Narrow width of a glueball decay into two mesons

Jun Cao, Tao Huang, and Hui-fang Wu
CCAST (World Laboratory) P.O. Box 8730, Beijing, 100080, People's Republic of China
*Institute of High Energy Physics, P.O. Box 918(4), Beijing, 100039, People's Republic of China**
(Received 2 September 1997; published 19 February 1998)

The widths of a glueball decay to two pions or kaons are analyzed in the perturbative QCD (PQCD) framework. Our results show that the glueball ground state has a small branching ratio for the two-meson decay mode, which is around 10^{-2}. The predicted values are consistent with the data of $\xi \to \pi\pi, KK$ if the ξ particle is a 2^{++} glueball. The applicability of PQCD to the glueball decay and a comparison with χ_{cJ} decay are also discussed. [S0556-2821(98)04705-5]

PACS number(s): 12.39.Mk, 12.38.Bx, 13.25.Gv

I. INTRODUCTION

The existence of glueballs and hybrids is a direct consequence of QCD. Since J/ψ is predicted to have appreciable decays to γgg where two gluon formation is expected to enhance the production of tensor and scalar glueballs, radiative J/ψ and ψ' decays have long been realized to be a favored area for glueball searches. At present one pays particular attention to three states: $f_0(1500)$, $f_J(1710)$ ($J=0$ or 2), and $\xi(2230)$ ($J \geqslant 2$). They are glueball candidates of 0^{++} or 2^{++} states, motivated by lattice QCD. The UKQCD Collaboration [1] reported their lattice prediction for glueball masses, which is 1550 ± 50 MeV for the 0^{++} state and 2270 ± 100 MeV for the 2^{++} state. At the same time the IBM group reported their mass [2] for the 0^{++} glueball as 1740(71) MeV and predicted a total width [3,4] for glueball decay to pseudoscalar pairs of 108(29) MeV. The coupling to two η's seems to be the largest, followed by K and π meson pairs. The decay width to $\eta + \eta'$ is 6(3) MeV.

In particular, the BES Collaboration discovered new, nonstrange decay modes of the $\xi(2230)$ state, such as $\xi \to \pi\pi$ and $\xi \to p\bar{p}$ [5]. Compared with other mesons, $\xi(2230)$ has many distinctive properties [6].

(1) Flavor-symmetric decays to light hadrons. After removal of the phase space factor, the probability for $\xi \to \pi^+\pi^-$ is of the same order as that for $\xi \to K^+K^-$.

(2) Copious production in radiative J/ψ decays. From the upper limit of 1×10^{-4} for $B(\xi \to p\bar{p})B(\xi \to K\bar{K})$ [7,8], where $K\bar{K}$ includes all kaon pairs, a lower bound 3×10^{-3} for $B(J/\psi \to \gamma\xi)$ can be estimated.

(3) Narrow width. Both results from Mark III and BES show that the width of $\xi(2230)$ is only about 20 MeV. Assuming $\Gamma_\xi = 20$ MeV, one can easily estimate from (2) that $B(\xi \to K^+K^-)$ and $B(\xi \to \pi^+\pi^-)$ are smaller than 2%, resulting in partial widths $\Gamma_{\pi^+\pi^-}$ and $\Gamma_{K^+K^-}$ smaller than 400 KeV [6].

As a consequence, the $q\bar{q}$, multiquark, and hybrid models cannot easily reproduce the three observations above. On the other hand, these properties are naturally explained by identifying $\xi(2230)$ as a glueball state with $J^{PC} = 2^{++}$.

Since the observed decay modes into $\pi\pi$, $K\bar{K}$, and $p\bar{p}$ are expected to be only a small portion of the decay modes of $\xi(2230)$, searches for other decay modes are very important. From a theoretical point of view, the narrow width of a glueball follows from a loose interpretation of the Okubo-Zweig-Iizuka (OZI) rule. The gluons in the glueball would annihilate and create a $q\bar{q}$ pair that would form the lighter hadrons. Since this suppression only acts at one vertex, it is called the $\sqrt{\text{OZI}}$ rule [9]. For example, the total width of a 2^{++} glueball can be estimated from the $\sqrt{\text{OZI}}$ rule to be about $\sqrt{\Gamma_{f_2(1270)}\Gamma_{\chi_{c2}}} \simeq 20$ MeV.

