# Atmospheric neutrino oscillation and a phenomenological lepton mass matrix

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We propose simple phenomenological lepton mass matrices which describe the three neutrinos almost degenerate in mass, leading to a very large mixing angle between $\nu_\mu$ and $\nu_\tau$, as consistent with a recent report on atmospheric neutrino oscillations from the SuperKamiokande Collaboration. Our matrix model also gives $\nu_e-\nu_\mu$ mixing in agreement with the value required for neutrino oscillation to explain the solar neutrino problem. [S0556-2821(98)03707-2]

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A recent report on the atmospheric neutrino from the SuperKamiokande Collaboration [1] has presented convincing evidence that the long-standing problem of the muon neutrino deficit in underground detectors [2] is indeed due to neutrino oscillation. The most surprising feature for theorists is a very large mixing angle close to maximal between $\nu_\mu$ and its oscillating partner in contrast with the quark sector for which mixing among different generations is all small. This points towards the lepton mass matrix being governed by a rule significantly different from the one that is relevant in the quark sector.

Accepting this atmospheric neutrino result from SuperKamiokande and assuming also that the solar neutrino problem is ascribed to neutrino oscillation (either matter enhanced [3] or usual oscillation in vacuum [4]), we may think of two distinct possibilities for the neutrino mass: i.e., (i) hierarchical massive neutrinos,

$$m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau}, \tag{1}$$

or (ii) almost degenerate massive neutrinos,

$$m_{\nu_e} \approx m_{\nu_\mu} \approx m_{\nu_\tau}, \tag{2}$$

where in both cases the $\nu_\mu-\nu_e$ mass difference is prescribed by oscillation for solar neutrinos, and $\nu_\tau-\nu_\mu$ by the atmospheric neutrino oscillation experiment.

In this paper we explore the possibility of whether the experimentally indicated lepton masses and mixings can be derived from a lepton mass matrix that is consistent with some simple symmetry principle, hopefully as parallel as possible to that for the quark sector. The problem of quark-lepton mass is one of the most difficult problems in particle physics, and we have no theory that predicts the mass matrix from a known principle. The best thing one can do now is to find a successful description of the mass matrix and look for some symmetry principle behind it. The most successful full mass matrix description in describing quark mass and mixing, at least at a phenomenological level, is the approach initiated by Fritzsch [5] and a number of its variants [6,7] (we call them phenomenological mass matrix approaches), although the basic physics of the dictated matrix is often not quite clear.

In our earlier paper we have presented a Fritzsch matrix type model that describes the case (i), in which the $\nu_\tau-\nu_\mu$ mixing angle comes out large when the $\nu_\mu-\nu_e$ mixing angle is small [8]. Indeed, the new SuperKamiokande result together with a small angle solution of the Mikheyev-Smirnov-Wolfenstein (MSW) scenario for the solar neutrino problem fits very well with the model, once one admits hierarchical massive neutrinos. In this paper we focus on the more heterodox possibility of the almost degenerate case, and discuss whether any simple, natural-looking mass matrix exists that leads to this unusual mass pattern together with a large mixing angle that explains atmospheric neutrino oscillation. We consider that the large difference between lepton and quark mixings should be ascribed to the Majorana character of the neutrino.

One of the most attractive descriptions of the quark sector in the phenomenological mass matrix approach starts with an $S_3(R)\times S_3(L)$ symmetric mass term (often called ''democratic'' mass matrix) [6], and adds a small term that breaks this symmetry [7], i.e.,

$$M_q=\frac{K_q}{3}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}+\begin{bmatrix}\delta_1^q&0&0\\0&\delta_2^q&0\\0&0&\delta_3^q\end{bmatrix}, \tag{3}$$

where $q=$up and down, and quarks belong to $\mathbf{3}=\mathbf{2}\oplus\mathbf{1}$ of $S_3(L)$ or $S_3(R)$ as discussed in [6]. The first term is a unique representation of the $S_3(R)\times S_3(L)$ symmetric matrix. This matrix is diagonalized as

