Probing penguin coefficients with the lifetime ratio $\tau(B_s)/\tau(B_d)$

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We calculate penguin contributions to the lifetime splitting between the B_s and the B_d meson. In the standard model the penguin effects are found to be opposite in sign, but of similar magnitude as the contributions of the current-current operators, despite the smallness of the penguin coefficients. We predict $\tau(B_s)/\tau(B_d)-1=(-1.2\pm 10.0)\times 10^{-3}(f_{B_s}/190\ \mathrm{MeV})^2$, where the error stems from hadronic uncertainties. Since penguin coefficients are sensitive to new physics and poorly tested experimentally, we analyze the possibility to extract them from a future precision measurement of $\tau(B_s)/\tau(B_d)$. Anticipating progress in the determination of the hadronic parameters ε_1, ε_2, and f_{B_s}/f_{B_d} we find that the coefficient C_4 can be extracted with an uncertainty of order $|\Delta C_4|\simeq 0.1$ from the double ratio $[\tau(B_s)-\tau(B_d)]/[\tau(B^+)-\tau(B_d)]$, if $|\varepsilon_1-\varepsilon_2|$ is not too small. [S0556-2821(98)04707-9]

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I. INTRODUCTION

The theoretical achievement of heavy quark expansion (HQE) [1] has helped a lot to understand the inclusive properties of B mesons. The measurements of lifetime differences among the b-flavored hadrons test HQE at the order $(\Lambda_{\mathrm{QCD}}/m_b)^3$. Today's experimental information on the B-meson lifetimes is in agreement with the expectations from HQE, but the present theoretical predictions still depend on four poorly known hadronic parameters B_1, B_2, ε_1, and ε_2 [2,3]. Recently they have been obtained by QCD sum rules [4]. Lattice results are expected soon from the Rome group [5] and will allow for significantly improved theoretical predictions of the lifetime ratios.

Weak decays are triggered by a Hamiltonian of the form

$$H=\frac{G_F}{\sqrt{2}}\left[V_{\mathrm{CKM}}\sum_{j=1}^{2} C_jQ_j - V'_{\mathrm{CKM}}\left(\sum_{k=3}^{6} C_jQ_j + C_8Q_8\right)\right]. \tag{1}$$

Here Q_1 and Q_2 are the familiar current-current operators, $Q_3 \ldots Q_6$ are penguin operators, and Q_8 is the chromomagnetic operator. Their precise definition is given below in Eq. (3). The factors V_{CKM} and V'_{CKM} represent the factors stemming from the Cabibbo-Kobayashi-Maskawa matrix and are specific to the flavor structure of the decay. Feynman diagrams in which the spectator quark participates in the weak decay amplitude induce differences among the various b-flavored hadrons. Such nonspectator effects have been addressed first by Bigi *et al.* in [6] evaluating the matrix elements in the factorization approximation in which $\varepsilon_1=\varepsilon_2=0$. Then Neubert and Sachrajda [2] have found that even small deviations of $\varepsilon_1,\varepsilon_2$ from zero drastically weaken the prediction of [6] for the lifetime ratio $\tau(B^+)/\tau(B_d)$, which can sizeably differ from 1. On the other hand, the deviation of $\tau(B_s)/\tau(B_d)$ from unity has been estimated to be below 1% in [6,2] and the detailed analysis of Beneke, Buchalla, and Dunietz [3]. Here $\tau(B_s)$ is the average lifetime of the two CP eigenstates of B_s.

Experimentally the ratio $\tau(B_s)/\tau(B_d)$ can also be addressed by the measurements of the corresponding semileptonic branching fractions. Since spectator effects in the semileptonic decay rate are negligible, one may use $\tau(B_s)/\tau(B_d)=B_{SL}(B_s)/B_{SL}(B_d)$.

So far only the effect of Q_1 and Q_2 has been considered in [6,2,3]. Taking into account the present experimental uncertainty and the fact that C_1 and C_2 are much larger than C_{3-8} in the standard model this is justified. Yet once the lifetime ratio $\tau(B_s)/\tau(B_d)$ is measured to an accuracy of a few permille, the situation will change: The smallness of $|\tau(B_s)/\tau(B_d)-1|$ is caused by the fact that the *weak annihilation* contribution of $Q_{1,2}$ depicted in Fig. 1 almost yields the same contribution to the decay rates of B_s and B_d. The difference in the Cabibbo-Kobayashi-Maskawa (CKM) factors is negligible and the lifetime difference is induced by the small difference of the $(c,\overline{c})$ vs $(c,\overline{u})$ phase space and by $\mathrm{SU}(3)_F$ violations of the hadronic parameters. These effects

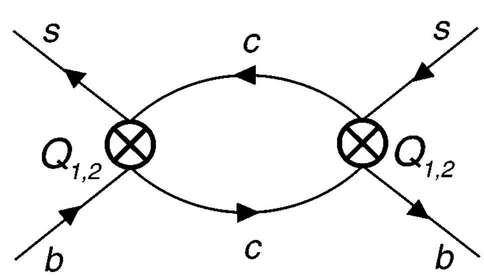

FIG. 1. Nonspectator (*weak annihilation*) contribution to the B_s decay rate involving two current-current operators. The corresponding diagram for the B_d decay is obtained by replacing s by d and the upper c by u.

