Space-time description of neutrino flavor oscillations

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Recently the issue of EPR-like correlations in the mutual probability of detecting a neutrino together with an accompanying charged lepton has received a new impetus. In this paper we describe this effect using the propagators of the particles involved in Schwinger's parametric integral representation. We find this description more simple and more suitable to the purpose than the usual momentum-space analysis. We consider the cases of a monochromatic neutrino source, wave packet source, and neutrino creation in a localized space-time region. In the latter case we note that the space-time oscillation amplitude depends on the values of the neutrino masses, and becomes rather small for large relative mass differences (mass hierarchy). We obtain the expressions for the oscillation and coherence lengths in various circumstances. In the region of overlap our results confirm those of Dolgov *et al.* [S0556-2821(98)03307-4]

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I. INTRODUCTION

The space-time oscillation of neutrino flavor (see [1,2]) is considered to be the most promising effect of observation which might indirectly establish a nonzero neutrino mass. By its very nature it requires a spatiotemporal description of the processes of neutrino creation, propagation, and detection, and of the similar processes that occur with the accompanying particles. Such a description has been performed in [3] and further developed in [4,5] without the ambiguities that sometimes accompany noncritical use of neutrino flavor eigenstates.

Recently [6,7] the issue of Einstein-Podolsky-Rosen-(EPR)-like correlations in the mutual probability of detecting both a neutrino and an accompanying charged lepton has received a new impetus. (It was previously considered in [3].) In order to simplify the derivation of the basic effects the authors of [7] combined together descriptions in configuration space and in momentum space, of the same relevant processes, using simultaneously such mutually exclusive notions as sharp wave packets in momentum space and definite space-time *a posteriori* trajectories of particles. EPR-like experiments of the same type involving neutral kaon and B meson oscillations were considered in [8]. In this paper the authors also adopted a simple approach using the action values on the particle classical trajectories to evaluate the relevant phase factors in the probability amplitude. Although the results obtained in such a simplified approach are correct, they also might call for a more careful derivation. This will be the aim of the present paper in which we consider the problem of neutrino flavor oscillations.

In this paper we try to analyze the phenomenon in a consistent way using the propagators of the particles involved in Schwinger's parametric integral representation [see Eq. (3) below]. We find this description more simple and more suitable to the purpose than the usual one which employs propagators in momentum space representation. The latter involves rather complicated momentum integrations (see, e.g., [4,9]) and, it seems, frequently obscures the physical picture of the phenomenon. Our treatment will be general and will contain the analysis of the EPR-like experiments of detecting a neutrino together with the accompanying charged lepton, as well as the standard textbook examples of neutrino flavor space-time oscillations.

After preliminaries in the following section, in Sec. III we consider the case of a monochromatic neutrino source and the probability of mutual detection of the neutrino and of the accompanying charged lepton. In Sec. IV the effect of a wave packet neutrino source is analyzed. We obtain the expressions for the oscillation and coherence lengths in various circumstances. The case of a neutrino source in a strongly localized space-time region will then be considered in Sec. V. In this case the space-time oscillation amplitude depends rather strongly on the values of the neutrino masses, and becomes rather small for large relative mass differences (neutrino mass hierarchy). We summarize our results in Sec. VI. In the Appendix we provide an alternative derivation of the probability amplitude for the case of a monochromatic neutrino source, in order to elucidate the difference between this case and the case of a neutrino source strongly localized in space-time.

II. PRELIMINARIES

Throughout this paper we consider a process in which a neutrino is created together with an accompanying charged lepton, and afterwards both particles are detected. The charged weak currents $\overline{l}O_\alpha\nu$ are involved in the description of this process, where $O_\alpha=\gamma_\alpha(1+\gamma_5)$, and $\gamma_5=i\gamma_0\gamma_1\gamma_2\gamma_3$. The amplitude of the creation of an l-ν pair at space-time point x is proportional to $O_\alpha J_S^\alpha(x)$, with $J_S^\alpha(x)$ being the source current responsible for this process. In the case of a pion we would have $J_S^\alpha(x)\propto\partial^\alpha\phi_\pi(x)$, where $\phi_\pi(x)$ is the pion wave function. The charged lepton produced at space-time point x_c in a flavor state a can propagate to space-time point x_l, and the neutrino to space-time point x_n, at which points these particles may be detected. At the space-time

*Permanent address. Email address: shtanov@ap3.gluk.apc.org point x_n one may detect neutrino-induced charged lepton production of flavor b. The amplitude of such a process in which a neutrino and the corresponding antilepton are created and subsequently detected will contain the factor

$$O_\beta J_D^\beta \sum_j U_{bj}^\dagger U_{ja} \int dx_c S_j(x_n, x_c) O_\alpha J_S^\alpha(x_c) S_a(x_c, x_l), \tag{1}$$

where U_{ja} is the unitary matrix of neutrino mass-flavor mixing amplitudes, $U_{bj}^\dagger$ is its Hermitian conjugate, a and b numerate flavors, j numerates the neutrino mass eigenstates, S_j and S_a are, correspondingly, the Feynman propagators of neutrino mass specie j with mass m_j and of charged lepton flavor a, and J_D^β is the current involved in the neutrino detection process, localized around the space-time point x_n.

The total amplitude that will describe the detection of charged lepton and neutrino events will contain, besides the factor (1), also positive and negative energy wave functions of different finite particles involved in the detection process. These factors are of a particular nature, they do not affect the dependence of the amplitude on the space-time coordinates x_l and x_n, hence they will be omitted as irrelevant to the main topic of this paper. Due to these factors, however, as well as, in typical cases, due to the positive-frequency character of the source, the integration region over x_c in (1) will be effectively restricted to the causal past of both points x_n and x_l.

The Feynman propagator $S(x,y) \equiv S_m(x-y)$ for the Dirac field of mass m has the form

$$S_m(x) = (i\gamma^\alpha \partial_\alpha + m) D_m(x), \tag{2}$$

where $D_m(x)$ is the Feynman propagator for the Klein-Gordon field of mass m. This propagator has the parametric integral representation (first considered by Schwinger, Dyson, and Feynman in the papers collected in [10])

$$D_m(x) = -\frac{1}{8\pi^2} \lim_{\epsilon \to +0} \int_0^\infty d\lambda \exp\left[-\frac{i}{2}\left(\lambda x^2 + \frac{1}{\lambda}[m^2 - i\epsilon] \right) \right], \tag{3}$$

where $x^2 = x \cdot x = x^\alpha x_\alpha$ is the Lorentz interval squared. The factors of type O_α in the amplitude Eq. (1) will have an effect that in the neutrino propagator the term proportional to a unit matrix will not contribute, and only that proportional to the Dirac gamma matrices will remain. This general property is due to the equality $O_\alpha O_\beta = 0$. Thus in (1) we can replace $S_j(x_n, x_c)$ by $\widetilde{S}_j(x_n, x_c) \equiv \widetilde{S}_{m_j}(x_n - x_c)$, where

$$\widetilde{S}_m(x) = i\gamma^\alpha \partial_\alpha D_m(x). \tag{4}$$

III. MONOCHROMATIC SOURCE

In this section we investigate the case of a monochromatic source current $J_S^\alpha(x)$ that can arise, for instance, in the process of pion decay. Let

$$J_S^\alpha(x) \propto e^{-ip \cdot x}, \tag{5}$$

with constant four-momentum p. In the case of a pion we would have $J_S^\alpha(x) \propto \partial^\alpha \phi_\pi(x) \propto p^\alpha \exp(-ip \cdot x)$, where $\phi_\pi(x)$ is the pion wave function. We make the notation

$$x_{nc} = x_n - x_c, \quad x_{lc} = x_l - x_c, \quad x_{nl} = x_n - x_l. \tag{6}$$

In the amplitude (1) we represent the propagators using (2)–(4), first perform integration over x_c, then over the parameters λ_l and λ_n that appear in the representation (3), respectively, for charged lepton and neutrino propagators. The integral over x_c is Gaussian, hence it can be evaluated exactly; preexponential factors can be obtained after integration over x_c by taking partial derivatives with respect to x_l and x_n according to (2). The remaining integral over λ_l, λ_n will be evaluated afterwards in the stationary phase approximation.

