Exclusive rare radiative decays of B mesons

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The exclusive rare radiative B decays are studied in the relativistic independent quark model based on the confining potential in the scalar-vector harmonic form. The relevant form factors as well as the branching ratios for the processes $B^0 \to K^{*0}\gamma$ and $B^{\pm} \to K^{*\pm}\gamma$ have been estimated in reasonable agreement with the available experimental data. The result compares well with several other model predictions. The calculation has been extended to the CKM-favored process $B_s \to \phi\gamma$ and CKM-suppressed processes $B_{u,d} \to \rho\gamma$ and $B_s \to K^*\gamma$. [S0556-2821(97)00923-5]

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I. INTRODUCTION

Theoretical interest in the rare radiative decay $B \to K^*\gamma$ as a test of the standard model (SM) has been renewed after the CLEO experiment [1] which positively identified the process in 1993 and gave the preliminary determination of the exclusive branching ratio $B(B \to K^*\gamma) = (4.0 \pm 1.7 \pm 0.8) \times 10^{-5}$. Subsequently the measurement of the inclusive photon energy and the branching ratio was also reported by the CLEO Collaboration [2] yielding $B(B \to X_s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$. The rare radiative decays of B mesons are remarkable for several reasons. The $B \to K^*\gamma$ decay arises from the quark level process $b \to s\gamma$ via penguin-type diagrams at one loop level. Hence it is not only a significant test for standard model flavor-changing neutral-current dynamics, but also sensitive to new physics appearing through virtual particles such as the top quark and W boson in the internal loop. The study of this process provides valuable information about the Cabibbo-Kobayashi-Maskawa (CKM) parameters $\mathcal{V}_{td}$, $\mathcal{V}_{ts}$, $\mathcal{V}_{tb}$ and the top quark mass. From the existing bounds on the $b \to s\gamma$ branching ratio, it is possible to place constraints on new physics such as the supersymmetry and other extensions of the standard model.

The inclusive decay $B \to X_s\gamma$ is predominantly a short-distance process and can be treated perturbatively in the spectator approximation. In contrast with the exclusive channel, these decay modes allow a less model-dependent comparison with theory, since no specific bound-state model is needed for the final state. This opens the road to a rigorous comparison with theory. The theoretical analysis of these decays is based on the effective Hamiltonian, which is obtained by integrating out the heavier degrees of freedom. The renormalization of the Wilson coefficients in the effective Hamiltonian has been calculated to leading orders [3] showing an increase of the decay amplitude approximately by a factor of 2. Some of the next-to-leading order corrections have also been calculated [4,5]. The data not only agree with the SM-based theoretical computations [6] but almost overlap with the estimation of [7]. The matrix element of the effective Hamiltonian for the inclusive rare decays has also been calculated in the heavy quark effective theory (HQET) using heavy quark expansion to leading and next-to-leading-order terms [8,9] in good agreement with data.

The decays of B mesons to exclusive final states of the type $B \to K^*(892)\gamma$ is a topic of tremendous interest and activity in recent years [10]. The theoretical analysis of this type of decay requires a long distance QCD contribution, which can hardly be determined perturbatively. It is also not straightforward to calculate the exclusive decays by the first-principle QCD application due to the complications inherent in the nonperturbative QCD. Therefore, workers resort to various phenomenological models to get some reliable predictions in this sector. In fact, there are several methods available in the literature to study the exclusive process. Some of them include the QCD sum rule [11–13], lattice QCD [14], nonrelativistic and relativistic quark models [15–17]. The heavy quark mass limit [18] has also been applied to exclusive $B \to K^*\gamma$ decay even though the s quark in the final K^* meson is certainly not heavy, contrary to the requirement of HQET. The mass of s quark is of the order of Λ parameter, which determines the scale of $1/m_Q$ corrections in HQET [9], for which substantial corrections to this limit come from the whole series in $1/m_b$. Nevertheless heavy quark expansion is applied to $B \to K^*\gamma$, since kinematically the final K^* meson is found to have a large relativistic recoil momentum of the order of $m_b/2$ and energy of the same order. So it is possible to expand the matrix element of the effective Hamiltonian both in inverse powers of b-quark mass from the initial B meson and in inverse powers of the recoil momentum of the final K^* meson yielding an expansion in powers of $1/m_b$. With this assumption Faustov and Galkin in the relativistic quark model based on the quasipotential approach in quantum field theory, calculated the exclusive rare radiative B decays to the leading and next-to-leading order terms of $1/m_b$ expansion [19,20].

