to make a color-singlet state with total angular momentum $J = \frac{1}{2}$

$$\begin{aligned} |(B_8G)^\uparrow\rangle &= -\frac{1}{\sqrt{3}}|B_8; S = \frac{1}{2}, S_z = +\frac{1}{2}\rangle \otimes |G; S = 1, S_z = 0\rangle \\ &+ \sqrt{\frac{2}{3}}|B_8; S = \frac{1}{2}, S_z = -\frac{1}{2}\rangle \\ &\otimes |G; S = 1, S_z = +1\rangle. \end{aligned} \quad (17)$$

Here

$$\begin{aligned} |B_8(xy); S = \frac{1}{2}, S_z = +\frac{1}{2}\rangle &= \frac{1}{\sqrt{3}}(|x^\uparrow x^\uparrow y^\downarrow\rangle + |x^\uparrow x^\downarrow y^\uparrow\rangle \\ &+ |x^\downarrow x^\uparrow y^\uparrow\rangle) \end{aligned} \quad (18)$$

for the xy baryons and

$$|\Lambda_8(uds); S = \frac{1}{2}, S_z = +\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}(|u^\uparrow d^\downarrow s^\uparrow\rangle - |u^\downarrow d^\uparrow s^\uparrow\rangle) \quad (19)$$

for the Λ baryon.

B. Wave functions with quark-diquark mixing

An alternative to the quark-gluon mixing as a source of configuration mixing is given by a model with quark-diquark mixing. We call this model the chiral quark model with quark-diquark mixing (χ QMd).

The diquark model is a modification of the usual quark model by considering two quarks glued together to form a diquark. There are SU(3) sextet axial-vector diquarks and SU(3) triplet scalar diquarks. We will only consider scalar diquarks. The symbol $(q_1 q_2)_d$ will denote a scalar diquark consisting of the quarks q_1 and q_2 .

It has been suggested in Ref. [19], that a quark-diquark model can be used to calculate strong and electromagnetic properties of baryons. In such a model, the diquark, although formed as a bound state of two quarks, is regarded as essentially elementary in its interaction with a quark to form a baryon.

In this model, the wave function for the octet baryons is a mixture of the usual SU(6) wave function and a quark-diquark wave function [10]

$$|B^\uparrow\rangle = \cos\theta_B |B_1^\uparrow\rangle + \sin\theta_B |B_d^\uparrow\rangle. \quad (20)$$

Thus, in this case we use $|B_1^{\prime\uparrow}\rangle = |B_d^\uparrow\rangle$ in Eq. (13).

The octet baryon color-singlet wave function for the xy baryons is again given by Eq. (15) and for the Λ baryon by Eq. (16). The quark-diquark octet baryon wave function is

$$|B_d^\uparrow(xy)\rangle \equiv |x^\uparrow\rangle \otimes |(xy)_d\rangle = |x^\uparrow(xy)_d\rangle \quad (21)$$

for the xy baryons and

$$|\Lambda_d^\uparrow(uds)\rangle = \frac{1}{\sqrt{6}}(|u^\uparrow(ds)_d\rangle - |d^\uparrow(us)_d\rangle - 2|s^\uparrow(ud)_d\rangle) \quad (22)$$

for the Λ baryon [20].

TABLE I. Parameter values obtained in the different fits. The subscript $\alpha\beta$ in a model name indicates that the parameters α and β were allowed to vary in the fit. Hyphen (-) indicates that the parameter was not defined in the fit. (1) means that the parameter was not free in the fit, but put to 1. The magnetic moment of the d quark, μ_d , is given in units of the nuclear magneton, μ_N .

Parameter	NQM	NQMg	NQMd	χ QM	$\chi_{\alpha\beta}$ QM	χ QMg	$\chi_{\alpha\beta}$ QMg	χ QMd	$\chi_{\alpha\beta}$ QMd
μ_d	-0.91	-1.15	-1.09	-1.35	-1.23	-1.40	-1.24	-1.42	-1.27
α	-	-	-	(1)	0.52	(1)	0.70	(1)	0.69
β	-	-	-	(1)	0.99	(1)	0.73	(1)	0.55
$\sin^2\theta_N$	-	0.18	0.39	-	-	0.00	0.00	0.00	0.11
$\sin^2\theta_\Sigma$	-	0.20	0.65	-	-	0.05	0.04	0.25	0.34
$\sin^2\theta_\Xi$	-	0.24	0.46	-	-	0.11	0.11	0.33	0.41

C. Discussion of parameters

In our further calculations, we will use the following parameters in the Lagrangian (1): $c_{\pi^0}=c_{\pi^+}=c_{\pi^-}=1$, $c_{K^+}=c_{K^-}=c_{K^0}=c_{\bar{K}^0}=\alpha$, $c_\eta=\beta$, and $c_{\eta'}=\zeta$. In some of our calculations we will use an SU(3) symmetric Lagrangian with $\alpha=\beta=1$. See Appendix A for a detailed discussion of the Lagrangian.

The parameter a , describing the probability of GB emission, and the parameter ζ can be estimated from the $\bar{u}-\bar{d}$ asymmetry. The New Muon Collaboration (NMC) experiment has measured the isospin asymmetry difference of the quark sea in the proton to be [21,22]

$$\bar{u}-\bar{d}\simeq-0.15. \quad (23)$$

In the χ QM this difference is given by

$$\bar{u}-\bar{d}=a\left(\frac{2\zeta+\beta}{3}-1\right). \quad (24)$$

The expressions for the antiquark numbers \bar{u} and \bar{d} are given in Appendix A. Combining Eqs. (23) and (24) we obtain

$$a\simeq\frac{0.44}{3-2\zeta-\beta}. \quad (25)$$

Similarly to Eq. (24) we have for the antiquark density ratio

$$\bar{u}/\bar{d}=\frac{21+2(2\zeta+\beta)+(2\zeta+\beta)^2}{33-2(2\zeta+\beta)+(2\zeta+\beta)^2} \quad (26)$$

with the experimental value $\bar{u}/\bar{d}=0.51\pm 0.09$ [23].