Consider the integral over x_c of one of the terms in the sum of (1). The phase in the exponent of the integrand will stem from the expression (3) for propagators, and from the source current in (1). It will be given by

$$\phi = -\frac{1}{2}\lambda_l x_{lc}^2 - \frac{1}{2}\lambda_n x_{nc}^2 - p \cdot x_c. \tag{7}$$

Its extremal point $x_c = x_c(\lambda_l, \lambda_n)$ is determined from the equation

$$\frac{\partial \phi}{\partial x_c} \equiv \lambda_l x_{lc} + \lambda_n x_{nc} - p = 0. \tag{8}$$

We also have for the matrix of the second derivatives

$$\frac{\partial^2 \phi}{\partial x_c^\alpha \partial x_c^\beta} = -(\lambda_l + \lambda_n) g_{\alpha\beta}, \tag{9}$$

so that integration over x_c will produce a factor

$$\int dx_c e^{i\phi} = \frac{4i\pi^2}{(\lambda_l + \lambda_n)^2} e^{i\phi_*}, \tag{10}$$

where ϕ_* is the value of the phase ϕ at the extremal point:

$$\phi_* = -\frac{\lambda_l \lambda_n x_{nl}^2 - m^2 + 2p \cdot (\lambda_l x_l + \lambda_n x_n)}{2(\lambda_l + \lambda_n)}. \tag{11}$$

Now consider the integral over the λ's. It will be evaluated in the stationary phase approximation. The phase of the integrand is given by

$$\Phi = \phi_* - \frac{1}{2}\left(\frac{m_l^2}{\lambda_l} + \frac{m_n^2}{\lambda_n} \right), \tag{12}$$

where m_l and m_n are the masses, respectively, of the charged lepton and of the neutrino. The stationary point is determined by differentiating (12) using (11), or by the equivalent conditions in the convenient form obtained using (7) and (8):

$$\frac{\partial \Phi}{\partial \lambda_l} \equiv -\frac{1}{2}x_{lc}^2 + \frac{m_l^2}{2\lambda_l^2} = 0, \quad \frac{\partial \Phi}{\partial \lambda_n} \equiv -\frac{1}{2}x_{nc}^2 + \frac{m_n^2}{2\lambda_n^2} = 0. \tag{13}$$

In these equations $x_c = x_c(\lambda_l, \lambda_n)$ is the solution of Eq. (8). From (13) we have the relation

$$\lambda_l x_{lc} = p_l\,, \quad \lambda_n x_{nc} = p_n\,, \tag{14}$$

satisfied by the extremal values of λ's, where p_l and p_n are the four-momenta that the charged lepton and the neutrino respectively would have were they free classical particles moving from the space-time creation point x_c respectively to the registration points x_l and x_n. Then Eq. (8) expresses the energy-momentum conservation law, the condition from which the extremal point x_c with extremal λ's can be found most easily.

We also need the matrix of the second derivatives of Φ over λ's at the extremal point. Differentiating the identity (8) we find

$$\frac{\partial x_c}{\partial \lambda_l} = \frac{x_{lc}}{\lambda_l + \lambda_n}, \quad \frac{\partial x_c}{\partial \lambda_n} = \frac{x_{nc}}{\lambda_l + \lambda_n}, \tag{15}$$

and, differentiating (13),

$$\frac{\partial^2 \Phi}{\partial \lambda_l^2} = -\frac{x_{lc}^2}{\lambda_l + \lambda_n} - \frac{m_l^2}{\lambda_l^3} = -m_l^2 \frac{2\lambda_l + \lambda_n}{\lambda_l^3(\lambda_l + \lambda_n)}, \tag{16}$$

$$\frac{\partial^2 \Phi}{\partial \lambda_n^2} = -\frac{x_{nc}^2}{\lambda_l + \lambda_n} - \frac{m_n^2}{\lambda_n^3} = -m_n^2 \frac{2\lambda_n + \lambda_l}{\lambda_n^3(\lambda_l + \lambda_n)}, \tag{17}$$

$$\frac{\partial^2 \Phi}{\partial \lambda_l \partial \lambda_n} = -\frac{x_{lc}\cdot x_{nc}}{\lambda_l + \lambda_n} = -\frac{p_l \cdot p_n}{\lambda_l \lambda_n (\lambda_l + \lambda_n)}. \tag{18}$$

Let

$$m = \sqrt{p \cdot p} \tag{19}$$

be the effective mass of the source. In the case of pion decay this will be equal to the pion mass m_π. In a realistic case

$$m - m_l \gg m_n\,, \quad m_l \gg m_n\,. \tag{20}$$

Below we will see [cf. Eq. (29)] that in the limit $m_n \to 0$ the extremal values of λ's remain finite. Thus we can approximate the determinant of the matrix $\partial^2\Phi/\partial\lambda_i\partial\lambda_j$ by its limit as $m_n \to 0$. The result is

$$\det\left(\frac{\partial^2 \Phi}{\partial \lambda_i \partial \lambda_j}\right) \approx -\left(\frac{\partial^2 \Phi}{\partial \lambda_l \partial \lambda_n}\right)^2 = -\left(\frac{p_l \cdot p_n}{\lambda_l \lambda_n (\lambda_l + \lambda_n)}\right)^2. \tag{21}$$

Therefore the integral over λ's will produce a factor

$$\frac{2\pi \lambda_l \lambda_n (\lambda_l + \lambda_n)}{p_l \cdot p_n} e^{i\Phi_*}, \tag{22}$$

where Φ_* is the extremal value of the phase Φ, which is given by

$$\Phi_* = -m_l \sqrt{x_{lc}^2} - m_n \sqrt{x_{nc}^2} - p \cdot x_c\,, \tag{23}$$

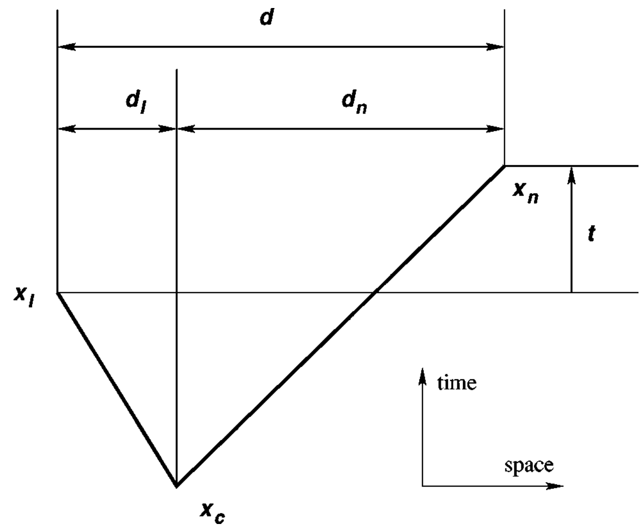

FIG. 1. Time and length definitions in the neutrino source rest frame.

with x_c being the extremal point of ϕ at extremal values of λ's—the solution to (8),(13). Using the extremality conditions (8),(13) we easily find

$$\frac{\partial \Phi_*}{\partial x_l} = -p_l\,, \quad \frac{\partial \Phi_*}{\partial x_n} = -p_n\,, \tag{24}$$

and

$$\Phi_* = -p_l \cdot x_l - p_n \cdot x_n = -p \cdot x_l - p_n \cdot x_{nl}\,. \tag{25}$$

Note that the four-momenta p_l and p_n lie in the plane formed by the four-vectors p and x_{nl}, and are determined by energy-momentum conservation.