In order to understand the narrow decay width quantitatively, we study a pure 0^{++} or 2^{++} glueball decay to two light mesons in perturbative QCD (PQCD). Our results show that a pure glueball decay to two light mesons has a small branching ratio or a narrow width, and this conclusion can be generalized to be valid for any two mesons. As a consequence, there is no dominant decay channel for a pure 0^{++} or 2^{++} glueball.

The paper is organized as follows. As a comparison with the glueball, the formula for χ_{cJ} decay to two mesons in PQCD is reviewed briefly in Sec. II. Glueball decays into two mesons are calculated in Sec. III. In Sec. IV the numerical results and applicability of PQCD to glueball decays are discussed. The last section is reserved for summary and conclusions.

II. HEAVY QUARKONIUM WITH $J^{PC} = 0^{++}$ OR 2^{++}

It is interesting to compare a glueball decay with the χ_{cJ}. A brief review for χ_{cJ} decay will be given in the PQCD framework. For a heavy quarkonium coupling to gluons, the vertex can be obtained with its radial wave function at origin, or its differentiation. Since the wave function is sharply peaked at small internal momentum for heavy quarkonium, the nonrelativistic limit is a good approximation.

As an example, we review the derivation of $B(\chi_{cJ} \to \pi\pi)$ in PQCD. To leading order, the Fermi movement of quarks in the pion can be neglected comparing with the momentum flow of order m_c, the mass of the c quark, in the hard scattering amplitude. Thus, the amplitude of χ_{cJ} decay to two pions can be expressed in a factorized form [10] as a π form factor:

*Mailing address.

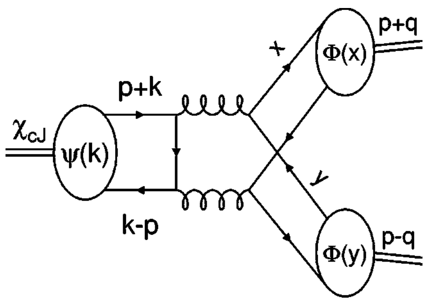

FIG. 1. The hard scattering diagram for $\chi_{cJ}\rightarrow\pi\pi$.

$$A=\int dxdy\,\phi_\pi(x)T_H(x,y,Q^2)\phi_\pi(y), \tag{1}$$

where $\phi_\pi(x)$ is the distribution amplitude of pion obtained by integrating the transverse momentum of the Bethe-Salpeter wave function [11]. The hard scattering amplitude $T_H(x,y,Q^2)$ (see Fig. 1) will include information of a heavy quarkonium coupling to two gluons:

$$T_H=\left\{\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\hat{O}_{\mu\nu}\chi(p,k)]\right\}T^{\mu\nu}\frac{ig^4}{l_1^2l_2^2}+\{x\leftrightarrow y\}, \tag{2}$$

with l_1 and l_2 the momentum of gluons,

$$\hat{O}^{\mu\nu}=\gamma^\mu\frac{1}{\not p+\not k-\not l_1-m}\gamma^\nu+\{\mu\leftrightarrow\nu\}, \tag{3}$$

and the wave function of heavy quarkonium in the nonrelativistic quark model (NRQM) [12],

$$\chi(p,k)=\sum_{M,S_z}(2\pi)\delta\left(k^0-\frac{\vec{k}^2}{2m}\right)\times\langle LM;SS_z|JJ_z\rangle\psi_{LM}(\vec{k})P_{SS_z}(p,k), \tag{4}$$

where $\psi_{LM}(\vec{k})$ and $P_{SS_z}(p,k)$ is the spatial and spin part, respectively. $T^{\mu\nu}$ involve the spin part of pion wave function and present the coupling of $gg\rightarrow\pi\pi$:

$$T^{\mu\nu}=\mathrm{Tr}\left[\gamma^\mu\frac{\gamma^5}{\sqrt{2}}(\not p-\not q)\gamma^\nu\frac{\gamma^5}{\sqrt{2}}(\not p+\not q)\right]=4(p^\mu p^\nu-q^\mu q^\nu-g^{\mu\nu}m_c^2). \tag{5}$$