If we set $\beta=1$, then Eq. (26) reduces to

$$\bar{u}/\bar{d}|_{\beta=1}=\frac{6+2\zeta+\zeta^2}{8+\zeta^2}. \quad (27)$$

From this we obtain $-4.3<\zeta<-0.7$. Following Cheng and Li [3], we choose the value $\zeta=-1.2$. The value of a is now given by Eq. (25) to be $a\approx 0.10$, which is in good agreement with Ref. [2]. However, when β is a free parameter in the calculations, we have to use the relation $2\zeta+\beta\simeq-1.4$, which comes from Eq. (25), in order to keep $a\approx 0.10$. We therefore make the assumption that

$$\zeta=-0.7-\frac{\beta}{2}. \quad (28)$$

This also fixes the value of \bar{u}/\bar{d} to 0.53.

In what follows, we consider the case where the magnetic moments of the quarks satisfy the relations

$$\mu_u=-2\mu_d \quad (29)$$

and

$$\mu_s=\frac{2}{3}\mu_d. \quad (30)$$

D. Numerical results

As we have seen, when $\theta_N=\theta_\Sigma=\theta_\Xi=0$, $\Sigma_\mu=0$ for every choice of the parameters c_i . Also when the mixing angles are the same, but not equal to zero, $\Sigma_\mu=0$. However, when at least one of the mixing angles θ_B is different from the others, the value of Σ_μ will be non-zero.

In the models, that we will discuss, all three mixing angles θ_N , θ_Σ , and θ_Ξ will be free parameters. The magnetic moment of the d quark, μ_d , will also be a free parameter and the other quark magnetic moments are then given by the relations (29) and (30). In order to calculate the magnetic moments of the octet baryons we will also need the quark spin polarizations, which are obtained from the quark spin structures. A detailed derivation of the spin polarizations starting from the Lagrangian (1) can be found in Appendices A and B. The baryon magnetic moments are given by Eq. (6).

We fit the experimental data for the octet baryon magnetic moments and the weak axial-vector form factor g_A . Since the magnetic moments depend on the products of quark magnetic moments and quark spin polarizations, the use of g_A serves as a normalization of the parameters. The parameter values obtained from the different fits can be found in Table I.

Let us first say a few words about the NQM with configuration mixing, *i.e.* no GB emission ($a=0$).

In the case with quark-gluon mixing, we will get the NQMg, an extension of the model for the proton suggested by Lipkin [9]. The NQMg gives $\Sigma_\mu\approx 0.17\mu_N$. However, the NQMg does not give rise to any $\bar{u}-\bar{d}$ asymmetry, because of lack of vacuum polarization.

TABLE II. Octet baryon magnetic moments, g_A , and Σ_μ . The subscript $\alpha\beta$ in a model name indicates that the parameters α and β were allowed to vary in the fit. The octet baryon magnetic moments and Σ_μ are given in units of the nuclear magneton, μ_N . The experimental values have been obtained from Ref. [25].

Quantity	Expt. values	NQM	NQMg	NQMd	χ QM	$\chi_{\alpha\beta}$ QM	χ QMg	$\chi_{\alpha\beta}$ QMg	χ QMd	$\chi_{\alpha\beta}$ QMd
χ^2		0.28	0.14	0.081	0.12	0.075	0.082	0.055	0.031	0.0032
$\mu(p)$	2.79 ± 0.00	2.72	2.77	2.85	2.67	2.65	2.76	2.74	2.80	2.76
$\mu(n)$	-1.91 ± 0.00	-1.81	-1.89	-1.76	-1.86	-1.94	-1.92	-1.96	-1.95	-1.95
$\mu(\Sigma^+)$	2.46 ± 0.01	2.61	2.56	2.53	2.57	2.52	2.52	2.49	2.48	2.46
$\mu(\Sigma^-)$	-1.16 ± 0.03	-1.01	-0.95	-1.14	-1.05	-1.15	-1.02	-1.07	-1.07	-1.15
$\mu(\Xi^0)$	-1.25 ± 0.01	-1.41	-1.38	-1.25	-1.45	-1.41	-1.35	-1.35	-1.24	-1.25
$\mu(\Xi^-)$	-0.65 ± 0.00	-0.50	-0.41	-0.66	-0.55	-0.49	-0.48	-0.48	-0.61	-0.67
$\mu(\Lambda)$	-0.61 ± 0.00	-0.60	-0.56	-0.45	-0.65	-0.62	-0.63	-0.64	-0.59	-0.61
g_A	1.26 ± 0.00	$\frac{5}{3}$	1.35	1.41	1.12	1.24	1.12	1.26	1.12	1.24
Σ_μ	0.49 ± 0.05	0	0.17	0.36	0	0	0.28	0.27	0.55	0.52

In the case with quark-diquark mixing, we will get the NQMd, an extension of the model for the proton considered by Noda *et al.* [10]. This model gives a much better value on Σ_μ , than the NQMg. The value obtained is $\Sigma_\mu \approx 0.36\mu_N$, which is still not within the experimental errors. As in the NQMg, there is no $\bar{u}-\bar{d}$ asymmetry in the NQMd. The mixings also become unrealistically large, for example $\sin^2\theta_\Sigma \approx 0.65$.

We now continue with the χ QM. We will discuss two cases, one where we put $\alpha = \beta = 1$, and one where we let α and β vary independently. Thus in the first case, we have the original SU(3) symmetric Lagrangian and we can study the effect of the mixing angles alone. The second case makes it possible to see how the combination of symmetry breaking and wave function mixing improves the results.

In the first calculation with $\alpha = \beta = 1$, we obtain the mixings $\sin^2\theta_N \approx 0.00$, $\sin^2\theta_\Sigma \approx 0.05$, and $\sin^2\theta_\Xi \approx 0.11$ in the quark-gluon model, and $\sin^2\theta_N \approx 0.00$, $\sin^2\theta_\Sigma \approx 0.25$, and $\sin^2\theta_\Xi \approx 0.33$ in the quark-diquark model. In Table II the values of the octet baryon magnetic moments, g_A , and Σ_μ are presented together with the experimental values. The over all fit is obviously better than in the case without mixing, as we have more parameters, but the important result is that we are now able to obtain non-zero values of the function Σ_μ . In the quark-gluon model we obtain $\Sigma_\mu \approx 0.28\mu_N$, which still differs from the experimental value, but in the quark-diquark model we obtain $\Sigma_\mu \approx 0.55\mu_N$, which is very close to experiment. Note, in Table III, that the total spin polarization $\Delta\Sigma$ is the same in the χ QMg and χ QMd as in the χ QM, simply because the mixing angle θ_N is zero in these models.

