Hadronic spectral moments in semileptonic B decays with a lepton energy cut

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We compute the first two moments $\langle (s_H-\overline{m_D}^2)^{1,2}\rangle$ of the final hadronic invariant mass in the inclusive decay $B\to X_c\ell\overline{\nu}$, in the presence of a cut $E_\ell^{\rm min}$ on the charged lepton energy. These moments may be measured directly by experiments at the $\Upsilon(4S)$ using the neutrino reconstruction technique, which requires such a cut. Measurement of these moments will place constraints on the nonperturbative parameters $\overline{\Lambda}$ and λ_1, which are relevant for extracting the quark masses m_b and m_c, as well as the CKM angle V_{cb}. We include terms of order $\alpha_s^2\beta_0$ and $1/m_b^3$ in the operator product expansion, and use the latter to estimate the theoretical uncertainty in the extraction of $\overline{\Lambda}$ and λ_1. [S0556-2821(98)00701-2]

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I. INTRODUCTION

Semileptonic B decays are a rich laboratory in which to study the bound state structure of the B meson. Of particular importance are the nonperturbative parameters $\overline{\Lambda}$, λ_1 and λ_2, which arise in the heavy quark expansion for the meson mass m_B to relative order $1/m_b^2$,

$$m_B=m_b+\overline{\Lambda}-\frac{\lambda_1+3\lambda_2}{2m_b}+\mathcal{O}(\Lambda_{\rm QCD}^3/m_b^2), \tag{1.1}$$

where

$$\lambda_1=\langle B|\,\overline{h_v}(iD)^2h_v|B\rangle/2m_B\,,$$

$$\lambda_2=\langle B|\,\overline{h_v}\frac{g}{2}\sigma^{\mu\nu}G_{\mu\nu}h_v|B\rangle/6m_B\,. \tag{1.2}$$

Here h_v is the heavy quark field, and m_b is the b quark pole mass. Since these quantities also parametrize the inclusive semileptonic and radiative decay rates of the B meson to order $1/m_b^2$ [1–5], an accurate determination of them is essential for a reliable extraction of the CKM Cabibbo-Kobayashi-Maskawa (CKM) angle $|V_{cb}|$ from inclusive semileptonic B decays. The parameter λ_2 is the expectation value of the leading operator which breaks heavy quark spin symmetry and therefore may be determined from the $B-B^*$ mass difference, yielding $\lambda_2\approx 0.12$ GeV^2. However, $\overline{\Lambda}$ and λ_1 cannot be determined solely from mass measurements.

There has been much recent interest in using inclusive observables other than the full semileptonic decay width to extract $\overline{\Lambda}$ and λ_1. An analysis of the decay based on the heavy quark expansion yields an expression for the doubly differential decay rate $d\Gamma/dq^2d(v\cdot q)$, where q^μ is the total momentum of the leptons and v^μ is the four-velocity of the B meson. Any observable which may be constructed from this doubly differential rate is sensitive to a linear combination of $\overline{\Lambda}$ and λ_1. Actually, the simplest differential distribution to measure experimentally is the energy spectrum of the charged lepton, $d\Gamma/dE_\ell$, which is somewhat more complicated because it depends on more than just q^μ. Study of this distribution has already yielded useful constraints on $\overline{\Lambda}$ and λ_1 [6–9].

In two previous publications [10], we suggested that moments of the hadronic invariant mass in semileptonic B decay would also be interesting to study. If s_H is the invariant mass of the hadrons produced in the semileptonic decay, and $\overline{m_D}=(m_D+3m_{D*})/4$ is the spin-averaged D meson mass, then positive moments of $s_H-\overline{m_D^2}$ vanish in the parton model at tree level. Hence they are particularly sensitive to the power corrections proportional to $\overline{\Lambda}$ and λ_1. However, it is difficult to measure these moments directly. Until recently, only fairly weak bounds on $\langle (s_H-\overline{m_D^2})^n\rangle$ could be obtained, by combining information on various exclusive decay channels [10]. With the introduction of the technique of neutrino reconstruction, this situation is changing [11]. Soon it will be possible to measure s_H directly and inclusively, by reconstructing the neutrino momentum p_ν^μ and using it to find q^2 and $v\cdot q$. The only complication is that this reconstruction requires a number of constraints on the phase space of the leptons, most importantly a lower cut on the charged lepton energy of about 1.5 GeV [12].

