Final state interactions and new physics in $B \to \pi K$ decays

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Within the standard model, and if one assumes that soft rescattering effects are negligible, the CP asymmetry $\mathcal{A}_{CP}^{\rm dir}(B^\pm \to \pi^\pm K)$ is predicted to be very small and the ratio $R = B(B_d \to \pi^\mp K^\pm)/B(B^\pm \to \pi^\pm K)$ provides a bound on the angle γ of the unitarity triangle, $\sin^2\gamma \leq R$. We estimate the corrections from soft rescattering effects using an approach based on Regge phenomenology, and find effects of order 10% with large uncertainties. In particular, we conclude that $\mathcal{A}_{CP}^{\rm dir} \sim 0.2$ and $\sin^2\gamma \sim 1.2R$ could not be taken unambiguously to signal new physics. Using SU(3) relations, we suggest experimental tests that could constrain the size of the soft rescattering effects thus reducing the related uncertainty. Finally, we study the effect of various models of new physics on $\mathcal{A}_{CP}^{\rm dir}$ and on R. [S0556-2821(98)01509-4]

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I. INTRODUCTION

Heavy quark decays serve as a powerful tool for testing the standard model and provide invaluable possibilities to study CP violation. However, the interpretation of experimental observables in terms of fundamental parameters is often less than clear. Rare hadronic decays of B mesons, for example, proceed through both tree level Cabibbo-suppressed amplitudes and through one loop penguin amplitudes. On the one hand, this situation allows direct CP violating effects that may give the first evidence for CP violation outside the neutral kaon system. On the other, these competing contributions complicate the extraction of Cabibbo-Kobayashi-Maskawa (CKM) angles and, in particular, the angle γ,

$$\gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{1.1}$$

The CLEO Collaboration has presented combined branching ratios for $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$ [1] making these modes of particular interest. In the Standard Model, these decays are mediated by the $\Delta B = 1$ Hamiltonian, which takes the form

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[V_{cb}V_{cs}^*\left(\sum_{i=1}^{2} C_i Q_i^{cs} + \sum_{i=3}^{6} C_i Q_i^s + \sum_{i=7}^{10} C_i Q_i^s\right) + V_{ub}V_{us}^*\left(\sum_{i=1}^{2} C_i Q_i^{us} + \sum_{i=3}^{6} C_i Q_i^s + \sum_{i=7}^{10} C_i Q_i^s\right)\right] + \text{H.c.} \tag{1.2}$$

The flavor structures of the current-current, QCD penguin, and electroweak penguin operators are, respectively, $Q_{1,2}^{qs} \sim \overline{s}q\overline{q}b$, $Q_{3,\ldots,6}^{s} \sim \overline{s}b\Sigma\overline{q}'q'$, and $Q_{7,\ldots,10}^{s} \sim \overline{s}b\Sigma e_{q'}\overline{q}'q'$, where the sum is over light quark flavors [2]. The Wilson coefficients C_i are renormalization scale dependent; at a low scale $\mu \sim 1$ GeV, they roughly satisfy $C_{1,2} = \mathcal{O}(1)$, $C_{3,\ldots,6,9} = \mathcal{O}(10^{-2})$, and $C_{7,8,10} \leq \mathcal{O}(10^{-3})$. In many extensions of the standard model, the effect of new physics at low energies is simply to modify the values of the Wilson coefficients.

A *scale-independent* way of decomposing the decay amplitudes of interest is to do so not naively according to contributions of the operators Q_i^{qs}, but rather according to their dependence on the elements of the CKM matrix,

$$\begin{aligned} A(B^+ \to \pi^+ K^0) &= A_{cs}^+ - A_{us}^+ e^{i\gamma} e^{i\delta_+}, \\ A(B^- \to \pi^- \overline{K^0}) &= A_{cs}^+ - A_{us}^+ e^{-i\gamma} e^{i\delta_+}, \\ A(B^0 \to \pi^- K^+) &= A_{cs}^0 - A_{us}^0 e^{i\gamma} e^{i\delta_0}, \\ A(\overline{B^0} \to \pi^+ K^-) &= A_{cs}^0 - A_{us}^0 e^{-i\gamma} e^{i\delta_0}, \end{aligned} \tag{1.3}$$

where δ_0 and δ_+ are CP-conserving phases induced by the strong interaction, and the dependence on the CKM phases is shown explicitly. The first and second terms in each amplitude correspond to matrix elements of the first and second terms in $\mathcal{H}_{\rm eff}$ (or their Hermitian conjugates), respectively. Note that each term is by itself scheme and renormalization scale independent.

We will avoid, as much as possible, the common terminology of "tree" versus "penguin" contributions, which can lead to much unnecessary confusion. The standard convention is to take "tree" contributions to a given decay to be those which are mediated by the current-current operators $Q_{1,2}^{qs}$, and "QCD penguin" contributions to be those mediated by $Q_{3,\ldots,6}^{qs}$. However, this is not a scale-invariant decomposition. If the computation of physical matrix elements could be accomplished perturbatively, this would not be such a serious failing, as the scale dependence of the matrix elements would cancel explicitly against that of the Wilson coefficients. But this is not the case; rather, the matrix elements must be modeled phenomenologically, and the manifest scale independence of the result is lost. The greatest difficulty is found when one is interested, as we will be, in a significant contribution to some process which arises essentially at long distances, where the the physics is intrinsically nonperturbative. There one can dispute endlessly, and pointlessly, about whether what one is computing is ''really'' tree or penguin in nature. The question obviously has no unique answer, but its resolution is, fortunately, of no practical consequence.

That having been said, one can still make some general statements about the expected relative contributions of the operators in $\mathcal{H}_{\rm eff}$ to a given exclusive decay mode. The electroweak penguin operators are commonly neglected, since the contributions with a sizable Wilson coefficient, $C_9Q_9^s$, are color suppressed or require rescattering from intermediate states. In this case isospin symmetry of the strong interactions leads to the simplification $A_{cs}^0=A_{cs}^+$. It is now believed that the current-current operator contributions to $A_{cs}^{0,+}$ are roughly of same order as the QCD penguin operator contributions [3,4]. To be specific, this observation is based on the fact that at next-to-leading order in QCD perturbation theory, it holds for the corresponding parton model decays $b\to u\bar{u}s$ and $b\to d\bar{d}s$. The contribution of the current-current operators to A_{us}^0 is also expected to be of the same order, despite the CKM suppression, because of the large value of C_2, namely, $V_{ub}V_{us}^*C_2\sim V_{cb}V_{cs}^*C_{3,\ldots,6}$. However, since for $B^\pm\to\pi^\pm K$ the relevant quark transition is $b\to d\bar{d}s$, one might expect the size of A_{us}^+ relative to A_{cs}^+ to be highly suppressed by the small ratio $|V_{ub}V_{us}^*/V_{cb}V_{cs}^*|\sim 0.02$. This would hold equally for the current-current and penguin operators. If, indeed, $r_+=A_{us}^+/A_{cs}^+\sim|V_{ub}V_{us}^*/V_{cb}V_{cs}^*|$ is a good approximation, then there are two important consequences:

(i) Direct CP violation could be observed, in principle, through the CP asymmetry $\mathcal{A}_{CP}^{\rm dir}\equiv\mathcal{A}_{CP}^{\rm dir}(B^+\to\pi^+K^0)$:

$$\mathcal{A}_{CP}^{\rm dir}=\frac{B(B^+\to\pi^+K^0)-B(B^-\to\pi^-\overline{K^0})}{B(B^+\to\pi^+K^0)+B(B^-\to\pi^-\overline{K^0})}=\frac{2r_+\sin\gamma\sin\delta_+}{1-2r_+\cos\gamma\cos\delta_++r_+^2}. \tag{1.4}$$

However, it would be small:

$$\mathcal{A}_{CP}^{\rm dir}(B^+\to\pi^+K^0)\leqslant\mathcal{O}(\lambda^2), \tag{1.5}$$

where $\lambda\simeq 0.22$ is the Wolfenstein parameter. For example, ''hard'' final-state interaction estimates [5–8], where the u quarks in $Q_{1,2}^{us}$ are treated as a perturbative loop, give $\mathcal{A}_{CP}^{\rm dir}\sim 1\%$.