In the second calculation we also let α and β be free parameters. In this fit we have to use $\zeta = -0.7 - \beta/2$, in or-

der to keep $a \approx 0.10$. The values of the magnetic moments are over all improved compared to the above case with $\alpha = \beta = 1$, especially χ^2 decreases with a factor of about 10 in the χ QMd, see Table II. The value of Σ_μ in the χ QMg is about the same as in the case with $\alpha = \beta = 1$, but in the χ QMd we obtain $\Sigma_\mu \approx 0.52\mu_N$, which lies within the experimental errors. The symmetry breaking in the Lagrangian becomes relatively large, $\alpha \approx 0.70$ and $\beta \approx 0.73$ in the quark-gluon model and $\alpha \approx 0.69$ and $\beta \approx 0.55$ in the quark-diquark model. The values obtained for α , which is a suppression factor for kaon GB emission, are reasonable as it can be argued that α is proportional to $m/m_s = 2/3$ [6–8]. On the other hand, the mixing angles are not changed very much compared to the fits with $\alpha = \beta = 1$, except for θ_N , which gets a non-zero value in the quark-diquark model. The mixings obtained are $\sin^2\theta_N \approx 0.00$, $\sin^2\theta_\Sigma \approx 0.25$, and $\sin^2\theta_\Xi \approx 0.33$ in the χ QMg and $\sin^2\theta_N \approx 0.11$, $\sin^2\theta_\Sigma \approx 0.34$, and $\sin^2\theta_\Xi \approx 0.41$ in the χ QMd.

By letting α and β vary, the major improvement we obtain is very good values for the weak axial-vector form factor, $g_A \approx 1.26$ in the χ QMg and $g_A \approx 1.24$ in the χ QMd. On the other hand, the total spin polarization $\Delta\Sigma$ becomes somewhat large (see Table III).

How does the choice of parametrization of the Lagrangian influence the results? We have chosen to introduce the SU(3) symmetry breaking parameters α and β in the same spirit as has been done by other authors [6]. There are of course other options. For example, it is possible that the probability of $d \rightarrow K^0 + s$ is different from that for $s \rightarrow \bar{K}^0 + d$ due to the different phase space. Taking this into account would require a set of new parameters. Although this would give small corrections to the results, it would not change the main con-

TABLE III. Spin polarizations for the proton. The subscript $\alpha\beta$ in a model name indicates that the parameters α and β were free in the fit.

Quantity	NQM	NQMg	NQMd	χ QM	$\chi_{\alpha\beta}$ QM	χ QMg	$\chi_{\alpha\beta}$ QMg	χ QMd	$\chi_{\alpha\beta}$ QMd
Δu^p	$\frac{4}{3}$	1.05	1.20	0.79	0.89	0.79	0.91	0.79	0.91
Δd^p	$-\frac{1}{3}$	-0.29	-0.20	-0.32	-0.35	-0.32	-0.35	-0.32	-0.33
Δs^p	0	0	0	-0.10	-0.03	-0.10	-0.05	-0.10	-0.05
$\Delta\Sigma$	1	0.76	1	0.37	0.52	0.37	0.51	0.37	0.53

clusions. As has been pointed out, there is no way to break the Coleman–Glashow sum-rule in the χ QM*d* by introducing more symmetry breaking parameters in the Lagrangian.

To investigate how a different set of parameters would influence the results, we have considered the case where the substitution $\varphi_{sq} \rightarrow \epsilon \varphi_{sq}$, $q = u, d$ has been carried out in the last row in the matrix (A2). The parameter ϵ accounts for the difference in probability of an s quark emitting a GB and a u or d quark emitting a GB, as discussed above. We have made a fit including ϵ in the model χ QM*d*, when α and β are considered as free parameters. For ϵ we obtained the value 1.27. This results in minor changes of the parameters α and β . The mixing angles are $\sin^2 \theta_N \approx 0.10$, $\sin^2 \theta_\Sigma \approx 0.32$, and $\sin^2 \theta_\Xi \approx 0.40$, which are almost identical to the fit with $\epsilon = 1$ (see the last column in Table I). This shows that the exact choice of parametrization in the Lagrangian does not affect the main conclusions, and verifies that the introduction of further SU(3) breaking parameters in the χ QM Lagrangian cannot reduce the size of configuration mixing needed to break the Coleman–Glashow sum-rule.

E. A simple mechanism for configuration mixing

We will here describe a simple mechanism in the form of a toy model for configuration mixing in the wave functions for the octet baryons.

In this simple toy model, we assume that we have a two level system of mass states for the octet baryons, such that these are mixings of (1) the usual three quark mass states, where the u and d quarks have mass m and the s quark has mass m_s , and (2) quark-diquark mass states. The mass of the $(ud)_d$ diquark is M and the mass of $(us)_d$ and $(ds)_d$ diquarks is M_s . The diquarks are only singlets. When these states mix we obtain the two physical mass states, the ground state and the first excited state. The first excited state is simply assumed to be the mass state next in order to the ground state with the same quantum numbers as the ground state. Thus, we interpret the excitation to be a quark-diquark excitation rather than a radial excitation.

The wave functions Ψ_- and Ψ_+ , corresponding to the physical mass states, can be expressed in the wave functions Ψ_0 and Ψ_1 , corresponding to the unphysical mass states, as

$$\begin{cases} \Psi_- = \Psi_0 \cos \vartheta + \Psi_1 \sin \vartheta \\ \Psi_+ = -\Psi_0 \sin \vartheta + \Psi_1 \cos \vartheta \end{cases} \quad (31)$$

where ϑ is the configuration mixing angle. The wave function Ψ_- should be compared to Eq. (13).

We then introduce the Hamiltonian

$$\hat{H} = \begin{pmatrix} m_0 & H \\ H & m_1 \end{pmatrix}, \quad (32)$$

where m_0 is the lower unphysical mass state and m_1 is the higher unphysical mass state. The parameter H corresponds to the transition probability between the unphysical mass states m_0 and m_1 , and it is assumed to be the same for N , Λ , Σ , and Ξ .