While one might consider extrapolating the data to lower E_ℓ and measuring the moments of s_H that way, it is clearly preferable to update the computation of the moments of s_H to include this lepton energy cut *ab initio*. It is the purpose of this paper to present the results of such an analysis. There are few new theoretical issues which arise in this case, although

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the actual calculation is considerably more complicated than without the cut. We will refer the reader to our previous papers for a more complete exposition of the theory [10], and only stress points where the new analysis differs in interesting ways from the old. We will also extend our earlier analyses by including the complete power corrections to the moments to $O(1/m_b^3)$. Dimensional estimates of the size of these corrections will help us to estimate the error in the extraction of $\overline{\Lambda}$ and λ_1 due to $1/m_b^3$ effects. They will turn out to be under control for the first moment, but very large for the second moment, compromising its usefulness for obtaining constraints on $\overline{\Lambda}$ and λ_1.

II. THE CALCULATION

A. Power corrections

The analysis of the moments of s_H begins with the doubly differential spectrum $d\Gamma/d\hat{q}^2 d(v\cdot\hat{q})$, where $\hat{q}^2=q^2/m_b^2$ and $v\cdot\hat{q}=v\cdot q/m_b$. This distribution may be written as a product of a hadron tensor $T^{\mu\nu}$ and a lepton tensor $L^{\mu\nu}$,

$$\frac{d\Gamma}{d\hat{q}^2 d(v\cdot\hat{q})}=T^{\mu\nu}(v,\hat{q})L_{\mu\nu}(v,\hat{q}). \tag{2.1}$$

Each of these tensors has a decomposition in terms of scalar invariants, such as

$$L^{\mu\nu}=\left(-g^{\mu\nu}+\frac{\hat{q}^\mu\hat{q}^\nu}{\hat{q}^2}\right)L_1+\left(v^\mu v^\nu+\frac{\hat{q}^\mu\hat{q}^\nu(v\cdot\hat{q})^2}{\hat{q}^4}-\frac{v\cdot\hat{q}(v^\mu\hat{q}^\nu+v^\nu\hat{q}^\mu)}{\hat{q}^2}\right)L_2-i\epsilon^{\mu\nu\alpha\beta}v_\alpha\hat{q}_\beta L_3. \tag{2.2}$$

The L_i are scalar functions of $\hat{q}^2$ and $v\cdot\hat{q}$. Here we neglect the masses of the leptons. If we impose a cut $E_\ell\geqslant x m_b$ on the charged lepton energy, then the effect is simply to modify the components L_i,

$$L_1=\frac{\hat{q}^2}{24\pi[(v\cdot\hat{q})^2-\hat{q}^2]^{3/2}}(4(v\cdot\hat{q})^3-3\hat{q}^2 v\cdot\hat{q}+4[(v\cdot\hat{q})^2-\hat{q}^2]^{3/2}+6x\hat{q}^2-12x(v\cdot\hat{q})^2+12x^2 v\cdot\hat{q}-8x^3),$$

$$L_2=\frac{\hat{q}^4}{8\pi[(v\cdot\hat{q})^2-\hat{q}^2]^{5/2}}(2x-v\cdot\hat{q})(\hat{q}^2-4xv\cdot\hat{q}+4x^2),$$

$$L_3=\frac{\hat{q}^2}{8\pi[(v\cdot\hat{q})^2-\hat{q}^2]^{3/2}}(\hat{q}^2-4xv\cdot\hat{q}+4x^2). \tag{2.3}$$

The cut may be removed by setting it equal to the minimum charged lepton energy, $x\to\frac{1}{2}[(v\cdot\hat{q})^2-\hat{q}^2]^{1/2}$, in which case $L_1=\hat{q}^2/3\pi$, $L_2=L_3=0$. Expressions for the analogous hadron tensor components T_i may be found in Refs. [2,3,8]. The hadron tensor is independent of the cut x.

Since the hadron tensor $T^{\mu\nu}$ is computed with operator product expansion techniques which assume parton-hadron duality, the calculation must be smeared by integration over at least one of $\hat{q}^2$ or $v\cdot\hat{q}$ before meaningful observables may be extracted [2,3]. A nonzero cut x has the effect of restricting the available phase space for the leptons, which controls the range of integration in $\hat{q}^2$ and $v\cdot\hat{q}$. If x is too large, the integration is too restricted, and the operator product expansion breaks down. This is known to happen, for example, near the lepton endpoint in charmless semileptonic B decays, for E_ℓ above about 2.2 GeV [2,3,13,14]. However, our calculations of the coefficients of the $1/m_b^3$ corrections indicate that the lepton energy cut of 1.5 GeV which is required for the neutrino reconstruction technique is not severe enough to cause such problems.

