Isoscalar resonances with $J^{PC}=1^{--}$ in e^+e^- annihilation

N. N. Achasov* and A. A. Kozhevnikov†
Laboratory of Theoretical Physics, S.L. Sobolev Institute for Mathematics, 630090, Novosibirsk 90, Russian Federation

(Received 25 March 1997; published 9 March 1998)

The analysis of the vector isoscalar excitations in the energy range between 1 and 2 GeV of the e^+e^- annihilation is presented for the final states $\pi^+\pi^-\pi^0$, $\omega\pi^+\pi^-$, K^+K^-, $K^0_SK^\pm\pi^\mp$, and $K^{*0}K^-\pi^+$ + c.c. The effects of both resonance mixing and the successive opening of multiparticle channels, with energy-dependent partial widths, are taken into account. The work extends our previous analysis of vector isovector excitations and is aimed at comparing the existing data with the predictions of the $q\bar{q}$ model. It is shown that this hypothesis does not contradict the data. [S0556-2821(98)01905-5]

PACS number(s): 13.65.+i, 13.25.Jx, 14.40.Cs

I. INTRODUCTION

The issue of excitations with quantum numbers $J^{PC}=1^{--}$ [1] in the energy range between 1 and 2 GeV of e^+e^- annihilation still remains, to a large extent, an unresolved one. Are they recurrences of the ground state nonet of $\rho(770)$, $\omega(782)$, and $\varphi(1020)$, or do they have an exotic nature [2–8]? In the former case, to what extent are the flavor SU(3) predictions good for them? In the latter case, are they completely exotic, and if not, what is the admixture of the exotic non-$q\bar{q}$ component? To answer these and similar questions, one should extract the masses and coupling constants of bare resonances in order to compare them with various models. As we have shown earlier [9] in the case of the vector-isovector ρ-like excitations, taking into account both the effects of resonance mixing and the fast energy growth of the partial widths of successively opened multiparticle channels affects the specific masses and coupling constants extracted from the data.

The present paper is aimed at extending a similar treatment to the case of vector-isoscalar ω- and φ-like excitations. To this end we analyze the data on the reactions

$$e^+e^-\to\pi^+\pi^-\pi^0 \quad [10\text{–}12], \tag{1.1}$$

$$e^+e^-\to\omega\pi^+\pi^- \quad [11], \tag{1.2}$$

$$e^+e^-\to K^+K^- \quad [13,14], \tag{1.3}$$

$$e^+e^-\to K^{*0}\overline{K^0}+\text{c.c.}\to K^0_SK^\pm\pi^\mp \quad [15], \tag{1.4}$$

$$e^+e^-\to K^{*0}K^-\pi^+(\overline{K^{*0}}K^+\pi^-) \quad [16], \tag{1.5}$$

allowing for contributions of the $\omega'_{1,2}$ $\varphi'_{1,2}$ resonances, in order to extract the masses and coupling constants of these excitations to various channels. The main result is that, within very large errors determined by poor data samples, the extracted parameters do not contradict the simple $q\bar{q}$ model of the ω- and φ-like resonances. The magnitudes of the $q\bar{q}$ bound state wave function at the origin are extracted from the magnitudes of the leptonic widths, yet the accuracy still does not permit one to verify the traditional $q\bar{q}$ assignment [1,17] of heavier excitations and to draw any definite conclusions about the interquark potential.

The paper is organized as follows. Section II contains the expressions for the cross sections and a discussion of the assumptions made about the interaction vertices and the coupling constants. Section III is devoted to the presentation of the results of our analysis, which are discussed in Sec. IV. Section V sketches possible further work necessary for the improvement of the situation with excitations with masses above 1 GeV.

II. BASIC FORMULAS REQUIRED FOR THE ANALYSIS

A. Expressions for the cross sections

An exact application of the explicitly unitary method [18] of taking the mixing of resonances into account is computationally time consuming in the present case, since it demands the inversion of the 9×9 matrix of inverse propagators whose elements are complex numbers. Instead, we take into account the mixing inside each sector, $\rho(770)$ -ρ'_1 -ρ'_2, $\omega(782)$ -ω'_1 -ω'_2, or $\varphi(1020)$ -φ'_1 -φ'_2 to all orders, while the terms which break the Okubo-Zweig-Iizuka (OZI) rule are taken into account to first order. Then the cross section of production of the final state f in e^+e^- annihilation can be represented as

*Email address: achasov@math.nsc.ru
†Email address: kozhev@math.nsc.ru

$$\sigma_f=\frac{(4\pi\alpha)^2}{s^{3/2}}\Bigg|(g_{\gamma\rho},g_{\gamma\rho_1'},g_{\gamma\rho_2'})G_\rho^{-1}(s)\begin{pmatrix}g_{\rho f}\\ g_{\rho_1' f}\\ g_{\rho_2' f}\end{pmatrix}+(g_{\gamma\omega},g_{\gamma\omega_1'},g_{\gamma\omega_2'})G_\omega^{-1}(s)\begin{pmatrix}g_{\omega f}\\ g_{\omega_1' f}\\ g_{\omega_2' f}\end{pmatrix}+(g_{\gamma\varphi},g_{\gamma\varphi_1'},g_{\gamma\varphi_2'})G_\varphi^{-1}(s)\begin{pmatrix}g_{\varphi f}\\ g_{\varphi_1' f}\\ g_{\varphi_2' f}\end{pmatrix}$$

$$+(g_{\gamma\omega},g_{\gamma\omega_1'},g_{\gamma\omega_2'})G_{\rm OZI}^{(1)}(s)\begin{pmatrix}g_{\varphi f}/D_\varphi\\ g_{\varphi_1' f}/D_{\varphi_1'}\\ g_{\varphi_2' f}/D_{\varphi_2'}\end{pmatrix}+(g_{\gamma\varphi},g_{\gamma\varphi_1'},g_{\gamma\varphi_2'})G_{\rm OZI}^{(2)}(s)\begin{pmatrix}g_{\omega f}/D_\omega\\ g_{\omega_1' f}/D_{\omega_1'}\\ g_{\omega_2' f}/D_{\omega_2'}\end{pmatrix}\Bigg|^2 P_f, \qquad (2.1)$$

where $f=\pi^+\pi^-\pi^0$, $\omega\pi^+\pi^-$, K^+K^-, $K_S^0K^+\pi^-$, $K^{*0}K^-\pi^+$; s is the total center-of-mass energy squared, $\alpha=1/137$. The leptonic widths on the mass shell of the unmixed states are expressed through the $\gamma\to V$ transition amplitudes $g_{\gamma V}$ and the leptonic coupling constants f_V as usual:

$$\Gamma_{Ve^+e^-}=\frac{4\pi\alpha^2 g_{\gamma V}^2}{3m_V^3},$$

$$g_{\gamma V}=\frac{m_V^2}{f_V}. \qquad (2.2)$$

The matrices entering into Eq. (2.1) are, respectively,

$$G_V(s)=\begin{pmatrix}D_V & -\Pi_{VV_1'} & -\Pi_{VV_2'}\\ -\Pi_{VV_1'} & D_{V_1'} & -\Pi_{V_1'V_2'}\\ -\Pi_{VV_2'} & -\Pi_{V_1'V_2'} & D_{V_2'}\end{pmatrix} \qquad (2.3)$$