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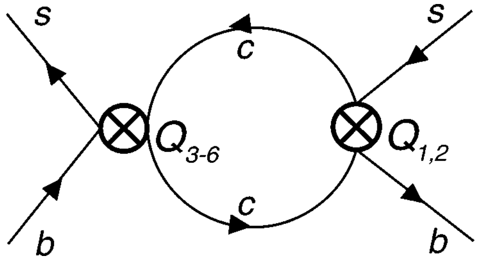

FIG. 2. Weak annihilation diagram involving one penguin operator Q_{3-6}. Penguin contributions to the nonspectator rate of the B_d meson are CKM suppressed and therefore negligible.

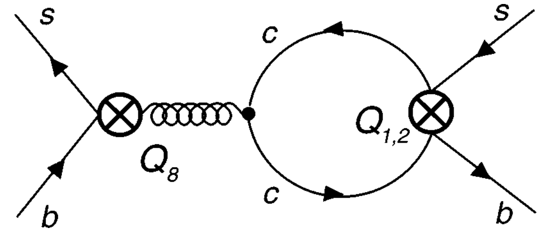

FIG. 3. Contribution of Q_8 to $\Gamma^{\rm nonspec}(B_s)$. In the standard model the diagram is of the same order of magnitude as radiative corrections to Fig. 2 and therefore negligible. Yet in models in which quark helicity flips occur in flavor-changing vertices $|C_8|$ can easily be ten times larger than in the standard model [7]. The contribution of Q_1 vanishes.

suppress $|\tau(B_s)/\tau(B_d)-1|$ by roughly an order of magnitude compared to $|\tau(B^+)/\tau(B_d)-1|$. The contributions stemming from the penguin operators and the chromomagnetic operator, however, do not exhibit such a cancellation. Their contribution to the nonspectator rate of B_s comes with the same power of the Wolfenstein parameter $\lambda=0.22$ as the contribution of $Q_{1,2}$. In contrast the effects of Q_{3-8} to the nonspectator rate of B_d or B^+ are suppressed by two powers of λ and are therefore negligible. Hence one expects the contributions of Q_{3-6} and Q_8 to $|\tau(B_s)/\tau(B_d)-1|$ to be of the same order as those of Q_1 and Q_2. $\tau(B^+)/\tau(B_d)$ is not modified, so that the phenomenological conclusions drawn from this ratio in [2] are unchanged. Observables sensitive to C_{3-8} like $\tau(B_s)/\tau(B_d)$ are phenomenologically highly welcome. The smallness of C_{3-8} is a special feature of the helicity structure of the corresponding diagrams in the standard model. In many of its extensions the values of these coefficients can easily be much larger. Such an enhancement due to supersymmetric contributions has been discussed in [7]. Up to now the focus of the search for new physics has been on new contributions to C_8 [7]. Yet many interesting possible nonstandard effects modify C_{3-6} rather than C_8: New heavy particles mediating flavor changing neutral currents (FCNCs) at the tree level or modifications of the b-s-g chromoelectric form factor affect C_{3-6}, but not C_8. Likewise new heavy colored particles yield extra contributions to C_{3-6}, e.g., in supersymmetry box diagrams with gluinos modify C_{3-6}.

It is especially difficult to gain experimental information on the numerical values of the penguin coefficients C_{3-6}. Even penguin-induced decays to final states solely made of d and s quarks do not provide a clean environment to extract C_{3-6}: Any such decay also receives sizeable contributions from Q_2 via CKM-unsuppressed loop contributions [8,9]. In exclusive decay rates these ‘‘charming penguins’’ preclude the clean extraction of the effects of penguin operators [8]. In semi-inclusive decay rates like $B\to X_s\Phi$ the situation is expected to be similar. In inclusive decay rates such as the total charmless b decay rate the effect of ‘‘charming penguins’’ can be reliably calculated in perturbation theory. Yet these rates are much more sensitive to new physics contributions in C_8 rather than in C_{3-6}, because Q_8 triggers the two-body decay $b\to sg$, while the effects of Q_{3-6} involve an integration over three-body phase space [9]. Notice from Fig. 2 and Fig. 3, however, that this phase space suppression of the terms involving C_{3-6} is absent in the nonspectator diagrams inducing the lifetime differences.

This work is organized as follows: In the following section we calculate the contributions to $\tau(B_s)/\tau(B_d)$ involving Q_{3-6} or Q_8. Here we also obtain the dominant part of the radiative corrections to order α_s. In Sec. III we discuss the phenomenological consequences within the standard model and with respect to a potential enhancement of C_{3-8} by new physics.

II. PENGUIN CONTRIBUTIONS

For the nonspectator contributions to the B_s decay rate we need the $|\Delta B|=|\Delta S|=1$-Hamiltonian

$$H=\frac{G_F}{\sqrt{2}}\,V_{cb}V_{cs}^{*}\left[\sum_{j=1}^{6}C_jQ_j+C_8Q_8\right]+\text{H.c.} \tag{2}$$

with

$$Q_1=(\overline{s}c)_{V-A}\cdot(\overline{c}b)_{V-A}\cdot\widetilde{\mathbb{1}},$$

$$Q_2=(\overline{s}c)_{V-A}\cdot(\overline{c}b)_{V-A}\cdot\mathbb{1},$$

$$Q_3=\sum_{q=u,d,s,c,b}(\overline{s}b)_{V-A}\cdot(\overline{q}q)_{V-A}\cdot\mathbb{1},$$

$$Q_4=\sum_{q=u,d,s,c,b}(\overline{c}b)_{V-A}\cdot(\overline{q}q)_{V-A}\cdot\widetilde{\mathbb{1}},$$

$$Q_5=\sum_{q=u,d,s,c,b}(\overline{s}b)_{V-A}\cdot(\overline{q}q)_{V+A}\cdot\mathbb{1},$$

$$Q_6=\sum_{q=u,d,s,c,b}(\overline{s}b)_{V-A}\cdot(\overline{q}q)_{V+A}\cdot\widetilde{\mathbb{1}},$$

$$Q_8=-\frac{g}{8\pi^2}\,m_b\overline{s}\sigma^{\mu\nu}(1+\gamma_5)T^ab\cdot G^a_{\mu\nu}. \tag{3}$$