The matrix element of the effective Hamiltonian for $B \to K^*\gamma$ type decays is covariantly expressed in terms of two-transition form factors. However, in such decays only one form factor $f_1(q^2)$ effectively contributes. The study of exclusive rare decay is, therefore, reduced to an extraction of the transition form factor $f_1(q^2)$. The extraction of the form factor from a model dynamics entails a large uncertainty. A survey of the existing treatments reveals sizable model dependence, which reflects the difficulty in treating the large recoil involved in these decays besides the difficulty in calculating the hadronic matrix element in between two bound-state hadrons within the scope of a constituent quark model. It is not surprising, therefore, to find the predictions in this sector dispersed over a wide range with an order of magnitude variation such as $B(B\to K^*\gamma)\simeq(2-16)\times 10^{-5}$ depending on the model used. Therefore a model, be it QCD-inspired or purely phenomenological, used to study these exclusive channels, should be one which reflects adequately the true bound-state character of the participating hadrons with the relativistic constituent quarks confined within.

We have developed a relativistic independent quark model based on a confining potential with a scalar-vector harmonic form which has been providing consistently reliable predictions for a fairly large spectrum of hadronic phenomena such as static hadronic properties [21,22], radiative [23,24], weak radiative [25], leptonic [26], weak leptonic [27], and semileptonic [28,29] decays of light as well as heavy mesons. Therefore, we intend here to study the exclusive rare radiative decays of B and B_s meson in the framework of such a model to test the applicability of the model in yet another interesting area of rare decays.

The paper is organized in the following manner. In Sec. II we describe the effective Hamiltonian and transition matrix element for the exclusive rare decays. Section III provides a brief outline of the independent quark model. We extract the transition from factor $f_1(0)$ and derive an expression for the decay width from the model dynamics in Sec. IV. Finally Sec. V embodies our results and discussion.

II. EFFECTIVE HAMILTONIAN AND TRANSITION MATRIX ELEMENT

In order to prepare the ground for the treatment of the exclusive rare radiative decays, we start with the QCD-corrected treatments of the decays at the quark level. The loop diagrams giving rise to $b\to(s,d)+\gamma$ decays have quite significant QCD corrections, as pointed out some time ago [16,17]. These perturbative QCD corrections are, in fact, incorporated in a systematic manner by integrating out the heavier degrees of freedom. In the standard model, B decays are described by the effective Hamiltonian obtained by integrating out the top quark and W boson and then using the Wilson expansion [3]. For the $b\to s\gamma$ transition [3–5]

$$\mathcal{H}_{\rm eff}(b\to s)=\frac{-4}{\sqrt{2}}\,\mathcal{V}_{tb}\mathcal{V}_{ts}^{*}\sum_{j=1}^{8}C_j(\mu)O_j(\mu), \tag{1}$$

where $\mathcal{V}_{ij}$ are the corresponding CKM parameters, $\{O_j\}$ are a complete set of renormalized dimension six operators involving light fields which govern $b\to s$ transition. They consist of two current operators $O_{1,2}$ and four strong penguin operators O_{3-6}, which determine the nonleptonic decays, the electromagnetic dipole operator O_7