The next step is to compute parton level moments of the form $\langle\hat{E}_0^m(\hat{s}_0-\hat{m}_c^2)^n\rangle$, where $\hat{s}_0$ and $\hat{E}_0$ are the invariant mass and total energy of the strongly interacting partons produced in the semileptonic decay of the b quark. The energy is computed in the b rest frame. The partonic variables are related implicitly to the hadronic variable s_H by

$$\hat{E}_0=1-v\cdot\hat{q},\quad \hat{s}_0=1-2v\cdot\hat{q}+\hat{q}^2;$$

$$s_H=m_B^2-2m_B v\cdot q+q^2. \tag{2.4}$$

Since these expressions involve both m_B and m_b, they must be inverted order by order in the heavy quark expansion using Eq. (1.1). The final result will be moments of the form $\langle(s_H-\overline{m_D^2})^m\rangle$, for $m=1,2$. In fact, we will see that only the first moment is really reliable, where the terms of order $\Lambda_{\rm QCD}$ and $\Lambda_{\rm QCD}^2$ in the heavy quark expansion are known. By contrast, the second moment starts only at order $\Lambda_{\rm QCD}^2$, and hence is extremely sensitive to the large number of unknown parameters which arise at order $\Lambda_{\rm QCD}^3$.

B. Radiative corrections

While it is possible to calculate the radiative corrections by calculating the $O(\alpha_s)$ contributions to the T_i's themselves, it is much simpler to calculate directly the leading corrections to the parton model rate. The only subtlety in the calculation arises in determining the boundaries of phase space when the electron cut is imposed. Since the limits of integration of the electron energy depend on the parton level invariant mass s_0 and lepton invariant mass squared q^2, for given values of s_0 and q^2 the electron cut may either lie below the lower limit of integration, in the region of integration, or above the upper limit of integration. Let $f(s_0,q^2)$ be any smooth weighting function. Then the phase space integral is divided into kinematic regions, depending on the values of x, s_0 and q^2,

for $x<\frac{1}{2}(1-m_c)$,

$$\langle f(s_0,q^2)\rangle=\int_{m_c^2}^{(1-2x)^2}ds_0\left[\int_0^{2x(1-s_0-2x)/1-2x}dq^2 f(s_0,q^2)\,\frac{d\Gamma_1}{dq^2ds_0}+\int_{2x(1-s_0-2x)/1-2x}^{(1-\sqrt{s_0})^2}dq^2 f(s_0,q^2)\,\frac{d\Gamma_2}{dq^2ds_0}\right]$$
$$+\int_{(1-2x)^2}^{1-2x}ds_0\int_0^{2x(1-s_0-2x)/1-2x}dq^2 f(s_0,q^2)\,\frac{d\Gamma_1}{dq^2ds_0},$$

for $x>\frac{1}{2}(1-m_c)$,

$$\langle f(s_0,q^2)\rangle=\int_{m_c^2}^{1-2x}ds_0\int_0^{2x(1-s_0-2x)/1-2x}dq^2 f(s_0,q^2)\,\frac{d\Gamma_1}{dq^2ds_0}.$$

Here $d\Gamma_1/dq^2ds_0$ is the differential rate calculated with the electron energy cut imposed, and $d\Gamma_2/dq^2ds_0$ is the differential rate calculated with no cut, corresponding to x lying below the lower limit of integration for the electron energy. In the regions of phase space omitted from the expressions above, the cut lies above the upper limit of integration for the electron energy.

The ''Brodsky-Lepage-Mackenzie- (BLM-) enhanced'' [15] two-loop corrections are those which are proportional to $\alpha_s^2\beta_0$, where $\beta_0=11-2n_f/3$ is the first term in the QCD beta function. These corrections dominate the two-loop corrections to many processes in QCD, and their effects in b decays have been discussed extensively in the literature [16]. They are straightforward to calculate numerically using the techniques of Ref. [17], and no new subtleties are introduced into the calculation when an electron energy cut is added. Because of the renormalon ambiguity in its definition [18], $\overline{\Lambda}$ is only defined order by order in perturbation theory. Since we are including the $\alpha_s^2\beta_0$ terms in our extraction of $\overline{\Lambda}$, the resulting value is the ''two-loop'' $\overline{\Lambda}$, and should only be compared with other extractions of $\overline{\Lambda}$ at the same order.