$$U_q^\dagger M_q U_q=\mathrm{diag}(m_1^q,m_2^q,m_3^q), \tag{4}$$

where

 

$$m_1^q=(\delta_1^q+\delta_2^q+\delta_3^q)/3-\xi^q/6$$

$$m_2^q=(\delta_1^q+\delta_2^q+\delta_3^q)/3+\xi^q/6$$

$$m_3^q=K_q+(\delta_1^q+\delta_2^q+\delta_3^q)/3 \tag{5}$$

with

$$\xi=[(2\delta_3^q-\delta_2^q-\delta_1^q)^2+3(\delta_2^q-\delta_1^q)^2]^{1/2}, \tag{6}$$

where terms of $O(\delta/K)$ are ignored. The matrix that diagonalizes $U_q=AB_q$ reads

$$A=\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{bmatrix}, \tag{7}$$

$$B_q=\begin{bmatrix} \cos\theta^q & -\sin\theta^q & \lambda^q\sin 2\theta^q \\ \sin\theta^q & \cos\theta^q & \lambda^q\cos 2\theta^q \\ -\lambda^q\sin 2\theta^q & \lambda^q\cos 2\theta^q & 1 \end{bmatrix}, \tag{8}$$

with

$$\tan 2\theta^q\simeq-\sqrt{3}\,\frac{\delta_2^q-\delta_1^q}{2\delta_3^q-\delta_2^q-\delta_1^q}, \tag{9}$$

and $\lambda_q=(1/\sqrt{2})(1/3K)\,\xi_q$. $A$ is the matrix that diagonalizes the first term of Eq. (3). It has been shown [7] that all quark masses and mixing angles are successfully given by taking $\delta_1=-\epsilon_q$, $\delta_2=\epsilon_q$ and $\delta_3=\delta_q$ in Eq. (3), and adjusting these parameters in a way $K_q\gg\delta_q>\epsilon_q$.

We assume the same structure for the charged leptons, and denote the matrices with the script $\ell$ instead of $q$. Analogous to the quark sector we choose $\delta_1^\ell=-\epsilon_\ell$, $\delta_2^\ell=\epsilon_\ell$ and $\delta_3^\ell=\delta_\ell$. The three mass eigenvalues [see Eq. (5)] are then

$$m_1\simeq-\epsilon_\ell^2/2\delta_\ell,\quad m_2\simeq 2\delta_\ell/3+\epsilon_\ell^2/2\delta_\ell,\quad m_3\simeq K_\ell+\delta_\ell/3, \tag{10}$$

and the angle that appears in Eq. (8) is

$$\sin\theta_\ell\simeq-\sqrt{|m_1/m_2|}. \tag{11}$$

Let us turn to the neutrino sector. Assuming that the neutrinos are of the Majorana type, we have two invariant mass terms $\mathbf{2_L}\times\mathbf{2_L}$ and $\mathbf{1_L}\times\mathbf{1_L}$. Then, there are two candidate matrices that are invariant under $S_3(L)$:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\quad \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}. \tag{12}$$

Here we take the first form as the main mass term $M_\nu^{(0)}$ with a coefficient $K_\nu$, deferring discussion about the second matrix until later in this paper. We then break symmetry by adding a small term with two adjustable parameters. As a simple parametrization we take

$$M_\nu^{(1)}=\begin{bmatrix} 0 & \epsilon_\nu & 0 \\ \epsilon_\nu & 0 & 0 \\ 0 & 0 & \delta_\nu \end{bmatrix}. \tag{13}$$

An alternative natural choice to lift the mass degeneracy may be diag$(-\epsilon_\nu,\epsilon_\nu,\delta_\nu)$, which we shall also discuss later. The mass eigenvalues of $M_\nu=M_\nu^{(0)}+M_\nu^{(1)}$ are $K_\nu\pm\epsilon_\nu$, and $K_\nu+\delta_\nu$, and the matrix that diagonalizes $M_\nu$ ($U^TM_\nu U=$diagonal) is

$$U_\nu=\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{14}$$

That is, our $M_\nu$ represents three degenerate neutrinos, with the degeneracy lifted by small parameters. In the literature [9] degenerate neutrinos are discussed starting with $M_\nu=$diag(1,1,1) as an assumption. Our argument provides a reason for degenerate neutrinos by treating quarks and leptons in an equal-footing way.