$$O_7=\frac{e}{16\pi^2}\,\overline{s\sigma}_{\mu\nu}(m_bP_R+m_sP_L)bF^{\mu\nu}$$

with

$$P_{R,L}\equiv(1\pm\gamma_5)/2, \tag{2}$$

and the chromomagnetic dipole oprator O_8, which are responsible for the rare B decays $B\to X_s\gamma$ and $b\to s+g$, respectively [3]. Here e and $F^{\mu\nu}$ denote the electromagnetic coupling constant and the field strength tensor. The Wilson coefficients $C_j(\mu)$ are evaluated perturbatively at the electroweak scale $\mu\sim M_W$ and then they are evolved down to the renormalization scale $\mu\sim m_b$ by the renormalization group equations. The coefficient C_7 of the electromagnetic dipole operator O_7 has been calculated to the leading logarithmic order [3]. The next-to-leading-order corrections to the anomalous dimension matrix are also partially known [4,5]. Since the dominant contribution to the decay width $\Gamma(B\to K^*\gamma)$ comes from the magnetic moment term $C_7(\mu)O_7(\mu)$, it is necessary to evaluate the matrix element of this operator only.

In the same argument, one can start from the S-matrix element with the effective Hamiltonian for the exclusive rare decay of the type $B(P)\to K^*(k)+\gamma(q)$ to obtain the on-shell invariant matrix element in the form

$$\mathcal{M}=\frac{eG_Fm_b}{2\sqrt{2}\pi^2}C_7(m_b)\mathcal{V}_{tb}\mathcal{V}_{ts}^{*}\eta^{*\mu}(q,\delta)\sqrt{4E_BE_{K^*}}\times\langle K^*(k)|\overline{si}\sigma_{\mu\nu}q^\nu b_R|B(P)\rangle, \tag{3}$$

where q and η are the momentum and polarization of the emitted photon. We keep only the b_R component, which has a major effect on this decay. The hadronic matrix element has the covariant decomposition

$$\begin{aligned}&\sqrt{4E_BE_{K^*}}\langle K^*(k,\epsilon^*)|\overline{si}\sigma_{\mu\nu}q^\nu b_R|B(P)\rangle\\&\quad\equiv\sqrt{4E_BE_{K^*}}\langle K^*(k,\epsilon^*)|V_\mu+A_\mu|B(P)\rangle\\&\quad=i\in_{\mu\nu\rho\sigma}\epsilon^{*\nu}P^\rho k^\sigma f_1(q^2)\\&\qquad+[\epsilon^*_\mu(M_B^2-M_{K^*}^2)+(\epsilon^*\cdot q)(P+k)_\mu]f_2(q^2).\end{aligned} \tag{4}$$

Here ϵ^* is the polarization vector of the final K^* meson; $q=P-k$ denotes the four-momentum transfer; V_μ and A_μ are the vector and the axial vector part of the effective current; (M_B,E_B) and (M_{K^*},E_{K^*}) are the mass, energy of the initial and final meson, respectively.

From the angular momentum conservation, K^* in $B\to K^*\gamma$ appears in a spin-up or a spin-down state due to the spin flip ($\Delta M_s=\pm 1$). Assuming the K^* momentum to lie in the z direction, we expand the hadronic matrix element in Eq. (4) in the B-meson rest frame for the allowed spin states: $S_V=\pm 1$ of K^* and then sum over the photon polarization index δ so as to find the decay width $\Gamma(B\to K^*\gamma)$ in its generic expression

$$\Gamma(B\to K^*\gamma)=\frac{1}{(2\pi)^2}\int\frac{d\vec{k}d\vec{q}}{2M_B2E_{K^*}2E_\gamma}\times\delta^{(4)}(P-k-q)\overline{\sum_{\delta,S_V}}|\mathcal{M}|^2, \tag{5}$$

where

$$\sum_{\delta,S_V}|\mathcal{M}|^2=\frac{\alpha G_F^2 m_b^2}{\pi^3}|C_7(m_b)|^2|\mathcal{V}_{tb}\mathcal{V}_{ts}^*|^2 \times M_B^2 E_\gamma^2[|f_1(0)|^2+4|f_2(0)|^2], \qquad (6)$$