III. RESULTS AND DISCUSSION

The expansion of a moment of s_H takes the following general form, up to terms of relative order $1/m_b^3$:

$$\langle(s_H-\overline{m_D^2})^m\rangle=m_B^{2m}\left\{C_1^m\,\frac{\alpha_s(m_b)}{\pi}+[C_{22}^{(m)}\beta_0+C_{21}^{(m)}]\,\frac{\alpha_s^2(m_b)}{\pi^2}+\mathcal{O}(\alpha_s^3(m_b))+D_1^{(m)}\frac{\overline{\Lambda}}{m_B}+\left[D_{20}^{(m)}\frac{\overline{\Lambda}^2}{m_B^2}+D_{21}^{(m)}\frac{\lambda_1}{m_B^2}+D_{22}^{(m)}\frac{\lambda_2}{m_B^2}\right]\right.$$
$$\left.+\left[D_{30}^{(m)}\frac{\overline{\Lambda}^3}{m_B^3}+D_{31}^{(m)}\frac{\overline{\Lambda}\lambda_1}{m_B^3}+D_{32}^{(m)}\frac{\overline{\Lambda}\lambda_2}{m_B^3}+D_{33}^{(m)}\frac{\rho_1}{m_B^3}+D_{34}^{(m)}\frac{\rho_2}{m_B^3}+\sum_{i=1}^{4}T_{3i}^{(m)}\frac{\mathcal{T}_i}{m_B^3}\right]\right\}. \tag{3.1}$$

All the coefficients which appear are functions of the lepton energy cut $E_\ell^{\min}$. The parameters ρ_1 and ρ_2 are expectation values of local operators of dimension six which arise at order $1/m_b^3$ in the heavy quark expansion,

$$\langle B|\overline{h_v}(iD_\alpha)(iD_\mu)(iD_\beta)h_v|B\rangle\equiv\frac{1}{3}\rho_1(g_{\alpha\beta}-v_\alpha v_\beta)v_\mu,$$

$$\langle B^{(*)}|\overline{h_v}(iD_\alpha)(iD_\mu)(iD_\beta)\gamma_\delta\gamma_5 h_v|B^{(*)}\rangle$$
$$\equiv\frac{1}{6}d_H\rho_2 i\epsilon_{\nu\alpha\beta\delta}v^\nu v_\mu, \tag{3.2}$$

where $d_H=3$ and $d_H=-1$, respectively, for matrix elements between B and B^* states. The $\mathcal{T}_i$ are related to nonlocal time ordered products of $1/m_b^2$ terms in the operator product expansion with $1/m_b$ terms in the Lagrangian [14],

$$\langle B^{(*)}|\overline{h_v}(iD_\perp)^2 h_v i\int d^3x\int_{-\infty}^0 dt\mathcal{L}_I(x)|B^{(*)}\rangle+\text{H.c.}$$
$$\equiv\frac{\mathcal{T}_1+d_H\mathcal{T}_2}{m_b},$$

$$\langle B^{(*)}|\overline{h_v}\frac{g}{2}\sigma_{\mu\nu}G^{\mu\nu}h_v i\int d^3x\int_{-\infty}^0 dt\mathcal{L}_I(x)|B^{(*)}\rangle+\text{H.c.}$$
$$\equiv\frac{\mathcal{T}_3+d_H\mathcal{T}_4}{m_b}. \tag{3.3}$$

The parameters ρ_i and $\mathcal{T}_i$ are determined by nonperturbative QCD, and their values are not known; however, we compute their coefficients to ensure that none of them are anomalously large. We will use dimensional analysis to estimate ρ_i and $\mathcal{T}_i$, to obtain a rough estimate of the error in the extraction of $\overline{\Lambda}$ and λ_1 induced by $1/m_b^3$ effects.

The coefficients D_{ij} are themselves functions of α_s, although we will not compute radiative corrections to any of these coefficients. At present, $D_1^{(m)}$ are known to order α_s only in the absence of an electron energy cut. While it would certainly be desirable to include terms of order $\alpha_s\overline{\Lambda}/m_b$ for general $E_\ell^{\rm min}$, the calculation is quite difficult and we have not attempted it here. This omission is particularly important for the second moment, since $D_1^{(2)}=0$. Without the cut, the term in question is numerically as large as the leading terms proportional to $C_1^{(2)}$ and $D_{20}^{(2)}$ [8].[1] (In the case of the first moment, by contrast, the radiative correction to $D_1^{(1)}$ is only a few percent [10].) For this reason, as well as because of the large $1/m_b^3$ corrections, it is dangerous to use the second moment in a measurement of $\overline{\Lambda}$ and λ_1. We will include our ignorance of these terms in the estimate of the theoretical error in our results below.