$(V=\rho,\omega,\varphi)$ and

$$G_{\rm OZI}^{(1)}(s)=\begin{pmatrix}\frac{\Pi_{\varphi\omega}}{D_\omega} & \frac{\Pi_{\varphi_1'\omega}}{D_\omega} & \frac{\Pi_{\varphi_2'\omega}}{D_\omega}\\ \frac{\Pi_{\varphi\omega_1'}}{D_{\omega_1'}} & \frac{\Pi_{\varphi_1'\omega_1'}}{D_{\omega_1'}} & \frac{\Pi_{\varphi_2'\omega_1'}}{D_{\omega_1'}}\\ \frac{\Pi_{\varphi\omega_2'}}{D_{\omega_2'}} & \frac{\Pi_{\varphi_1'\omega_2'}}{D_{\omega_2'}} & \frac{\Pi_{\varphi_2'\omega_2'}}{D_{\omega_2'}}\end{pmatrix}, \qquad (2.4a)$$

$$G_{\rm OZI}^{(2)}(s)=\begin{pmatrix}\frac{\Pi_{\varphi\omega}}{D_\varphi} & \frac{\Pi_{\varphi\omega_1'}}{D_\varphi} & \frac{\Pi_{\varphi\omega_2'}}{D_\varphi}\\ \frac{\Pi_{\varphi_1'\omega}}{D_{\varphi_1'}} & \frac{\Pi_{\varphi_1'\omega_1'}}{D_{\varphi_1'}} & \frac{\Pi_{\varphi_1'\omega_2'}}{D_{\varphi_1'}}\\ \frac{\Pi_{\varphi_2'\omega}}{D_{\varphi_2'}} & \frac{\Pi_{\varphi_2'\omega_1'}}{D_{\varphi_2'}} & \frac{\Pi_{\varphi_2'\omega_2'}}{D_{\varphi_2'}}\end{pmatrix}, \qquad (2.4b)$$

where the inverse propagators of the bare states, $D_V\equiv D_V(s)$, and the nondiagonal polarization operators $\Pi_{VV'}\equiv\Pi_{VV'}(s)$ responsible for the mixing are discussed below in Sec. II C. In what follows we will often use also the notation V_i $(i=1,2,3)$ such that V_1,V_2,V_3 corresponds to V,V_1',V_2', and $V=\rho,\omega,\varphi$.

The factor P_f for the final states $f=\pi^+\pi^-\pi^0$, $\omega\pi^+\pi^-$, K^+K^-, $K_S^0K^+\pi^-$, $K^{*0}K^-\pi^+$ reads, respectively,

$$P_f\equiv P_f(s)=\frac{W_{3\pi}(\sqrt{s})}{4\pi},W_{VP\pi}(\sqrt{s},m_\omega,m_\pi),$$

$$\frac{q_{KK}^3}{6\pi s},\frac{1}{2}\frac{q_{K^*\bar{K}}^3}{12\pi},W_{VP\pi}(\sqrt{s},m_{K^*},m_K), \qquad (2.5)$$

where

$$\langle q_{K^*\bar{K}}^3\rangle=\int_{(m_K+m_\pi)^2}^{(\sqrt{s}-m_K)^2}\frac{dm^2 m_{K^*}\Gamma_{K^*}/\pi}{(m^2-m_{K^*}^2)^2+m_{K^*}^2\Gamma_{K^*}^2}q^3(\sqrt{s},m,m_{K^*}) \qquad (2.6)$$

stands for the smearing implied by the finite width of the K^* meson. Hereafter,

$$q_{ij}\equiv q(M,m_i,m_j)=\frac{1}{2M}\{[M^2-(m_i-m_j)^2]\times[M^2-(m_i+m_j)^2]\}^{1/2} \qquad (2.7)$$

is the magnitude of the momentum of either particle i or j, in the rest frame of the decaying particle. The origin of the multiplier 1/2 in the case of $f=K_S^0K^+\pi^-$ is explained below. See Sec. II B. Assuming pointlike dynamics for the vertex vector$\to VP\pi$, where V stands for the vector meson, and $P=K,\pi$, the factor of the $VP\pi$ final state can be written as

$$W_{VP\pi}(\sqrt{s},m_V,m_P)=\frac{1}{(2\pi)^3 4s}\int_{m_P+m_\pi}^{\sqrt{s}-m_V}dm\times\left(1+\frac{q^2(\sqrt{s},m_V,m)}{3m_V^2}\right)\times q(\sqrt{s},m_V,m)q(m,m_P,m_\pi). \qquad (2.8)$$

The factor

$$W_{3\pi}(\sqrt{s})=\frac{g_{\rho\pi\pi}^2}{12\pi^2}\int_{2m_\pi}^{\sqrt{s}-m_\pi}dm q_\rho^3(m)q_\pi^3(m)\int_{-1}^{1}dx(1-x^2)\times\left|\frac{1}{D_\rho(m^2)}+\frac{1}{D_\rho(m_-^2)}+\frac{1}{D_\rho(m_+^2)}\right|^2, \quad (2.9)$$

where

$$m_\pm^2=\frac{1}{2}(s+3m_\pi^2-m^2)\pm\frac{2\sqrt{s}}{m}q_\pi(m)q_\rho(m)x,$$

$$q_\rho(m)=q(\sqrt{s},m,m_\pi),$$

$$q_\pi(m)=q(m,m_\pi,m_\pi) \quad [19],$$

stands for the phase space volume of the $\pi^+\pi^-\pi^0$ final state. The general form of all propagators including the $\rho(770)$ one, $D_\rho(m^2)$, whose imaginary part is determined by the $\pi^+\pi^-$ partial width

$$\Gamma_\rho(m^2)=\frac{g_{\rho\pi\pi}^2}{6\pi m^2}q_\pi^3(m),$$

is given in Sec. II C.

B. Discussing the coupling constants

Before writing down explicit expressions for the various matrix elements entering into the matrices above, let us comment on the coupling constants of the vector mesons with various final states f.