where α is the electromagnetic fine structure constant and E_γ is the energy of the emitted photon. Then one obtains

$$\Gamma(B\to K^*\gamma)=\frac{\alpha G_F^2}{8\pi^4}|\mathcal{V}_{tb}\mathcal{V}_{ts}^*|^2|C_7(m_b)|^2 \times m_b^2\overline{E_\gamma^3}[|f_1(0)|^2+4|f_2(0)|^2]. \qquad (7)$$

Here $\overline{E_\gamma}=(M_B^2-M_{K^*}^2)/2M_B$ is the photon energy. This is obtained in the argument factorization of the energy delta function and is fixed at the meson level. In view of the fact that $\mathcal{V}_{ub}$ is very small, from the unitarity relation one usually replaces the product $|\mathcal{V}_{tb}\mathcal{V}_{ts}^*|$ by $|\mathcal{V}_{cb}\mathcal{V}_{cs}^*|$ so as to write the decay width expression in the form

$$\Gamma(B\to K^*\gamma)=\frac{\alpha G_F^2 m_b^2}{64\pi^4}|\mathcal{V}_{cb}\mathcal{V}_{cs}^*|^2|C_7(m_b)|^2 \times M_B^3\left(1-\frac{M_{K^*}^2}{M_B^2}\right)^3[|f_1(0)|^2+4|f_2(0)|^2]. \qquad (8)$$

Calculating the form factors $f_1(0)$ and $f_2(0)$ from within the dynamical scheme of a suitable model one can estimate the decay width as well as the branching ratio using Eq. (8).

III. MODEL DESCRIPTION

In the present model a meson, in general, is pictured as a color-singlet assembly of a quark and an antiquark independently confined within the meson by an effective average potential [21–29]:

$$U(r)=\frac{1}{2}(1+\gamma^0)(ar^2+V_0), \quad a>0. \qquad (9)$$

The potential taken in this form represents phenomenologically the confining interaction expected to be generated by the nonperturbative multigluon mechanism. The quark-gluon interaction at short distance originating from one-gluon exchange and the quark-pion-like interaction required in the non-strange sector to preserve chiral symmetry are presumed to be residual interactions compared to the dominant confining interaction. Although the residual interactions treated perturbatively in the model are crucial in generating mass splitting [13,18] in the hadron spectroscopy, their role in hadronic decay processes are considered less significant. Therefore, to the first approximation, it is believed that the zeroth-order quark dynamics inside the meson core, generated by the confining part of the interaction phenomenologically represented by the potential $U(r)$ in Eq. (9), can provide an adequate description for the exclusive rare radiative decay of the type $B\to K^*\gamma$. In this picture the independent quark Lagrangian density in the zeroth order is given by

$$\mathcal{L}_q^0(x)=\overline{\psi_q}(x)\left(\frac{i}{2}\gamma^\mu\overleftrightarrow{\partial}_\mu-m_q-U(r)\right)\psi_q(x). \qquad (10)$$

The ensuing Dirac equation with $E_q'=(E_q-V_0/2)$, $m_q'=(m_q+V_0/2)$, $\lambda_q=(E_q'+m_q')$, and $r_{0q}=(a\lambda_q)^{-1/4}$ admits a static solution of positive and negative energy in zeroth order, which for the ground-state meson can be obtained in the form

$$\phi_{q_\lambda}^{(+)}(\vec{r})=\frac{1}{\sqrt{4\pi}}\begin{pmatrix} ig_q(r)/r \\ \vec{\sigma}.\hat{r}f_q(r)/r\end{pmatrix}\chi_\lambda,$$

$$\phi_{q_\lambda}^{(-)}(\vec{r})=\frac{1}{\sqrt{4\pi}}\begin{pmatrix} i(\vec{\sigma}.\hat{r})f_q(r)/r \\ g_q(r)/r\end{pmatrix}\tilde{\chi}_\lambda. \qquad (11)$$