For quarkonium in the P-wave state, only 0^{++} and 2^{++} can decay to two pions. In the case of 2^{++} state, the polarization tensor $\epsilon^{\alpha\beta}$ is composed of the spin and orbital polarization vectors $\epsilon^\alpha(S_z)$ and $\epsilon^\beta(M)$ of χ_{c2} as

$$\epsilon^{\alpha\beta}(J_z)=\sum_{M,S_z}\langle 1M;1S_z|2J_z\rangle\epsilon^\alpha(M)\epsilon^\beta(S_z). \tag{6}$$

The polarization tensor satisfies $\epsilon^{\alpha\beta}=\epsilon^{\beta\alpha}$, $p^\alpha\epsilon^{\alpha\beta}=0$, $\epsilon^\alpha{}_\alpha=0$ and

$$\sum_{J_z}\epsilon^{\alpha\beta}(J_z)\epsilon^{\mu\nu}(J_z)=\frac{1}{2}(\mathcal{P}^{\alpha\mu}\mathcal{P}^{\beta\nu}+\mathcal{P}^{\alpha\nu}\mathcal{P}^{\beta\mu})-\frac{1}{3}\mathcal{P}^{\alpha\beta}\mathcal{P}^{\mu\nu}, \tag{7}$$

where

$$\mathcal{P}^{\mu\nu}=-g^{\mu\nu}+\frac{p^\mu p^\nu}{m_c^2}. \tag{8}$$

Now we arrive at the decay width if $\alpha_s(x_1x_2Q^2)\simeq\alpha_s(Q^2)$,

$$\Gamma(\chi_{c2}\rightarrow\pi\pi)=\frac{2C^2}{5\pi^2m_c^8}[(4\pi\alpha_s)^2\phi_2'(0)I_2]^2, \tag{9}$$

where $C^2=\frac{4}{27}$ is the color factor and the integral I_2 is given by

$$I_2=\int\frac{dx\,\phi_\pi(x)}{2x(1-x)}\int\frac{dy\,\phi_\pi(y)}{2y(1-y)}\frac{1}{2(x+y-2xy)}\times\left(1-\frac{(x-y)^2}{x+y-2xy}\right). \tag{10}$$

Similarly, the 0^{++} decay width can be expressed as

$$\Gamma(\chi_{c0}\rightarrow\pi\pi)=\frac{4C^2}{\pi^2m_c^8}[(4\pi\alpha_s)^2\phi_0'(0)I_0]^2, \tag{11}$$

with the overlap integral I_0 given by

$$I_0=\int\frac{dx\,\phi_\pi(x)}{2x(1-x)}\int\frac{dy\,\phi_\pi(y)}{2y(1-y)}\frac{1}{2(x+y-2xy)}\times\left(1+\frac{(x-y)^2}{2(x+y-2xy)}\right). \tag{12}$$

The total width of χ_{cJ} can be obtained by calculating the width of its decay to two real gluons in a similar way. In NRQM, it is

$$\Gamma_{\chi_{c2}}=\frac{8\alpha_s^2}{5m_c^4}|\phi_2'(0)|^2, \tag{13a}$$

$$\Gamma_{\chi_{c0}}=\frac{6\alpha_s^2}{m_c^4}|\phi_0'(0)|^2. \tag{13b}$$

Finally, we get the branching ratio

$$B(\chi_{c2}\rightarrow\pi\pi)=\frac{16}{27}\frac{(4\pi\alpha_sI_2)^2}{m^4}, \tag{14a}$$

$$B(\chi_{c0}\rightarrow\pi\pi)=\frac{144}{81}\frac{(4\pi\alpha_sI_0)^2}{m^4}, \tag{14b}$$

which is independent of the χ_{cJ} wave function.

Numerical calculations can be easily done. However, the obtained results are much smaller than the data [13]. Even when stretching all parameters to their extreme values, the predictions stay a factor 3–6 below the data [14]. Moreover, the obtained branching ratio dominantly come from the end-point region, where PQCD is not available. As stated by Bodwin, Braaten, and Lepage [15], the color-octet decay contribution arising from the higher Fock component $|c\overline{c}g\rangle$ of the χ_{cJ} wave function is actually not suppressed with respect to the usual color-singlet decay, owing to the angular momentum barrier in $|c\overline{c}\rangle$ contribution, and is necessary to separate rigorously short-distance effects and long-distance effects. Including the color-octet contributions, the branching ratio of $\chi_{cJ}\to\pi\pi$ can be compared with the experimental data [14].