The lepton mixing angle (Cabibbo-Kobayashi-Maskawa matrix) as defined by $V_\ell=(U_\ell)^\dagger U_\nu$ is thus given by

$$V_\ell=(AB_\ell)^\dagger U_\nu\simeq\begin{bmatrix} 1 & -(1/\sqrt{3})\sqrt{(m_e/m_\mu)} & (2/\sqrt{6})\sqrt{(m_e/m_\mu)} \\ \sqrt{(m_e/m_\mu)} & 1/\sqrt{3} & -2/\sqrt{6} \\ 0 & 2/\sqrt{6} & 1/\sqrt{3} \end{bmatrix}, \tag{15}$$

where $m_1$ and $m_2$ in Eq. (10) are identified with $m_e$ and $m_\mu$. We note that the neutrino mass parameters do not appear in this mixing matrix. The parameters $K_\ell$, $\delta_\ell$ and $\epsilon_\ell$ are determined so that the charged lepton analogue of Eq. (5) gives electron, $\mu$ and $\tau$ masses for the charged lepton sector, and $\epsilon_\nu K_\nu$ and $\delta_\nu K_\nu$ are fixed by the neutrino mass difference explored by the oscillation effect: $\Delta m_{32}^2=m_{\nu_3}^2-m_{\nu_2}^2\approx 0.5\times10^{-2}$ eV$^2$ [1] and $\Delta m_{21}^2=m_{\nu_2}^2-m_{\nu_1}^2\approx 0.8\times10^{-5}$ eV$^2$ [3] are obtained from the atmospheric and solar neutrino oscillation (we take the small angle solution of the MSW scenario for the solar neutrino problem [10]). The normalization $K_\nu$ is not fixed unless one of the neutrino masses is known, but it is not important for our argument, since the lepton mixing matrix is almost independent of the details of these parameters except for the $m_e/m_\mu$ ratio, as we see in Eq. (15) where small terms are ignored. If we retain all small terms, the lepton mixing angle is predicted to be

$$V_{\ell}=\begin{bmatrix} 0.998 & -0.045 & 0.05 \\ 0.066 & 0.613 & -0.787 \\ 0.005 & 0.789 & 0.614 \end{bmatrix} \tag{16}$$

instead of Eq. (15), where $K_{\ell}=1719$ MeV, $\delta_{\ell}=163$ MeV, $\epsilon_{\ell}=11$ MeV, $\delta_{\nu}=0.0025$ eV and $\epsilon_{\nu}=2\times10^{-6}$ eV are used and $K_{\nu}=1$ eV is assumed (the matrix depends very little on the assumption of $K_{\nu}$).

$\nu_{\mu}-\nu_{\tau}$ oscillation is then given by

$$P(\nu_{\mu}\to\nu_{\tau})\simeq 4V_{23}^2V_{33}^2\sin^2\left(\frac{\Delta m_{32}^2}{4E}L\right)\simeq\frac{8}{9}\sin^2\left(\frac{\Delta m_{32}^2}{4E}L\right), \tag{17}$$

which represents that mixing is close to maximal. With a more accurate matrix (16) the factor 8/9 is modified to 0.93. This means that the survival probability of $\nu_{\mu}$ is 54% for average neutrino oscillation, in very good agreement with the finding at the SuperKamiokande [1] (and also the result from Kamiokande [2]). For the $\nu_e-\nu_{\mu}$ oscillation $\sin^2 2\theta\simeq 8\times10^{-3}$, which also agrees with the neutrino mixing corresponding to the small angle solution of the MSW scenario for the solar neutrino problem [3,4].