The color singlet and nonsinglet structure are indicated by $\mathbb{1}$ and $\widetilde{\mathbb{1}}$ and $V\pm A$ is the Dirac structure. For more details see [9,10]. In Eq. (2) we have set $V_{ub}V_{us}^{*}=O(\lambda^4)$ to zero. The diagram of Fig. 1 has been calculated in [2,3] and yields contributions to the nonspectator part $\Gamma^{\rm nonspec}$ of the B_s decay rate proportional to C_2^2, $C_1\cdot C_2$, and C_1^2. The result involves four hadronic matrix elements, which are parametrized by the B factors B_1, B_2, ε_1, and ε_2 [2]:

$$\langle B_s|\overline{s}\gamma_\mu(1-\gamma_5)b\overline{b}\gamma^\mu(1-\gamma_5)s|B_s\rangle = f_{B_s}^2 M_{B_s}^2 B_1,$$

$$\langle B_s|\overline{s}(1+\gamma_5)b\overline{b}(1-\gamma_5)s|B_s\rangle = f_{B_s}^2 M_{B_s}^2 B_2,$$

$$\langle B_s|\overline{s}\gamma_\mu(1-\gamma_5)T^a b\overline{b}\gamma^\mu(1-\gamma_5)T^a s|B_s\rangle = f_{B_s}^2 M_{B_s}^2 \varepsilon_1,$$

$$\langle B_s|\overline{s}(1+\gamma_5)T^a b\overline{b}(1-\gamma_5)T^a s|B_s\rangle = f_{B_s}^2 M_{B_s}^2 \varepsilon_2. \quad (4)$$

Here T^a is the color SU(3) generator, $M_{B_s}=5369\pm 2$ MeV, and f_{B_s} are the mass and decay constant of the B_s meson. $\tau(B_s)/\tau(B_d)-1$ is proportional to $\Gamma^{\rm nonspec}(B_d) - \Gamma^{\rm nonspec}(B_s)$. The main differences between the result of Fig. 1 for these two rates are due to the different mass of u and c and the difference between f_{B_d} and f_{B_s}. Hence the current-current parts of $\tau(B_s)/\tau(B_d)-1$ proportional to C_2^2, $C_1\cdot C_2$, or C_1^2 are suppressed by a factor of z or Δ with

$$z=\frac{m_c^2}{m_b^2}=0.085\pm 0.023, \quad \Delta = 1-\frac{f_{B_d}^2 M_{B_d}}{f_{B_s}^2 M_{B_s}} = 0.23\pm 0.11. \quad (5)$$

The result for Δ in Eq. (5) is the present world average of lattice calculations [11]. There are also $SU(3)_F$ violations in the B factors, but they are expected to be small from the experience with those appearing in B^0-$\overline{B}^0$ mixing. We want to achieve an accuracy of 2 permille in our prediction for $\tau(B_s)/\tau(B_d)$, which corresponds to an accuracy of 20–30 % in $\tau(B_s)/\tau(B_d)-1$. Therefore we use the same B_1, B_2, ε_1, and ε_2 in $\tau(B_s)$ and $\tau(B_d)$. Likewise there is $SU(3)_F$ breaking in the matrix elements of the b-quark kinetic energy operator and the chromomagnetic moment operator. These effects are suppressed by a factor of $m_b/(\Lambda_{\rm QCD}16\pi^2)$ with respect to those discussed above. In [3] they have been estimated from heavy meson spectroscopy to be an effect of order one permille in $\tau(B_s)/\tau(B_d)$.

We are now interested in the diagram of Fig. 2 involving one large coefficient $C_{1,2}$ and one small penguin coefficient C_{3-6}. Diagrams with two insertions of penguin operators yield smaller contributions proportional to C_{3-6}^2 and are neglected here. To order λ^2 in H we have $V'_{\rm CKM}=0$ in Eq. (1) for the B_d system and penguin effects are only relevant in $\tau(B_s)$. Hence the penguin contributions to $\tau(B_s)/\tau(B_d)-1$ do not suffer from the suppression factors z and Δ. Next we want to evaluate the diagram of Fig. 3 which encodes the

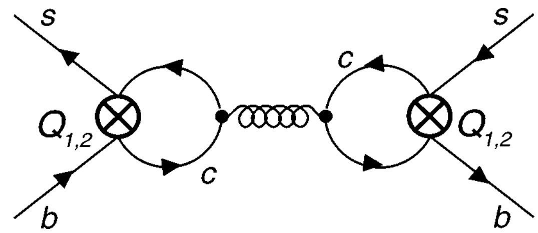

FIG. 4. Penguin diagram contribution to $\Gamma^{\rm nonspec}(B_s)$. The final state corresponds to a cut through either of the $(\overline{c},c)$ loops. The contributions of Q_1 vanish by color. This is the only NLO contribution to $\tau(B_s)/\tau(B_s)-1$ involving $Q_{1,2}$ without suppression factors of Δ or z.

interference of $Q_{1,2}$ with the chromagnetic operator Q_8. This part of $\Gamma^{\rm nonspec}$ already belongs to the order α_s and is small in the standard model, but it can be sizeable in the new physics scenarios discussed in [7].