III. GLUEBALL

Similar to the $Q\overline{Q}$ state, we apply PQCD to the glueball decay. The amplitude for its decay to two pions can be written as Eq. (1), too. Now T_H contain the glueball wave function. The coupling of a J^{++} state to two gluons can be obtained from requirements such as being linear in the polarization vectors of two gluons ϵ_1 and ϵ_2 and Lorentz and gauge invariant [12,16]. Keeping only the leading twist term, the wave function can be written phenomenologically as

$$\Psi(2^{++})=\frac{\delta^{ab}}{\sqrt{8}}\epsilon^{\mu\nu}G^1_{\mu\rho}G^2_{\nu\rho}\phi_2(k), \tag{15}$$

$$\Psi(0^{++})=\frac{\delta^{ab}}{\sqrt{8}}\frac{\mathcal{P}^{\mu\nu}}{\sqrt{3}}G^1_{\mu\rho}G^2_{\nu\rho}\phi_0(k), \tag{16}$$

where $\delta^{ab}/\sqrt{8}$ is the color wave function, $\epsilon^{\mu\nu}$ is the polarization tensor of the 2^{++} glueball, and $\mathcal{P}^{\mu\nu}$ is defined in Eq. (8). $G^i_{\mu\nu}=\epsilon^i_\mu k^i_\nu-k^i_\nu\epsilon^i_\mu,(i=1,2)$, ϵ^i_μ and k^i_ν is the polarization vector and momentum of the ith gluon, respectively.

The leading order contribution to a 2^{++} glueball decay to two pions in PQCD is shown in Fig. 2. In center-of-mass frame, the amplitude can be written as

$$A=g_s^2\int dz\,\phi_\pi(z)dz'\,\phi_\pi(z')\,\phi_2(k)\frac{1}{k_1^2k_2^2}$$

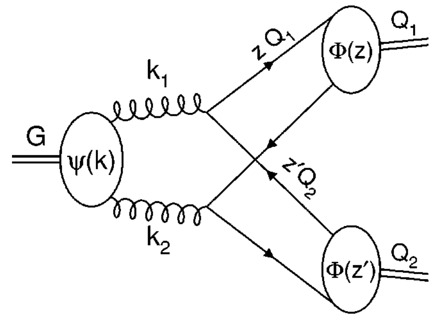

FIG. 2. The hard scattering diagram for the glueball decay to $\pi\pi$.

$$\times\sum_{s_1s_2}\epsilon_{\mu\nu}(s)G^1_{\mu\rho}(s_1)G^2_{\nu\rho}(s_2)\epsilon^1_\sigma(s_1)\epsilon^2_\eta(s_2)T^{\sigma\eta}, \tag{17}$$

where $T^{\sigma\eta}$ is the same as in Eq. (5) and the color factor will be included in the width formula. The momenta of gluons are fixed by that of quarks in pions, i.e., $k_1=zQ_1+z'Q_2$ and $k_2=\overline{z}Q_1+\overline{z'}Q_2$. The width can be obtained as

$$\Gamma(2^{++}\to\pi\pi)$$
$$=C\frac{\pi}{30M}\left|\int dzdz'\,\phi_\pi(z)\phi_\pi(z')\phi_2(k_i)\alpha_s\frac{z\overline{z}+z'\overline{z'}}{z\overline{z}z'\overline{z'}}\right|^2, \tag{18}$$

where C is the color factor $\frac{2}{9}$ and $\overline{z}=(1-z)$. The total width of 2^{++}, can be obtained by its decay width to two on-shell gluons with $\vec{k}_i^2=M^2/4$:

$$\Gamma_{2^{++}}=\frac{M^3}{320\pi}|\phi_2(k_i)|^2. \tag{19}$$

Similarly, we can obtain the width of 0^{++} decay to two pions:

$$\Gamma(0^{++}\to\pi\pi)=C\frac{\pi}{12M}\left|\int dzdz'\,\phi_\pi(z)\phi_\pi(z')\phi_0(k_i)\alpha_s\frac{(z-z')^2+2z\overline{z'}+2z'\overline{z}}{z\overline{z}z'\overline{z'}}\right|^2, \tag{20}$$