Combining together the factors calculated in (10) and (22), dropping the resulting overall numerical constant $i/8\pi$, and using (25) we obtain the expression for the amplitude (1) in the case of a monochromatic source

$$e^{-ip\cdot x_l} \sum_j \frac{\lambda_l \lambda_n}{(\lambda_l + \lambda_n) p_l \cdot p_n} O_D \gamma_\alpha p_n^\alpha O_S (m_l - \gamma_\beta p_l^\beta) \times U_{bj}^\dagger U_{ja} e^{-ip_n \cdot x_{nl}}, \tag{26}$$

where $O_D = O_\alpha J_D^\alpha$, $O_S = O_\alpha J_S^\alpha$. Note that the coordinate dependence of the current $J_S(x)$ has transformed to the phase of (26). Also note that the extremal values of λ's as well as the four-momenta p_l and p_n under the sum (26) depend on the neutrino specie j. However, the prefactors in our expression (26), as well as in (30) below, are calculated only up to terms proportional to m_n^2; with this precision they can be taken in the limit $m_n = 0$.

The extremal values of λ_l and λ_n determined by the system of Eqs. (8),(13) can be easily obtained from the kinematics of the problem. Let us denote by t and by d correspondingly the time difference and the absolute spatial distance between the events x_n and x_l in the rest frame of the source [in which $p^\alpha = (m, \mathbf{0})$], and by v_l and v_n the velocities, respectively, of the charged lepton and of the neutrino in this frame (see Fig. 1). In the rest frame of the source one has

$$t_{lc}=\frac{d-v_n t}{v_l+v_n},\quad d_l:=|\mathbf{x}_{lc}|=v_l t_{lc},\quad t_{nc}=\frac{d+v_l t}{v_l+v_n},$$

$$d_n:=|\boldsymbol{x}_{nc}|=v_n t_{nc}, \tag{27}$$

where also d_l denotes the spatial distance in the source rest frame between the point x_l of charged lepton detection and the extremal point x_c, and d_n has the same meaning for a neutrino. Then

$$x_{lc}^2=\left(\frac{d-v_n t}{v_l+v_n}\right)^2(1-v_l^2),\quad x_{nc}^2=\left(\frac{d+v_l t}{v_l+v_n}\right)^2(1-v_n^2), \tag{28}$$

and, using (13), we obtain

$$\lambda_l=\frac{m_l}{\sqrt{x_{lc}^2}}=\frac{v_l E_l}{d_l},\quad \lambda_n=\frac{m_n}{\sqrt{x_{nc}^2}}=\frac{v_n E_n}{d_n}, \tag{29}$$

where E_l and E_n are the energies, respectively, of the charged lepton and of the neutrino in the rest frame of the source. In this notation and in the approximation of $m_n=0$ for the *prefactors* (but not for the phase) the amplitude (26) will acquire the form

$$\frac{e^{-ip\cdot x_l}}{md}O_D\gamma_\alpha p_n^\alpha O_S(m_l-\gamma_\beta p_l^\beta)\sum_j U_{bj}^\dagger U_{ja}e^{-ip_n\cdot x_{nl}}. \tag{30}$$

By the way, from the expressions (29) it is clear that the extremal values of λ's remain finite in the limit of $m_n\to 0$, as was stated above.

We shall now estimate the applicability limits of the stationary phase approximation used. Our approximation will be good when the extremal values of λ's are much larger than their dispersions determined by the matrix $\partial^2\Phi/\partial\lambda_i\partial\lambda_j$. Using (29) and (16)–(18) we obtain after straightforward analysis the conditions

$$d_l\leqslant d_n\ll md_l^2 \quad\text{or}\quad d_n\leqslant d_l\ll md_n^2, \tag{31}$$

under which our approximation is valid. They imply also the condition

$$md\gg 1, \tag{32}$$

which is quite reasonable.

In the limit of (20) the four-momenta p_l and p_n change relatively very slightly with the neutrino mass species j, and, we remember, our prefactors in (30) were actually calculated in this limit. In this case the space-time behavior of the l-ν pair detection probability is given by

$$P_{ba}(x_n,x_l)\propto\frac{1}{d^2}\Bigg[\sum_j |U_{bj}^\dagger U_{ja}|^2 + \sum_{j\neq k}|U_{bj}^\dagger U_{ja}U_{ak}^\dagger U_{kb}|\cos(p_{jk}\cdot x_{nl}+\varphi_{jk}^{ab})\Bigg], \tag{33}$$

where p_j and p_k are neutrino four-momenta of mass species, correspondingly, j and k, $p_{jk}=p_j-p_k$, and φ_{jk}^{ab} are constant phases that stem from the product of matrices U and $U^\dagger$. The second sum in the square brackets of (33) describes space-time oscillations of the probability. If one is interested in the *conditional* probability—that a neutrino event of flavor b is detected provided *some* neutrino event has been detected—this is given by the expression

$$P_{ba}^{(\mathrm{cond})}(x_n,x_l)=\frac{P_{ba}(x_n,x_l)}{\Sigma_b P_{ba}(x_n,x_l)}=\sum_j |U_{bj}^\dagger U_{ja}|^2 + \sum_{j\neq k}|U_{bj}^\dagger U_{ja}U_{ak}^\dagger U_{kb}|\cos(p_{jk}\cdot x_{nl}+\varphi_{jk}^{ab}). \tag{34}$$

In other words, this is the relative frequency of detecting a neutrino event of flavor b. It is normalized to unity as $\Sigma_b P_{ba}^{(\mathrm{cond})}(x_n,x_l)=1$.

From Eq. (33) or (34) one obtains the oscillation length and oscillation time of the probability considered. Using the energy-momentum conservation in the source rest frame of reference one has

$$p_{jk}^0=\frac{\Delta_{jk}}{2m},\quad |\boldsymbol{p}|_{jk}\equiv|\boldsymbol{p}_j|-|\boldsymbol{p}_k|\approx-\frac{\Delta_{jk}}{2v_l m}=-\frac{\Delta_{jk}E_l}{2v_n m E_n}, \tag{35}$$

where approximation uses the assumption (20) that a neutrino has very small mass, and

$$\Delta_{jk}=m_j^2-m_k^2. \tag{36}$$

Thus, oscillation length L_{osc} and oscillation time T_{osc} of the (jk) component of (33) in this frame of reference are given, respectively, by (we use the limit of $v_n=1$)

$$L_{\mathrm{osc}}=\frac{m}{E_l}L,\quad T_{\mathrm{osc}}=\frac{m}{E_n}L, \tag{37}$$

where

$$L=\frac{2E_n}{|\Delta_{jk}|} \tag{38}$$

is the standard expression. To proceed to any other reference frame what one has to do is to transform the four-vector components p_{jk}^α obtained in (35) to this new frame. The expressions (37) and the relevant expressions in the laboratory frame of reference have been obtained in [7].