Let $y=E_\ell^{\rm min}/m_B$ be the scaled lepton energy cut. When $y=0$, we reproduce the known results [8,10],

$$\begin{aligned}
&C_1^{(1)}=0.051 \quad C_{22}^{(1)}=0.096\\
&D_1^{(1)}=0.23 \quad D_{20}^{(1)}=0.26 \quad D_{21}^{(1)}=1.0 \quad D_{22}^{(1)}=-0.32\\
&D_{30}^{(1)}=0.33 \quad D_{31}^{(1)}=2.2 \quad D_{32}^{(1)}=-0.56 \quad D_{33}^{(1)}=2.3 \quad D_{34}^{(1)}=-1.2\\
&T_{31}^{(1)}=1.6 \quad T_{32}^{(1)}=0.80 \quad T_{33}^{(1)}=1.5 \quad T_{34}^{(1)}=0.41
\end{aligned} \tag{3.4}$$

and

$$\begin{aligned}
&C_1^{(2)}=0.0054 \quad C_{22}^{(2)}=0.0078\\
&D_1^{(2)}=0 \quad D_{20}^{(2)}=0.066 \quad D_{21}^{(2)}=-0.14 \quad D_{22}^{(2)}=0\\
&D_{30}^{(2)}=0.14 \quad D_{31}^{(2)}=0.32 \quad D_{32}^{(2)}=-0.31 \quad D_{33}^{(2)}=-0.85 \quad D_{34}^{(2)}=0.23\\
&T_{31}^{(1)}=-0.14 \quad T_{32}^{(1)}=-0.41 \quad T_{33}^{(1)}=0 \quad T_{34}^{(1)}=0.
\end{aligned} \tag{3.5}$$

The leading radiative correction to $D_1^{(1)}$ is $0.099\,\alpha_s(m_b)/\pi$, and to $D_1^{(2)}$ is $0.038\,\alpha_s(m_b)/\pi$. For the preferred experimental cut of 1.5 GeV, for which $y=0.28$, we find

$$\begin{aligned}
&C_1^{(1)}=0.028 \quad C_{22}^{(1)}=0.058\\
&D_1^{(1)}=0.21 \quad D_{20}^{(1)}=0.19 \quad D_{21}^{(1)}=1.4 \quad D_{22}^{(1)}=0.19\\
&D_{30}^{(1)}=0.19 \quad D_{31}^{(1)}=3.2 \quad D_{32}^{(1)}=1.4 \quad D_{33}^{(1)}=4.3 \quad D_{34}^{(1)}=-0.56\\
&T_{31}^{(1)}=2.0 \quad T_{32}^{(1)}=1.8 \quad T_{33}^{(1)}=1.7 \quad T_{34}^{(1)}=0.91
\end{aligned} \tag{3.6}$$

and

$$\begin{aligned}
&C_1^{(2)}=0.0015 \quad C_{22}^{(2)}=0.0026\\
&D_1^{(2)}=0 \quad D_{20}^{(2)}=0.054 \quad D_{21}^{(2)}=-0.12 \quad D_{22}^{(2)}=0\\
&D_{30}^{(2)}=0.10 \quad D_{31}^{(2)}=0.51 \quad D_{32}^{(2)}=-0.045 \quad D_{33}^{(2)}=-1.2 \quad D_{34}^{(2)}=0.0032\\
&T_{31}^{(2)}=-0.12 \quad T_{32}^{(2)}=-0.36 \quad T_{33}^{(2)}=0 \quad T_{34}^{(2)}=0.
\end{aligned} \tag{3.7}$$

We present plots of the coefficients C_i and D_i for arbitrary $E_\ell^{\rm min}$ in Figs. 1–3. Note that the power corrections tend to blow up as the electron cutoff approaches its maximum value; fortunately, for a cut of 1.5 GeV the coefficients are not dramatically larger than without a cut.

The effects of the $1/m_b^3$ corrections to the moments in the extraction of $\overline{\Lambda}$ and λ_1 are displayed in Fig. 4. For the purpose of illustration, we first assume perfect experimental measurements of $\langle s_H-\overline{m_D^2}\rangle=0.30\ {\rm GeV}^2$ and $\langle(s_H-\overline{m_D^2})^2\rangle=0.96\ {\rm GeV}^4$, and then extract $\overline{\Lambda}$ and λ_1 using the theoretical expressions at order $1/m_b^2$. We take $\alpha_s(m_b)=0.22$. These values for the first and second moments restrict $\overline{\Lambda}$ and λ_1 to lie on the solid and dashed curves, respectively, meeting at the point $(\overline{\Lambda},\lambda_1)=(0.31\ {\rm GeV},-0.16\ {\rm GeV}^2)$. The hypothesized data have been chosen so that this intersection coincides with the central values obtained from an existing analysis of the lepton energy spectrum [7]. We then may estimate the theoretical uncertainty in our hypothetical result by following the approach of Ref. [8]. By dimensional analysis, the parameters ρ_i and $\mathcal{T}_i$ are all of order $\Lambda_{\rm QCD}^3$, and an estimate of their effect on the extraction of $\overline{\Lambda}$ and λ_1 is obtained by varying their magnitudes independently in the range 0

[1] The authors of Ref. [8] correct a numerical error in this term in Ref. [10].