(ii) Model-independent bounds could be obtained for the angle γ using only the *combined* branching ratios $B(B^\pm\to\pi^\pm K)$ and $B(B_d\to\pi^\mp K^\pm)$ [9,10]. One can construct the ratio

$$R=\frac{B(B^0\to\pi^-K^+)+B(\overline{B^0}\to\pi^+K^-)}{B(B^+\to\pi^+K^0)+B(B^-\to\pi^-\overline{K^0})}=\left(\frac{A_{cs}^0}{A_{cs}^+}\right)^2\frac{1-2r_0\cos\gamma\cos\delta_0+r_0^2}{1-2r_+\cos\gamma\cos\delta_++r_+^2}, \tag{1.6}$$

where $r_0=A_{us}^0/A_{cs}^0$. Assuming that A_{us}^+ and electroweak penguin operator contributions are negligible, the ratio (1.6) takes the simple form

$$R=1-2r_0\cos\gamma\cos\delta_0+r_0^2. \tag{1.7}$$

The observable R may be minimized with respect to the unknown hadronic parameter r_0, yielding the inequality

$$R\geqslant 1-\cos^2\gamma\cos^2\delta_0. \tag{1.8}$$

Since $\cos^2\delta_0\leqslant 1$, this leads to the bound

$$\sin^2\gamma\leqslant R. \tag{1.9}$$

The bound including electroweak penguin operators is obtained by substituting

$$R\to R(A_{cs}^+/A_{cs}^0)^2. \tag{1.10}$$

It has recently been observed that the modified bound could differ by as much as $\pm 10\%$ [11]. If true, a stringent bound on γ would be obtained if the experimental errors in the presently reported $R_{\rm exp}=0.65\pm 0.40$ [1] were to be reduced, with the central value unchanged.

The $B\to\pi K$ transitions are suppressed in the standard model by either CKM matrix elements or small Wilson coefficients. As a consequence, these decays are potentially sensitive to new physics. In particular, in the presence of new physics, a large CP asymmetry can be induced, thus violating the bound (1.5), and R can be modified in a way that violates the bound (1.9). The analyses leading to (1.5) and to (1.9), however, explicitly assume that the CKM angle γ does not enter the theoretical expression for the charged decay amplitudes (1.3). As it is usually expressed, one requires the absence of significant contributions from the current-current operators $Q_{1,2}^{us}$ to this decay channel. As noted above, this assumption is based on the observation that the quark level decay $b\to d\bar{d}s$ is not mediated direcly by $Q_{1,2}^{us}$. However, this naive treatment of the dynamics ignores the effects of soft rescattering effects at long distances, which can include the exchange of global quantum numbers such as charge and strangeness. It is the purpose of the next section to discuss an explicit model for such a rescattering, and to consider its effect on the bounds (1.5) and (1.9). Before presenting the model, however, we must make some general comments about what we do, and what we do *not,* expect to accomplish with this exercise.

We will consider the contributions to $B^+\to\pi^+K^0$ from rescattering through coupled channels such as $B^+\to\pi^0(\eta)K^+$ or corresponding multi-body decays. In fact, we will treat only the two body intermediate states explicitly. The model we will employ will be based on Regge phenomenology, including the exchange of the ρ trajectory and others related by SU(3) flavor symmetry. The coupling of the trajectory to the final state will be extracted from data on πp and pp scattering cross sections. A few points are in order. First, we have little confidence in the quantitative predictions of the model *per se.* In fact, we believe that it would be irresponsible to claim an accuracy of better than a factor of two for the size of soft rescattering effects, using *any* model currently available. Neither our nor any other model should be taken as a canonical framework for the estimate of final state interactions in B decays. Rather, the purpose of our calculation is to be illustrative: our model will predict r_+ at the level of ten percent, with no fine tuning or unnatural enhancements. We will use this result to argue that such a value of r_+ is entirely generic within the standard model. At such a level, the effect of final state interactions on total branching fractions is small, but the effect on quantities which are most interesting for $r_+=0$, such as $\mathcal{A}_{CP}^{\rm dir}(B^\pm\to\pi^\pm K)$ and R, can be more dramatic.

Second, the rescattering effects which we will consider are *not* already included in an analysis of the strong Bander-Silverman-Soni (BSS) phases induced when the virtual particles in a penguin loop go onto the mass shell [12]. The fact that such an analysis typically yields small corrections cannot be used to argue that rescattering contributions to r_+ must be small. On the contrary, we will consider intermediate states with on-shell pseudoscalar mesons, rather than on-shell quarks. In the absence of an argument that parton-hadron duality should hold in exclusive processes involving pions and kaons (for which there is scant evidence), one must conclude that the long distance physics of meson rescattering is not probed by the BSS analysis. (The recent proposal that rescattering effects must be small [13] does *not* go outside the BSS framework in obtaining quantitative estimates.)

Third, the issue of whether the processes we consider are of the ''tree'' or ''penguin'' type is a dangerous red herring. As discussed above, the question has no scale-invariant meaning, and also no important implications. Our model addresses contributions to the well-defined amplitude A_{us}^+. Within the model, we will first use the current-current operators $Q_{1,2}^{us}$ to generate the transition $B\to\pi^0(\eta)K^-$, which will then rescatter to $\pi^-\overline{K^0}$. However, we will deliberately refrain from referring to this as a ''tree'' contribution, in order to avoid unnecessary confusion.

Fourth, if one decomposes the matrix element for $B^-\to\pi^-\overline{K^0}$ into amplitudes of definite isospin, the rescattering process which we will consider is non-trivially embedded in their sum. Recent isospin analysis of this decay [13,14] have stressed the importance of the strong phases associated with the isospin amplitudes. (For previous isospin analyses of $B\to\pi K$ decays, see [15,16].) In the isospin language the magnitude of A_{us}^+ depends on the differences between these phases and vanishes in the limit that they vanish. However, the isospin decomposition sheds no light on the sizes of these phase differences, hence on the magnitude of A_{us}^+, leaving the former as free parameters. The literature presently contains a variety of quite divergent opinions concerning the ''natural'' size of rescattering effects in this decay [9,11–14]. The purpose of our calculation, however crude, is to address the issue more quantitatively by providing the first analysis to model the *magnitude* of rescattering in this decay. Finally, we note that the rescattering in question is *inelastic*, despite its quasi-elastic kinematics, and cannot be studied adequately in any model of purely *elastic* final state phases.

The calculation of the standard model predictions is given in Sec. II. First, we describe how the final state interactions (FSI) affect the CP asymmetry and the bound on γ. Then we calculate, using the phenomenological Regge model, FSI corrections for specific two body states. In Sec. III we suggest experimental tests that could potentially give an upper bound on these contributions, independent of hadronic models for the final state rescattering. In Sec. IV we analyze which types of new physics models can significantly affect the relevant $B\to\pi K$ decays, and whether there are relations between such new contributions to the charged and neutral modes. We summarize our results in Sec. V.