1. Final state $\pi^+\pi^-\pi^0$

The $\rho(770)$ -ρ_1' -ρ_2' sector does not contribute by G-parity conservation, and hence $g_{\rho_i f}=0$. The $\omega_i\rho(770)\pi$ coupling constant should be inserted in place of $g_{\omega_i f}$. In particular, $g_{\omega\rho\pi}$ and the OZI-suppressed coupling constant $g_{\varphi\rho\pi}$ are determined from fitting the data on the present final state and are kept fixed in fitting the remaining final states. It should be emphasized that the existing data still cannot distinguish between the two mechanisms of the $\pi^+\pi^-\pi^0$ decay of the $\varphi(1020)$, namely, a sizable $\varphi\omega$ mixing, $\varphi\to\omega\to\pi^+\pi^-\pi^0$ and the direct transition $\varphi\to\pi^+\pi^-\pi^0$ [20]. In principle, a careful experimental study of the $\varphi\omega$ interference minimum in the reaction $e^+e^-\to\pi^+\pi^-\pi^0$ could discriminate between the above models [21]. However, such subtleties are inessential in the present case, since both models give a similar behavior of the cross section. So we take here for definiteness the purely $s\bar{s}$ quark content of the $\varphi(1020)$, thus attributing its $\pi^+\pi^-\pi^0$ decay mode solely to the direct coupling constant $g_{\varphi\rho\pi}$. The masses and coupling constants of the $\omega(782)$ -$\varphi(1020)$ complex extracted from the fit will turn out to be

$$m_\omega=783.4^{+2.7}_{-3.0}\ \text{MeV},\quad m_\varphi=1019.8^{+1.6}_{-1.4}\ \text{MeV},$$

$$g_{\omega\rho\pi}=14.3\pm1.6\ \text{GeV}^{-1},\quad g_{\varphi\rho\pi}=0.63\pm0.17\ \text{GeV}^{-1}, \quad (2.10)$$

$$f_\omega=16.6^{+1.7}_{-1.3}.$$

They coincide, within errors, with the parameters obtained earlier [19], and so we will not discuss them further. Note that we will assume hereafter the quark model relation

$$f_{\varphi_i}=-\frac{f_{\omega_i}}{\sqrt{2}} \quad (2.11)$$

between the leptonic coupling constants f_{ω_i} and f_{φ_i}.

The accuracy of existing data in the energy range of $\sqrt{s}=1.1-2$ GeV is still insufficient (see below) for the introduction of the nonzero coupling constants $\varphi_{1,2}'\rho\pi$; hence they are fixed to zero, so that the OZI rule breaking in the sectors, which include the heavier excitations, is attributed solely to the mixing via the OZI-allowed two-step processes proceeding through common decay modes.

2. Final state $\omega\pi^+\pi^-$

The ρ-like resonances do not contribute by G-parity conservation, similar to the previous case. Our analysis of pure isovector channels of e^+e^- annihilation [9] reveals the negligible contribution of the off-mass-shell coupling $\rho\to\rho\pi^+\pi^-$ in the energy range $\sqrt{s}\geqslant1$ GeV. Guided by the planar quark diagram approach, a similar $\omega\to\omega\pi^+\pi^-$ coupling is neglected in the present case. The coupling constants of the φ-like resonances to the state under consideration are suppressed by the OZI rule and hence can be set to zero, bearing in mind the poor accuracy of the present data. Note also that the $\omega\pi^+\pi^-$ mode takes into account effectively the $b_1\pi$, etc., modes. In fact, the chain $\omega_{1,2}'\to b_1(1235)\pi\to\omega\pi^+\pi^-$ includes the decay of the axial b_1 whose decay amplitude contains two independent partial waves. This results, in general, in a structureless angular distribution of final pions and could be modelled by the effective pointlike $\omega\pi^+\pi^-$ vertex, which includes also possible intermediate states containing the scalarlike mesons, $\omega'\to\omega\sigma\to\omega\pi^+\pi^-$.

The $\varphi_{1,2,3}\to\varphi\pi\pi$ coupling constant is expected to be suppressed due to the OZI rule and hence is omitted. This guess is supported by the fact that the final state $\varphi\pi\pi$ is not observed in e^+e^- annihilation [1].

3. Final state K^+K^-

The contribution of the ρ- and ω-like resonances is taken into account via SU(3) relations for their coupling, assuming a $q\bar{q}$ quark content:

$$g_{\rho^0_{1,2,3}K^+K^-}=-\frac{1}{\sqrt{2}}g_{\varphi_{1,2,3}K^+K^-},$$

$$g_{\omega_{1,2,3}K^+K^-}=-\frac{1}{\sqrt{2}}g_{\varphi_{1,2,3}K^+K^-}, \quad (2.12)$$

and the SU(2) related to the above. As a further fit shows,

$$g_{\varphi K^+K^-}=4.7\pm0.5, \quad (2.13)$$

and so we will not discuss this coupling constant further anymore. The parameters of the ρ excitations are chosen as follows. The analysis [9] of these excitations gives a number of variants of the best description of the specific final state, and the parameters extracted from various final states agree within errors. We plot the $\rho(770)+\omega(782)+\varphi(1020)+\rho'_1+\rho'_2$ resonance contribution to the production cross section of the K^+K^- final state and convince ourselves that, surprisingly, the $\rho'_{1,2}$ parameters from the variant of the description of the reaction $e^+e^-\to\pi^+\pi^-\pi^0\pi^0$ (with the subtraction of the $\omega\pi^0$ events) [9] falls closer to the data [13,14] than other variants do, and by that reason this variant is adopted in the present case, hereafter dubbed the set A. In the meantime, another set of the $\rho'_{1,2}$ parameters from [9] is briefly discussed at the proper place below.

4. Final state $K^0_SK^\pm\pi^\mp$

This final state originates from the $K^{*0}\bar{K}^0+\bar{K}^{*0}K^0$ intermediate state, so that the production cross sections are related as $\sigma(K^0_SK^+\pi^-)=\frac{1}{2}\sigma(K^{*0}\bar{K}^0)$, analogously for the charge conjugated states. Hence, the coupling constant $g_{VK^*\bar{K}}$ should be inserted instead of g_{Vf}, and a factor of 1/2 appears in the corresponding expression for P_f. The coupling constants of the ρ-like, ω-like, and φ-like resonances are supposed to obey the $q\bar{q}$ model relations

$$g_{\rho_{1,2,3}K^{*+}\bar{K}^-}=\frac{1}{2}g_{\omega_{1,2,3}\rho\pi},$$
$$g_{\omega_{1,2,3}K^{*+}\bar{K}^-}=\frac{1}{2}g_{\omega_{1,2,3}\rho\pi}, \tag{2.14}$$
$$g_{\varphi_{1,2,3}K^{*+}\bar{K}^-}=\frac{1}{\sqrt{2}}g_{\omega_{1,2,3}\rho\pi},$$

and the SU(2) related to them. The parameters of the ρ-like excitations are the same as for the final state K^+K^-.