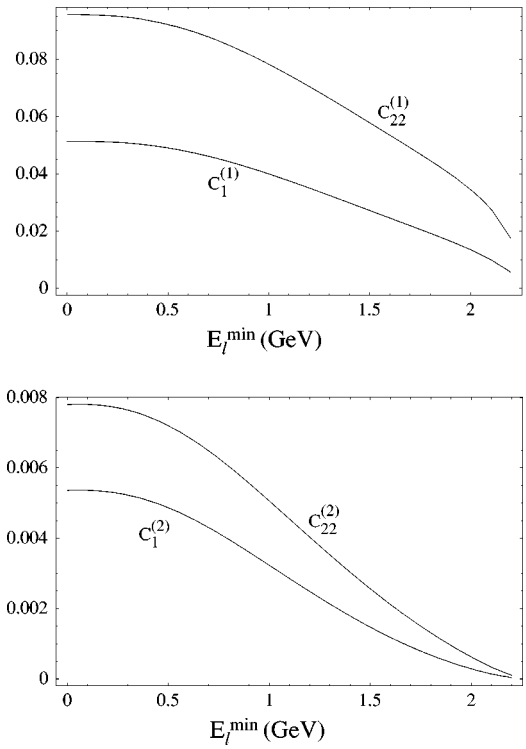

FIG. 1. Coefficients $C_{ij}^{(k)}$ of the radiative corrections to the first and second moments, as functions of the electron energy cutoff in GeV.

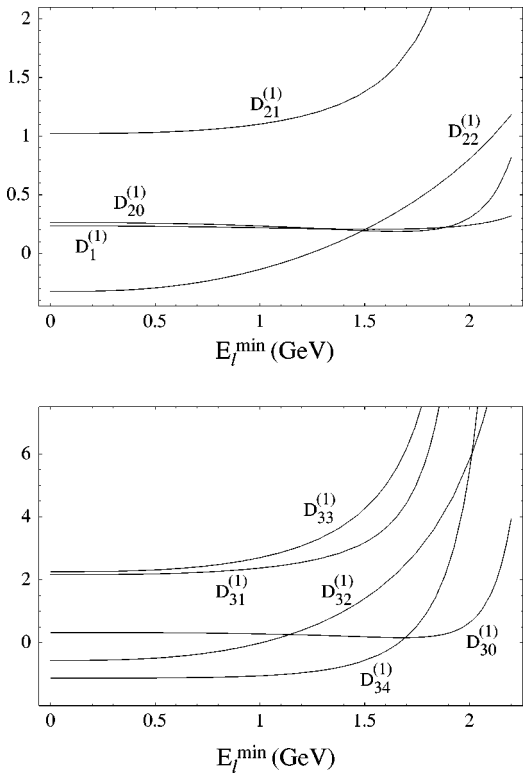

FIG. 2. Coefficients $D_{ij}^{(1)}$ of the power corrections to the first moment of the hadronic invariant mass spectrum, as functions of the electron energy cutoff in GeV.

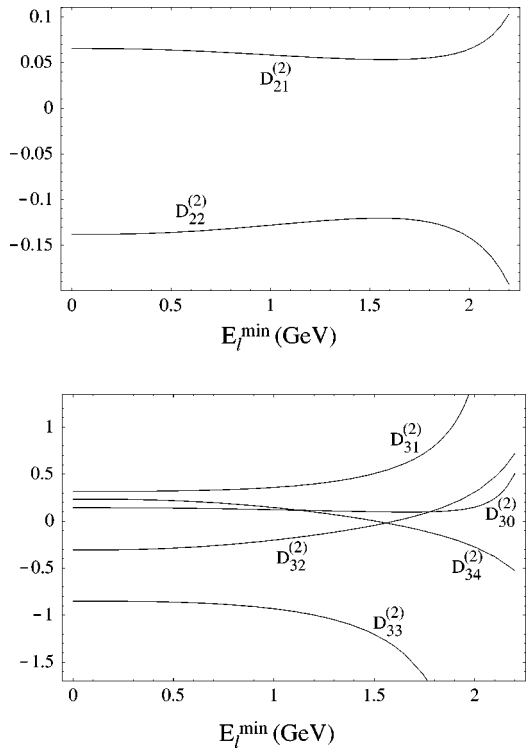

FIG. 3. Coefficients $D_{ij}^{(2)}$ of the power corrections to the second moment of the hadronic invariant mass spectrum, as functions of the electron energy cutoff in GeV.