For the lower unphysical mass states we use a simple mass formula with a hyperfine coupling term [24]

$$m_0(B(q_1 q_2 q_3)) = m_{q_1} + m_{q_2} + m_{q_3} + h \left(\frac{\mathbf{s}_{q_1} \cdot \mathbf{s}_{q_2}}{m_{q_1} m_{q_2}} + \frac{\mathbf{s}_{q_1} \cdot \mathbf{s}_{q_3}}{m_{q_1} m_{q_3}} + \frac{\mathbf{s}_{q_2} \cdot \mathbf{s}_{q_3}}{m_{q_2} m_{q_3}} \right), \quad (33)$$

where \mathbf{s}_{q_i} is the spin of the quark q_i . The parameter h is the QCD hyperfine coupling parameter. Since the diquarks are scalars, there is no hyperfine coupling in the higher unphysical mass states. The mass formulas for the higher unphysical mass states are $m_1(N) = m + M$, $m_1(\Lambda) = (m + M_s)/3 + 2(m_s + M)/3$, $m_1(\Sigma) = m + M_s$, and $m_1(\Xi) = m_s + M_s$.

Solving the eigenvalue problem for the Schrödinger equation $\hat{H}\Psi = E\Psi$, where

$$\Psi = \begin{pmatrix} \Psi_0 \\ \Psi_1 \end{pmatrix},$$

we get the eigenvalues

$$E_{\pm} = \frac{m_0 + m_1}{2} \pm \frac{1}{2} \sqrt{(m_0 - m_1)^2 + 4H^2}, \quad (34)$$

which should correspond to the physical mass states.

The quantity $\sin^2 \vartheta$ measures the part of the total mass state which is of quark-diquark origin and is given by

$$\sin^2 \vartheta = \frac{2x^2}{1 + 4x^2 + \sqrt{1 + 4x^2}}, \quad (35)$$

where $x = H/(m_1 - m_0)$.

Choosing the illustrative values $m = 400$ MeV, $m_s = 590$ MeV, $M = 920$ MeV, $M_s = 1000$ MeV, $h/(4m^2) = 60$ MeV, and $H = 190$ MeV, we obtain an octet baryon mass spectrum, which is in good agreement with the measured spectrum. For the mixings we get $\sin^2 \vartheta_N \approx 0.19$, $\sin^2 \vartheta_\Lambda \approx 0.22$, $\sin^2 \vartheta_\Sigma \approx 0.36$, and $\sin^2 \vartheta_\Xi \approx 0.36$. The mixing for Σ and Ξ is the same, since $m_1(\Sigma) - m_0(\Sigma)$ is equal to $m_1(\Xi) - m_0(\Xi)$ in this simple model.

Since we have assumed that the mixing angles for Σ and Λ should be equal in the χ QM*d*, the corresponding mixing $\sin^2 \vartheta_\Sigma$ should be compared to the harmonic mean of $\sin^2 \vartheta_\Sigma$ and $\sin^2 \vartheta_\Lambda$ in the toy model, which is $\sin^2 \vartheta_{\Sigma\Lambda} \equiv 2\sin^2 \vartheta_\Sigma \sin^2 \vartheta_\Lambda / (\sin^2 \vartheta_\Sigma + \sin^2 \vartheta_\Lambda) \approx 0.27$.

Comparing the mixings in the toy model ($\sin^2 \vartheta_N \approx 0.19$, $\sin^2 \vartheta_{\Sigma\Lambda} \approx 0.27$, and $\sin^2 \vartheta_\Xi \approx 0.36$) with the ones obtained from the χ QM*d* with α and β free ($\sin^2 \theta_N \approx 0.11$, $\sin^2 \theta_\Sigma \approx 0.34$, and $\sin^2 \theta_\Xi \approx 0.41$), we see that they are of the same order of magnitude and they also appear in increasing order.

IV. SUMMARY AND CONCLUSIONS

In this paper, we have studied the octet baryon magnetic moments in the χ QM with configuration mixing. In particular, the experimentally well established violation of the Coleman–Glashow sum-rule cannot be reproduced in the χ QM, no matter how many SU(3) symmetry breaking parameters one introduces in the Lagrangian (1).

As discussed, there are in principle two ways of overcom-

ing this problem, one is to let the quark magnetic moments vary between the isomultiplets, and the other is to introduce symmetry breaking in the wave functions of the octet baryons. Taking the view, that the quarks should have the same properties independently of in which baryon they are, we are lead to choose the second alternative.

We considered two extensions of the χ QM, one with quark-gluon configuration mixing (χ QMg), and one with quark-diquark configuration mixing (χ QMd). The χ QMd with symmetry breaking in the Lagrangian ($\alpha \approx 0.69$ and $\beta \approx 0.55$) led to $\Sigma_\mu \approx 0.52\mu_N$, a value which lies within the experimental errors. The experimental value is $\Sigma_\mu = (0.49 \pm 0.05)\mu_N$. The values of the octet baryon magnetic moments and the weak axial-vector form factor g_A are also in very good agreement with experiments. The amount of quark-diquarks lies between 11% and 41% in this model.

APPENDIX A: A SURVEY OF THE CHIRAL QUARK MODEL

The Lagrangian \mathcal{L}_I in Eq. (1), giving rise to GB emission, will be specialized by putting $c_{\pi^0} = c_{\pi^+} = c_{\pi^-} = 1$, $c_{K^+} = c_{K^-} = c_{K^0} = c_{\bar{K}^0} = \alpha$, $c_\eta = \beta$, and $c_{\eta'} = \zeta$. To find the quark polarizations, we replace the GBs with their quark contents. The Lagrangian of the effective interaction is then given by

$$\hat{\mathcal{L}}_I = \sum_{q=u,d,s} \hat{\mathcal{L}}_q, \quad (\text{A1})$$

where

$$\hat{\mathcal{L}}_q = g_8 \bar{q} \sum_{q'=u,d,s} \hat{\Phi}_{qq'} q'.$$

The transition of $q \rightarrow \text{GB} + q' \rightarrow (q\bar{q}')_0 + q'$, where $q' = u, d, s$, is described by the Lagrangian $\hat{\mathcal{L}}_q$. The matrix $\hat{\Phi}$ is