If one of the particles, a charged lepton or a neutrino, is not observed, then the probability of detecting the other one is uniform in space and time. This is quite obvious without any calculations and is due to the fact that the l-ν pair creation probability for a monochromatic source is homogeneous in space and time. If a neutrino is not detected, oscillations in the charged lepton detection probability disappear also due to orthogonality of neutrino mass eigenstates. This last cause will operate with any type of source, not necessarily monochromatic. Specifically, it is the necessity of summing the probability over the neutrino flavor index b that will eliminate the oscillatory terms in this case. A detailed discussion of these issues is presented in [7].

IV. WAVE-PACKET SOURCE

First of all let us analyze in a little more detail the *effective* region of integration over x_c in (1) in the case of a monochromatic source considered in the previous section. In other words, it is the region of constructive interference from which most of the contribution to the integral in (1) comes. The extension of this region in space-time around the extremal point x_c is determined by the covariance matrix (9) for given values of λ's, and by the variation of λ's that are determined by the covariance matrix $\partial^2\Phi/\partial\lambda_i\partial\lambda_j$ with the components (16)–(18). First, using (9) and (29) we obtain the estimate of the linear dimensions δx of the effective region of integration for fixed extremal values of λ's as

$$\delta x \simeq (\lambda_l+\lambda_n)^{-1/2} = \sqrt{\frac{d_l d_n}{E_n d}} \leqslant \sqrt{\frac{d}{E_n}}. \tag{39}$$

Next, we must estimate the linear dimensions δx_c of the spread of the extremal value $x_c(\lambda_l,\lambda_n)$ caused by the spread $\delta\lambda$ of the values of λ's. This latter spread can be estimated using (21) as

$$\delta\lambda \simeq \left|\det\left(\frac{\partial^2\Phi}{\partial\lambda_i\partial\lambda_j}\right)\right|^{-1/4} \approx \sqrt{\frac{\lambda_l\lambda_n(\lambda_l+\lambda_n)}{p_l\cdot p_n}}. \tag{40}$$

Then using (15),(27),(29) and the condition $\delta\lambda \ll \lambda_{l,n}$ provided by Eq. (31), we will have the estimate

$$\delta x_c \simeq \frac{d\,\delta\lambda}{\lambda_l+\lambda_n} \simeq d\sqrt{\frac{\lambda_l\lambda_n}{p_l\cdot p_n(\lambda_l+\lambda_n)}} \approx \sqrt{\frac{d}{m}}. \tag{41}$$

Approximations ''≈'' in (40) and (41) use smallness of the neutrino mass and become equalities in the limit $m_n \to 0$. The last value in (39) is larger than that in (41) hence the dimension of the region of constructive interference will be estimated by (39).

In realistic situations the source of neutrinos can often be approximated by a wave packet with sharp energy and momentum distribution (therefore small relative energy-momentum spread). Let us suppose that in the rest frame of the source the spread in the coordinate space is σ_x, in momentum space σ_p, so that $\sigma_x\sigma_p \sim 1$. The source also has finite coherence time; in the case of the pion this will be determined by its lifetime, or by its collision time with the environment. Typically, however, the time spread σ_t of the wave packet is much larger than its spatial spread and the latter will determine most of the interesting effects.

If the source is to a high precision monochromatic so that its spatial and temporal spread is sufficiently large, namely, if

$$\delta x \ll \sigma_x, \sigma_t, \tag{42}$$

then we can use the expressions from the previous section for the detection probability amplitude whenever the effective region of integration over x_c (region of constructive interference) lies well within the source wave packet. Because of Eq. (39) the conditions (42) essentially imply

$$d \ll E_n \times \min(\sigma_x^2, \sigma_t^2). \tag{43}$$

Spatiotemporal oscillations in the mutual detection probability can be observed only up to certain relative distances between the detection points of a charged lepton and a neutrino. We are going to determine such maximal distances, called *coherence lengths*, beyond which oscillations cease to occur. The reason for such distances to exist is that for different neutrino masses m_j and m_k the corresponding centers (extremal points) x_j and x_k of the integration region in x_c are different. If they become sufficiently separated in space-time they may no longer be able to lie simultaneously within the wave packet of the source; thus components in the probability amplitude that correspond to different neutrino mass species will not be able to interfere.

Consider this effect quantitatively. The shift four-vector $x_{jk}=x_j-x_k$ lies in the plane of the four-momenta p_l and p_n, and can be decomposed into components $x_{jk}^{(l)}$ and $x_{jk}^{(n)}$ that go, respectively, along the directions of p_l and p_n. These components can be easily estimated. Using Eq. (27) we obtain, for the time components (approximation uses $v_n \approx 1$ and $\Delta v_l, \Delta v_n \ll v_l, v_n$),

$$t_{jk}^{(l)} \approx \frac{\Delta v_n}{(v_l+v_n)^2}(d+v_l t) \approx -\frac{\Delta_{jk}E_l}{2mE_n^2}d_n, \tag{44}$$

$$t_{jk}^{(n)} \approx \frac{\Delta v_l}{(v_l+v_n)^2}(d-v_n t) \approx -\frac{\Delta_{jk}m_l^2}{2m^2E_n^2}d_l, \tag{45}$$

where Δ_{jk} is given by (36). The distances d_l and d_n then change as

$$\Delta d_l = -\Delta d_n \approx v_n t_{jk}^{(n)} - v_l t_{jk}^{(l)} \approx -\frac{\Delta_{jk}m_l^2}{2m^2E_n^2}d_l + \frac{\Delta_{jk}}{2mE_n}d_n. \tag{46}$$

For large enough absolute values of $t_{jk}=t_{jk}^{(l)}+t_{jk}^{(n)}$ and/or Δd_l the centers x_j and x_k will not be able to lie simultaneously within the source wave packet. Components in the probability amplitude which correspond to different neutrino mass species will not be able to interfere, and probability will cease to oscillate. This will determine coherence lengths of the detection probability oscillations in various cases.