and the total width

$$\Gamma_{0^{++}}=\frac{3M^3}{256\pi}|\phi_0(k_i)|^2. \tag{21}$$

Comparing Eqs. (18) and (20) with Eqs. (9)–(13), it can be shown that there are two different ingredients between the glueball decay and the P-wave quarkonium decay. (i) There is no end-point singularity in expressions (18) and (20) for glueball decay since gluons in the glueball are directly related to the glueball wave function. (ii) Since we get the total width by the decay to two on-shell gluons, it depends on the glueball wave function at a particular momentum point. The point is located at $\vec{k}^2=M^2/4$. Intuitively, the wave function must be peaked at low momentum, since a composite particle has little amplitude for existing while its constituents are flying apart with large momentum. While the mass M is large, the obtained width decreases fast and depends on the shape of the wave function drastically. On the other side, the next order contributions to the total width, whose amplitudes are expected to have the form $\alpha_s\int d^4k\,\phi(k)f(k)$ in general, will be the lack of the suppression of the wave function and not sensitive to the shape of the wave function as long as $f(k)$ is smooth. It means that the higher order contributions are important to the total width if the mass is large, since the zeroth order contribution is suppressed strongly by wave function. In another word, the PQCD evolution of the wave function to a particular scale will be nontrivial.

The formula for decay to a pair of kaons can be easily obtained by substituting the distribution amplitude of pions by that of kaons. As a result of isospin symmetry, the branching ratio to $\pi^0\pi^0$ is half of the charged channel except that $\pi^0\pi^0$ in the final state can be formed via QCD anomaly. This contribution is negligible due to the minor difference between the mass of the u and d quarks. The anomaly contributions to $\eta\eta$, $\eta\eta'$, and $\eta'\eta'$ channels are not necessarily small, since two soft gluons have a large possibility to form a η (η') meson [17]:

$$\left\langle 0\left|\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}\right|\eta\right\rangle=\sqrt{\frac{3}{2}}f_\pi m_\eta^2. \tag{22}$$

However, it is a low energy theorem. If the energy scale of the decay process is high enough, the anomaly contribution is also negligible as it emerges via QCD renormalization and is essentially a higher order contribution in α_s. For example, the branching ratio of $\chi_{cJ}\to\eta\eta$ has no apparent enhancement compared to the $\pi^0\pi^0$ channel. So the branching ratios of the $\eta\eta$, $\eta\eta'$, and $\eta'\eta'$ channels are expected to have the same order of the $\pi\pi$ channel if PQCD dominates the decay process.

IV. APPLICABILITY OF PQCD AND NUMERICAL RESULTS

Evaluation of Eqs. (18)–(21) will require the glueball wave function, which we have little information about up to now. In the following context we will use an oscillator wave function:

$$\phi(k)=A_g\exp(-b_g^2\vec{k}^2), \tag{23}$$

with the parameters appropriately chosen. If 0^{++} and 2^{++} glueballs have similar wave functions in momentum space as argued in Ref. [18], a constraint on b_g can be obtained from their total decay width. The 0^{++} glueball candidates $f_0(1500)$ and $f_0(1700)$ have widths 100–150 MeV. The 2^{++} glueball candidate $\xi(2230)$ has width $\sim$ 20 MeV. From our width formulas (19) and (21) we have

$$\frac{\Gamma_{\xi(2230)}}{\Gamma_{f_0(1500)}}=\frac{4M_\xi^3A_2^2}{15M_{f_0}^3A_0^2}\,\frac{\exp(-2b_g^2M_\xi^2)}{\exp(-2b_g^2M_{f_0}^2)}\simeq\frac{20\ \text{MeV}}{120\ \text{MeV}}. \tag{24}$$

If the masses of scalar and tensor states are equal, the ratio will be simply $\frac{4}{15}$, which is in accord with Ref. [16]. Taking into account the mass gap, we get $b_g^2\sim 0.4\ \text{GeV}^{-2}$ if $A_2\simeq A_0$. In general, A_g is larger for a higher mass state. For example, A_K is larger than A_π (see below). So we get $b_g^2>0.4$. According to the discussion in the previous section, higher order corrections may be large and should be larger for a higher mass state. Thus we can say that the lower limit of b_g^2 is 0.4 GeV^{-2}. Furthermore, the parameter b_g is related to the radius of the hadron. We know that $b^2\simeq 0.8\ \text{GeV}^{-2}$ for a well-established pion oscillator wave function [19]. Assuming that all hadrons have a similar size, b^2 will be in the same range for the glueball. The parameter b^2 for the glueball will be taken in the range $0.6\leqslant b^2\leqslant 0.8$ in the following context.