We also must discuss radiative corrections to the contributions involving the large coefficients C_1 and C_2. Dressing the diagram in Fig. 1 with gluons gives contributions to $\Gamma^{\rm nonspec}$ for both B_d and B_s and therefore yields small corrections of order $C_2^2\Delta\alpha_s/\pi$ or less. The penguin diagram of Fig. 4, however, contributes only to $\Gamma^{\rm nonspec}(B_s)$ in the order λ^4. Hence Fig. 4 yields an unsuppressed contribution of order $C_2^2\alpha_s/\pi$ to $\tau(B_s)/\tau(B_d)-1$ and cannot be neglected. The result of these penguin loop diagrams can easily be absorbed into the penguin coefficients C_{3-6}: In the result of the diagram of Fig. 2 one must simply replace C_j by

$$C'_j = C_j^{\rm NLO} + \frac{\alpha_s}{4\pi} C_2 {\rm Re}[r_{2j}(1,\sqrt{z},\mu/m_b)], \quad j=3,\ldots,6. \quad (6)$$

Here r_{2j} encodes the result of the penguin diagram and can be found in [9] in the naive dimensional regularization (NDR) scheme. To cancel the scheme dependence of r we must also include the next-to-leading order (NLO) corrections to C_j as indicated in Eq. (6). More precisely, we must include the NLO mixing of C_2 into C_j in $C_j^{\rm NLO}$, $j=3,\ldots,6$, but the penguin-penguin mixing only to the LO. The difference between these partial NLO coefficients, which are tabulated in [9], and the full $C_j^{\rm NLO}$'s has a negligible impact on our result. Here we bypass this technical aspect of scheme independence by tabulating the C'_j's in Table I. Our result for the nonspectator part of the B_s decay rate reads

TABLE I. The effective Wilson coefficients C'_j defined in Eq. (6) for $z=0.085$, $\alpha_s(M_Z)=0.118$, and $m_b=4.8$ GeV. Varying z within the range given in Eq. (5) affects the C'_j by 3–4 % and is negligible for our purposes. The $C_j^{(0)}$'s are the LO Wilson coefficients.

j	1	2	3	4	5	6	8
$C_j^{(0)}$ $(\mu=m_b)$	-0.249	1.108	0.011	-0.026	0.008	-0.031	-0.149
$C'_j(\mu=m_b)$			0.014	-0.041	0.014	-0.047	
$C_j^{(0)}(\mu=m_b/2)$	-0.361	1.169	0.017	-0.036	0.010	-0.048	-0.166
$C'_j(\mu=m_b/2)$			0.017	-0.045	0.015	-0.058	
$C_j^{(0)}(\mu=2m_b)$	-0.167	1.067	0.007	-0.018	0.005	-0.020	-0.135
$C'_j(\mu=2m_b)$			0.012	-0.036	0.013	-0.039	

$$\Gamma^{\text{nonspec}}(B_s) = -\frac{G_F^2 m_b^2}{12\pi} |V_{cb}V_{cs}|^2 \sqrt{1-4z} f_{B_s}^2 M_{B_s} \times [a_1\varepsilon_1 + a_2\varepsilon_2 + b_1 B_1 + b_2 B_2], \tag{7}$$

with

$$a_1 = [2C_2^2 + 4C_2C_4'][1-z] + 12zC_2C_6' + [1+2z]\frac{\alpha_s}{\pi} C_2C_8,$$

$$a_2 = -[1+2z]\left[2C_2^2 + 4C_2C_4' + \frac{\alpha_s}{\pi} C_2C_8\right],$$

$$b_1 = [C_2 + N_cC_1]\left\{(1-z)\left[\frac{C_2}{N_c} + C_1 + 2C_3' + 2\frac{C_4'}{N_c}\right] + 6z\left[C_5' + \frac{C_6'}{N_c}\right]\right\},$$

$$b_2 = -[1+2z][C_2 + N_cC_1]\left\{\frac{1}{N_c}[C_2 + N_cC_1] + 2\left[C_3' + \frac{C_4'}{N_c}\right]\right\}. \tag{8}$$

Here $N_c = 3$ is the number of colors. By setting C_j', $j = 3, \ldots, 6$, and C_8 in Eq. (8) to zero one recovers the result[1] of [2]. The result for the nonspectator contributions to the B_d decay rate reads [2]

$$\Gamma^{\text{nonspec}}(B_d) = \frac{G_F^2 m_b^2}{12\pi} |V_{cb}V_{ud}|^2 (1-z)^2 f_{B_s}^2 M_{B_s}(\Delta - 1) \times [a_1^d \varepsilon_1 + a_2^d \varepsilon_2 + b_1^d B_1 + b_2^d B_2], \tag{9}$$

with[2]