II. FINAL STATE INTERACTIONS

A. The effects of FSI corrections

We would like to investigate the impact of final state rescattering on the CP asymmetry $\mathcal{A}_{CP}^{\rm dir}(B^\pm\to\pi^\pm K)$ and the ratio of branching fractions R. The rescattering process involves an intermediate on-shell state X, such that $B\to X\to K\pi$. In particular, we assume that there exists a generic (multibody) state $Kn\pi$. In a straightforward generalization of Eq. (1.3), the charged and neutral channel amplitudes can be written as

$$\begin{aligned} A(B^+\to Kn\pi)&=A_{cs}^{n+}-A_{us}^{n+}e^{i\gamma}e^{i\delta_+^n},\\ A(B^0\to Kn\pi)&=A_{cs}^{n0}-A_{us}^{n0}e^{i\gamma}e^{i\delta_0^n}. \end{aligned} \tag{2.1}$$

Rescattering contributions, again decomposed according to their dependence on CKM factors, are given by

$$\begin{aligned} A(B^+\to Kn\pi\to\pi^+K^0)&=S_1^nA_{cs}^{n+}-S_2^nA_{us}^{n+}e^{i\gamma},\\ A(B^0\to Kn\pi\to\pi^-K^+)&=S_3^nA_{cs}^{n0}-S_4^nA_{us}^{n0}e^{i\gamma}, \end{aligned} \tag{2.2}$$

where S_i^n is the complex amplitude for rescattering from a given multibody final state to the channel of interest. Analogous contributions arise in the conjugated channels. In the limit where one neglects electroweak penguin operator contributions, isospin symmetry requires $A_{cs}^+=A_{cs}^0$, and this equality is not spoiled by rescattering effects. The $i=1,3,4$ rescattering amplitudes can, for our purposes, be absorbed into the unknown amplitudes in Eq. (1.3).

We are interested in the possibility that the rescattering of transitions mediated by $Q_{1,2}^{us}$ are significant enough to dominate A_{us}^+, so we make the approximation

$$A_{us}^+e^{i\delta_+}=\sum_n S_2^nA_{us}^{n+}, \tag{2.3}$$

and define $\epsilon=A_{us}^+/A_{cs}^+$. Let us assume that rescattering effects do not dominate the overall decay, so we may retain just terms linear in ϵ. In that case, $\mathcal{A}_{CP}^{\rm dir}$ of Eq. (1.4) and R of Eq. (1.6) take the form

$$\mathcal{A}_{CP}^{\rm dir}=\frac{2\epsilon\sin\gamma\sin\delta_+}{1-2\epsilon\cos\gamma\cos\delta_+}, \tag{2.4}$$

$$R=\frac{1-2r_0\cos\gamma\cos\delta_0+r_0^2}{1-2\epsilon\cos\gamma\cos\delta_+}. \tag{2.5}$$

Once again, we may extremize R with respect to the unknown r_0,

$$R\geqslant\frac{1-\cos^2\gamma\cos^2\delta}{1-2\epsilon\cos\gamma\cos\delta_+}. \tag{2.6}$$

Following the same line of reasoning as before with respect to the unknown strong phases δ_0 and δ_+, we find the new bound

$$\sin^2\gamma\leqslant R(1+2\epsilon\sqrt{1-R}), \tag{2.7}$$

or solving for cos γ,

$$|\cos\gamma|\geqslant\sqrt{1-R}-\epsilon R. \tag{2.8}$$

It is clear that even a small rescattering amplitude $\epsilon\sim0.1$ could induce a significant shift in the bound on γ deduced from R, effectively diminishing the model-independent bound of Ref. [9] as $R\to1$. It is also clear from (2.4) that a small rescattering effect, again $\epsilon\sim0.1$, could in principle generate an $\mathcal{O}(10\%)$ CP asymmetry which is significantly larger than the bound (1.5) on $\mathcal{A}_{CP}^{\rm dir}(B^\pm\to\pi^\pm K)$. Therefore, in order to understand whether a large CP asymmetry signals new physics, and whether it is possible to obtain a bound on γ, it is useful to employ a particular model of soft FSI to obtain an order of magnitude estimate of the effect. In the next section we will estimate the amplitude ϵ using a phenomenological approach based on the exchange of Regge trajectories [17–19].

B. The two body rescattering contribution

We will estimate the contribution to ϵ from the rescattering of certain two body intermediate states. While these channels alone are not expected to dominate rescattering [17], we might expect them to provide a conservative lower bound on the size of the effect. At any rate, it would be peculiar for the total effect of rescatttering to be significantly *smaller* than the two body contribution. The most important channels in charged B decays that might rescatter to the final states of interest are $B^-\to\pi^0K^-$ and $B^-\to\eta K^-$. [To be conservative, we will neglect the $\eta' K$ channel, which is unrelated to the others by SU(3) symmetry. Its inclusion would likely enhance the effect which we will find.] To estimate the contribution to ϵ of these channels, it is necessary to estimate both the relative amplitude A_{us}^{2+}/A_{cs}^{+} for producing the intermediate state, and the amplitude $\epsilon_2=|S_2|$ for it to rescatter to the final state of interest. In our model, then, $\epsilon=\epsilon_2(A_{us}^{2+}/A_{cs}^{+})$, and we will extract only the magnitude ϵ. We will not attempt to predict the strong phase δ_+, which is even more model-dependent.

The amplitude ratio A_{us}^{2+}/A_{cs}^{+} may be estimated using factorization [20–23] and the Bauer-Stech-Wirbel [24] model, starting from $\mathcal{H}_{\rm eff}$. We will ignore the electroweak penguin operators. In addition to the soft FSI contributions which we will estimate using the leading order Wilson coefficients, there are also ''hard'' FSI phases which can be generated via quark rescattering [12] and are estimated at next-to-leading order [6]. As discussed earlier, the final state interactions which we consider are distinct from these ''BSS'' phases.

We will impose SU(3) symmetry on all aspects of our estimate of ϵ. Corrections to the SU(3) limit are typically at the level of 30%, small compared with other uncertainties in the calculation. Hence we must consider both the intermediate states π^0K^+ and ηK^+. For normalization, we will also need to compute A_{cs}^{+}, which in the BSW model is induced by the QCD penguin operators. The computation is straightforward, and we find

$$\begin{aligned}A_{us}^{2+}&(B^+\to P^0K^+)\\&=-G_Fm_B^2|V_{ub}V_{us}^*|\{(C_1+C_2/3)F_{P^0}^{uu}f_+^K(m_{P^0}^2)L_K(\mu_{P^0})\\&\quad+(C_2+C_1/3)F_K^{su}f_+^{P^0}(m_K^2)L_\pi(\mu_{P^0})\},\end{aligned} \tag{2.9}$$

$$\begin{aligned}A_{cs}^{+}&(B^+\to\pi^+K^0)\\&=-G_Fm_B^2|V_{cb}V_{cs}^*|\Bigg\{(C_4+C_3/3)F_{K^0}^{sd}f_+^{\pi}(m_K^2)L_K(\mu_\pi)\\&\quad\times(C_6+C_5/3)F_{K^0}^{sd}f_+^{\pi}(m_K^2)\frac{2m_K^2}{m_sm_b}M_\pi(\mu_K)\Bigg\},\end{aligned}$$