Before getting the numerical results it is necessary to discuss whether the derived formula is applicable or reliable. The applicability of the PQCD theory to exclusive processes at the present experimental energy region was argued by Isgur and Smith [20] by excluding the contributions of the end-point regions where subleading (higher twist) terms are *a priori* likely to be greater than the perturbative contribution. Recently, the applicability of PQCD to the pionic form factor has been examined by cutting end-point contribution [21] and by including the effects of Sudakov form factor [22]. The first approach argues that the PQCD results are self-consistent in the energy region where the contributions after cutting dominate. The second one attempts to explain the suppression in the end-point regions by including the effects of Sudakov form factor of the quarks, which serves as a natural filter to pick out the hard contributions. Two approaches give similar conclusion that PQCD is applicable to the pion form factor as $Q^2>4\ \text{GeV}^2$. In fact, the applicability of PQCD to exclusive processes differs from one process to another and depends on the end-point singularities. For example, the hard scattering amplitude of the χ_{cJ} decay to two pions is more singular than the case of the pion form factor. The obtained rate of $\chi_{cJ}\to\pi\pi$ depends strongly on the end-point behavior of the pion wave function [23]. In the case of glueball decay, the hard scattering amplitude of glueball decay to two pions is less singular than that of the pion form factor, and has a good behavior to ensure that the dominant contributions come from the hard part. We can argue that PQCD is applicable for the glueball, particularly, for the glueball candidate $\xi(2230)$. We will adopt the first method which is simpler.

In order to realize the condition $\alpha_s(z\bar{z}M^2)<1$ in Eqs. (18) and (20), we extend the parametrization of $\alpha_s(Q^2)$ by replacing [24]

$$\alpha_s(Q^2)=\frac{12\pi}{(33-2n_f)\ln(Q^2+M_0^2)/\Lambda^2} \tag{25}$$

to reflect the fact that at low Q^2 the transverse momentum intrinsic to the bound state wave function flows through all the propagators. The parameter M_0 can be determined to ensure $\alpha_s(0)=1$. The cut contribution, obtained by cutting the integral limit to keep only hard contributions, say, $\alpha_s<0.7$, will be compared with the uncut contribution.

In order to calculate the decay width we take the wave function of the pion and the kaon as [24]

$$\psi_\pi(x,k_\perp)=A_\pi\exp\left[-b^2\frac{k_\perp^2+m_q^2}{x(1-x)}\right], \tag{26a}$$

$$\psi_K(x,k_\perp)=A_K\exp\left[-b^2\left(\frac{k_\perp^2+m_q^2}{x}+\frac{k_\perp^2+m_s^2}{1-x}\right)\right], \tag{26b}$$

where m_q means the mass of u or d quark and m_s the mass of s quark. The distribution amplitude $\phi(x)$ can be obtained by integrating $k_\perp$:

TABLE I. Numerical results.

	b_g^2	$\Gamma_{\rm tot}$	$\Gamma_{\pi^+\pi^-}$	$B(\pi^+\pi^-)$	$B(K^+K^-)$
	0.6	$2.82\times10^{-5}A_g^2$	$1.08\times10^{-7}A_g^2$	0.38×10^{-2}	0.69×10^{-2}
$\xi(2230)$	0.7	$1.04\times10^{-5}A_g^2$	$1.01\times10^{-7}A_g^2$	0.97×10^{-2}	1.75×10^{-2}
	0.8	$3.86\times10^{-6}A_g^2$	$0.95\times10^{-7}A_g^2$	2.45×10^{-2}	4.45×10^{-2}
	0.6	$8.46\times10^{-4}A_g^2$	$6.00\times10^{-6}A_g^2$	0.71×10^{-2}	1.11×10^{-2}
$f_0(1500)$	0.7	$5.39\times10^{-4}A_g^2$	$5.64\times10^{-6}A_g^2$	1.05×10^{-2}	1.65×10^{-2}
	0.8	$3.44\times10^{-4}A_g^2$	$5.32\times10^{-6}A_g^2$	1.55×10^{-2}	2.47×10^{-2}