$$a_1^d = 2C_2^2\left(1 + \frac{z}{2}\right), \quad a_2^d = -2C_2^2(1+2z),$$

$$b_1^d = \frac{1}{N_c}(C_2 + N_cC_1)^2\left(1 + \frac{z}{2}\right),$$

$$b_2^d = -\frac{1}{N_c}(C_2 + N_cC_1)^2(1+2z). \tag{10}$$

When we combine Eqs. (7)–(10) in order to predict $\tau(B_s)/\tau(B_d) - 1$,

$$\frac{\tau(B_s)}{\tau(B_d)} - 1 = \frac{\Gamma^{\text{nonspec}}(B_d) - \Gamma^{\text{nonspec}}(B_s)}{\Gamma^{\text{total}}} + O(10^{-3})$$

$$= K(z)\left\{\Delta\left[2C_2^2(\varepsilon_1 - \varepsilon_2) + \frac{(C_2 + N_cC_1)^2}{N_c}(B_1 - B_2)\right]\right. \tag{11a}$$

$$-3C_2^2 z\varepsilon_1 - \frac{3}{2}\frac{(C_2 + N_cC_1)^2}{N_c} zB_1 \tag{11b}$$

$$+\Delta z\left[C_2^2(\varepsilon_1 - 4\varepsilon_2) + \frac{(C_2 + N_cC_1)^2}{2N_c} \times (B_1 - 4B_2)\right] \tag{11c}$$

$$+\left[4C_2C_4' + (1+2z)\frac{\alpha_s}{\pi} C_2C_8\right](\varepsilon_1 - \varepsilon_2) \tag{11d}$$

$$+2(C_2 + N_cC_1)\left(C_3' + \frac{C_4'}{N_c}\right)(B_1 - B_2) \tag{11e}$$

$$-4zC_2C_4'(\varepsilon_1 + 2\varepsilon_2) + 12zC_2C_6'\varepsilon_1 + 2z(C_2 + N_cC_1) \times \left[-\left(C_3' + \frac{C_4'}{N_c}\right)(B_1 + 2B_2) + 3\left(C_5' + \frac{C_6'}{N_c}\right)B_1\right]\Bigg\} + O(2\times 10^{-3}).$$

Here $K(z)$ reads

$$K(z) = \frac{16\pi^2 |V_{ud}|^2 B_{SL}}{m_b^2 f_1(z)[1 + \alpha_s(\mu)/(2\pi) h_{SL}(\sqrt{z})]} f_{B_s}^2 M_{B_s}[1-2z] \tag{12}$$

$$\simeq \frac{0.060}{1 - 4(\sqrt{z} - 0.3)}(1-2z)\frac{B_{SL}}{0.105}\left(\frac{4.8}{m_b}\right)^3 \times \left(\frac{f_{B_s}}{190\ \text{MeV}}\right)^2. \tag{13}$$

In Eq. (12) we have used the common trick to evaluate the total width Γ^{total} in terms of the semileptonic rate and the measured semileptonic branching ration B_{SL} via $\Gamma^{\text{total}} = \Gamma_{SL}/B_{SL}$. f_1 and h_{SL} are the phase space and QCD correction factor of Γ_{SL} calculated in [12]. We use the notation of [9]. The approximation in Eq. (13) reproduces $K(z)$ to an accuracy of 3%. The numerical value of h_{SL} entering Eq. (13) corresponds to the use of the one-loop pole mass ($\simeq$4.8 GeV) for m_b. For simplicity we have expanded $K(z)$ and the terms in the curly braces in Eqs. (11a)–(11e) up to the first order in z. The size of the error in Eqs. (11a)–(11e) is estimated as 2×10^{-3}. Its main source is the $SU(3)_F$

[1]Notice that our notation of C_1 and C_2 is opposite to the one in [2].

[2]In the large N_c limit one finds Γ^{nonspec} helicity suppressed in analogy to the leptonic decay rate. This shows that one cannot neglect the $O(1/N_c)$ terms.

breaking in the kinetic energy and chromomagnetic moment matrix elements appearing at order $\Lambda_{\rm QCD}^2/m_b^2$ of the HQE, which has been calculated to equal $(0-1)\times 10^{-3}$ in [3]. Then terms of order $16\pi^2\Lambda_{\rm QCD}^4/m_b^4$ can maximally be of the same order of magnitude. Conversely the remaining NLO correction of order $C_2^2\Delta\alpha_s/\pi$ and the CKM-suppressed contributions are much smaller. Likewise the $SU(3)_F$ breaking in ε_1, ε_2, B_1, and B_2 is expected to be at the level of a few percent and therefore smaller than the present uncertainty in Δ.

The first three lines [(11a)–(11c)] contain the result of the current-current operators calculated in [2,3]. The remaining lines comprise the penguin effects. Note that the terms in Eqs. (11c) and (11e) are neither suppressed by Δ nor by z. For $z=0$ the hadronic parameters in Eqs. (11a)–(11e) only appear in the combinations $\varepsilon_1-\varepsilon_2$ and B_1-B_2, both of which are of order $1/N_c$. The coefficients of B_1-B_2 suffer from numerical cancellations, e.g., $0.09\leqslant C_2+3C_1\leqslant 0.57$ (cf. Table I), so that for most values of the input parameters only the terms involving ε_1 and ε_2 in (11a), (11b), and (11d) are important.