$-(0.5\ \mathrm{GeV})^3$. Since the vacuum saturation approximation suggests that $\rho_1 > 0$, we take it to be positive, and we eliminate ρ_2 by making use of the relation between ρ_2, $\mathcal{T}_2$ and $\mathcal{T}_4$ and the $D^* - D$ and $B^* - B$ mass splittings presented in Ref. [8]. Finally, we vary the unknown coefficient of the $\alpha_s \overline{\Lambda}/m_b$ term for both moments between half and twice its value with the cut removed. Varying the unknown parameters randomly in the allowed ranges, we find that the shaded ellipse shown in the figure and centered about the mean of the distribution contains 68% of the points. (Because the $\overline{\Lambda^3}$, $\overline{\Lambda\lambda_1}$ and $\overline{\Lambda\lambda_2}$ terms bias the determination of $\overline{\Lambda}$ and λ_1 in a known way, the distribution is not centered about the point extracted from the theory at order $1/m_b^2$.) The region inside the ellipse gives a reasonable estimate of the theoretical error in the extraction of $\overline{\Lambda}$ and λ_1 due to higher order effects. Note that, as expected, the constraints from the second moment are strongly affected by higher order corrections, whereas the constraints from the first moment are quite tightly distributed about the leading result. Thus, as discussed earlier, only the linear combination of $\overline{\Lambda}$ and λ_1 given by the first moment is significantly constrained.

Constraints on $\overline{\Lambda}$ and λ_1 also have been obtained from moments of the lepton energy spectrum above 1.5 GeV [7]. To compare the theoretical errors in this approach to ours, we have performed an analysis analogous to that of the previous paragraph. This is similar to what was done in Ref. [8], but we also include the terms proportional to $\alpha_s^2 \beta_0$ [9]. The result is shown in Fig. 5. The size of the ellipse from the lepton energy analysis is slightly larger than, but comparable to, that which we obtained from the hadronic mass moments. Of course, the relative position of the ellipses is meaningless,

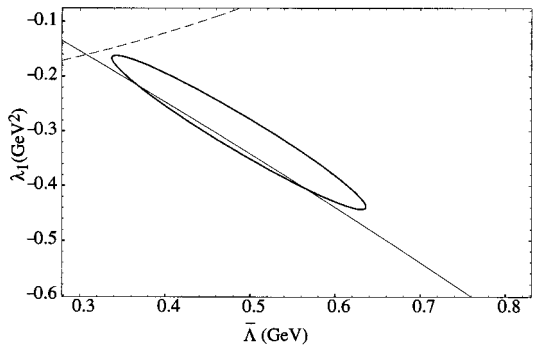

FIG. 4. Estimate of the theoretical uncertainty in $\overline{\Lambda}$ and λ_1 due to unknown $1/m_b^3$ and $\alpha_s\overline{\Lambda}/m_b$ contributions to the moments (see text for description). For the purpose of illustration, perfect experimental measurements of $\langle s_H-\overline{m}_D^2\rangle=0.30$ GeV^2 and $\langle(s_H-\overline{m}_D^2)^2\rangle=0.96$ GeV^4 have been assumed. The solid and dashed lines show the constraints on $\overline{\Lambda}$ and λ_1 from the first and second moments, respectively, while the area in the shaded ellipse shows the estimated allowed range.

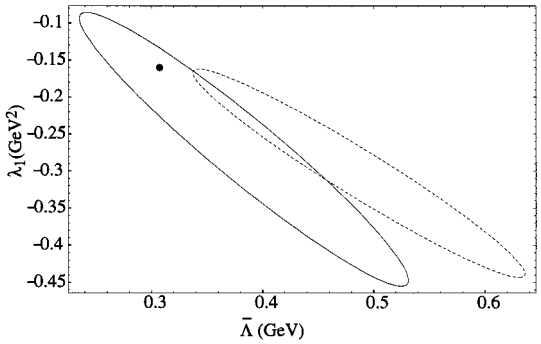

FIG. 5. Estimate of the theoretical uncertainty in $\overline{\Lambda}$ and λ_1 due to unknown $1/m_b^3$ contributions from the shape of the electron spectrum, using the same approach as in the previous plot. The central value at $O(1/m_b^2)$ is given by the black dot, while the dashed ellipse is the plot from the previous figure, shown here for comparison. Only the relative sizes and orientations of the two ellipses are meaningful in this figure, not their relative positions.