$$\phi_\pi=\frac{A_\pi}{16\pi^2b^2}x(1-x)\exp\left[-b^2\frac{m_q^2}{x(1-x)}\right], \qquad (27a)$$

$$\phi_K=\frac{A_K}{16\pi^2b^2}x(1-x)\exp\left[-b^2\left(\frac{m_q^2}{x}+\frac{m_s^2}{1-x}\right)\right]. \qquad (27b)$$

The parameters can be adjusted by using the constraints from decays of $\pi\to\gamma\gamma$ and $\pi\to\mu\nu$ ($K\to\mu\nu$) and the average quark transverse momentum $\langle k_\perp^2\rangle_\pi\simeq\langle k_\perp^2\rangle_K\simeq$ (360 MeV)^2 [19,25]. It turns out that

$$m_q=0.25\ \text{GeV},\quad b^2=0.80\ \text{GeV}^{-2},\quad A_\pi=27.7\ \text{GeV}^{-1},$$

$$m_s=0.55\ \text{GeV},\quad b^2=0.80\ \text{GeV}^{-2},\quad A_K=51.0\ \text{GeV}^{-1}.$$

The numerical results are listed in Table I. The parameter b_g is taken as 0.6, 0.7, and 0.8. Here we have not fixed the normalization constant and the branching ratio is independent of A_g^2. But it is dependent on the parameter b_g. The main uncertainty comes from the calculation of total width, which has been discussed in Sec. III. Due to the lack of end-point singularities, the partial width is stable for different b_g and should be stable for different wave function models, if the wave function is peaked at low momentum. For different b_g, the branching ratios of 2^{++} and 0^{++} to $\pi^+\pi^-$ or K^+K^-, although they are not quantitatively accurate, all are around 10^{-2} for a pure glueball. The numerical results are qualitatively consistent with $\xi\to\pi\pi$ if ξ is a 2^{++} glueball.

Also we examine the applicability of PQCD to the glueball decay by cutting end-point contributions. Our numerical results show that the hard part ($\alpha_s<0.7$) contributes about 81% to $\Gamma(2^{++}\to\pi^+\pi^-)$, whose value is listed in Table I, and the ratio is nearly the same for different b_g^2 because the partial width is not sensitive to b_g^2. For 0^{++} the ratio is about 50%. Another kind of distribution amplitude of the pion, the asymptotic form, is also used. We find that the branching ratio increases to about 10% for the 2^{++} glueball and 20% for the 0^{++} one and the hard contributions occupy about 61% in the width of the 2^{++} decay to $\pi^+\pi^-$ and 30% in the width of the 0^{++} decay.

V. CONCLUSIONS AND SUMMARY

We have analyzed the decay width of a glueball decay to two pions and kaons in the PQCD framework and it can be generalized to other two-meson decay channels. The numerical results of the decays show that the branching ratio is small and the decay width is very narrow, compared to the $q\bar{q}$ bound state. The branching ratios are consistent with the data of $\xi\to2\pi,2K$ if the ξ particle is a 2^{++} glueball.

Our conclusions are as follows.

(1) Applicability of PQCD to the glueball decay is discussed in our paper. We show that the hard scattering amplitude of glueball decay has a good behavior at the end-point region and is favored by PQCD while χ_{cJ} is opposite. The reason is that gluons are directly related to the wave function for glueball decay. The extra quark propagator (and the differentiation on it) produces the end-point singularity in χ_{cJ} exclusive decay. More specifically speaking, PQCD is applicable for $\xi(2230)$ decay to two mesons if it is a 2^{++} glueball, but is not reliable for $f_0(1500)$.

(2) The numerical results for decay widths depend on the choice of wave function, which is of genuinely nonperturbative origin, and the parameter of it. Although not quantitatively accurate, we found that a pure glueball has small branching ratios for the two-meson decay mode in a PQCD framework, which are all around 10^{-2} for different parameters b_g in a reasonable region. However, if the mass of the glueball is not high enough to ensure that PQCD dominates the decay process, the QCD anomaly will play an important role and the $\eta\eta$, $\eta\eta'$, and $\eta'\eta'$ channels will enhance apparently.