Finally we discuss a potential systematic uncertainty: The derivation of Eqs. (11a)–(11e) has assumed quark-hadron duality (QHD) for the sum over the final states. QHD means that inclusive observables are unaffected by the hadronization process of the quarks and gluons in the final state. The new results for inclusive observables in B decays presented at the 1997 Summer Conferences are consistent with QHD [13]. There are two potential sources of QHD violation in our problem: First it may be possible that the spectator decay rate of the b quark is affected by the hadronization process. Yet the ballpark of this effect is independent of the flavor of the spectator quark and cancels out in the ratio $\tau(B_s)/\tau(B_d)$. $SU(3)_F$ breaking can only appear in the hadronization of the final state antiquark which picks up the spectator quark and we do not expect the $SU(3)_F$ breaking in the spectator decay rate to be larger than the $SU(3)_F$ breaking in the $(\Lambda_{\rm QCD}/m_b)^2$ terms of the HQE. This effect should further not depend on whether the hadron containing the spectator quark recoils against other hadrons or against a lepton pair. Hence one can control the $SU(3)_F$ breaking in the spectator decay rate by comparing the hadron energy in semileptonic B_d and B_s decays. More serious is a potential violation of QHD in the nonspectator contribution $\Gamma^{\rm nonspec}$ itself. In a theoretical analysis for the similar case of the width difference $\Delta\Gamma_{B_s}$ of the two B_s eigenstates the size of QHD violation has been estimated to be moderate, maximally of order 30%. We can incorporate this into Eqs. (11a)–(11e) by assigning an additional error of ± 0.3 to Δ. In any case the issue of QHD violation in lifetime differences will be experimentally tested in the forthcoming years, when high precision measurements of $\tau(B^+)/\tau(B_d)$ and of $\Delta\Gamma_{B_s}$ are confronted with accurate lattice results for the hadronic parameters.

III. PHENOMENOLOGY

In the following we want to investigate the numerical importance of the penguin contribution. Then we analyze which accuracy is necessary to detect or constrain new physics contributions to C_{3-6} by a precision measurement of $\tau(B_s)/\tau(B_d)$.

The three main hadronic parameters entering Eqs. (11a)–(11e) are Δ, f_{B_s}, and $\varepsilon_1-\varepsilon_2$, while B_1 and B_2 come with small coefficients. The canonical sizes of the B factors are $\varepsilon_i=O(1/N_c)$ and $B_i=1+O(1/N_c)$. An important constraint on the ε_i's is given by the measured value of $\tau(B^+)/\tau(B_d)$ [2]. The result of [2] for $\Gamma^{\rm nonspec}(B^+)$ is obtained from Eq. (9) by replacing the a_i^d, b_i^d's with

$$a_1^u=-6(C_1^2+C_2^2),\quad b_1^u=-\frac{3}{N_c}(C_2+N_cC_1)^2+3N_cC_1^2,$$

$$a_2^u=b_2^u=0. \tag{14}$$

The experimental world average [14]

$$\frac{\tau(B^+)}{\tau(B_d)}=1.07\pm 0.04 \tag{15}$$

leads to the following constraint:

$$\varepsilon_1\simeq(-0.2\pm 0.1)\left(\frac{0.17\ \text{GeV}}{f_B}\right)^2\left(\frac{m_b}{4.8\ \text{GeV}}\right)^3+0.3\varepsilon_2+0.05. \tag{16}$$

In [4] the ε_i's and B_i's have been calculated with QCD sum rules within the heavy quark effective theory (HQET). The results are $\varepsilon_1(\mu=m_b)=-0.08\pm 0.02$ and $\varepsilon_2(\mu=m_b)=-0.01\pm 0.03$ and $B_{1,2}=1+O(0.01)$. In view of the smallness of the ε_i's, however, it is conceivable that other neglected effects are numerically relevant. For example, a NLO calculation of the matching between HQET and full QCD amplitudes replaces ε_i in Eqs. (7) and (9) by $\varepsilon_i+d_iB_i$, where d_i is a coefficient of order $\alpha_s(m_b)/\pi$. Here we will consider the range $|\varepsilon_1|$, $|\varepsilon_2|\leqslant 0.3$, and further obey Eq. (15).

In Table II we have tabulated $\tau(B_s)/\tau(B_d)-1$ for various values of Δ and $\varepsilon_1,\varepsilon_2$. We have further split $\tau(B_s)/\tau(B_d)-1$ into its current-current part consisting of Eqs. (11a)–(11c) and the new penguin part involving C'_{3-6}, C_8. These results can be found in Table III. From Table III we realize that the penguin contributions calculated in this work are comparable in size, but opposite in sign to the current-current part obtained in [3]. This makes the experimental detection of any deviation of $\tau(B_s)/\tau(B_d)$ from 1 even more difficult, if the penguin coefficients are really dominated by standard model physics. The results of Table II can be summarized as

$$\frac{\tau(B_s)}{\tau(B_d)}-1=(-1.2\pm 8.0\pm 2.0)\times 10^{-3}\left(\frac{f_{B_s}}{190\ \text{MeV}}\right)^2\left(\frac{4.8\ \text{GeV}}{m_b}\right)^3. \tag{17}$$

Here the first error stems from the uncertainty in ε_1 and ε_2 and will be reduced once lattice results for the hadronic parameters are available. The second error summarizes the remaining uncertainties. If Δ and ε_2 simultaneously acquire

TABLE II. Standard model prediction for $10^3\times[\tau(B_s)/\tau(B_d)-1]$ obtained from Eqs. (11a)–(11e) for $f_{B_s}=190$ MeV, $\mu=m_b=4.8$ GeV, $z=0.085$, $\alpha_s(M_Z)=0.118$, and $B_1=B_2=1$. The entries marked with an asterisk are in conflict with the experimental constraint (15), which also implies $\varepsilon_1\lesssim 0$. There is an overall error of ± 2.0 [See Eqs. (11a)–(11e)] for all entries.