Here the two component spinors χ_λ and $\tilde{\chi}_\lambda$ stand for

$$\chi_\uparrow=\begin{pmatrix}1\\0\end{pmatrix},\ \chi_\downarrow=\begin{pmatrix}0\\1\end{pmatrix} \text{ and } \tilde{\chi}_\uparrow=\begin{pmatrix}0\\-i\end{pmatrix},\ \tilde{\chi}_\downarrow=\begin{pmatrix}i\\0\end{pmatrix},$$

respectively. The reduced radial parts in the upper and lower component solutions corresponding to the quark flavor q are

$$g_q(r)=\mathcal{N}_q\left(\frac{r}{r_{0q}}\right)\exp(-r^2/2r_{0q}^2),$$

$$f_q(r)=-\frac{\mathcal{N}_q}{\lambda_q r_{0q}}\left(\frac{r}{r_{0q}}\right)^2\exp(-r^2/2r_{0q}^2), \qquad (12)$$

where the normalization factor $\mathcal{N}_q$ is given by

$$\mathcal{N}_q^2=\frac{8\lambda_q}{\sqrt{\pi}r_{0q}}\frac{1}{(3E_q'+m_q')}. \qquad (13)$$

The quark binding energy E_q of zeroth order in the meson ground state is derivable from the bound-state condition

$$\sqrt{\frac{\lambda_q}{a}}(E_q'-m_q')=3. \qquad (14)$$

From the quark-antiquark eigenmodes in Eq. (11) obtainable from the model, it is possible to derive the effective momentum distribution amplitude for constructing the meson ground state with a definite momentum $\vec{P}$ and spin projection S_V as

$$|M(\vec{P},S_V)\rangle=\frac{1}{\sqrt{N(\vec{P})}}\sum_{\lambda_1\lambda_2\in S_V}\zeta_{q_1q_2}^M(\lambda_1,\lambda_2)\int d\vec{p}_1 d\vec{p}_2\,\delta^{(3)}(\vec{p}_1+\vec{p}_2-\vec{P})G_M(\vec{p}_1,\vec{p}_2)\hat{b}_{q_1}^\dagger(\vec{p}_1,\lambda_1)\hat{\tilde{b}}_{q_2}^\dagger(\vec{p}_2,\lambda_2)|0\rangle. \qquad (15)$$

Here $\hat{b}^{\dagger}_{q_1}(\vec{p}_1,\lambda_1)$ and $\hat{\tilde{b}}^{\dagger}_{q_2}(\vec{p}_2,\lambda_2)$ are, respectively, the quark and antiquark creation operator. $\zeta^{M}_{q_1 q_2}(\lambda_1,\lambda_2)$ stands for the appropriate SU(6) spin flavor coefficient for the meson $M(q_1,\overline{q_2})$. $N(\vec{P})$ represents the overall normalization factor, which can be expressed in an integral form as

$$N(\vec{P})=\int d\vec{p}_1 |G_M(\vec{p}_1,\vec{P}-\vec{p}_1)|^2. \tag{16}$$

This is obtainable from the meson-state normalization considered here in the form

$$\langle M(\vec{P})|M(\vec{P}')\rangle=\delta^{(3)}(\vec{P}-\vec{P}'). \tag{17}$$

Finally $G_M(\vec{p}_1,\vec{p}_2)$ provides the effective momentum distribution amplitude for the quark and antiquark inside the meson. In the independent particle picture of the present model, $G_M(\vec{p}_1,\vec{p}_2)$ can be expressed as the geometric mean of the momentum probability amplitudes of the constituent quark-antiquark [24,26–30] as

$$G_M(\vec{p}_1,\vec{p}_2)=\sqrt{g_{q_1}(\vec{p}_1)\tilde{g}_{q_2}(\vec{p}_2)}. \tag{18}$$