5. Final state $K^{*0}K^-\pi^+(\bar{K}^{*0}K^+\pi^-)$

The production amplitude of this final state includes, in principle, both the effective pointlike, in the sense explained earlier in the case of the $\omega\pi^+\pi^-$ decay channel, $V\to K^{*0}K^-\pi^+$, and the triple vector, $V\to K^{*0}\bar{K}^{*0}$, vertices. The latter is the SU(3) related to the vertex $\rho_i\to\rho^+\rho^-$. It was shown earlier [9] that the contribution of the $\rho^0\to\rho^+\rho^-$ tail is negligible at $\sqrt{s}>1$ GeV, while the $\rho'_{1,2}\to\rho^+\rho^-$ coupling constants are compatible with zero. Guided by SU(3), it is reasonable to neglect the triple vector couplings in the present case, too. Note that a zero isospin of the vector mesons involved in the effective pointlike four-particle vertices allows us to relate various charge combinations of pions and kaons, resulting in a ratio of the coupling constants:

$$K^{*0}K^-\pi^+:\bar{K}^{*0}K^+\pi^-:K^{*-}K^+\pi^0:K^{*+}K^-\pi^0$$
$$=1:1:\frac{1}{\sqrt{2}}:\frac{1}{\sqrt{2}}. \tag{2.15}$$

Hence, we retain here only the coupling constants $g_{\omega'_{1,2}K^{*0}K^-\pi^+}$, $g_{\varphi'_{1,2}K^{*0}K^-\pi^+}$ and keep them free. The $\rho'_{1,2}\to K^*\bar{K}\pi$ couplings contain two isotopic states of the $K^*\bar{K}+\bar{K}^*K$ system, $I=0,1$. Then the ratios of the coupling constants of the neutral $\rho'_{1,2}$ states to various $K^*\bar{K}\pi$ charge states are expressed through the coupling constants $c^{(I)}_{\rho'_{1,2}}$ with definite isospin according to

$$K^{*+}K^-\pi^0:K^{*0}\bar{K}^0\pi^0:K^{*+}\bar{K}^0\pi^-:K^{*0}K^-\pi^+$$
$$=c^{(0)}_{\rho'_{1,2}}+\frac{c^{(1)}_{\rho'_{1,2}}}{\sqrt{2}}:c^{(0)}_{\rho'_{1,2}}-\frac{c^{(1)}_{\rho'_{1,2}}}{\sqrt{2}}:c^{(1)}_{\rho'_{1,2}}:c^{(1)}_{\rho'_{1,2}}, \tag{2.16}$$

and the charge conjugated to the above. These couplings were neglected in [9]. In principle, they should be included in the future, after obtaining good consistent data on various channels. In the following analysis of the final states containing strange mesons, we will set upper bounds on the $\rho'_{1,2}\to K^*\bar{K}\pi$ couplings with definite isospin of the $K^*\bar{K}$ states.

C. Propagators and the nondiagonal polarization operators

The propagator of the bare vector meson V and the imaginary part of the nondiagonal polarization operator describing the mixing between the bare states V_i and V_i ($V=\omega,\varphi$) are, respectively,

$$D_{V_i}\equiv D_{V_i}(s)=m^2_{V_i}-s-i\sqrt{s}\Gamma_{V_i}(s) \tag{2.17}$$

and

$$\begin{aligned}\mathrm{Im}\Pi_{V_iV_j}(s)=\sqrt{s}(&g_{V_i\rho\pi}g_{V_j\rho\pi}P_{\pi^+\pi^-\pi^0}\\&+2g_{V_iK^+K^-}g_{V_jK^+K^-}P_{K^+K^-}\\&+4g_{V_iK^{*+}K}g_{V_jK^{*+}K^-}P_{K^0_SK^+\pi^-}\\&+6g_{V_iK^{*0}K^-\pi^+}g_{V_jK^{*0}K^-\pi^+}P_{K^{*0}K^-\pi^+}\\&+g_{V_iV_1\pi^+\pi^-}g_{V_jV_1\pi^+\pi^-}W_{VP\pi}),\end{aligned} \tag{2.18}$$

where $W_{VP\pi}\equiv W_{VP\pi}(\sqrt{s},m_{V_1},m_\pi)$. Here the factors of 2, 4, and 6 multiplying, respectively, the $K\bar{K}$, $K^*\bar{K}$, and $K^*\bar{K}\pi$ contributions allow for various charge combinations, taken with the proper SU(2) coefficients [see Eq. (2.15)], and the phase space factors of different final states are given in Eq. (2.5). The width of the bare state V_i can be represented, in this notation, as

$$\Gamma_{V_i}(s)=\mathrm{Im}\Pi_{V_iV_i}(s)/\sqrt{s}. \tag{2.19}$$

The sector $V=\rho$ was described earlier [9]. Here we should add the partial width

$$\begin{aligned}\Gamma(\rho'_{1,2}\to K^*\bar{K}\pi+\text{c.c.})=(&4c^{(0)2}_{\rho'_{1,2}}+6c^{(1)2}_{\rho'_{1,2}})\\&\times W_{VP\pi}(\sqrt{s},m_{K^*},m_K)\end{aligned} \tag{2.20}$$

to the full width of the $\rho'_{1,2}$ and the contribution

$$\sqrt{s}(4c^{(0)}_{\rho'_1}c^{(0)}_{\rho'_2}+6c^{(1)}_{\rho'_1}c^{(1)}_{\rho'_2})W_{VP\pi}(\sqrt{s},m_{K^*},m_K)$$

TABLE I. The parameters of the ω_1' and φ_1' resonances giving the best description of the data on various final states. The error bars are determined from the χ^2 function. The symbol $\sim$ means that χ^2 varies insignificantly upon large variations around the corresponding parameter. The modulus of the corresponding parameter is implied in the case of the upper bound.

Final state	$\pi^+\pi^-\pi^0$	$\omega\pi^+\pi^-$	K^+K^-	$K_S^0K^\pm\pi^\mp$	$K^{*0}K^-\pi^+$
$m_{\omega_1'}$ [GeV]	$1.4^{+0.1}_{-0.2}$	~ 1.4	~ 1.46	$1.5^{+0.3}$ [a]	$1.4^{+0.7}$ [a]
$\Gamma_{\omega_1' e^+e^-}$ [keV]	$5^{+18}_{-5}\times 10^{-2}$	$<1\times 10^{-2}$	$6^{+200}_{-6}\times 10^{-3}$	$8^{+1500}_{-8}\times 10^{-3}$	$5^{+95}_{-5}\times 10^{-2}$
$g_{\omega_1'\rho\pi}$ [GeV^{-1}]	$-21^{+19}_{-32}\times 10^{-1}$	<12	<50	-9^{+4}_{-2}	~ -2
$g_{\omega_1' K^{*0}K^-\pi^+}$	~ 0	$-7^{+4}_{-50}\times 10^2$	~ -2000	~ 0	<500
$g_{\omega_1'\omega\pi^+\pi^-}$	-120^{+60}_{-90}	-110 ± 110	~ -70	~ 0	~ -100
$m_{\varphi_1'}$ [GeV]	~ 1.5	~ 1.5	~ 1.5	~ 1.9	~ 1.4
$g_{\varphi_1' K\bar{K}}$	$\equiv 0$	$\equiv 0$	$<15\times 10^{-1}$	$\equiv 0$	$\equiv 0$
$g_{\varphi_1' K^{*0}K^-\pi^+}$	~ -300	$\equiv -300$	~ -400	<1200	$\equiv 0.$

[a] χ^2 is insensitive to lower values of the mass.

to the imaginary part of the nondiagonal polarization operator $\Pi_{\rho_1'\rho_2'}$ [9].