since the hadronic mass moments have not yet been measured. Unfortunately, the two experiments effectively constrain the same linear combination of $\overline{\Lambda}$ and λ_1, so that the measurements cannot be combined to determine both parameters simultaneously. Instead, an observable sensitive to a different linear combination of $\overline{\Lambda}$ and λ_1, such as the first moment in the photon spectrum in $B\to X_s\gamma$ [19], will be required. In the meantime, consistency of the various allowed regions in the $\overline{\Lambda}-\lambda_1$ plane will provide a powerful test of the heavy quark expansion for these decays.

IV. CONCLUSIONS

We have presented a calculation of the first two hadronic invariant mass moments in semileptonic B decay, in the presence of a moderate cut on the energy of the charged lepton. We included effects up to order $\alpha_s^2\beta_0$ and $1/m_b^3$. These moments may be used to measure a linear combination of the HQET parameters $\overline{\Lambda}$ and λ_1, with a theoretical accuracy which is comparable to, or slightly better than, the accuracy obtained from an analysis of the charged lepton energy spectrum. The consistency of the results obtained from these approaches will provide a test of the heavy quark expansion as applied to semileptonic B decays. To extract $\overline{\Lambda}$ and λ_1 simultaneously, it will be necessary to combine this analysis with that of a quantity sensitive to a different linear combination of the two parameters.

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[1] J. Chay, H. Georgi and B. Grinstein, Phys. Lett. B **247**, 399 (1990).
[2] I. I. Bigi, N. G. Uraltsev, and A. I. Vainshtein, Phys. Lett. B **293**, 430 (1992); **297**, 477(E) (1993); I. I. Bigi *et al.*, Phys. Rev. Lett. **71**, 496 (1993); B. Blok *et al.*, Phys. Rev. D **49**, 3356 (1994); **50**, 3572(E) (1994).
[3] A. V. Manohar and M. B. Wise, Phys. Rev. D **49**, 1310 (1994).
[4] T. Mannel, Nucl. Phys. **B413**, 396 (1994).
[5] A. F. Falk, M. Luke, and M. J. Savage, Phys. Rev. D **49**, 3367 (1994); M. Neubert *ibid.* **49**, 4623 (1994).
[6] M. B. Voloshin, Phys. Rev. D **51**, 4934 (1995).
[7] M. Gremm *et al.*, Phys. Rev. Lett. **77**, 20 (1996).
[8] M. Gremm and A. Kapustin, Phys. Rev. D **55**, 6924 (1997).
[9] M. Gremm and I. Stewart, Phys. Rev. D **55**, 1226 (1997).
[10] A. F. Falk, M. Luke, and M. J. Savage, Phys. Rev. D **53**, 2491 (1996); **53**, 6316 (1996).
[11] See, for example, CLEO Collaboration, J. Alexander *et al.*, Phys. Rev. Lett. **77**, 5000 (1996).
[12] Ed Thorndike (private communication).
[13] M. Neubert, Phys. Rev. D **49**, 3392 (1994); **49** 4623 (1994); I. I. Bigi *et al.*, Int. J. Mod. Phys. A **9**, 2467 (1994).
[14] T. Mannel, Phys. Rev. D **50**, 428 (1994); I. I. Bigi *et al.*, *ibid.* **52**, 196 (1995).
[15] S. Brodsky, G. P. Lepage, and P. Mackenzie, Phys. Rev. D **28**, 228 (1983).
[16] M. Luke, M. Savage, and M. B. Wise, Phys. Lett. B **343**, 329 (1995); **345**, 301 (1995); P. Ball, M. Beneke, and V. M. Braun, Phys. Rev. D **52**, 3929 (1995); M. Neubert, Phys. Rev. D **51**, 5924 (1995).

[17] B. H. Smith and M. B. Voloshin, Phys. Lett. B **340**, 176 (1994).

[18] M. Beneke and V. M. Braun, Nucl. Phys. **B426**, 301 (1994); I. Bigi *et al.*, Phys. Rev. D **50**, 2234 (1994); M. Beneke, V. M. Braun, and V. I. Zakharov, Phys. Rev. Lett. **73**, 3058 (1994); M. Luke, A. V. Manohar, and M. J. Savage, Phys. Rev. D **51**, 4924 (1995); M. Neubert and C. T. Sachrajda, Nucl. Phys. **B438**, 235 (1995).

[19] A. Kapustin and Z. Ligeti, Phys. Lett. B **355**, 318 (1995).