In the case

$$d_n = \frac{m_l^2}{mE_n}d_l \tag{47}$$

from (46) it follows that $\Delta d_l \approx 0$, and the shift x_{jk} is in the temporal direction in the source rest frame. For a sufficiently large value of $d=d_l+d_n$ the absolute value of the shift t_{jk} becomes larger than the extension σ_t of the source wave packet in the temporal direction. This determines coherence length L_c—the largest value of d at which oscillations can be observed. In the case considered (47) it is determined by using (44), (45), and (47) as

$$L_c=\sigma_t E_n L\left(1+\frac{mE_n}{m_l^2}\right), \tag{48}$$

with L given by (38). The condition (43) necessary for our approximation will imply that the formula (48) is valid for

$$\sigma_t,\frac{\sigma_x^2}{\sigma_t}\gg L\left(1+\frac{mE_n}{m_l^2}\right). \tag{49}$$

Equation (47) implies rather special experimental coincidence conditions. In a less special case

$$d_n>\frac{m_l^2}{mE_n}d_l, \tag{50}$$

the second term in the last expression of (46) dominates. In this case the coherence length will be determined by the condition that either $|t_{jk}|$ becomes larger than σ_t, or/and $|\Delta d_l|$ becomes larger than σ_x. Using (44) and (46) we obtain in this case

$$L_c=Lm\times\min\left(\sigma_x,\sigma_t\frac{E_n}{E_l}\right). \tag{51}$$

The condition (43) will determine the validity limit of (51) to be [see Eq. (37)]

$$\sigma_x,\frac{\sigma_t^2}{\sigma_x}\gg\frac{m}{E_n}L=T_{\rm osc};\quad \sigma_t,\frac{\sigma_x^2}{\sigma_t}\gg\frac{m}{E_l}L=L_{\rm osc}. \tag{52}$$

The case

$$d_n<\frac{m_l^2}{mE_n}d_l \tag{53}$$

can describe the situation when the momentum of the charged lepton is measured to a good accuracy by measuring its time of flight. Its analysis is quite similar to that of the previous cases. The oscillations in the probability will disappear at the length

$$L_c=\frac{m^2E_n}{m_l^2}L\times\min(\sigma_x,\sigma_t), \tag{54}$$

and the analysis is valid as long as the estimate (43) is satisfied, which gives

$$\sigma_x,\sigma_t,\frac{\sigma_x^2}{\sigma_t},\frac{\sigma_t^2}{\sigma_x}\gg\frac{m^2}{m_l^2}L. \tag{55}$$

If the source wave packet is sufficiently broad in space and if one of the particles, a charged lepton or a neutrino, is not observed, the probability of observing the other one will not oscillate in space and time. This is because after integrating the probability (33) over one of the variables $\{\boldsymbol{x}_l,\boldsymbol{x}_n\}$ the oscillatory terms are averaged to approximately zero. However, with the source wave packet sufficiently narrow in space [but still such that the condition (42) or its equivalent (43) holds], namely,

$$\sigma_x\ll L, \tag{56}$$

even if the accompanying charged lepton is not observed the neutrino flavor oscillations can be observed relative to the source spatial position. Indeed, when integrating the detection probability $P_{ba}(x_n,x_l)$ over $\boldsymbol{x}_l$ in the source rest frame we notice that only in a small region of $\boldsymbol{x}_l$ the probability will be nonzero and will be given by (33). This will be the region for which the extremal values of x_c lie within the wave packet of the source. The linear dimensions of this region are similar to $\sigma_x m/E_l$, as can be inferred from Fig. 1, and Eq. (56) follows from the condition that these dimensions be much smaller than the oscillation length $L_{\rm osc}$ which is given by (37). Then, after integration over $\boldsymbol{x}_l$ the phases of the oscillatory terms in (33) will be fixed and their dependence on the value of $\boldsymbol{x}_n$ will remain. Detection of a neutrino alone is the case most frequently discussed in the literature.

Consider this effect more thoroughly. The phase $p_{jk}\cdot x_{nl}$ in the oscillating term in the probability (33) can be written in terms of the distances d_l and d_n introduced in (27). Choosing the z axis in the direction of $\boldsymbol{p}_n$ in the source rest frame one will have

$$p_{jk}=(p_{jk}^0,0,0,|\boldsymbol{p}|_{jk}),\quad x_{nl}=(t,0,0,d). \tag{57}$$

Taking into account the values $p_{jk}^0=\Delta_{jk}/2m$, $|\boldsymbol{p}|_{jk}\approx-\Delta_{jk}E_l/2v_nmE_n$ [see Eq. (35)] and using (27), one obtains the standard expression for the phase

$$p_{jk}\cdot x_{nl}=\frac{\Delta_{jk}}{2v_nm}\left(d_n-\frac{E_l}{E_n}d_l\right)+\frac{\Delta_{jk}E_l}{2v_nmE_n}(d_n+d_l)=\frac{d_n}{v_nL}$$
$$=\frac{t_{nc}}{L}. \tag{58}$$

From this expression it is again clear that if the distance d_n is fixed with accuracy better than L by the position of the source wave packet relative to the neutrino detector, the phases of the probability oscillations will remain fixed even after integration of (33) over the unobserved point $\boldsymbol{x}_l$, and the oscillations will be observed with respect to the value of d_n. This condition again leads to the estimate (56).

Remarkably, the phase (58) does not depend on d_l. This fact can also be explained as follows. If d_n is fixed and d_l is changing, this means that the four-vector x_{nl} changes (say, by the amount Δx_{nl}) in the direction of the charged lepton's four-momentum p_l (this is clear from Fig. 1). Then the oscillation phase change is $p_{jk}\cdot\Delta x_{nl}\propto p_{jk}\cdot p_l=-\Delta p_l\cdot p_l=-\Delta(p_l^2)/2=0$. The expression (58) for the phase coincides with that derived in [7].

As noted already at the end of the previous section, and as it was discussed in [7], if a neutrino is not detected oscillations in the charged lepton detection probability disappear in any case due to orthogonality of neutrino mass eigenstates. Specifically, it is the summation of (33) over the neutrino flavor index b that will eliminate the oscillatory terms.

V. SOURCE IN A LOCALIZED SPACE-TIME REGION

First consider a hypothetical process in which a neutrino, together with a charged lepton, is created at a fixed space-time point x_c. Note that in this case the energy and momentum of the neutrino created is totally undetermined. The probability amplitude of detecting neutrino-induced charged lepton production of flavor b at the space-time point x_n will contain the factor

$$A_{ba}(x_n,x_c)=\sum_j U^\dagger_{bj}U_{ja}\widetilde{S}_j(x_{nc}), \tag{59}$$

if the charged lepton created together with the neutrino at point x_c is of flavor a.

Again, as was already discussed in Sec. II, the total amplitude that will describe the detection of a neutrino event will contain, besides the factor (59), also those related to the processes of creation, propagation, and detection of other particles involved. These factors, however, are of particular nature, and do not affect the dependence of the probability amplitude of neutrino detection on the space-time points x_n and x_c; hence they will be omitted.