for $P^0=\pi^0,\eta$. The form factors $f_+^P(q^2)$, the decay constants $F_P^{q_1q_2}$, and the kinematic functions $L_P(\mu_i)$ and $M_P(\mu_i)$ are defined as follows:

$$\langle P|\overline{q}\gamma^\mu b|B\rangle=f_+^P(q^2)(p_B+p_P)^\mu+f_-^P(q^2)(p_B-p_P)^\mu,$$

$$\langle P|\overline{q_1}\gamma^\mu\gamma_5q_2|0\rangle=-i\sqrt{2}F_P^{q_1q_2}p_P^\mu,$$

$$L_P(\mu)=1-\mu_P+\frac{f_-^P(q^2)}{f_+^P(q^2)}\mu,$$

$$\begin{aligned}M_P(\mu)=\frac{1}{2}\Bigg[&(3-y+(1-3y)\mu_P-(1-y)\mu)\\&+\frac{f_-^P(q^2)}{f_+^P(q^2)}(1-y+(1-y)\mu_P\\&+(1+y)\mu)\Bigg],\end{aligned} \tag{2.10}$$

where $\mu_i=m_i^2/m_B^2$, q^2 is the momentum carried away by the current, and $y\approx1/2-1$ is related to the distribution of quark momenta in the pseudoscalar mesons. Note that in the Bauer-Stech-Wirbel (BSW) model, the dominant contribution to $A_{us}^{2+}(B^+\to P^0K^+)$ is from the current-current operators $Q_{1,2}^{us}$, while the dominant contribution to $A_{cs}^{2+}(B^+\to\pi^+K^0)$ is from the QCD penguins $Q_{3,\ldots,6}^{cs}$.

The ratios $(A^{2+}_{us}/A^{+}_{cs})_{\pi^0,\eta}$ simplify significantly if SU(3) symmetry is imposed on the quantities which appear in Eq. (2.9). In particular, we make use of the SU(3) relations

$$F^{uu}_{\pi^0}=F^{sd}_K/\sqrt{2},\quad f^{\pi^0}_+(0)=f^K_+(0)/\sqrt{2},\tag{2.11}$$

$$F^{uu}_{\eta}=F^{sd}_K/\sqrt{6},\quad f^{\eta}_+(0)=f^K_+(0)/\sqrt{6}.$$

We have checked that the inclusion of $\eta-\eta'$ mixing, which violates SU(3), would change our final answer by no more than 20%. The ratios of interest then may be written

$$\left(\frac{A^{2+}_{us}}{A^{+}_{cs}}\right)_{\pi^0}=\frac{|V_{ub}V^*_{us}|}{|V_{cb}V^*_{cs}|}\frac{(1+1/3)(C_1+C_2)/\sqrt{2}}{(C_4+C_3/3)+(C_6+C_5/3)(2m_K^2/m_sm_b)(M_\pi(\mu_K)/L_\pi(\mu_K))},$$
$$\left(\frac{A^{2+}_{us}}{A^{+}_{cs}}\right)_{\eta}=\frac{1}{\sqrt{3}}\left(\frac{A^{2+}_{us}}{A^{+}_{cs}}\right)_{\pi^0},\tag{2.12}$$

where $\eta-\eta'$ mixing is neglected. With values for the coefficients C_i taken from Ref. [25], at leading order and with $\Lambda^{(5)}_{\overline{\rm MS}}=225$ MeV, we find $|(A^{2+}_{us}/A^{+}_{cs})_{\pi^0}|\simeq 0.35$.

We now turn to an estimate of the rescattering amplitude ϵ_2. Our technique is described in detail in Ref. [17], and here we only outline the procedure. We begin by writing an expression for the discontinuity of the amplitude for the charged B decay,

$$\begin{aligned}\text{Disc }\, &A(B^-\to\overline{K^0}\pi^-)_{\rm FSI}\\ &=\frac{1}{2}\int\frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1}\frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^{(4)}(p_B-p_1-p_2)\\ &\quad\times A(B^-\to\pi^0(p_1)K^-(p_2))\\ &\quad\times\mathcal{M}^\dagger(\pi^0(p_1)K^-(p_2)\to\overline{K^0}\pi^-).\end{aligned}\tag{2.13}$$

The rescattering matrix element $\mathcal{M}(\pi^0(p_1)K^-(p_2)\to\overline{K^0}\pi^-)$ has a well known parametrization inspired by Regge phenomenology [17]. For the exchange of the leading ρ trajectory, it may be written as

$$\mathcal{M}^+(\pi^0K^-\to\overline{K^0}\pi^-)=\gamma(t)\frac{e^{-i\pi\alpha(t)/2}}{\cos(\pi\alpha(t)/2)}\left(\frac{s}{s_0}\right)^{\alpha(t)},\tag{2.14}$$

where $\gamma(t)$ is a residue function, s and t are the Mandelstam variables, and $s_0=1$ GeV^2 is an arbitrary hadronic scale. We take a linear Regge trajectory, $\alpha(t)=\alpha_0+\alpha' t$, with $\alpha_0=0.44$ and $\alpha'=0.94$ GeV^{-2}. Since α_0 is approximately the same for the ρ, K^* and ω trajectories, it is convenient to consider a single ''octet'' trajectory which carries a representation of SU(3) and contains all the vector mesons. Taking $\gamma(t)=\gamma(0)\equiv\gamma$ for simplicity, the discontinuity can be calculated,

$$\text{Disc }\, A(B^-\to\overline{K^0}\pi^-)_{\rm FSI}=\gamma\overline{\epsilon_2}(s)\left(\frac{s}{s_0}\right)^{\alpha_0-1}A(B^-\to\pi^0K^-),\tag{2.15}$$

where

$$\overline{\epsilon_2}(s)=\frac{1}{16\pi}\frac{1}{\cos(\pi\alpha_0/2)}\frac{e^{-i\pi\alpha_0/2}}{s_0\alpha'(\ln(s/s_0)-i\pi/2)}.\tag{2.16}$$

We restore the FSI contribution to $A(B^-\to\overline{K^0}\pi^-)$ by use of a dispersion relation. The dispersion integral may be evaluated in closed form with the approximations $\alpha_0=\frac{1}{2}$ and $\ln(s/s_0)=\ln(m_B^2/s_0)$:

$$\begin{aligned}A(B^-\to\overline{K^0}\pi^-)_{\rm FSI}&=\gamma\overline{\epsilon_2}(m_B^2)A(B^-\to\pi^0(\eta)K^-)\\ &\quad\times\frac{1}{\pi}\int_{(m_\pi+m_K)^2}^{\infty}\frac{ds}{s-m_B^2}\left(\frac{s}{s_0}\right)^{\alpha_0-1}\\ &=i\gamma\overline{\epsilon_2}(m_B^2)\frac{\sqrt{s_0}}{m_B}A(B^-\to\pi^0K^-),\end{aligned}\tag{2.17}$$

where we have taken the limit $m^2_{\pi(K)}/m_B^2\to 0$. For rescattering through the ηK^- channel, the only difference is an SU(3) group theory factor in the residue function γ. The magnitude of the contribution of a given channel to the soft rescattering amplitudes defined in the previous section is then

$$\epsilon_2=\gamma\frac{\sqrt{s_0}}{m_B}|\overline{\epsilon_2}(m_B^2)|,\tag{2.18}$$

where the residue γ depends on the channel.