The expressions for the partial widths could include the energy-dependent factors $C_f(s)$ which, analogously to the well-known Blatt-Weiskopf centrifugal factors, are aimed at restricting a too fast growth of the partial widths with the energy rise. They are somewhat arbitrary under the demand of $\sqrt{s}\Gamma(s)\to$const at $\sqrt{s}\to\infty$. In practice, the only mode with a strong dependence is the vector $(V)+$pseudoscalar (P) one, and our choice for the factor multiplying corresponding coupling constant is

$$C_{VP}(s)=\frac{1+(R_{VP}m_0)^2}{1+(R_{VP}\sqrt{s})^2}, \tag{2.21}$$

where m_0 is the mass of the resonance and R_{VP} is the so-called range parameter.

Contrary to the imaginary parts fixed by the unitarity relation, the real parts of *all* nondiagonal polarization operators cannot be evaluated at present and hence should be taken as free parameters. However, some information about the mass spectrum of the ground state mesons can provide a reasonable guess about the real parts of the nondiagonal polarization operators describing the mixing of the ground state mesons with the heavier ones. In fact, it was shown earlier [19] in the case of two mixed states 1 and 2 that the masses of both these states acquire shifts in the opposite directions. In particular, the shift of the lower state is

$$\delta m_1\simeq -\mathrm{Re}\frac{\Pi_{12}^2(m_1^2)/2m_1}{m_2^2-m_1^2-im_1[\Gamma_2(m_2^2)-\Gamma_1(m_1^2)]}, \tag{2.22}$$

and it can be large. However, a fit by the minimization of the χ^2 function fixes only the combination $m_1+\delta m_1$. Hence, it is natural to assume that the dominant contribution to the mass renormalization, Eq. (2.22), coming from $(\mathrm{Re}\Pi_{12}^2)$ is already subtracted, so that the mass of the lower state minimizing χ^2 differs from the actual position of peak 1 in the cross section by a quantity quadratic in $\mathrm{Im}\Pi_{1,2}$. Then the minimum of χ^2 is provided by the values of $\mathrm{Re}\Pi_{1,2}$ falling close to zero, of course, with very large error bars. So it is reasonable to fix the latter to zero from the very start. These considerations, justifiable in the case of the ground state mesons $\rho(770)$, $\omega(782)$, and $\varphi(1020)$ whose $q\bar{q}$ nature is firmly established, cannot be applied to the higher excitations. The latter may contain an appreciable admixture of an exotic component like $q\bar{q}g$, $q^2\bar{q}^2$, etc. [6], and so it is a matter of principle to extract from the data the parameters of unmixed states. By this reason the real parts of the nondiagonal polarization operators describing the mixing among heavier excitations should be kept free. We consider them to be independent of energy.

III. RESULTS

The procedure of the extraction of the resonance parameters is the same as in [9]. We fit the data on each reaction, Eqs. (1.1)–(1.5), separately by minimizing the χ^2 function. In principle, the specific set of parameters giving the best description of the specific cross section unnecessarily gives a good description of other channels. Hence, a final choice is made on the demand that the sets of the parameters obtained from various channels should differ by no more than one to two standard deviations. This looks reasonable, especially if one bears in mind the desirable possibility of gathering good consistent data on different channels and on the single facility.

It should be emphasized that the usual representation of the resonance parameters as masses and partial widths evaluated at these masses is inadequate in the case of strongly mixed resonances and the strong energy dependence of the partial widths. As will be clear later on, actual peaks in the cross sections are displaced considerably from the input masses of the heavier excitations. This is the reason for our choice of the masses and *coupling constants* of bare states, not their partial widths, to represent the results. Furthermore, a large number of free parameters pushes us to invoke some hypotheses on the relations between the coupling constants. The assumption adopted in the present paper is the $q\bar{q}$ nature of the isoscalar excitations. Corresponding relations among the hadronic coupling constants are given in Sec. II B.

Our results are collected in Tables I, II, and in Figs. 1–5. In Table III we quote the values of χ^2/n_{DOF} for each channel considered in the work. It is seen that all the parameters agree within large error bars. The range parameter $R_{\rho\pi}$ for

TABLE II. The same as in Table I, but for the ω_2' and φ_2' resonances.

Final state	$\pi^+\pi^-\pi^0$	$\omega\pi^+\pi^-$	K^+K^-	$K_S^0K^\pm\pi^\mp$	$K^{*0}K^-\pi^+$
$m_{\omega_2'}$ [GeV]	$1.82^{+0.19}_{-0.15}$	$1.84^{+0.10}_{-0.07}$	$1.78^{+0.17}_{-0.30}$	~ 2.1	$1.88^{+0.60}_{-1.00}$
$\Gamma_{\omega_2' e^+e^-}$ [keV]	$10^{+9}_{-8}\times 10^{-2}$	$10^{+5}_{-3}\times 10^{-2}$	$<6\times 10^{-2}$	$19^{+112}_{-19}\times 10^{-2}$	$120^{+40}_{-50}\times 10^{-2}$
$g_{\omega_2'\rho\pi}$ [GeV^{-1}]	-7^{+4}_{-3}	-7^{+3}_{-2}	<50	-11 ± 2	-14 ± 6
$g_{\omega_2' K^{*0}K^-\pi^+}$	<3000	<1000	~ 0	~ 0	$31^{+40}_{-29}\times 10^{1}$
$g_{\omega_1'\omega\pi^+\pi^-}$	90^{+100}_{-50}	90^{+40}_{-30}	~ 0	~ 0	<2000
$\mathrm{Re}\Pi_{\omega_1'\omega_2'}$ [GeV]^2	$<6\times 10^{-1}$	$<6\times 10^{-1}$	~ 0	~ 0	<20
$m_{\varphi_2'}$ [GeV]	$2.8_{-1.1}$ [a]	~ 1.9	~ 2.5	$2.6_{-0.3}$ [a]	$2.52^{+0.35}_{-0.27}$
$g_{\varphi_2' K\bar{K}}$	$\equiv 0$	$\equiv 0$	<3	$\equiv 0$	$\equiv 0$
$g_{\varphi_2' K^{*0}K^-\pi^+}$	~ -100	~ -20	~ -400	<600	-80^{+40}_{-60}
$\mathrm{Re}\Pi_{\varphi_1'\varphi_2'}$ [GeV]^2	~ 0.4	~ 0	~ 0.1	<5	~ 0

[a] χ^2 is insensitive to greater values of the mass.

the $\rho\pi$ decay mode [and the SU(3) related to it] turns out to be 0.4 ± 1.0 GeV^{-1} in the case of the $\omega(782)$, and is not fixed by the fit in the case of the ω_1'. We choose it to be zero for the latter excitation and for the ω_2' one, too.