$$\hat{\Phi} = (\hat{\Phi}_{qq'}) = \begin{pmatrix} \phi_{uu}u\bar{u} + \phi_{ud}d\bar{d} + \phi_{us}s\bar{s} & \varphi_{ud}u\bar{d} & \varphi_{us}u\bar{s} \\ \varphi_{du}d\bar{u} & \phi_{du}u\bar{u} + \phi_{dd}d\bar{d} + \phi_{ds}s\bar{s} & \varphi_{ds}d\bar{s} \\ \varphi_{su}s\bar{u} & \varphi_{sd}s\bar{d} & \phi_{su}u\bar{u} + \phi_{sd}d\bar{d} + \phi_{ss}s\bar{s} \end{pmatrix}, \quad (\text{A2})$$

where

$$\begin{aligned} \phi_{uu} = \phi_{dd} = \frac{1}{2} + \frac{\beta}{6} + \frac{\zeta}{3}, \quad \phi_{du} = \phi_{ud} = -\frac{1}{2} + \frac{\beta}{6} + \frac{\zeta}{3}, \quad \phi_{us} = \phi_{ds} = \phi_{su} = \phi_{sd} = -\frac{\beta}{3} + \frac{\zeta}{3}, \\ \phi_{ss} = \frac{2\beta}{3} + \frac{\zeta}{3}, \quad \varphi_{ud} = \varphi_{du} = 1, \quad \text{and} \quad \varphi_{us} = \varphi_{ds} = \varphi_{su} = \varphi_{sd} = \alpha. \end{aligned}$$

The transition probability of u , d , and s quarks can then be expressed by the functions

$$|\psi(u)|^2 = a[(2\phi_{uu}^2 + \phi_{ud}^2 + \phi_{us}^2 + \varphi_{ud}^2 + \varphi_{us}^2)\hat{u} + \phi_{uu}^2\hat{\bar{u}} + (\phi_{ud}^2 + \varphi_{ud}^2)(\hat{d} + \hat{\bar{d}}) + (\phi_{us}^2 + \varphi_{us}^2)(\hat{s} + \hat{\bar{s}})], \quad (\text{A3})$$

$$|\psi(d)|^2 = a[(\phi_{du}^2 + 2\phi_{dd}^2 + \phi_{ds}^2 + \varphi_{du}^2 + \varphi_{ds}^2)\hat{d} + \phi_{dd}^2\hat{\bar{d}} + (\phi_{du}^2 + \varphi_{du}^2)(\hat{u} + \hat{\bar{u}}) + (\phi_{ds}^2 + \varphi_{ds}^2)(\hat{s} + \hat{\bar{s}})], \quad (\text{A4})$$

and

$$|\psi(s)|^2 = a[(\phi_{su}^2 + \phi_{sd}^2 + 2\phi_{ss}^2 + \varphi_{su}^2 + \varphi_{sd}^2)\hat{s} + \phi_{ss}^2\hat{\bar{s}} + (\phi_{su}^2 + \varphi_{su}^2)(\hat{u} + \hat{\bar{u}}) + (\phi_{sd}^2 + \varphi_{sd}^2)(\hat{d} + \hat{\bar{d}})], \quad (\text{A5})$$

where $a \propto |g_8|^2$ and the coefficients of the \hat{q} and $\hat{\bar{q}}$ should be interpreted as the number of q and \bar{q} quarks, respectively.

The total probabilities of emission of a GB from u , d , and s quarks are given by

The introduction of a different set of symmetry breaking parameters in the Lagrangian does not change the results significantly. The violation of the Coleman–Glashow sum-rule is, in our models, solely due to the configuration mixing parameters.

In conclusion, extensions of the χ QM with configuration mixing of quark-diquarks can explain the experimentally observed violation of the Coleman–Glashow sum-rule for the octet baryon magnetic moments.

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$$\Sigma P_u = a(\phi_{uu}^2 + \phi_{ud}^2 + \phi_{us}^2 + \varphi_{ud}^2 + \varphi_{us}^2) = a\left(\frac{9 + \beta^2 + 2\xi^2}{6} + \alpha^2\right), \quad (\text{A6})$$

$$\Sigma P_d = a(\phi_{du}^2 + \phi_{dd}^2 + \phi_{ds}^2 + \varphi_{du}^2 + \varphi_{ds}^2) = a\left(\frac{9 + \beta^2 + 2\xi^2}{6} + \alpha^2\right), \quad (\text{A7})$$

and

$$\Sigma P_s = a(\phi_{su}^2 + \phi_{sd}^2 + \phi_{ss}^2 + \varphi_{su}^2 + \varphi_{sd}^2) = a\left(\frac{2\beta^2 + \xi^2}{3} + 2\alpha^2\right). \quad (\text{A8})$$

The total probability of no emission of GB from a q quark is then given by

$$P_q = 1 - \Sigma P_q. \quad (\text{A9})$$

The antiquark numbers of the proton can be obtained from the expression $2P_u\hat{u} + P_d\hat{d} + 2|\psi(u)|^2 + |\psi(d)|^2$. They are

$$\bar{u} = \frac{1}{12}[(2\xi + \beta + 1)^2 + 20]a, \quad (\text{A10})$$

$$\bar{d} = \frac{1}{12}[(2\xi + \beta - 1)^2 + 32]a, \quad (\text{A11})$$

and

$$\bar{s} = \frac{1}{3}[(\xi - \beta)^2 + 9\alpha^2]a. \quad (\text{A12})$$

The spin structure of a baryon B is described by the function \hat{B} , which is defined by

$$\hat{B} \equiv \langle B^\dagger | \mathcal{N} | B^\dagger \rangle, \quad (\text{A13})$$

where $|B^\dagger\rangle$ is the wave function and \mathcal{N} is the number operator

$$\mathcal{N} = N_{u^\dagger}\hat{u}^\dagger + N_{u^\dagger}\hat{u}^\dagger + N_{d^\dagger}\hat{d}^\dagger + N_{d^\dagger}\hat{d}^\dagger + N_{s^\dagger}\hat{s}^\dagger + N_{s^\dagger}\hat{s}^\dagger.$$

In the model with quark-gluon mixing (χ QMg) the wave function for xy baryons is

$$|B^\dagger(xxy)\rangle = \cos\theta_B |B_1^\dagger(xxy)\rangle + \sin\theta_B |(B_8(xxy)G)^\dagger\rangle. \quad (\text{A14})$$