Finally, we must make a numerical estimate of γ, which parametrizes the coupling of the ρ trajectory to the pseudoscalar mesons. These couplings may be estimated by considering pp and π^+p scattering data and using SU(3) relations. The exchange of the ρ trajectory requires a coupling at each vertex, so

$$\gamma_\rho(pp\to pp)=\gamma^2_{pp\rho},\quad\gamma_\rho(\pi p\to\pi p)=\gamma_{\pi\pi\rho}\gamma_{pp\rho}.\tag{2.19}$$

The optical theorem gives

$$\sigma_{tot}=\frac{1}{s}\mathcal{M}_{f\to f} \tag{2.20}$$

for both pp and πp scattering. The forward scattering amplitude is obtained from Eq. (2.14), written for pp and πp scattering, by setting $t=0$ (that is, $\cos\theta=1$). The residue functions (2.19) entering the expression for the forward scattering amplitudes are fixed from the Particle Data Group parametrizations of pp and πp scattering data [26],

$$\sigma_{\rm tot}^{ik}=X_{ik}\left(\frac{s}{s_0}\right)^{0.08}+Y_{ik}\left(\frac{s}{s_0}\right)^{-0.56}. \tag{2.21}$$

The first term represents the Pomeron contribution. The second comes from the ρ trajectory for π^+p scattering, and is a combined contribution from the ρ and a trajectories for pp scattering. The Particle Data Group fit gives $Y_{pp}=56.08$ mb and $Y_{\pi^+p}=27.56$ mb. Assuming that the ρ and a trajectories contribute equally in the pp channel, and using again the approximation $\alpha_0=\frac{1}{2}$, we find

$$\gamma_0^2\equiv\gamma_{\pi\pi\rho}^2=\frac{2s_0Y_{\pi p}^2}{Y_{pp}}\approx 72. \tag{2.22}$$

As defined, γ_0 is the $\pi^+\pi^-\rho^0$ coupling, since of the vector meson octet, only the ρ_0 contributes in the π^+p channel. The residue functions which we will require may be found by applying SU(3) symmetry. For ρ exchange in the πK channel, we find $\gamma_{\pi\pi\rho}=\gamma_0$, $\gamma_{KK\rho}=-\gamma_0/\sqrt{2}$, and $\gamma_{\pi\eta\rho}=0$. As a consequence, we have for the ρ trajectory,

$$\gamma_\rho(\pi^0K^-\to\pi^-\overline{K^0})=-\frac{1}{\sqrt{2}}\gamma_0^2,$$
$$\gamma_\rho(\eta K^-\to\pi^-\overline{K^0})=0, \tag{2.23}$$

and similarly for K^* exchange,

$$\gamma_{K^*}(\pi^0K^-\to\pi^-\overline{K^0})=-\frac{1}{2}\sqrt{\frac{1}{2}}\gamma_0^2,$$
$$\gamma_{K^*}(\eta K^-\to\pi^-\overline{K^0})=\frac{1}{2}\sqrt{\frac{3}{2}}\gamma_0^2. \tag{2.24}$$

With these residues, we complete the computation of the rescattering amplitude ϵ_2.

We are now in a position to estimate the contribution to ϵ from a given two body rescattering channel, by combining the rescattering amplitudes (2.18), the production amplitude ratios (2.12), and the residues (2.23) and (2.24). For example, for the intermediate π^0K^- state, rescattering via ρ exchange, we find a term

$$\epsilon_{\pi K\rho}=\left(\frac{A_{us}^{n+}}{A_{cs}^{+}}\right)_{\pi^0}\frac{\sqrt{s_0}}{m_B}|\overline{\epsilon_2}(m_B^2)|\gamma_\rho(\pi^0K^-\to\pi^-\overline{K^0}). \tag{2.25}$$

We do not know the strong phase which multiplies $\epsilon_{\pi K\rho}$. In adding the three contributions $\epsilon_{\pi K\rho}$, $\epsilon_{\pi KK^*}$, and $\epsilon_{\eta KK^*}$ (recall that $\epsilon_{\eta K\rho}$ vanishes), we must make some assumption about their relative phases.[1] For the purpose of this estimate we might imagine adding them incoherently, so $\epsilon^2=\epsilon_{\pi K\rho}^2+\epsilon_{\eta KK^*}^2+\epsilon_{\eta K\rho}^2$. Our estimates for the various channels are $\epsilon_{\pi K\rho}\simeq 0.044$, $\epsilon_{\pi KK^*}\simeq 0.022$, and $\epsilon_{\eta KK^*}\simeq 0.022$, which by this prescription would give $\epsilon\sim 0.06$. Alternatively, adding them coherently would yield $\epsilon\sim 0.09$. These unknown relative strong phases are one important source of uncertainty in this estimate of ϵ.

Of course, there are many other sources of uncertainty as well. Perhaps the most severe of these is the neglect of multibody intermediate states, which we have omitted because we have no good model for them. For this reason, we are likely to have, if anything, underestimated the effect of FSI. More model dependence arises from the use of factorization to estimate the ratios A_{us}^{2+}/A_{cs}^{+}, given that this ansatz is known at times to fail in B decays to light pseudoscalar mesons. Smaller uncertainties arise from the phenomenological extraction of the residues and from the use of SU(3) symmetry. Finally, we note that in some (but not all) models of Regge exchange, there may be cancellations which further suppress the small contribution from the K^* exchange to both the ηK^- and π^0K^- intermediate states. (For recent discussions of these questions, see Refs. [27–29].) Similar effects may enhance the contribution from[2] $\eta' K^-$. Our neglect of the η' has been conservative, in the sense that it has likely caused us to underestimate the total effect of rescattering. We have done so because, in the presence of the anomaly, the η' is not related by unitary symmetry to any other meson. Hence we have little guidance, other than from the quark model, for how to include it. In the end, it certainly would be unwise to trust our estimate of ϵ to better than a factor of two, and even that much confidence would be optimistic. The same caveat should be applied to *any* phenomenological model of soft final state interactions. What is important here is that we have found neither a dominant effect of order one or larger, nor an insignificant effect of order one percent.

C. The effect of FSI on $\mathcal{A}_{CP}^{\rm dir}$ and R

We close by returning to the effect of FSI on the observables $\mathcal{A}_{CP}^{\rm dir}$ and R. We have seen from our model that rescattering effects as large as $\epsilon\sim 0.1-0.2$ easily could be consistent with the standard model. Therefore, unless γ were known independently to be very small, an observation of $\mathcal{A}_{CP}^{\rm dir}\sim 0.2$ could *not* be taken unambiguously to be a signal for new physics. Note that we do not claim to predict such a large asymmetry; we simply observe that it would be neither unnatural nor surprising for it to be generated by final state interactions. On the other hand, we would maintain that a larger asymmetry, such as $\mathcal{A}_{CP}^{\rm dir}\sim\mathcal{O}(1)$, would still be an exciting sign of a source of CP violation beyond the CKM matrix.

[1]We prefer not to impose constraints from SU(3) symmetry on the phases because of the substantial model-dependence already present in our calculation. Doing so would not change substantially the magnitude of the rescattering effects, but it would imply a smaller uncertainty than we would advocate.

[2]We are grateful to H. Lipkin for discussions of this point, and for stressing to us the important role played by tensor meson exchange.