Let us comment on the visible disagreements in Tables I and II. First, the small, compared to the other, value of $\Gamma_{\omega_{1,2}' e^+e^-}$ extracted from the K^+K^- data is an artifact of our particular choice of $\rho_{1,2}'$ resonance parameters. Another choice [9], with the parameters extracted from the $e^+e^-\to\pi^+\pi^-\pi^+\pi^-$ data hereafter dubbed as the set B, gives better values, $\Gamma_{\omega_1' e^+e^-}=(20^{+60}_{-20})\times 10^{-2}$ keV and $\Gamma_{\omega_2' e^+e^-}=(11^{+19}_{-11})\times 10^{-2}$ keV. This emphasizes the necessity of obtaining consistent data about various final states in e^+e^- annihilation. The visible disagreement of the central value of the leptonic width $\Gamma_{\omega_2' e^+e^-}$ extracted from the reaction (1.5) is due to the following. First, the error bars are so large that the disagreement is statistically insignificant. Second, the threshold of the reaction, Eq. (1.5), 1.53 GeV, is so high that the inclusion of additional multiparticle decay modes may be necessary, which could change the result towards better values. We postpone this task until more satisfactory experimental data will appear.

Notice that the peak positions of the heavier excitations are displaced towards lower values from the bare masses of resonances. The same phenomenon was observed in the case of isovector excitations [9] and is due, predominantly, to the growth of the partial widths with the energy,

$$\delta m_{V_{1,2}'}\sim -\Gamma(s)\frac{d\Gamma}{d\sqrt{s}}(\sqrt{s}=m_{V_{1,2}'}). \tag{3.1}$$

Unfortunately, available data do not put any serious restrictions on the real parts of the nondiagonal polarization

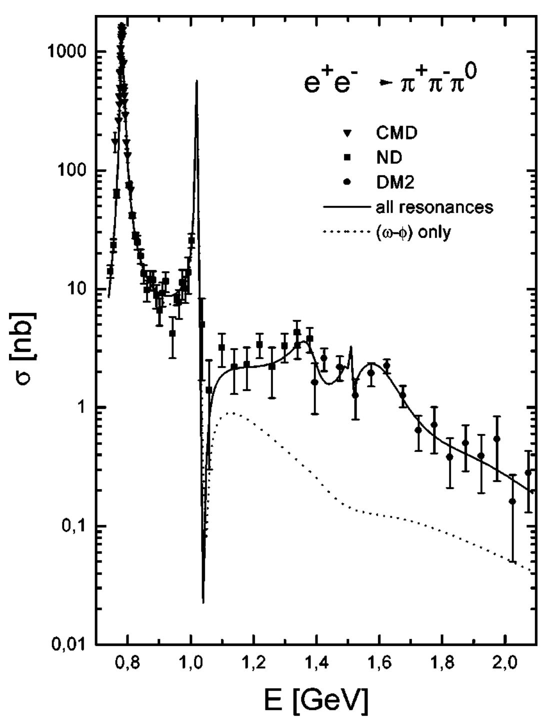

FIG. 1. Cross section of the reaction $e^+e^-\to\pi^+\pi^-\pi^0$. The data are from ND [10], DM2 [11], CMD [12].

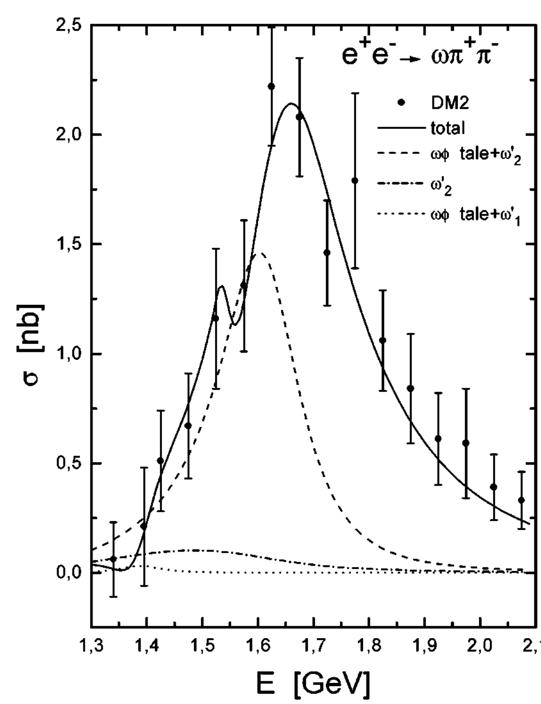

FIG. 2. Cross section of the reaction $e^+e^-\to\omega\pi^+\pi^-$. The data are from DM2 [11].

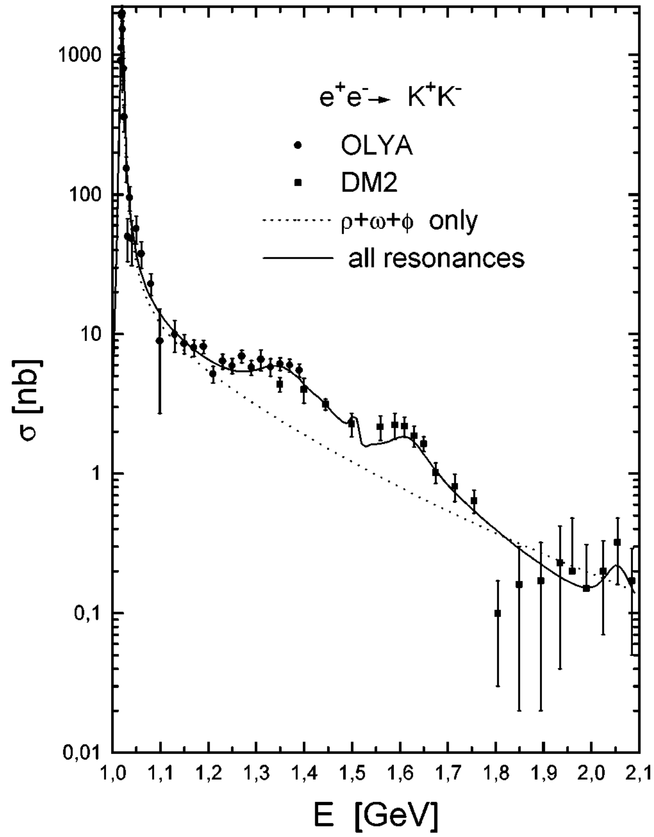

FIG. 3. Cross section of the reaction $e^+e^-\to K^+K^-$. The data are from OLYA [13], DM2 [14]. Lower error bars of two experimental points around 1.95 GeV are not shown because their lowest values are below 0.01 nb.

operators. The minimization procedure points to possible nonzero values which are quoted in Table II, but the error bars are very large.