Simple calculations, using Eqs. (15) and (17), give

$$\langle B_1^\dagger(xxy) | \mathcal{N} | B_1^\dagger(xxy) \rangle = \frac{5}{3}\hat{x}^\dagger + \frac{1}{3}\hat{x}^\dagger + \frac{1}{3}\hat{y}^\dagger + \frac{2}{3}\hat{y}^\dagger \quad (\text{A15})$$

and

$$\langle (B_8(xxy)G)^\dagger | \mathcal{N} | (B_8(xxy)G)^\dagger \rangle = \frac{8}{9}\hat{x}^\dagger + \frac{10}{9}\hat{x}^\dagger + \frac{4}{9}\hat{y}^\dagger + \frac{5}{9}\hat{y}^\dagger. \quad (\text{A16})$$

The coefficients of the $\hat{q}^{\dagger\downarrow}$ in the above formulas should be interpreted as the number of $q^{\dagger\downarrow}$ quarks.

Using Eqs. (A15) and (A16), and then making the substitution

$$\hat{q}^{\dagger\downarrow} \rightarrow P_q \hat{q}^{\dagger\downarrow} + |\psi(q^{\dagger\downarrow})|^2, \quad (\text{A17})$$

for every quark, $q = u, d, s$, in the obtained formula, we get the spin structure, after one interaction, as

$$\begin{aligned} \hat{B}(xy) = & \cos^2\theta_B \left[\frac{5}{3}(P_x\hat{x}^\dagger + |\psi(x^\dagger)|^2) + \frac{1}{3}(P_x\hat{x}^\dagger + |\psi(x^\dagger)|^2) + \frac{1}{3}(P_y\hat{y}^\dagger + |\psi(y^\dagger)|^2) + \frac{2}{3}(P_y\hat{y}^\dagger + |\psi(y^\dagger)|^2) \right] \\ & + \sin^2\theta_B \left[\frac{8}{9}(P_x\hat{x}^\dagger + |\psi(x^\dagger)|^2) + \frac{10}{9}(P_x\hat{x}^\dagger + |\psi(x^\dagger)|^2) + \frac{4}{9}(P_y\hat{y}^\dagger + |\psi(y^\dagger)|^2) + \frac{5}{9}(P_y\hat{y}^\dagger + |\psi(y^\dagger)|^2) \right], \quad (\text{A18}) \end{aligned}$$

where the functions $|\psi(q^\uparrow\downarrow)|^2$ describe the probability of emission of GBs, *i.e.* the probability of transforming a $q^\uparrow\downarrow$ quark.

The probabilities of transforming u , d , and s quarks with spin up by one interaction can be expressed by the functions

$$|\psi(u^\uparrow)|^2 = a[(\phi_{uu}^2 + \phi_{ud}^2 + \phi_{us}^2)\hat{u}^\uparrow + \varphi_{ud}^2\hat{d}^\uparrow + \varphi_{us}^2\hat{s}^\uparrow] = \frac{a}{6}(3 + \beta^2 + 2\zeta^2)\hat{u}^\uparrow + a\hat{d}^\uparrow + a\alpha^2\hat{s}^\uparrow, \quad (\text{A19})$$

$$|\psi(d^\uparrow)|^2 = a[(\phi_{du}^2 + \phi_{dd}^2 + \phi_{ds}^2)\hat{d}^\uparrow + \varphi_{du}^2\hat{u}^\uparrow + \varphi_{ds}^2\hat{s}^\uparrow] = a\hat{u}^\uparrow + \frac{a}{6}(3 + \beta^2 + 2\zeta^2)\hat{d}^\uparrow + a\alpha^2\hat{s}^\uparrow, \quad (\text{A20})$$

and

$$|\psi(s^\uparrow)|^2 = a[(\phi_{su}^2 + \phi_{sd}^2 + \phi_{ss}^2)\hat{s}^\uparrow + \varphi_{su}^2\hat{u}^\uparrow + \varphi_{sd}^2\hat{d}^\uparrow] = a\alpha^2\hat{u}^\uparrow + a\alpha^2\hat{d}^\uparrow + \frac{a}{3}(2\beta^2 + \zeta^2)\hat{s}^\uparrow. \quad (\text{A21})$$

As before, the coefficient of \hat{q}^\uparrow is the transition probability to q^\uparrow . We have here neglected the quark-antiquark pair created by the GB, since it will not contribute to the spin polarizations.

Similarly, in the model with quark-diquark mixing ($\chi\text{QM}d$), we replace the wave function $|(B_8(xxy)G)^\uparrow\rangle$ by $|B_d^\uparrow(xxy)\rangle$ in Eq. (A14). Using Eq. (21), we find

$$\langle B_d^\uparrow(xxy) | \mathcal{N} | B_d^\uparrow(xxy) \rangle = \hat{x}^\uparrow. \quad (\text{A22})$$

After one interaction we then have

$$\begin{aligned} \hat{B}(xxy) = & \cos^2\theta_B \left[\frac{5}{3}(P_x\hat{x}^\uparrow + |\psi(x^\uparrow)|^2) + \frac{1}{3}(P_x\hat{x}^\downarrow + |\psi(x^\downarrow)|^2) + \frac{1}{3}(P_y\hat{y}^\uparrow + |\psi(y^\uparrow)|^2) + \frac{2}{3}(P_y\hat{y}^\downarrow + |\psi(y^\downarrow)|^2) \right] \\ & + \sin^2\theta_B(P_x\hat{x}^\uparrow + |\psi(x^\uparrow)|^2). \end{aligned} \quad (\text{A23})$$