In fact, $g_{q_1}(\vec{p}_1)$ can be obtained by a suitable momentum-space projection of the orbital $\phi^{(+)}_{q_\lambda}(\vec{r})$ in Eq. (11) corresponding to the lowest eigenmode. If $g_{q_1}(\vec{p}_1;\lambda_1,\lambda_1')$ is the amplitude of the bound quark q_1 in its lowest eigenmode to be found in a state of definite momentum $\vec{p}_1$ and spin projection λ_1', then

$$g_{q_1}(\vec{p}_1;\lambda_1,\lambda_1')=\frac{u^{\dagger}_{q_1}(\vec{p}_1,\lambda_1')}{\sqrt{2E_{q_1}}}\int d\vec{r}\,\phi^{(+)}_{q_1\lambda_1}(\vec{r})\exp(-i\vec{p}_1.\vec{r}), \tag{19}$$

where $E_{p_1}=\sqrt{(\vec{p}_1^{\,2}+m_{q_1}^2)}$ and $u_{q_1}(\vec{p}_1,\lambda_1')$ is the usual free Dirac spinor. Using free Dirac spinor normalization and taking $\alpha_q=1/2r_{0q}^2$, $g_q(\vec{p};\lambda,\lambda_1')$ is reduced to

$$g_{q_1}(\vec{p}_1;\lambda_1,\lambda_1')=g_{q_1}(\vec{p}_1)\,\delta_{\lambda_1\lambda_1'} \tag{20}$$

with

$$g_{q_1}(\vec{p}_1)=\frac{i\pi\mathcal{N}_{q_1}}{2\alpha_{q_1}\lambda_{q_1}}\sqrt{\frac{(E_{p_1}+m_{q_1})}{E_{p_1}}}\,(E_{p_1}+E_{q_1})\times\exp\left(-\frac{\vec{p}_1^{\,2}}{4\alpha_{q_1}}\right). \tag{21}$$

Thus $g_{q_1}(\vec{p}_1)$ essentially provides the momentum probability amplitude for a quark q_1 in its eigenmode $\phi^{(+)}_{q\lambda}(\vec{r})$ to have a definite momentum $\vec{p}_1$ inside the meson. In the similar way one can find the momentum probability amplitude $\tilde{g}_{q_2}(\vec{p}_2)$ for the antiquark $\overline{q_2}$ in its eigenmode $\phi^{(-)}_{q\lambda}(\vec{r})$ as

$$\tilde{g}_{q_2}(\vec{p}_2)=\frac{-i\pi\mathcal{N}_{q_2}}{2\alpha_{q_2}\lambda_{q_2}}\sqrt{\frac{(E_{p_2}+m_{q_2})}{E_{p_2}}}\,(E_{p_2}+E_{q_2})\times\exp\left(-\frac{\vec{p}_2^{\,2}}{4\alpha_{q_2}}\right). \tag{22}$$

Thus using the effective momentum profile function $G_M(\vec{p}_1,\vec{p}_2)$ constructed suitably from model dynamics through Eqs. (21), (22), and (18), one can represent the meson in definite momentum state $\vec{P}$ and spin projection S_V as the appropriate momentum-wave packet in Eq. (15). With this phenomenological picture showing detail dynamics of the constituent particles inside the meson-bound state, we are in a convenient position to calculate the hadronic matrix element for the exclusive rare decays of the type $B\to K^*\gamma$ and hence the corresponding form factors contributing to the process.

IV. MODEL CALCULATION OF THE FORM FACTOR

As described in Sec. II, we expect the dominant mechanism for the $B\to K^*\gamma$ transition to be the quark subprocess $b\to s\gamma$, determined by the electromagnetic dipole operator O_7. Such a decay is known as the ''spectator decay.'' The decays, governed by seven other renormalized dimension six operators ($O_{1-6,8}$) of the effective Hamiltonian, include nonleptonic decays and $b\to s\gamma$ decay accompanied by the gluonic exchange with the spectator quark. These decays termed as the ''nonspectator decays'' are not considered in the QCD-corrected quark level calculations [15–17]. These are, in fact, very hard to calculate in the quark model. Therefore, we consider here the spectator decay only where we assume that the spectator quark has no role to play in the decay except in binding into the meson.