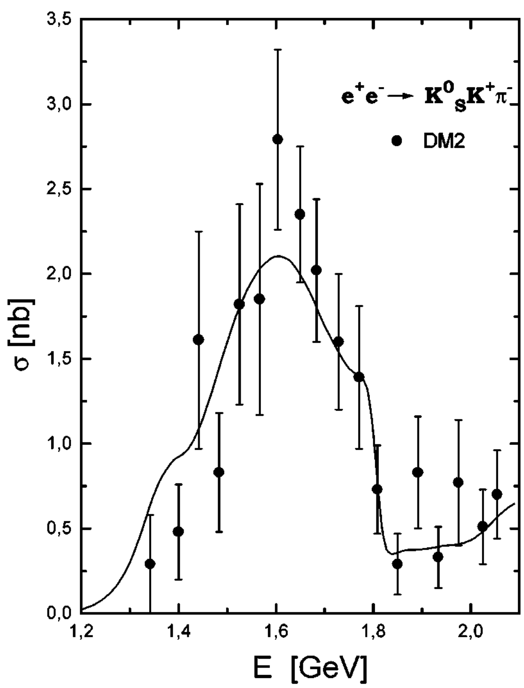

FIG. 4. Cross section of the reaction $e^+e^-\to K^0_SK^\pm\pi^\mp$. The data are from DM2 [15].

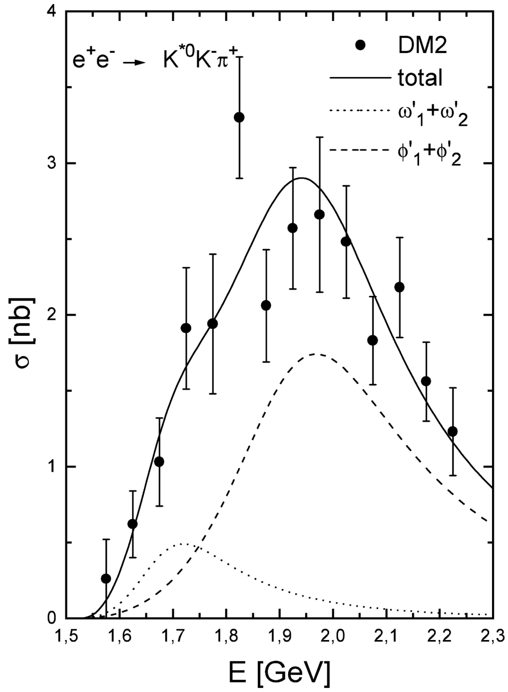

FIG. 5. Cross section of the reaction $e^+e^-\to K^{*0}K^-\pi^+$. The data are from DM2 [16].

The best upper bounds on the $\rho'_{1,2}K^*\bar{K}\pi$ couplings $c^{(I)}_{\rho'_{1,2}}$ with definite isospin of the $K^*\bar{K}$ system are as follows: (i) $|c^{(0)}_{\rho'_1}|<1500$ and $|c^{(0)}_{\rho'_2}|<1100$ come from fitting the cross section of the reaction $e^+e^-\to K_SK^\pm\pi^\mp$; (ii) $|c^{(1)}_{\rho'_2}|<390$ and $|c^{(1)}_{\rho'_2}|<210$ come from fitting the cross section of the reaction $e^+e^-\to K^{*0}K^-\pi^+$.

IV. DISCUSSION

The nonrelativistic quark model (NRQM) for bound states of light quarks cannot be easily justified in view of the expectedly large QCD and relativistic corrections. Yet the remarkable agreement of NRQM predictions with the data in the physics of light quarks is impressive, and by this reason the NRQM provides a conventional reference frame for representing the results of numerous analyses. In particular, conclusions about the possible non-$q\bar{q}$ component [6,7] are based on a comparison of calculations [8] with the ratios of coupling constants based on the NRQM. So we also will follow here the custom of expressing the results in terms of

TABLE III. The values of χ^2 per number of degrees of freedom ($n_{\rm DOF}$) for each fitted channel.

Channel	$\chi^2/n_{\rm DOF}$
$\pi^+\pi^-\pi^0$	40/36
$\omega\pi^+\pi^-$	6/5
K^+K^-	81/42
$K_SK^\pm\pi^\mp$	11/7
$K^{*0}K^\pm\pi^\mp$	14/3
K_LK_S	5/7

the interquark potential, bound state wave functions, etc., bearing in mind that the extent of the reliability to the models of such a kind is supported by their effectiveness in known cases rather than by a firm theoretical basis.

As is known, the leptonic widths, Eq. (2.2), are sensitive to the behavior of the wave function of the bound $q\bar{q}$ state at the origin [22,23]:

$$\Gamma(^3S_1\to e^+e^-)=\frac{4\alpha^2Q_V^2}{m^2}|R_S(0)|^2,$$

$$\Gamma(^3D_1\to e^+e^-)=\frac{200\alpha^2Q_V^2}{m^6}|R''_D(0)|^2, \tag{4.1}$$

where $R_L(0)$ is the radial wave function at the origin of the $q\bar{q}$ bound state with angular momentum L. Q_V is related to the quark content of the vector meson $V=\rho_i,\omega_i,\varphi_i$ and reads $1/\sqrt{2},1/3\sqrt{2},-1/3$ for, respectively, $\rho_i=(u\bar{u}-d\bar{d})/\sqrt{2},\omega_i=(u\bar{u}+d\bar{d})/\sqrt{2},\varphi_i=s\bar{s}$, assuming a $q\bar{q}$ nature of the heavier excitations. To make the comparison easier, we quote the magnitudes of the wave function and the second derivative at the origin averaged over the channels under consideration. The results for ρ-like excitations are evaluated with the help of [9]. One obtains, for ρ-like excitations,

$$|R_S(0,m_{\rho'_1})|^2=110^{+30}_{-20}\times10^{-3}\ \ \mathrm{GeV}^3,$$

$$|R_S(0,m_{\rho'_2})|^2=(140\pm20)\times10^{-3}\ \ \mathrm{GeV}^3,$$

$$|R''_D(0,m_{\rho'_1})|^2=80^{+20}_{-10}\times10^{-4}\ \ \mathrm{GeV}^7,$$

$$|R''_D(0,m_{\rho'_2})|^2=(300\pm50)\times10^{-4}\ \ \mathrm{GeV}^7, \tag{4.2}$$