The spin structure of the Λ baryon after one interaction can be obtained by a similar procedure like the one above for xy baryons. The result is

$$\begin{aligned} \hat{\Lambda}(uds) = & \cos^2\theta_\Sigma \left[P_u \left(\frac{1}{2}\hat{u}^\uparrow + \frac{1}{2}\hat{u}^\downarrow \right) + P_d \left(\frac{1}{2}\hat{d}^\uparrow + \frac{1}{2}\hat{d}^\downarrow \right) + P_s\hat{s}^\uparrow + \frac{1}{2}|\psi(u^\uparrow)|^2 + \frac{1}{2}|\psi(u^\downarrow)|^2 + \frac{1}{2}|\psi(d^\uparrow)|^2 + \frac{1}{2}|\psi(d^\downarrow)|^2 + |\psi(s^\uparrow)|^2 \right] \\ & + \sin^2\theta_\Sigma \left[P_u \left(\frac{1}{2}\hat{u}^\uparrow + \frac{1}{2}\hat{u}^\downarrow \right) + P_d \left(\frac{1}{2}\hat{d}^\uparrow + \frac{1}{2}\hat{d}^\downarrow \right) + P_s \left(\frac{1}{3}\hat{s}^\uparrow + \frac{2}{3}\hat{s}^\downarrow \right) + \frac{1}{2}|\psi(u^\uparrow)|^2 + \frac{1}{2}|\psi(u^\downarrow)|^2 + \frac{1}{2}|\psi(d^\uparrow)|^2 + \frac{1}{2}|\psi(d^\downarrow)|^2 \right. \\ & \left. + \frac{1}{3}|\psi(s^\uparrow)|^2 + \frac{2}{3}|\psi(s^\downarrow)|^2 \right] \end{aligned} \quad (\text{A24})$$

in the $\chi\text{QM}g$ and

$$\begin{aligned} \hat{\Lambda}(uds) = & \cos^2\theta_\Sigma \left[P_u \left(\frac{1}{2}\hat{u}^\uparrow + \frac{1}{2}\hat{u}^\downarrow \right) + P_d \left(\frac{1}{2}\hat{d}^\uparrow + \frac{1}{2}\hat{d}^\downarrow \right) + P_s\hat{s}^\uparrow + \frac{1}{2}|\psi(u^\uparrow)|^2 + \frac{1}{2}|\psi(u^\downarrow)|^2 + \frac{1}{2}|\psi(d^\uparrow)|^2 + \frac{1}{2}|\psi(d^\downarrow)|^2 + |\psi(s^\uparrow)|^2 \right] \\ & + \sin^2\theta_\Sigma \left(\frac{1}{6}P_u\hat{u}^\uparrow + \frac{1}{6}P_d\hat{d}^\uparrow + \frac{2}{3}P_s\hat{s}^\uparrow + \frac{1}{6}|\psi(u^\uparrow)|^2 + \frac{1}{6}|\psi(d^\uparrow)|^2 + \frac{2}{3}|\psi(s^\uparrow)|^2 \right) \end{aligned} \quad (\text{A25})$$

in the $\chi\text{QM}d$.

The spin polarization, Δq^B , where $q = u, d, s$, is defined as

$$\Delta q^B \equiv n_{q^\uparrow}(B) - n_{q^\downarrow}(B), \quad (\text{A26})$$

where in the spin structure formulas $n_{q^\uparrow}(B)$ and $n_{q^\downarrow}(B)$ are the coefficients of \hat{q}^\uparrow and \hat{q}^\downarrow , respectively, for the baryon B . The spin polarizations for the octet baryons are given in Appendix B.

The magnetic moment of a baryon B is determined from the expression

$$\mu(B) = \Delta u^B \mu_u + \Delta d^B \mu_d + \Delta s^B \mu_s. \quad (\text{A27})$$

The total spin polarizations of the proton (the spin fraction carried by the quarks in the proton) is given by

$$\Delta\Sigma = \Delta u^p + \Delta d^p + \Delta s^p. \quad (\text{A28})$$

For the weak decay $n \rightarrow p + e^- + \bar{\nu}_e$ we can express the weak axial-vector form factor, g_A , in terms of the spin polarizations as

$$g_A = \Delta u^p - \Delta d^p. \quad (\text{A29})$$

APPENDIX B: SPIN POLARIZATIONS

1. Spin polarizations in the χQMg

The spin polarizations for the proton are given by

$$\Delta u^p = \cos^2 \theta_N \left[\frac{4}{3} - \frac{a}{3} \left(7 + 4\alpha^2 + \frac{4}{3}\beta^2 + \frac{8}{3}\zeta^2 \right) \right] + \sin^2 \theta_N \left[-\frac{2}{9} + \frac{a}{9} \left(5 + 2\alpha^2 + \frac{2}{3}\beta^2 + \frac{4}{3}\zeta^2 \right) \right] \quad (\text{B1})$$

$$\Delta d^p = \cos^2 \theta_N \left[-\frac{1}{3} - \frac{a}{3} \left(2 - \alpha^2 - \frac{1}{3}\beta^2 - \frac{2}{3}\zeta^2 \right) \right] + \sin^2 \theta_N \left[-\frac{1}{9} + \frac{a}{9} \left(4 + \alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2 \right) \right] \quad (\text{B2})$$

$$\Delta s^p = \cos^2 \theta_N (-a\alpha^2) + \sin^2 \theta_N \left(\frac{a}{3}\alpha^2 \right). \quad (\text{B3})$$

The spin polarizations for Σ^+ are given by

$$\Delta u^{\Sigma^+} = \cos^2 \theta_\Sigma \left[\frac{4}{3} - \frac{a}{3} \left(8 + 3\alpha^2 + \frac{4}{3}\beta^2 + \frac{8}{3}\zeta^2 \right) \right] + \sin^2 \theta_\Sigma \left[-\frac{2}{9} + \frac{a}{9} \left(4 + 3\alpha^2 + \frac{2}{3}\beta^2 + \frac{4}{3}\zeta^2 \right) \right] \quad (\text{B4})$$

$$\Delta d^{\Sigma^+} = \cos^2 \theta_\Sigma \left(\frac{a}{3}(-4 + \alpha^2) \right) + \sin^2 \theta_\Sigma \left(\frac{a}{9}(2 + \alpha^2) \right) \quad (\text{B5})$$

$$\Delta s^{\Sigma^+} = \cos^2 \theta_\Sigma \left[-\frac{1}{3} - \frac{a}{3} \left(2\alpha^2 - \frac{4}{3}\beta^2 - \frac{2}{3}\zeta^2 \right) \right] + \sin^2 \theta_\Sigma \left[-\frac{1}{9} + \frac{a}{9} \left(4\alpha^2 + \frac{4}{3}\beta^2 + \frac{2}{3}\zeta^2 \right) \right]. \quad (\text{B6})$$