	$\Delta=0.12$			$\Delta=0.23$			$\Delta=0.34$		
ε_1	-0.3	-0.1	0	-0.3	-0.1	0	-0.3	-0.1	0
$\varepsilon_2=-0.3$	4.3	1.9	*	4.3	4.5	*	4.3	7.1	*
$\varepsilon_2=-0.1$	3.7	1.3	*	1.1	1.3	*	-1.5	1.3	*
$\varepsilon_2=0.1$	*	0.6	-0.6	-2.2	-2.0	-1.9	-7.4	-4.6	-3.2
$\varepsilon_2=0.3$	*	-0.1	-1.3	*	-5.3	-5.2	-13.3	-10.5	-9.1

extreme values, $\tau(B_s)/\tau(B_d)-1$ can be slightly outside the range in Eq. (17) (see Table II).

Today we have little experimental information on the sizes of the penguin coefficients. Their smallness in the standard model allows for the possibility that they are dominated by new physics. The total charmless inclusive branching fraction $B(B\to\text{no charm})$ is a candidate to detect new physics contributions to C_8 [7], but it is much less sensitive to C_{3-6} [9]. The decreasing experimental upper bounds on $B(B\to\text{no charm})$ [14] therefore constrain C_8 but leave room for a sizeable enhancement of C_{3-6}. Now Eqs. (11a)–(11e) reveal that $\tau(B_s)/\tau(B_d)$ is a complementary observable mainly sensitive to C_4, while C_8 is of minor importance. As mentioned in the Introduction, many interesting new physics scenarios affect C_{3-6}, but not necessarily C_8. We remark here that we constrain ourself to new physics scenarios, in which the CKM factors of the new contributions are the same as the ones of the standard model. This is fulfilled to a good approximation in most interesting models [7]. Now any new physics effect modifies C_{3-6} at some high scale of the order of the new particle masses, while the Wilson coefficients entering Eqs. (11a)–(11e) are evaluated at a low scale $\mu\approx m_b$. The renormalization group evolution down to $\mu\approx m_b$ mixes the new contributions to C_{3-6}. New physics contributions $\Delta C_{3-6}(\mu=200\text{ GeV})$ affect $C_4(\mu=4.8\text{ GeV})$ by

$$\begin{aligned}\Delta C_4(\mu=4.8\ \text{GeV}) = &-0.35\Delta C_3(200\ \text{GeV})\\ &+0.99\Delta C_4(200\ \text{GeV})\\ &-0.03\Delta C_5(200\ \text{GeV})\\ &-0.22\Delta C_6(200\ \text{GeV}).\end{aligned}$$

Observe that $\Delta C_4(200\text{ GeV})=-0.05$ already increases $C_4'(m_b)$ by more than a factor of 2.

Clearly the usefulness of $\tau(B_s)/\tau(B_d)$ to probe C_{3-6} crucially depends on the size of $|\varepsilon_1-\varepsilon_2|$ and f_{B_s}. We now investigate the sensitivity of $\tau(B_s)/\tau(B_d)$ to $\Delta C_4(\mu=m_b)$ in a possible future scenario for the hadronic parameters. We assume

$$\begin{gathered}\varepsilon_1=-0.10\pm0.05,\quad \varepsilon_2=0.20\pm0.05,\quad B_1,B_2=1.0\pm0.1,\\ f_{B_s}=(190\pm15)\ \ \text{GeV},\quad \Delta=0.23\pm0.05,\\ m_b=(4.8\pm0.1)\ \ \text{GeV}.\end{gathered}\tag{18}$$

The assumed accuracy for f_{B_s} will be achieved, once more experimental information on the B_s system is obtained, e.g., after the detection of $B_s-\overline{B}_s$ mixing. Also a more precise measurement of f_{D_s} is helpful, because lattice QCD predicts the ratio f_{B_s}/f_{D_s} much better than f_{B_s} [11]. The error bars of the other hadronic parameters likewise appear within reach, if one keeps in mind that information on ε_1 and ε_2 will not only be obtained from the lattice but also from other observables like $\tau(B^+)/\tau(B_d)$. Experimental progress in Eq. (15) and a next-to-leading order calculation of the coefficients in Eqs. (14) and (10) will significantly improve the constraint in Eq. (16). In Fig. 5 we show the dependence of $\tau(B_s)/\tau(B_d)-1$ on $\Delta C_4(\mu)$ for the scenario in Eq. (18). A cleaner observable is the double ratio

$$\frac{\tau(B_s)-\tau(B_d)}{\tau(B^+)-\tau(B_d)}=\frac{B_{SL}(B_s)-B_{SL}(B_d)}{B_{SL}(B^+)-B_{SL}(B_d)},\tag{19}$$

TABLE III. The columns labeled with ''peng'' list the penguin contribution to $10^3\times[\tau(B_s)/\tau(B_d)-1]$ as a function of $\varepsilon_1-\varepsilon_2$ and f_{B_s}. The other input parameters have little impact on the size of the penguin contribution. The current-current part of $10^3\times[\tau(B_s)/\tau(B_d)-1]$ is listed for $\varepsilon_1=-0.1$ and $\Delta=0.23$. For the remaining parameters see Table II.