While expanding the hadronic matrix element of Eq. (4) for such a spectactor decay ($B\to K^*\gamma$), we have taken $P\equiv(M_B,0,0,0)$ and $k\equiv(E_{K^*},0,0,|\vec{k}|)$. As a result, the hadronic matrix element is considered to be spacelike with the index μ having values $\mu=1,2$ only. Then it is trivial to find the form factors in terms of the matrix elements belonging to specific spin states. For $\mu=1$, the form factors are found to be

$$f_1(0)=\sqrt{\frac{E_{K^*}}{M_B}}\frac{\sqrt{2}}{E_\gamma}\left[\langle K^*(k,\epsilon^*)|(V_1+A_1)|B(0)\rangle_+ + \langle K^*(k,\epsilon^*)|(V_1+A_1)|B(0)\rangle_-\right],$$

$$2f_2(0)=\sqrt{\frac{E_{K^*}}{M_B}}\frac{\sqrt{2}}{E_\gamma}\left[\langle K^*(k,\epsilon^*)|(V_1+A_1)|B(0)\rangle_+ - \langle K^*(k,\epsilon^*)|(V_1+A_1)|B(0)\rangle_-\right]. \tag{23}$$

Next it is required to calculate, from the model, the matrix elements for spin states ($S_V=\pm 1$) to obtain appropriate model expressions for the form factors. For that we take the momentum-wave packets suitably constructed in this model in the form of Eq. (15) for the initial and final mesons and calculate the hadronic matrix element for specific K^* spin states in the B-meson rest frame to find

$$\langle K^*(k,\epsilon^*)|(V_1+A_1)|B(0)\rangle_\pm=\pm\frac{E_\gamma}{2\sqrt{N(0)N(\vec{k})}}\int\frac{d\vec{p}_b G_B(\vec{p}_b,-\vec{p}_b)G_{K^*}(\vec{p}_b+\vec{k},-\vec{p}_b)}{\sqrt{4E_{p_b}E_{p_b+k}}}\langle S_{K^*}|j_1(0)|S_B\rangle_\pm, \tag{24}$$

where $E_{p_b}=\sqrt{\vec{p}_b^{\,2}+m_b^2}$ and $E_{p_b+k}=\sqrt{(\vec{p}_b+\vec{k})^2+m_s^2}$ stand for the energy of the nonspectator quark of B and K^*, respectively; $\langle S_{K^*}|j_1(0)|S_B\rangle_\pm$ symbolically represents the spin matrix elements given by

$$\langle S_{K^*}|j_1(0)|S_B\rangle_+=\frac{1}{\sqrt{2}}[\overline{u_s}(\vec{p}_b+\vec{k},\uparrow)\tilde{\sigma}_+u_b(\vec{p}_b,\downarrow)],$$

$$\langle S_{K^*}|j_1(0)|S_B\rangle_-=\frac{1}{\sqrt{2}}[\overline{u_s}(\vec{p}_b+\vec{k},\downarrow)\tilde{\sigma}_+u_b(\vec{p}_b,\uparrow)], \tag{25}$$

where

$$\tilde{\sigma}_+\equiv\begin{pmatrix}\sigma_1+i\sigma_2 & \sigma_1+i\sigma_2\\ \sigma_1+i\sigma_2 & \sigma_1+i\sigma_2\end{pmatrix}.$$

Now a straightforward application of spin algebra to the spin matrix elements of Eq. (25) involving free Dirac spinors $(\overline{u_s},u_b)$ yields

$$\langle S_{K^*}|j_1(0)|S_B\rangle_+=\sqrt{2z}\left[1+\frac{E_\gamma}{E_{p_b+k}+m_s}+\frac{|\vec{p}_b|^2}{3z}\right],$$

$$\langle S_{K^*}|j_1(0)|S_B\rangle_-=0, \tag{26}$$

with $z=(E_{p_b}+m_b)(E_{p_b+k}+m_s)$.