The propagator $S_m(x)$ of Eq. (2) has the leading asymptotic behavior (see, e.g., [11])

$$S_m(x)\sim\left(\frac{e^{3\pi i/4}}{4\sqrt{2}\pi^{3/2}}\right)\frac{m^{3/2}}{(x^2)^{3/4}}\left(1+\frac{\gamma_\alpha x^\alpha}{\sqrt{x^2}}\right)\exp(-im\sqrt{x^2}),$$

$$\text{for}\quad m\sqrt{x^2}\gg 1, \tag{60}$$

$$S_m(x)\sim\frac{\gamma_\alpha x^\alpha}{2\pi^2(x^2)^2},\quad \text{for}\quad m\sqrt{x^2}\ll 1. \tag{61}$$

Hence, oscillations in the neutrino detection probabilities can develop in space and time around x_n only when $m_j\sqrt{x_{nc}^2}\gtrsim 1$ at least for the largest of the neutrino masses, since $\widetilde{S}_j(x_{nc})$ do not differ for different j in the opposite limit $m_j\sqrt{x_{nc}^2}\ll 1$. Consider, therefore, the case of $m_j\sqrt{x_{nc}^2}\gg 1$ for all j. In this limit the amplitude (59) up to one and the same factor will be given by

$$A_{ba}(x_n,x_c)\propto\sum_j m_j^{3/2}U^\dagger_{bj}U_{ja}\exp(-im_j\sqrt{x_{nc}^2}). \tag{62}$$

Note the mass dependence of the coefficients in the last equation. The space-time variation of the probabilities of the corresponding processes will be given by

$$\begin{aligned}P_{ba}(x_n,x_c)&=\mathrm{tr}[\cdots A^\dagger_{ba}(x_n,x_c)\cdots A_{ba}(x_n,x_c)\cdots]\\ &\propto\sum_j m_j^3|U^\dagger_{bj}U_{ja}|^2\\ &\quad+\sum_{j\neq k}(m_jm_k)^{3/2}|U^\dagger_{bj}U_{ja}U^\dagger_{ak}U_{kb}|\\ &\quad\times\cos(m_{jk}\sqrt{x_{nc}^2}+\varphi^{ab}_{jk}),\end{aligned} \tag{63}$$

where

$$m_{jk}=m_j-m_k, \tag{64}$$

and φ^{ab}_{jk} are constant phases that stem from the product of matrices U and $U^\dagger$. The last term in Eq. (63) describes space-time oscillations of the probabilities.

The conditional probability that a neutrino event of flavor b is detected provided *some* neutrino event has been detected is given by the expression

$$\begin{aligned}P^{(\mathrm{cond})}_{ba}(x_n,x_l)=N\Bigg[&\sum_j m_j^3|U^\dagger_{bj}U_{ja}|^2+\sum_{j\neq k}(m_jm_k)^{3/2}\\ &\times|U^\dagger_{bj}U_{ja}U^\dagger_{ak}U_{kb}|\cos(m_{jk}\sqrt{x_{nc}^2}+\varphi^{ab}_{jk})\Bigg],\end{aligned} \tag{65}$$

where the normalization factor N is given by

$$N^{-1}=\sum_j m_j^3U^\dagger_{aj}U_{ja}. \tag{66}$$

The conditional probability has the normalization property $\Sigma_bP^{(\mathrm{cond})}_{ba}(x_n,x_l)=1$ and gives the relative frequency of detection of the neutrino event of flavor b.

As an illustration consider mixing between two mass eigenstates ν_1 and ν_2, with two flavor eigenstates ν_μ and ν_e. Let, for definiteness, $m_1>m_2$,

$$\nu_\mu=\nu_1\cos\theta+\nu_2\sin\theta,\quad \nu_e=-\nu_1\sin\theta+\nu_2\cos\theta, \tag{67}$$

and let a neutrino be created in a flavor eigenstate ν_μ. The corresponding conditional probabilities of detecting muon and electron events at x_n will be

$$\begin{aligned}P^{(\mathrm{cond})}_\mu(x_n,x_c)=N[&m_1^3\cos^4\theta+m_2^3\sin^4\theta+2(m_1m_2)^{3/2}\\ &\times\sin^2\theta\cos^2\theta\cos(\Delta m\sqrt{x_{nc}^2})],\end{aligned} \tag{68}$$

$$\begin{aligned}P^{(\mathrm{cond})}_e(x_n,x_c)=N[&m_1^3+m_2^3-2(m_1m_2)^{3/2}\cos(\Delta m\sqrt{x_{nc}^2})]\\ &\times\sin^2\theta\cos^2\theta,\end{aligned} \tag{69}$$

where $\Delta m=m_1-m_2$, and

$$N^{-1}=m_1^3\cos^2\theta+m_2^3\sin^2\theta. \tag{70}$$

The last terms in the square brackets in Eqs. (68) and (69) describe space-time oscillations of the probabilities. Because of mass dependence of the coefficients in these expressions the amplitude of oscillations will be suppressed if $m_2\ll m_1$, and $\cos\theta$ is not too small.

The conditions $m_j\sqrt{x_{nc}^2}\gg 1$, together with the asymptotic form (60) of the propagator, imply that all the mass eigenstate neutrinos ν_j arrive at the space-time point x_n practically as on-mass-shell particles, with four-velocity $u=x_{nc}/\sqrt{x_{nc}^2}$. In the opposite limit $m\sqrt{x^2}\ll 1$ the Feynman propagator has the asymptotic behavior (61) independent of mass. Then, for instance, in our example of two neutrinos, in the region $m_2\sqrt{x_{nc}^2}\ll 1\ll m_1\sqrt{x_{nc}^2}$, which exists in the case of large relative mass difference, $m_2\ll m_1$, one obtains

$$P_\mu^{(\mathrm{cond})}(x_n,x_c)=N\left[\sin^4\theta+\frac{\pi}{8}\zeta^3\cos^4\theta+\sqrt{\frac{\pi}{2}}\zeta^{3/2}\sin^2\theta\cos^2\theta\right.$$
$$\left.\times\cos(\zeta-3\pi/4)\right], \tag{71}$$

$$P_e^{(\mathrm{cond})}(x_n,x_c)=N\left(1+\frac{\pi}{8}\zeta^3-\sqrt{\frac{\pi}{2}}\zeta^{3/2}\cos(\zeta-3\pi/4)\right)$$
$$\times\sin^2\theta\cos^2\theta, \tag{72}$$

where

$$\zeta=m_1\sqrt{x_{nc}^2},\quad N^{-1}=\sin^2\theta+\frac{\pi}{8}\zeta^3\cos^2\theta. \tag{73}$$

It is important to stress the difference between the cases of the sufficiently extended wave packet source and the fixed space-time point source. In the first case the probability of detecting a neutrino is given by Eq. (33) with the phase given by Eq. (58); in the second case the probability is given by Eq. (63). The origin of this difference lies in the fact that in the former case the amplitude (1) involves integration over x_c, whereas in the latter case the point x_c is fixed. In the asymptotic limit in which Eq. (60) is valid neutrino propagators have strong preexponential mass dependence that results in the peculiar mass dependence of the probability (63). With propagators in the asymptotic limit (60) it can be explicitly demonstrated that integration of the amplitude (1) over x_c in the case of a monochromatic source produces neutrino-mass-dependent factors that cancel out such preexponential neutrino-mass dependence of the probability amplitude and also modify the phase of the probability amplitude, leading to Eq. (30). In view of the derivation of Eq. (30) presented in Sec. III such a demonstration in the main text would be redundant. Therefore, in order to make this point clear, we perform it in the Appendix.

In reality, creation of a neutrino cannot occur at a fixed space-time point. However, a possible creation region might happen to be sufficiently localized by the nature of the source or by the experimenter, so that the phase differences between components of (62) will be well fixed. Thus the equations of this section will apply to the situation when a neutrino creation region is localized in space and time in such a way that

$$\delta[m_{jk}\sqrt{(x_n-x_c)^2}]\lesssim 1, \tag{74}$$

where by $\delta[f(x_c)]$ we signify characteristic variation of $f(x_c)$ due to variation of x_c over the creation region. As the probability oscillation phases $m_{jk}\sqrt{(x_n-x_c)^2}$ are symmetric with respect to $x_n\leftrightarrow x_c$ interchange, this means that the creation region is to be restricted in space and time by the *oscillation* time and length scales in the vicinity of the point x_n. Since for small variations

$$\delta[m_{jk}\sqrt{(x_n-x_c)^2}]\approx-\frac{m_{jk}x_{nc}\cdot\delta x_c}{\sqrt{x_{nc}^2}}, \tag{75}$$

these oscillation scales will be determined by the four-momentum differences $p_{jk}=m_{jk}x_{nc}/\sqrt{x_{nc}^2}$, with fixed four-vector $x_{nc}/\sqrt{x_{nc}^2}$ which is the neutrino four-velocity u at the detection event x_n.