	-0.5		-0.3		-0.1		0.1		0.2	
$\varepsilon_1-\varepsilon_2=$	peng	cc	peng	cc	peng	cc	peng	cc	peng	cc
$f_{B_s}=160$ MeV	3.9	-8.8	2.3	-4.9	0.8	-1.0	-0.8	2.8	-1.5	4.7
$f_{B_s}=190$ MeV	5.4	-12.4	3.3	-6.9	1.1	-1.5	-1.1	4.0	-2.2	6.7
$f_{B_s}=220$ MeV	7.3	-16.6	4.4	-9.3	1.5	-2.0	-1.4	5.3	-2.9	9.0

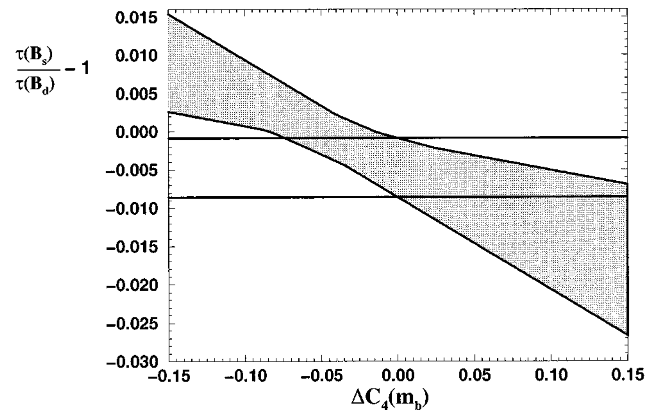

FIG. 5. Dependence of $\tau(B_s)/\tau(B_d)-1$ on a new physics contribution ΔC_4. The shaded area corresponds to a variation of the input parameters within the range of Eq. (18). The horizontal lines mark the standard model range corresponding to $\Delta C_4=0$.

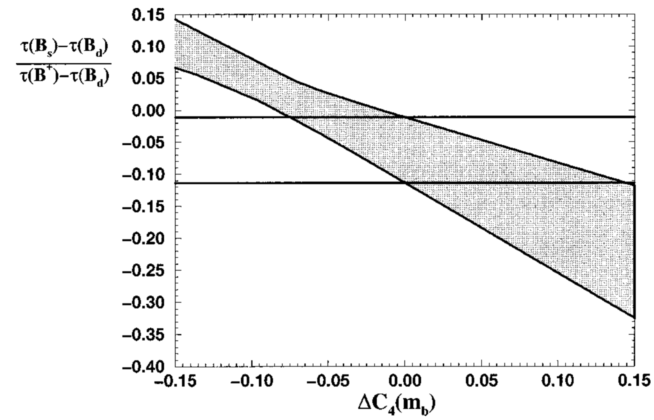

FIG. 6. Dependence of $[\tau(B_s)-\tau(B_d)]/[\tau(B^+)-\tau(B_d)]$ on ΔC_4 for the parameter set in Eq. (18). This double ratio depends on f_{B_s} and f_{B_d} only through Δ, and the factor of m_b^{-3} in Eqs. (11a)–(11e) cancels.

which depends on ε_1, ε_2, and Δ, while the dependence on f_B and m_b cancels. The corresponding plot for the parameter set of Eq. (18) can be found in Fig. 6.

We find a smaller error band for $[\tau(B_s)-\tau(B_d)]/[\tau(B^+)-\tau(B_d)]$ than for $\tau(B_s)/\tau(B_d)-1$. If $\Delta C_4<-0.075$ or $\Delta C_4>0.140$, we find the allowed range for $[\tau(B_s)-\tau(B_d)]/[\tau(B^+)-\tau(B_d)]$ incompatible with the standard model. An experimental lower bound $\tau(B_s)/\tau(B_d)>1.005$ would indicate a new physics contribution $\Delta C_4<-0.063$ in our scenario. Likewise the experimental detection of a sizeable negative lifetime difference $\tau(B_s)-\tau(B_d)$ may reveal nonstandard contributions to C_4' of similar size as its standard model value. Figure 6 shows that, e.g., the bound $\tau(B_s)-\tau(B_d)<-0.20[\tau(B^+)-\tau(B_d)]$ would indicate $\Delta C_4>0.051$. We conclude that the detection of new physics contributions to C_4 of order 0.1 is possible with precision measurements of $\tau(B_s)/\tau(B_d)$.

IV. CONCLUSIONS

We have calculated the contributions of the penguin operators Q_{3-6}, of the chromomagnetic operator Q_8, and of penguin diagrams with insertions of Q_2 to the lifetime splitting between the B_s and B_d meson. In the standard model the penguin effects are found to be roughly half as big as the contributions from the current-current operators Q_1 and Q_2, despite the smallness of the penguin coefficients. Yet they are opposite in sign, so that any deviation of $\tau(B_s)-\tau(B_d)$ from zero is even harder to detect experimentally. Assuming a reasonable progress in the determination of the hadronic parameters a precision measurement of $\tau(B_s)/\tau(B_d)$ can be used to probe the coefficient C_4 with an accuracy of $|\Delta C_4|=0.1$. Hence new physics can only be detected if C_4 is dominated by nonstandard contributions. The sensitivity to C_4 depends crucially on the difference of the hadronic parameters ε_1 and ε_2. For the extraction of C_4 the double ratio $[\tau(B_s)-\tau(B_d)]/[\tau(B^+)-\tau(B_d)]$ turns out to be more useful than $\tau(B_s)/\tau(B_d)$.

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