The spin polarizations for Ξ^0 are given by

$$\Delta u^{\Xi^0} = \cos^2 \theta_\Xi \left[-\frac{1}{3} + \frac{a}{3} \left(2 - 3\alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2 \right) \right] + \sin^2 \theta_\Xi \left[-\frac{1}{9} + \frac{a}{9} \left(2 + 3\alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2 \right) \right] \quad (\text{B7})$$

$$\Delta d^{\Xi^0} = \cos^2 \theta_\Xi \left(\frac{a}{3}(1 - 4\alpha^2) \right) + \sin^2 \theta_\Xi \left(\frac{a}{9}(1 + 2\alpha^2) \right) \quad (\text{B8})$$

$$\Delta s^{\Xi^0} = \cos^2 \theta_\Xi \left[\frac{4}{3} - \frac{a}{3} \left(7\alpha^2 + \frac{16}{3}\beta^2 + \frac{8}{3}\zeta^2 \right) \right] + \sin^2 \theta_\Xi \left[-\frac{2}{9} + \frac{a}{9} \left(5\alpha^2 + \frac{8}{3}\beta^2 + \frac{4}{3}\zeta^2 \right) \right]. \quad (\text{B9})$$

The spin polarizations for Λ are given by

$$\Delta u^\Lambda = \cos^2 \theta_\Sigma (-a\alpha^2) + \sin^2 \theta_\Sigma \left(\frac{a}{3}\alpha^2 \right) \quad (\text{B10})$$

$$\Delta d^\Lambda = \cos^2 \theta_\Sigma (-a\alpha^2) + \sin^2 \theta_\Sigma \left(\frac{a}{3}\alpha^2 \right) \quad (\text{B11})$$

$$\Delta s^\Lambda = \cos^2 \theta_\Sigma \left(1 - \frac{a}{3}(6\alpha^2 + 4\beta^2 + 2\zeta^2) \right) + \sin^2 \theta_\Sigma \left[-\frac{1}{3} + \frac{a}{3} \left(2\alpha^2 + \frac{4}{3}\beta^2 + \frac{2}{3}\zeta^2 \right) \right]. \quad (\text{B12})$$

The spin polarizations for the other octet baryons are found from isospin symmetry.

2. Spin polarizations in the χ QMd

The spin polarizations for the proton are given by

$$\Delta u^p = \cos^2 \theta_N \left[\frac{4}{3} - \frac{a}{3} \left(7 + 4\alpha^2 + \frac{4}{3}\beta^2 + \frac{8}{3}\zeta^2 \right) \right] + \sin^2 \theta_N \left[1 - a \left(2 + \alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2 \right) \right] \quad (\text{B13})$$

$$\Delta d^p = \cos^2 \theta_N \left[-\frac{1}{3} - \frac{a}{3} \left(2 - \alpha^2 - \frac{1}{3}\beta^2 - \frac{2}{3}\zeta^2 \right) \right] + \sin^2 \theta_N (-a) \quad (\text{B14})$$

$$\Delta s^p = -a\alpha^2. \quad (\text{B15})$$

The spin polarizations for Σ^+ are given by

$$\Delta u^{\Sigma^+} = \cos^2 \theta_\Sigma \left[\frac{4}{3} - \frac{a}{3} \left(8 + 3\alpha^2 + \frac{4}{3}\beta^2 + \frac{8}{3}\zeta^2 \right) \right] + \sin^2 \theta_\Sigma \left[1 - a \left(2 + \alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2 \right) \right] \quad (\text{B16})$$

$$\Delta d^{\Sigma^+} = \cos^2 \theta_\Sigma \left(\frac{a}{3} (-4 + \alpha^2) \right) + \sin^2 \theta_\Sigma (-a) \quad (\text{B17})$$

$$\Delta s^{\Sigma^+} = \cos^2 \theta_\Sigma \left[-\frac{1}{3} - \frac{a}{3} \left(2\alpha^2 - \frac{4}{3}\beta^2 - \frac{2}{3}\zeta^2 \right) \right] + \sin^2 \theta_\Sigma (-a\alpha^2). \quad (\text{B18})$$

The spin polarizations for Ξ^0 are given by

$$\Delta u^{\Xi^0} = \cos^2 \theta_\Xi \left[-\frac{1}{3} + \frac{a}{3} \left(2 - 3\alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2 \right) \right] + \sin^2 \theta_\Xi (-a\alpha^2) \quad (\text{B19})$$

$$\Delta d^{\Xi^0} = \cos^2 \theta_\Xi \left(\frac{a}{3} (1 - 4\alpha^2) \right) + \sin^2 \theta_\Xi (-a\alpha^2) \quad (\text{B20})$$

$$\Delta s^{\Xi^0} = \cos^2 \theta_\Xi \left[\frac{4}{3} - \frac{a}{3} \left(7\alpha^2 + \frac{16}{3}\beta^2 + \frac{8}{3}\zeta^2 \right) \right] + \sin^2 \theta_\Xi \left[1 - a \left(2\alpha^2 + \frac{4}{3}\beta^2 + \frac{2}{3}\zeta^2 \right) \right]. \quad (\text{B21})$$

The spin polarizations for Λ are given by

$$\Delta u^\Lambda = \cos^2 \theta_\Sigma (-a\alpha^2) + \sin^2 \theta_\Sigma \left[\frac{1}{6} - \frac{a}{6} \left(3 + 5\alpha^2 + \frac{\beta^2}{3} + \frac{2\zeta^2}{3} \right) \right] \quad (\text{B22})$$

$$\Delta d^\Lambda = \cos^2 \theta_\Sigma (-a\alpha^2) + \sin^2 \theta_\Sigma \left[\frac{1}{6} - \frac{a}{6} \left(3 + 5\alpha^2 + \frac{\beta^2}{3} + \frac{2\zeta^2}{3} \right) \right] \quad (\text{B23})$$

$$\Delta s^\Lambda = \cos^2 \theta_\Sigma \left(1 - \frac{a}{3} (6\alpha^2 + 4\beta^2 + 2\zeta^2) \right) + \sin^2 \theta_\Sigma \left[\frac{2}{3} - \frac{a}{3} \left(5\alpha^2 + \frac{8\beta^2}{3} + \frac{4\zeta^2}{3} \right) \right]. \quad (\text{B24})$$

The spin polarizations for the other octet baryons are found from isospin symmetry.

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