to be compared to $|R_S(0,m_\rho)|^2=37\times10^{-3}$ GeV^3. As is pointed out in Sec. III, the amplitude of the reaction $e^+e^-\to K^{*0}K^-\pi^+$ seems to be affected by the multiparticle intermediate states neglected in the present analysis, and so we exclude it from averaging in the case of the isoscalars. One gets

$$|R_S(0,m_{\omega'_1})|^2=40^{+620}_{-40}(140^{+620}_{-140})\times10^{-4}\ \ \mathrm{GeV}^3,$$

$$|R_S(0,m_{\omega'_2})|^2=430^{+1420}_{-250}(400^{+1080}_{-200})\times10^{-4}\ \ \mathrm{GeV}^3,$$

$$|R''_D(0,m_{\omega'_1})|^2=30^{+960}_{-30}(110^{+980}_{-110})\times10^{-5}\ \ \mathrm{GeV}^7,$$

$$|R''_D(0,m_{\omega'_2})|^2=140^{+570}_{-100}(120^{+430}_{-80})\times10^{-4}\ \ \mathrm{GeV}^7, \tag{4.3}$$

to be compared to $|R_S(0,m_\omega)|^2=33\times10^{-3}$ GeV^3. Here the numbers in the parentheses refer to the set B of the parameters of the ρ-like excitations mentioned earlier. Note that within error bars the numerical characteristics of the $q\bar{q}$ structure of the ρ-like and ω-like excitations are coincident, thus supporting their assignment to the same nonet.

A realistic interquark potential could include the sum of a Coulomb-like one, with a running QCD coupling constant and a confining potential [17]. However, because the errors in extracting the wave functions and second derivatives at the origin are still too large, one cannot draw any definite conclusions about the parameters of the potential or to verify the usual assignments $\rho'_1\equiv\rho(1450)\sim2^3S_1$ and $\rho'_2\equiv\rho(1600)\sim1^3D_1$, similar for ω'_1 and ω'_2.

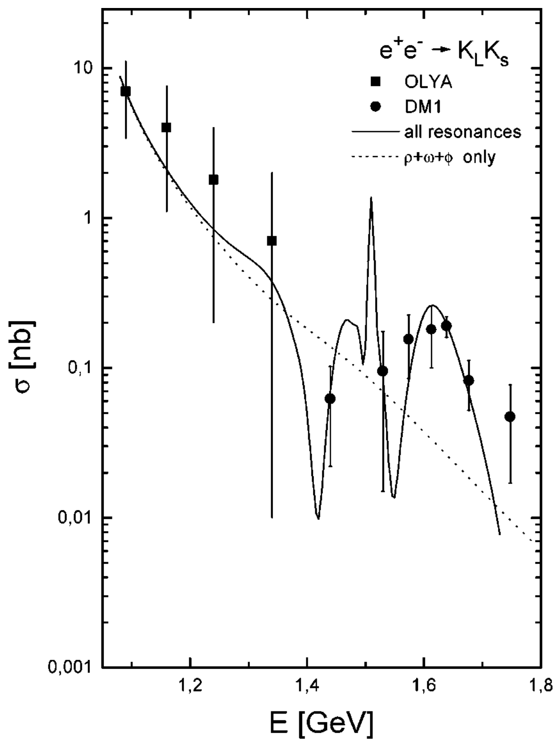

FIG. 6. Cross section of the reaction $e^+e^-\to K_LK_S$. Since the data of OLYA [26] and DM1 [27] have large error bars, the parameters of the resonances extracted also have large errors and hence are omitted from Tables I and II.

The present study differs basically from that of Ref. [7], where the simplest Breit-Wigner amplitude, with the neglect of mixing among the states, was used for fitting the cross sections. So our conclusions are different from those of [6–8] at the point that the $q\bar{q}$ nature of heavier resonances is not excluded by existing data. The $q\bar{q}$ model relation, Eq. (2.11), is fixed here from the very start, while the ratio $\Gamma(\omega'_i\to e^+e^-)/\Gamma(\rho'_i\to e^+e^-)$ spreads from zero to 0.3 in all the fits and does not contradict to the $q\bar{q}$ ratio $\sim1/9$. The vector-pseudoscalar and pseudoscalar-pseudoscalar couplings are also related via the $q\bar{q}$ model in our work. It should be recalled that our previous analysis [9] revealed the ratio $B(\rho'_1\to2\pi^+2\pi^-)/B(\rho'_1\to\omega\pi^0)$ to be consistent with zero. This is in a contrast with the hypothesis of the hybrid admixture [6–8] which predicts the dominance of final states containing the $L=1$ mesons [8]. Pseudoscalar-pseudoscalar final states appear to be suppressed in our analysis, in qualitative agreement with the considerations of Ref. [8]. Note, however, that there are reasons to expect the suppression of these final states for pure $q\bar{q}$ mesons [24].

V. CONCLUSION

The data on the isoscalar heavier excitations existing now are still of poor accuracy, not only in that the errors of the extracted parameters of these excitations are large, but in that these data are insufficient to discriminate between possible dynamical models of the resonances with masses greater than 1 GeV. In fact, we find an alternative variant of the description which does not at all demand the existence of ω_1' and φ_1' resonances, however, at the expense of abandoning the $q\bar{q}$ model relations among strong coupling constants. The corresponding curves look even better than those shown in Figs. 1–5 in that ω_1' and φ_1' peaks, which seem accidental, are absent in this variant.

Looking at the curves in the present paper convinces us that fitting the scarce data with expressions containing many free parameters can bring one to the trap of low χ^2 when the curve goes through the central points, each possessing large error bars. A narrow structure at $\sqrt{s}\simeq 1.5$ GeV seen in Figs. 1–3 and 6, is due to the φ_1' resonance. Nonzero coupling constants of the φ_1' resulting in its narrow width possess very large errors which make the former to be consistent with zero. However, simply dropping them makes χ^2 considerably larger. Nevertheless, we include just the present variant because it is coherent with the variant of the description of the isovector channels [9] based on the picture of two heavier resonances ρ_1' and ρ_2' [25]. An additional illustration of such a trap is the channel $e^+e^-\to K_LK_S$. The curve with parameters obtained from fitting the reaction $e^+e^-\to K^+K^-$ and with the proper reversing of sign of the isovector contribution goes through four low energy OLYA points [26] but fails to describe the higher energy DM1 data [27] consisting of eight experimental points. When we fit the data [26,27] on neutral kaons, we obtain χ^2 as low as 5. The curve shown in Fig. 6 goes through almost each experimental point, yet looks unnatural in view of very large error bars. The only way of escaping a trap of this sort and of proving or disproving any particular model of resonances in the mass range $1 < m < 2.5$ GeV is to collect good consistent data on all relevant channels. The ranges of admissible resonance parameters found in the present paper and in the earlier one [9] will be hopefully useful.

ACKNOWLEDGMENTS

We would like to thank G. N. Shestakov for discussions. The present work was supported in part by grant INTAS-94-3986.

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