Similarly, we can consider the effect of $\epsilon\sim 0.1$ on the bounds (2.5) and (2.7) on γ. For example, the fractional correction to the bound on $|\cos\gamma|$ is $\Delta\equiv\epsilon R/\sqrt{1-R}$. The value of Δ is a strong function of the experimentally observed $R_{\rm exp}$,

$$\Delta\simeq\epsilon \quad \text{for } R_{\rm exp}=0.65, \qquad \Delta\simeq 2\epsilon \quad \text{for } R_{\rm exp}=0.80. \tag{2.26}$$

The bound deteriorates quickly as $R_{\rm exp}\to 1$. In terms of $\sin^2\gamma$, we certainly would conclude that the observation $\sin^2\gamma\sim 1.2\, R_{\rm exp}$ would *not* constitute an unambiguous signal of new physics.

III. MODEL INDEPENDENT BOUNDS ON THE FSI CORRECTIONS

Our phenomenological model suggests that soft FSI contributions of the current-current operators $Q_{1,2}^{us}$ could account for $\mathcal{O}(10\%)$ of the $B^+\to K^0\pi^+$ amplitude. This has a dramatic consequence, namely making $\mathcal{A}_{CP}^{\rm dir}\sim 10-20$ % realistic within the standard model. In view of the large theoretical uncertainties involved, it would be extremely useful to find an experimental method by which to bound the magnitude of the FSI contribution. The observation of a larger asymmetry would then be a signal for new physics. In this section we describe such an attempt, along the lines proposed in Ref. [30] for the decay $B\to\phi K$. The idea is to find decay modes mediated by the quark level transition $b\to s\bar{s}d$, for which branching ratio measurements or upper bounds would, by application of flavor SU(3) flavor symmetry, imply a direct upper bound on ϵ.

The most interesting modes in our case turn out to be $B^\pm\to K^\pm K$. The effective Hamiltonian for $b\to d$ transitions may be obtained from (1.2) by the substitution of $s\to d$ in the operators and in the indices of the CKM matrix elements. In analogy with Eq. (1.3) the amplitudes may be decomposed according to their dependence on CKM factors, giving

$$A(B^+\to K^+\overline{K^0})=A_{cd}-A_{ud}e^{i\gamma}e^{i\delta}, \qquad A(B^-\to K^-K^0)=A_{cd}-A_{ud}e^{-i\gamma}e^{i\delta}. \tag{3.1}$$

Invariance under the SU(3) rotation $\exp(i\,(\pi/2)\,\lambda_7)$, i.e., interchange of s and d quark fields, implies equalities among operator matrix elements,

$$\langle K^-K^0|Q_i^{qd}|B^-\rangle=\langle K^0\pi^-|Q_i^{qs}|B^-\rangle,\quad q=u,c,;\quad i=1,2,$$
$$\langle K^-K^0|Q_i^{d}|B^-\rangle=\langle K^0\pi^-|Q_i^{s}|B^-\rangle,\quad i=3,\ \dots\ ,10. \tag{3.2}$$

These lead to the relations

$$A_{ud}e^{i\delta}=A_{us}^+e^{i\delta_+}\frac{V_{ud}}{V_{us}}(1+R_{ud}),\quad A_{cd}=A_{cs}^+\frac{V_{cd}}{V_{cs}}(1+R_{cd}), \tag{3.3}$$

where R_{ud} and R_{cd} parametrize SU(3) violation, which is typically of the order of 20–30 %. Note that it is only an SU(2) subgroup of SU(3), namely U spin, which is required to derive these relations. Since the B^- carries $U=0$ and the transition operators Q_i^{qd} and Q_i^d carry $U=\frac{1}{2}$, it is only the $U=\frac{1}{2}$ component of the K^-K^0 final state which couples to the decay channel. As a result, we have the freedom to add any pure $U=\frac{3}{2}$ combination to the final state, involving additional $\pi^-\pi^0$ and $\pi^-\eta$ pairs, without affecting the relations (3.2). As it turns out, it is the simple combination K^-K^0 which yields the most phenomenologically interesting bound.

An upper bound on ϵ follows from the ratio

$$R_K=\frac{B(B^+\to K^+\overline{K^0})+B(B^-\to K^-K^0)}{B(B^+\to K^0\pi^+)+B(B^-\to\overline{K^0}\pi^-)}. \tag{3.4}$$

After some algebra, we obtain

$$\epsilon<\lambda\sqrt{R_K}(1+{\rm Re}[R_{ud}])+\lambda^2(R_K+1)\cos\gamma\cos\delta_+ +\mathcal{O}(\lambda^3,\lambda^2R_{ud,cd}), \tag{3.5}$$

where $\lambda\simeq 0.22$ is the Wolfenstein parameter. [In deriving this relation, we assume that the full matrix element for $B^-\to K^-K^0$ is not smaller, in magnitude, than the partial contribution from rescattering. It is possible, if there is some fine tuned cancellation, for rescattering actually to lower the branching ratio; in this case, the relation (3.5) would not be valid.] This bound becomes more reliable if we set $\cos\gamma\cos\delta_+=1$. With Eq. (2.7), we find a bound on γ,

$$\sin^2\gamma<R+2\lambda R\sqrt{R_K(1-R)}+\mathcal{O}(\lambda^2,\lambda R_{ud,cd}), \tag{3.6}$$

ignoring electroweak penguin operators. One could include them, as before, with the substitution in (1.10).

We are also interested in obtaining an upper bound on $\mathcal{A}_{CP}^{\rm dir}$. Keeping $\cos\delta$ free gives

$$|\mathcal{A}_{CP}^{\rm dir}|<2\epsilon\sin\gamma+\mathcal{O}(\epsilon^3). \tag{3.7}$$

In the absence of a nontrivial bound on γ from Eq. (3.6), $\mathcal{A}_{CP}^{\rm dir}$ is maximized at $\sin\gamma=1$, leading to

$$|\mathcal{A}_{CP}^{\rm dir}|<2\lambda\sqrt{R_K}(1+{\rm Re}[R_{ud}])+\mathcal{O}(\lambda^3,\lambda^2R_{ud,cd}). \tag{3.8}$$

However, an independent bound on γ would place a tighter limit on the asymmetry,

$$|\mathcal{A}_{CP}^{\rm dir}|<2\lambda\sqrt{R_KR}(1+{\rm Re}[R_{ud}])+2\lambda^2\sqrt{R}(R_K\sqrt{1-R}+R_K+1)+\mathcal{O}(\lambda^3,\lambda^2R_{ud,cd}). \tag{3.9}$$

Again, electroweak penguin operators may be included with the substitution (1.10). Following Ref. [30], analogous bounds on $\mathcal{A}_{CP}^{\rm dir}(B^\pm\to\phi K^{(*)\pm})$ can be obtained by substituting for R_K the ratio $[\sqrt{B(B^\pm\to K^{0(*)}K^\pm)}+\sqrt{B(B^\pm\to\phi\pi^\pm(\rho^\pm))}]/\sqrt{B(B^\pm\to\phi K^\pm)}$.

The CLEO Collaboration has recently obtained the upper bound $R_K<0.95$ at 90% C.L., including only statistical errors, and approximately $R_K<1.9$ once systematic errors are included as well [31]. Unfortunately, this is not very restrictive, implying $\epsilon<0.4$ and $\mathcal{A}_{CP}^{\rm dir}<0.6$. It is possible that more interesting constraints on ϵ and $\mathcal{A}_{CP}^{\rm dir}$ could be obtained in the future, given that $R_K \sim \mathcal{O}(\lambda^2)$ in the limit of vanishing FSI. Ultimately, the utility of such a bound is limited by the fact that, for ϵ small enough, R_K becomes independent of ϵ, since rescattering channels are then negligible compared with other contributions. Conversely, a large observed value for R_K would not necessarily mean that FSI contributions are large, since this could also result from new physics which enhances the $b \to d$ transitions. Resolving the question of how to distinguish these two possibilities is left for the future.