The same is also true as regards the detection point: in order to observe space-time oscillations of the probability the detection point must be localized within the limits given by the oscillation time and length scales. If this is not so, that is, if the resolution of the detection point is insufficient, the oscillatory terms in the probabilities (68),(69),(71),(72) will be averaged, and only the so-called *global* effects of appearance and disappearance of neutrino flavor species will be observed.

Let us consider these criteria within the model of two neutrinos considered above. We have

$$p_1-p_2=u\Delta m=p_1\frac{\Delta m}{m_1}, \tag{76}$$

where p_1 is the four-momentum of the heavier neutrino mass specie at the detection point. In the case of close neutrino masses, $\Delta m\ll m_1$, the oscillation length and time will be given by

$$L_{\mathrm{osc}}\simeq T_{\mathrm{osc}}\simeq\frac{m_1}{E_n\Delta m}\approx\frac{m_1^2}{E_n^2}L, \tag{77}$$

where L is given by Eq. (38) and E_n is the neutrino energy at the detection point. Then Eqs. (68), (69) will be valid as long as

$$\sigma_x,\sigma_t<\frac{m_1^2}{E_n^2}L. \tag{78}$$

In the case of mass hierarchy, $m_1\gg m_2$, we have

$$p_1-p_2\simeq p_1, \tag{79}$$

so that

$$L_{\mathrm{osc}}\simeq T_{\mathrm{osc}}\simeq E_H^{-1}, \tag{80}$$

where E_H is the energy of the heavier neutrino at the detection point. The equations for the probabilities in this case are valid for

$$\sigma_x,\sigma_t<E_H^{-1}. \tag{81}$$

Due to incoherent distribution of the sources in realistic situations (in a supernova or in the Sun), or due to insufficient resolution of the space-time detection point as was discussed above, the probabilities (68),(69),(71),(72) will be averaged over the source and/or detection sites and the oscillatory terms are likely to be averaged to give zero. In this case the probabilities will describe the so-called *global* effects of appearance and disappearance of neutrino flavor species. To obtain the relevant expressions in this case one must consider the integral of the (unaveraged, unconditioned) probability $P_\mu(x_n,x_c)$ or $P_e(x_n,x_c)$ over the space-time points x_n, x_c using the asymptotic expressions (60) and (61) for the propagators and the property of the trace

$\mathrm{tr}\gamma^{\dagger}_{(\alpha}\gamma_{\beta)}=4\delta_{\alpha\beta}$. Thus the expressions for averaged unconditioned probabilities P_μ and P_e will be

$$P_\mu=C\int_\Delta dx_{nc}(\mathrm{tr}[\tilde{S}_1^\dagger(x_n,x_c)\tilde{S}_1(x_n,x_c)]\cos^4\theta + \mathrm{tr}[\tilde{S}_2^\dagger(x_n,x_c)\tilde{S}_2(x_n,x_c)]\sin^4\theta), \tag{82}$$

$$P_e=C\int_\Delta dx_{nc}(\mathrm{tr}[\tilde{S}_1^\dagger(x_n,x_c)\tilde{S}_1(x_n,x_c)] + \mathrm{tr}[\tilde{S}_2^\dagger(x_n,x_c)\tilde{S}_2(x_n,x_c)])\sin^2\theta\cos^2\theta, \tag{83}$$

where C is a constant, $S_j=S_{m_j}$ as before, the expression for $\tilde{S}_m$ is given by Eq. (4), and the integrals proceed over the effective region Δ of the values of x_{nc} determined by the distribution of sources and detection points.

In the case $m_j\sqrt{x_{nc}^2}\gg 1$ the averaged *conditional* probabilities will look like

$$P_\mu^{(\mathrm{cond})}=\frac{m_1^3\cos^4\theta+m_2^3\sin^4\theta}{m_1^3\cos^2\theta+m_2^3\sin^2\theta}, \tag{84}$$

$$P_e^{(\mathrm{cond})}=\frac{(m_1^3+m_2^3)\sin^2\theta\cos^2\theta}{m_1^3\cos^2\theta+m_2^3\sin^2\theta}. \tag{85}$$

In the case $m_2\sqrt{x_{nc}^2}\ll 1\ll m_1\sqrt{x_{nc}^2}$ the averaged probabilities are more complicated because of more complicated space-time dependence of the corresponding unaveraged probabilities. The result is

$$P_\mu^{(\mathrm{cond})}=\frac{\sin^4\theta+\eta\cos^4\theta}{\sin^2\theta+\eta\cos^2\theta}, \tag{86}$$

$$P_e^{(\mathrm{cond})}=\frac{(1+\eta)\sin^2\theta\cos^2\theta}{\sin^2\theta+\eta\cos^2\theta}, \tag{87}$$

where η is a weighted average of the value of $(\pi/8)\,\zeta^3=(\pi/8)\,m_1^3(x_{nc}^2)^{3/2}$ and is given by

$$\eta=\frac{\pi m_1^3\int_\Delta |x|^2(x^2)^{-5/2}dx}{8\int_\Delta |x|^2(x^2)^{-4}dx}. \tag{88}$$

In the last expression $|x|^2=(x^0)^2+|\mathbf{x}|^2$, the integrals proceed over the effective region Δ of the values of $x=x_{nc}$ determined by the distribution of sources and detection points, and the space-time components of x_{nc} are to be taken in the detection reference system.

To see whether the assumption of a well-localized neutrino source is realistic, let us make some estimates for the cases of solar and supernova neutrinos. We take all the data from the book [2]. In all these cases one has $\sigma_x\ll\sigma_t$, and it is the value of σ_t which will be important.[1] In the case of solar neutrinos

$$\sigma_t\sim 10^{-7}\ \mathrm{cm},\quad E_n\sim 10\ \mathrm{MeV}. \tag{89}$$

With $\Delta_{12}=m_1^2-m_2^2\sim 10^{-4}\ \mathrm{eV}^2$ in the case of close neutrino masses the condition (78), under which the formulas of this section will apply, will read

$$m_1\gtrsim 3\ \mathrm{eV}. \tag{90}$$

Since $E_n^{-1}\sim 10^{-12}$ cm, in the case of mass hierarchy the formulas of this section will not work.

For supernova neutrinos from the core

$$\sigma_t\sim 10^{-14}\text{–}10^{-13}\ \mathrm{cm},\quad E_n\sim 100\ \mathrm{MeV}, \tag{91}$$

so that $E_n^{-1}\sim 10^{-13}$ cm, the condition (81) will be on the edge of fulfillment and the formulas of this section might be applicable.