Let us now discuss constraints placed on this scenario. Since we have assumed the Majorana type of neutrinos, we must require the condition that the presence of the effective Yukawa term

$$\mathcal{L}=\frac{h}{M}\ell_L\ell_L HH \tag{18}$$

($\ell_L$ is the left handed lepton doublet, $H$ the Higgs field, $M$ an effective mass and $h$ is the Yukawa coupling) should not erase the baryon number of the universe above the weak mass scale [11]. Namely, the condition reads

$$h^2/M^2<g/(M_{\rm planck}T) \tag{19}$$

with $g$ the effective number of relativistic degrees of freedom at temperature $T$ and $T$ is set equal to $10^{12}$ GeV [12], above which sphalerons do not work to violate $B+L$. This yields

$$m_{\nu}<\frac{h}{M}\langle H\rangle^2\simeq 1 \text{ eV}. \tag{20}$$

It is obvious that the scenario requires all neutrino masses to be larger than $\approx 0.07$ eV, the limit set by $\Delta m_{23}^2$ itself.

A very important constraint comes from neutrinoless double beta decay experiments. The latest result on the lifetime of $^{76}$Ge$\to^{76}$Se, $\tau_{1/2}>1.1\times10^{25}$ yr [13] yields an upper limit on the Majorana neutrino mass 0.4 eV [14] to 1.1 eV [15] depending on which nuclear model is adopted for nuclear matrix elements (see [16] for a review of the matrix element). This limit coincides with what is derived from the survival of the baryon number of the universe. We are then left with quite a narrow window for the neutrino mass 0.1 eV$\leq m_{\nu_e}\simeq m_{\nu_{\mu}}\simeq m_{\nu_{\tau}}\leq 1$ eV for the present scenario to be viable. It will be most interesting to push down the lower limit of neutrinoless double beta decay lifetime; if the limit on neutrino mass is lowered by one order of magnitude the degenerate neutrino mass scenario as discussed in this paper will be ruled out.

The argument we have made above is of course by no means unique, and a different assumption on the matrix leads to a different mass-mixing relation. Let us briefly discuss the consequence of the other matrices we have encountered in the line of our argument above. If we adopt the symmetry breaking term alternative to Eq. (13),

$$M_{\nu}^{(1)}=\begin{bmatrix} -\epsilon_{\nu} & 0 & 0 \\ 0 & \epsilon_{\nu} & 0 \\ 0 & 0 & \delta_{\nu} \end{bmatrix} \tag{21}$$

in parallel to the charged lepton and quark sectors, we obtain the lepton mixing matrix

$$V_{\ell}\simeq\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}. \tag{22}$$

This is identical to the matrix presented by Fritzsch and Xing [17], where they *assumed* the neutrino mass matrix basically identical to the case discussed here. For this case we obtain

$$\sin^2 2\theta_{12}\simeq 1, \quad \sin^2 2\theta_{23}\simeq 8/9. \tag{23}$$

The maximal mixing is derived for the (1,2) sector, whereas the mixing angle for the (2,3) sector is unchanged, again irrespective of the details of neutrino masses. Namely, this case can accommodate the ''just-so'' scenario for the solar neutrino problem due to neutrino oscillation in a vacuum [4], instead of the small angle solution of the MSW scenario. The constraints from double beta decay, baryon number of the universe etc. discussed above all apply to this case in the same way.

There is another branch of the argument within our framework. If the second form is adopted for $M_{\nu}^{(0)}$ in Eq. (12), we are led to small mixing angles for all neutrinos. Therefore, the choice of a diagonal form in Eq. (12) was crucial to obtain a large mixing angle for the lepton sector. We do not discuss this case further here.

In this paper we have shown that there exist simple lepton mass matrices derived from some symmetry principle with a simple breaking term, which gives rise to almost degenerate neutrinos with the (2,3) component almost maximally mixed. Our lepton matrix also gives a mixing angle for the (1,2) sector consistent with either the small angle solution of the MSW neutrino conversion scenario or the maximal mixing solution included in the ''just-so'' scenario of neutrino oscillation in vacuum, as required from the solar neutrino problem. The prediction we discussed for double beta decay is interesting, but does not depend on our specific model. The allowed window of the neutrino mass in our scenario is very narrow: this motivates us to push hard the double beta decay experiment to set a more stringent limit on the Majorana neutrino mass.

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