IV. NEW PHYSICS

Assuming that a large CP asymmetry is observed in $B^\pm \to \pi^\pm K$ decays or that $\sin\gamma$ is measured independently and found to violate the bound (1.9), this could be explained either by large soft rescattering effects or by new physics. From our calculations above, it is clear that theory cannot at present exclude the possibility of large FSI effects. Yet, it is also possible that these effects are not large and that, in the future, the experimental tests proposed in the previous section will imply that new physics indeed is required. It is important, then, to understand which extensions of the standard model could contribute significantly (and with new CP violating phases) to the relevant $B \to \pi K$ modes. Furthermore, we would like to understand whether new physics contributions to the charged and neutral modes should be expected to have any special features (such as isospin symmetry relations) which will allow further tests.

The most general new physics effects on the amplitudes (1.3) can be parametrized by six new parameters,

$$\begin{aligned} A(B^0 \to \pi^- K^+) &= A^0_{\rm SM} - A^u_N e^{i\phi_u} e^{i\delta_u}, \\ A(\overline{B^0} \to \pi^+ K^-) &= \overline{A}^0_{\rm SM} - A^u_N e^{-i\phi_u} e^{i\delta_u}, \\ A(B^+ \to \pi^+ K^0) &= A^+_{\rm SM} - A^d_N e^{i\phi_d} e^{i\delta_d}, \\ A(B^- \to \pi^- \overline{K^0}) &= A^-_{\rm SM} - A^d_N e^{-i\phi_d} e^{i\delta_d}. \end{aligned} \tag{4.1}$$

Here $A_{\rm SM}$ are the standard model amplitudes, where A^0 and $\overline{A}^0$ carry the same strong phases and opposite weak phases, as do $A^\pm$. We introduce the new physics amplitudes $A^{u,d}_N$, with CP violating phases $\phi_{u,d}$ and strong phases $\delta_{u,d}$.

A first class of models are those which give potentially large tree-level contributions to $\Delta B = 1$ four quark operators. These include supersymmetry without R parity and models with extraquarks in vector-like representations of the SM gauge group. A second class of models gives new contributions to four quark operators through loop diagrams only, with new particles running in the loop. This includes various supersymmetric flavor models and a sequential fourth generation. A third class of models gives new contributions to the $\Delta B = 1$ chromomagnetic dipole operators. Finally, there are models where no significant effect is expected. These include models of extra scalars and left-right symmetric models.

A. Supersymmetry without R parity

In supersymmetric models without R_p there are new, slepton mediated tree diagrams, contributing at tree level to the $b \to u\bar{u}\bar{s}$ and $b \to d\bar{d}\bar{s}$ transitions.

First, consider the $\lambda'_{ijk} L_i Q_j \bar{d}_k$ couplings. For $b \to u\bar{u}\bar{s}$ transitions, the contributions come from charged slepton mediated diagrams, while for $b \to d\bar{d}\bar{s}$ transitions, the contributions come from sneutrino mediated diagrams:

$$A^u_N e^{i\phi_u} \propto \sum_{i=1}^{3} \frac{\lambda'_{i13}\lambda'^*_{i12}}{m^2(\tilde{l}^-_i)}, \tag{4.2}$$

$$A^d_N e^{i\phi_d} \propto \sum_{i=1}^{3} \frac{\lambda'_{i13}\lambda'^*_{i12} + \lambda'_{i23}\lambda'^*_{i11} + \lambda'_{i21}\lambda'^*_{i31} + \lambda'_{i11}\lambda'^*_{i32}}{m^2(\tilde{\nu}_i)}. \tag{4.3}$$

We learn that indeed the new physics introduces six new independent parameters. In the special case where (i) $m^2(\tilde{l}^-_i) = m^2(\tilde{\nu}_i)$ and (ii) $\lambda'_{i13}\lambda'^*_{i12}$ is much larger than the other three combinations that appear in (4.2), isospin is a good symmetry (similar to the SM QCD penguin diagrams). The first condition is fulfilled in many models, but the second is not. Generically, we have $A^d_N \neq A^u_N$, $\phi_d \neq \phi_u$ and $\delta_u \neq \delta_d$.

Second, consider the $\lambda''_{ijk} \bar{u}_i \bar{d}_j \bar{d}_k$ couplings and note that these couplings are antisymmetric in (j,k). For $b \to u\bar{u}\bar{s}$ transitions, the contributions come from the down squark mediated diagrams only, while for $b \to d\bar{d}\bar{s}$ transitions, the contributions come from up, charm and top squark mediated diagrams:

$$A^u_N e^{i\phi_u} \propto \frac{\lambda''_{113}\lambda''_{112}}{m^2(\tilde{d}_1)}, \tag{4.4}$$

$$A^d_N e^{i\phi_d} \propto \sum_{i=1}^{3} \frac{\lambda''_{i13}\lambda''^*_{i12}}{m^2(\tilde{u}_i)}. \tag{4.5}$$

Again, there are six new independent parameters unless (i) the $i=1$ contribution dominates A^d_N and (ii) $m^2(\tilde{d}_1) = m^2(\tilde{u}_1)$, in which case isospin is a good symmetry. The first condition is unlikely to be fulfilled. We expect $A^d_N \neq A^u_N$, $\phi_d \neq \phi_u$ and $\delta_u \neq \delta_d$.

B. Singlet down quarks

Models with additional $SU(2)_L$-singlet down quarks could lead to dramatic effects in CP violation in the interference between decays with and without mixing [32]. This is a result of Z-mediated tree level contributions to $B-\overline{B}$ mixing. Could there also be large effects in $\mathcal{A}^{\rm dir}_{CP}$?

Obviously, there are new contributions to the decay amplitudes from Z-mediated tree level diagrams,

$$A^u_N e^{i\phi_u} \propto \frac{U_{sb} g_{Zuu}}{m_Z^2}, \tag{4.6}$$

$$A_N^d e^{i\phi_d} \propto \frac{U_{sb} g_{Zdd}}{m_Z^2}, \tag{4.7}$$

where $g_{Zff} \propto (T_3^f - Q_f \sin^2\theta_W)$, and U_{sb} is the mixing angle in the Z couplings. The present experimental bound on $B(B \to X\mu^+\mu^-)$ gives $|U_{sb}/(V_{cb}V_{cs})| \leq 0.04$. This suppression roughly compensates for the loop suppression of the standard model QCD penguin. Thus, we learn that

(i) A_N could be of the same order as A_{SM};
(ii) $A_N^u \neq A_N^d$ because of the g_{Zqq} factor;
(iii) $\phi_u = \phi_d = \arg(U_{sb})$.