For supernova neutrinos from the neutrino sphere

$$\sigma_t\sim 10^{-9}\ \mathrm{cm},\quad E_n\sim 10\ \mathrm{MeV}, \tag{92}$$

in the case of mass hierarchy the expressions of this section will not be applicable. In the case of close neutrino masses for $\Delta_{12}=m_1^2-m_2^2\sim 10^{-4}\ \mathrm{eV}^2$ we will have the condition

$$m_1\gtrsim 0.3\ \mathrm{eV}, \tag{93}$$

for which the relevant expressions of this section will apply.

In connection with the above examples we must note that the probabilities (68)–(72) will refer to neutrinos as they appear from the source. Subsequent neutrino scattering off the particles of the solar or supernova media will result in the well-known Mikheyev-Smirnov-Wolfenstein effect (see [1,2]), which is not considered in this paper.

To end this section let us stress once again that the criteria under which the equations of this section will apply are given by Eq. (78) in the case of close neutrino masses, and by Eq. (81) in the case of neutrino mass hierarchy. In the previous sections we discussed the case of a neutrino source with sharp energy and momentum distribution; the formulas derived there are valid under conditions (31) and (43). Note that these two cases, namely, of broad wave-packet neutrino source and neutrino source localized in space-time, are exclusive but not exhaustive. Intermediate cases when neither (31) and (43) nor (78) or (81) are valid should be studied separately, perhaps by different methods.

[1]Note that in [2] as well as in some other literature σ_x usually stands for the emitted *neutrino* wave packet spread. In this paper both σ_x and σ_t denote the spread of the neutrino *source*. Also note that we put the speed of light as well as the Planck constant to unity and measure σ_t in units of length.

VI. SUMMARY

In this paper we treated the problem of neutrino flavor oscillations by consistently using space-time description of the relevant processes of particle creation and subsequent detection. We described the EPR-like experiments of detecting a neutrino together with the accompanying charged lepton, as well as the standard textbook examples of neutrino flavor space-time oscillations, without invoking *a priori* the notion of particle trajectories. From our analysis it is also clear why in fact it is possible to use such a notion. The effective integration region (the region of constructive interference) in the probability amplitude (1) over the space-time point x_c of particle creation is localized around the place determined by particle classical trajectories, and the contribution to the phase of the amplitude comes mainly from the action along these trajectories. In the case of a wave packet neutrino source our treatment enabled us to obtain in a rather simple way the coherence lengths of the oscillations. We also considered the case of a neutrino source strongly localized in space and time and in this case found out dependence of the probability oscillation amplitude on the neutrino masses.

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APPENDIX: ALTERNATIVE DERIVATION OF THE PROBABILITY AMPLITUDE

In this appendix we shall derive the expression for the probability amplitude (30) in the case of a monochromatic source, in the limit $m_n\sqrt{x_{nc}^2}\gg 1$, $m_l\sqrt{x_{lc}^2}\gg 1$, using the asymptotic expression (60) for the propagators in the coordinate representation. The integral over x_c in (1) will be evaluated in the stationary phase approximation, according to the assumption that the main contribution comes from the region of stationary phase of the integrand. This phase stems from the propagators and from the source current in (1) and is given by the expression

$$\Phi(x_l,x_n,x_c)=-m_l\sqrt{x_{lc}^2}-m_n\sqrt{x_{nc}^2}-p\cdot x_c, \tag{A1}$$

and its stationary point is determined from the condition

$$\frac{\partial\Phi}{\partial x_c^\alpha}\equiv m_l\hat{x}_{lc}^\alpha+m_n\hat{x}_{nc}^\alpha-p^\alpha=0, \tag{A2}$$

where the notation is used $\hat{x}=x/\sqrt{x^2}$. Since $\hat{x}_{lc}$ and $\hat{x}_{nc}$ are just four-velocities, respectively, of the charged lepton and the neutrino at their respective detection points, Eq. (A2) expresses the total energy-momentum conservation. Note that for different neutrino masses $m_n=m_j$ the value of x_c determined by Eq. (A2) will be different. Let x_j be the solution for x_c of Eq. (A2) with $m_n=m_j$. For further convenience we make the notation $x_l-x_j=x_{lj}$, $x_n-x_j=x_{nj}$, and $\hat{x}_{lj}=u_l$, $\hat{x}_{nj}=u_n$. The phase Φ can be developed in powers of $x=x_c-x_j$ around the stationary point $x=0$ with the result

$$\Phi(x_l,x_n,x_c)=\frac{1}{2}C_{\alpha\beta}x^\alpha x^\beta+\cdots, \tag{A3}$$

where

$$C_{\alpha\beta}=c_l(u_l)_\alpha(u_l)_\beta+c_n(u_n)_\alpha(u_n)_\beta-(c_l+c_n)g_{\alpha\beta}, \tag{A4}$$

$$c_l=\frac{m_l}{\sqrt{x_{lj}^2}},\quad c_n=\frac{m_j}{\sqrt{x_{nj}^2}}. \tag{A5}$$

Dropping the higher order terms in (A3), denoted by dots, we will be interested in the value of a Gaussian integral

$$\int d^4x\exp\left(\frac{i}{2}C_{\alpha\beta}x^\alpha x^\beta\right). \tag{A6}$$

This, up to a constant factor, is given by $|\det\{C_{\alpha\beta}\}|^{-1/2}$. In terms of the velocities v_l and v_n, respectively, of the charged lepton and the neutrino in the source rest frame the determinant is given by

$$\det\{C_{\alpha\beta}\}=c_lc_n(c_l+c_n)^2[(u_l\cdot u_n)^2-1]=c_lc_n(c_l+c_n)^2\times\frac{(v_l+v_n)^2}{(1-v_l^2)(1-v_n^2)}. \tag{A7}$$

The values of c_l and c_n given by (A5) can be easily seen to coincide with the extremal values, respectively, of λ_l and λ_n determined by the system of Eqs. (8), (13), and given by (29).

Combining all the factors in (1) together we obtain the expression for the amplitude in the limit of $m_n\to 0$ for the prefactors and up to an irrelevant constant as

$$\sum_j\frac{c_lc_n}{(c_l+c_n)}\frac{\sqrt{(1-v_l^2)(1-v_n^2)}}{v_l+v_n}\times O_D\gamma_\alpha u_n^\alpha O_S(1-\gamma_\beta u_l^\beta)U_{bj}^\dagger U_{ja}e^{i\Phi_j}=\frac{1}{md}O_D\gamma_\alpha p_n^\alpha O_S(m_l-\gamma_\beta p_l^\beta)\sum_j U_{bj}^\dagger U_{ja}e^{i\Phi_j}, \tag{A8}$$

where Φ_j is the value of the phase at the stationary point of $x_c=x_j$ that corresponds to neutrino mass specie j. In view of the expression (A1) for the phase we see that this value is identical to that of Eq. (23). Therefore the expression (A8) for the amplitude coincides with that of Eq. (30).

Note that the preexponential factors in the final expression (A8) for the amplitude remain finite in the limit of $m_n=0$ in spite of the fact that neutrino propagators (60) have preexponential factors $m_n^{3/2}$. The reason is that the strong neutrino-mass dependence of these factors has been counterbalanced by the neutrino-mass dependence of the value x_{nj}^2 as well as of the factor $|\det\{C_{\alpha\beta}\}|^{-1/2}$, with the determinant given by (A7). In the case of a fixed neutrino creation point there is no integration over x_c and factors $m_n^{3/2}$ remain in the probability amplitude, Eq. (62).

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