(3) Related to the end-point singularity in χ_{cJ} decay, the color octet contributions have the same order as the color singlet and may be the essential part for P-wave state quarkonium decay. Therefore, the dynamic mechanism will be very different for decays of glueball and quarkonium with the same J^{PC}.

(4) It is not difficult to generalize our calculation to other two-meson decay processes. The conclusion is expected to be similar. Therefore the branching ratio is small for each decay mode and there is no dominant decay channel for a pure glueball whose mass is high enough to ensure the PQCD contributions dominate.

ACKNOWLEDGMENTS

This work was supported by National Science Foundation of China (NSFC) and Grant No. LWTZ-1298 of Academia Sinica.

[1] G.S. Bali, A. Hulsebos, A. Irving, C. Michael, K. Schilling, and P.W. Stephenson, Phys. Lett. B **309**, 378 (1993).
[2] H. Chen, J. Sexton, A. Vaccarino, and D. Weingarten, in *Lattice '93*, Proceedings of the International Symposium, Dallas, Texas, edited by T. Draper *et al.* [Nucl. Phys. B, Proc. Suppl. **34**, 357 (1994)].
[3] J. Sexton, A. Vaccarino, and D. Weingarten, Phys. Rev. Lett. **75**, 4563 (1995).
[4] J. Sexton, A. Vaccarino, D. Weingarten, in *Lattice '95*, Proceedings of the International Symposium, Melbourne, Australia, edited by T. D. Kieu *et al.* [Nucl. Phys. B (Proc. Suppl.) **47**, 128 (1996)].
[5] BES collaboration, J. Bai *et al.*, Phys. Rev. Lett. **76**, 3502 (1996).
[6] T. Huang, S. Jin, D. Zhang, and K. Chao, Phys. Lett. B **380**, 189 (1996); K.T. Chao, Commun. Theor. Phys. **24**, 373 (1995).
[7] P.D. Barnes *et al.*, Phys. Lett. B **309**, 469 (1993).
[8] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).
[9] D. Robson, Nucl. Phys. **B130**, 328 (1977).
[10] V.L. Chernyak and A.R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).
[11] S.J. Brodsky and G.P. Lepage, Phys. Rev. D **22**, 2157 (1980).
[12] J.H. Kuhn, J. Kaplan, and E. Safiani, Nucl. Phys. **B157**, 125 (1979).
[13] T. Huang, Commun. Theor. Phys. **4**, 741 (1985).
[14] J. Bolz, P. Kroll, and G.A. Schuler, Phys. Lett. B **392**, 198 (1997).
[15] G.T. Bodwin, E. Braaten, and G.P. Lepage, Phys. Rev. D **46**, 1914 (1992); **51**, 1125 (1995).
[16] F.E. Close, G.R. Farrar, and Z. Li, Phys. Rev. D **55**, 5749 (1997).
[17] V.A. Shifman, Phys. Rep. **209**, 341 (1991).
[18] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. **B191**, 301 (1981).
[19] T. Huang, B.Q. Ma, and Q.X. Shen, Phys. Rev. D **49**, 1490 (1994).
[20] N. Isgur and C.H. Llewellyn Smith, Nucl. Phys. **B317**, 526 (1989).
[21] T. Huang and Q.X. Shen, Z. Phys. C **50**, 139 (1991).
[22] H.N. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992); F.G Cao, T. Huang, and C.W. Luo, Phys. Rev. D **52**, 5358 (1995).
[23] T. Huang, X.N. Wang, and X.D. Xiang, Commun. Theor. Phys. **5**, 123 (1986).
[24] S.J. Brodsky, T. Huang, and G.P. Lepage, in *Particle and Fields 2*, edited by A.Z. Capri and A.N. Kamal (Plenum, New York, 1983), p. 143; T. Huang, *Proceedings of XXth International Conference on high energy physics*, Madison, Wisconsin, 1980, edited by L. Durand and L.G. Pondrom (AIP, New York, 1981), p. 1000.
[25] X.H. Guo and T. Huang, Phys. Rev. D **43**, 2931 (1991).