C. Supersymmetric flavor models

Supersymmetric models (with R_p conserved) contribute to $b \to s$ transitions through penguin diagrams with squark-gluino loops. The ratio between this contribution and the standard model penguin amplitude may be estimated to be [33]

$$\frac{A^{\rm SUSY}}{A^{\rm SM}} \sim \frac{\alpha_3}{\alpha_2} \frac{m_W^2}{\max(m_{\tilde{b}}^2, M_3^2)} \frac{K_{32}^* K_{33}}{V_{tb}V_{ts}^*} \frac{\eta}{\ln(m_t^2/m_c^2)}, \tag{4.8}$$

where K is the mixing matrix for the gluino couplings and $\eta \sim (m_{\tilde{b}}^2 - m_{\tilde{s}}^2)/(m_{\tilde{b}}^2 + m_{\tilde{s}}^2)$ is a measure of the non-universality in the squark masses. We see that even in the absence of a super-Glashow-Iliopoulos-Maiani (GIM) mechanism, namely with $\eta \sim 1$, the supersymmetric contribution is comparable to the standard model one only if the relevant mixing angle, namely $K_{32}^* K_{33}$, is large. Such a situation is phenomenologically allowed. Furthermore, the fact that $m_s/m_b \sim |V_{cb}|$ implies that a large $\tilde{b}_R - s_R$ mixing $[K_{32} \sim \mathcal{O}(1)]$ is not unlikely [34]. Below we examine whether this possibility is indeed realized in various supersymmetric flavor models.

Most flavor problems of supersymmetry are solved (without giving up naturalness) in models where the first two squark generations are heavy [35–39]. Mixing angles with the third generation can be large. In these models, for $\tilde{b}_L$ and gluino masses in the range of 100–300 GeV, it was found [40] that the supersymmetric QCD penguins (namely, squark-gluino loops) can be twice as large as the standard model ones for $b \to s$ transitions, and can carry a new phase. Similar to the standard model QCD penguins, isospin is a good symmetry here, namely $A_N^d = A_N^u$, $\phi_d = \phi_u$ and $\delta_d = \delta_u$.

The situation is different in models of Abelian horizontal symmetries, where alignment of quark and squark mass matrices is the mechanism which suppresses supersymmetric contributions to flavor-changing neutral currents (FCNCs) [41,42]. The constraints from $K - \bar{K}$ mixing require that the (32) entries in the mass matrices are also small, leading to a suppression of K_{32}. A similar situation occurs in models of non-Abelian horizontal symmetries, where degeneracy of the first two squark generations suppresses FCNC [36,43–45]. Typically, the first two generations are in a doublet and the third in a singlet of the horizontal symmetry. Then, $\tilde{b}_{L,R} - s_{L,R}$ mixing does break the horizontal symmetry and is, therefore, suppressed: $K_{32}K_{33}$ is of order $V_{ts}V_{tb}$ and the supersymmetric contribution to the $b \to s$ transition is typically small [33,46].

Finally, if CP is an approximate symmetry of the new physics (which is a viable possibility in the supersymmetric framework), so that all CP violating phases are small, say $\phi_{u,d} = \mathcal{O}(10^{-3})$, then we expect new physics to contribute to $\mathcal{A}_{CP}^{\rm dir}$ at the level $\leq 10^{-3}$ regardless of the size of the supersymmetric contributions to the various $B \to \pi K$ decays.

D. Fourth quark generation

Models of four quark generation require, of course, that the fourth generation neutrino is rather heavy ($\geq m_Z/2$). If this possibility is realized in nature, then we expect large new contributions from QCD penguin diagrams with W-t' loops. The $m_{t'}$ dependence of this contribution may compensate for a possibly small CKM factor, $V_{t'b}V_{t's}^*$, and become a significant, if not dominant, contribution. Furthermore, as the 4×4 quark mixing matrix has three independent CP violating phases, this contribution is likely to carry a new phase. Isospin should be a good symmetry and $\delta_u = \delta_d$ is likely.

E. Models with enhanced $b \to sg$

Models with enhanced $b \to sg$ dipole operator coefficients, leading to $\mathcal{B}(b \to sg) \sim 10\%$, have been suggested as a possible resolution of several potential puzzles in inclusive B decays [47,48]. Examples have been discussed which employ squark-gluino loops, vectorlike quark-neutral scalar loops, or techniscalar exchange [48,49]. Despite the large overall rate, the dipole induced amplitudes for rare B decays are of same order as the standard model amplitudes and so the two can interfere substantially. Arbitrary new weak phases in the dipole operator coefficients can therefore lead to sizable $A_{CP}^{\rm dir} \geq 10\%$ [50]. More modest enhancement of $b \to sg$ can also lead to sizable CP asymmetries. Such an example has been discussed in the context of top color models [51]. Again isospin is a good symmetry.

F. Discussion

There exist, of course, extensions of the standard model where large new contributions to the relevant $B \to \pi K$ decays are unlikely. Below we give a few examples.

Charged Higgs mediated tree diagrams contribute only to A_N^u, so $A_N^d = 0$. The CKM combinations are similar to those in the standard model W mediated tree diagrams, namely we still have $\phi_u = \gamma$. The contribution is, however, small compared to the standard model A_{us}^+ because, while the CKM suppression persists, we now have in addition a strong suppression from small Yukawa couplings. Consequently, there is no effect on $\mathcal{A}_{CP}^{\rm dir}$ or on R.

Neutral Higgs exchange in models without natural flavor conservation (NFC) could contribute to both $b \to u\bar{u}s$ and $b \to d\bar{d}s$. But if the smallness of scalar mediated FCNC is explained by a horizontal symmetry, then we expect a suppression of order $\leq \mathcal{O}(m_b m_{u,d}/m_Z^2) \sim 10^{-5}$, which means that the effects are negligible.

In left-right symmetric models there is a new contribution from W_R-mediated decays, but it is suppressed by $\mathcal{O}(m_{W_L}^2/m_{W_R}^2) \leqslant 10^{-2}$. The CKM ratio is expected to be $\mathcal{O}(1)$. So, again, we expect no observable effects on either $\mathcal{A}_{CP}^{\text{dir}}$ or R.

To summarize the situation regarding new physics effects, we note the following points:

(i) There are several well-motivated extensions of the standard model which can significantly affect $B \to \pi K$ decays.

(ii) In models where there are new tree diagram contributions, there are no relations, in general, between the contributions to $B^+ \to \pi^+ K^0$ and the contributions to $B^0 \to \pi^- K^+$. The new physics effects in the amplitudes (1.3) introduce six new parameters.

(iii) In models where the new contributions are through QCD penguin or chromomagnetic dipole operators, isospin is a good symmetry, and the number of new parameters is therefore reduced from six to three.

(iv) We note that if a large CP asymmetry (1.4) is measured, that will invalidate the bound (1.9). In contrast, if a small CP asymmetry is measured, that would not provide an unambiguous confirmation for the validity of this bound, because it could be a result of small strong phases rather than a small magnitude of final state interactions.

V. CONCLUSIONS

We have analyzed the effect of final state interactions on the search for new physics in $B \to \pi K$ decays. Using a phenomenological model, we found that while such effects are unlikely to be large enough to dominate individual branching fractions, they can still complicate those avenues for identifying new physics which rely on a standard model suppression of weak phases in the matrix elements for $B^\pm \to \pi^\pm K$. As a result, and in contrast to previous expectations, we conclude that the observation of $\mathcal{A}_{CP}^{\text{dir}} \sim 0.2$ or $\sin^2 \gamma \sim 1.2R$ would *not* be an unambiguous sign of a source of CP violation beyond the CKM matrix. While we do not claim to compute the magnitude of final state interactions reliably, our model is sufficient to demonstrate that effects of this size are entirely generic and cannot be ruled out without independent *empirical* evidence. We propose a simple test which could probe this question experimentally. Finally, in anticipation of the observation of CP violation in these channels at a level which cannot be explained by the standard model, we discuss the features of various models of new physics as they would be manifested in these decays.

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