Using Eqs. (24) and (26), the model expression for transition form factors of Eq. (23) can be found in the form

$$f_1(0)=\sqrt{\frac{E_{K^*}}{M_B}}\frac{1}{\sqrt{N(0)N(\vec{k})}}\times\int\frac{d\vec{p}_b G_B(\vec{p}_b,-\vec{p}_b)G_{K^*}(\vec{p}_b+\vec{k},-\vec{p}_b)}{\sqrt{4E_{p_b}E_{p_b+k}}}Q(\vec{p}_b,\vec{k}) \tag{27}$$

$$f_2(0)=\frac{1}{2}f_1(0) \tag{28}$$

with

$$Q(\vec{p}_b,\vec{k})=\sqrt{z}\left[1+\frac{E_\gamma}{E_{p_b+k}+m_s}+\frac{|\vec{p}_b|^2}{3z}\right]. \tag{29}$$

One can proceed as well with $\mu=2$ in Eq. (4) and obtain an identical expression for the form factors $f_1(0)$ and $f_2(0)$. We must point out here that the invariant matrix element in this model is extracted out of the S-matrix element realized in the standard form with the energy momentum conservation through the appropriate four-momentum delta function at the mesonic level. But such a realization at the composite level starting from a picture at the constituent level has never been so straightforward. This is due to the fact that although three-momentum conservation is automatically guaranteed at the mesonic level through the appropriate delta function, it is not so transparent in case of energy conservation. The energy conservation at the mesonic level can, however, be realized by extracting out the energy delta function $\delta(E_{p_b}-E_{p_b+k}-E_\gamma)$ from within the quark level integral in the form $\delta(M_B-E_{K^*}-E_\gamma)$ with an approximation that $(E_{p_b}+E_{p_d})$ and $(E_{p_b+k}+E_{p_d})$ in the δ-function argument be equated, in an integrated sense, to the parent meson mass M_B and daughter meson energy E_{K^*}, respectively. Such an approximation known as the ''loose binding approximation'' is used here to describe the particle process manifested at the composite level through a constituent level dynamics. Next, when we integrate the amplitude squared $\Sigma_{\delta,S_V}|\mathcal{M}|^2$ over all

TABLE I. The quark mass m_q, quark binding energy E_q, λ_q, and α_q for the potential parameter $(a,V_0)\equiv(0.017166\ \mathrm{GeV}^3;-0.1375\ \mathrm{GeV})$.

Parameter Set	Quark q	m_q	E_q	λ_q	α_q
	u	0.07875	0.47125	0.55	0.04858
Set.1	d	0.07875	0.47125	0.55	0.04858
	s	0.31575	0.59100	0.90675	0.06238
	b	4.77659	4.76633	9.54292	0.20237
	u	0.010	0.45129	0.46129	0.04449
Set.2	d	0.010	0.45129	0.46129	0.04449
	s	0.240	0.54588	0.78588	0.05807
	b	4.77759	4.76732	9.54491	0.20239

the final particle momenta to arrive at the model expression for the decay width, a phase space factor of E_{K*}/M_B is obtained from the argument factorization of the energy delta function $\delta(M_B-E_{K*}-E_\gamma)$. Such a spurious factor is cancelled out by the counter factor M_B/E_{K*} already sitting in the expression yielding to the familiar expression for $\Gamma(B\to K^*\gamma)$ in Eq. (8) through Eqs. (27) and (28). There might be some amount of uncertainty crept into the calculation in the evolution of the spurious factor E_{K*}/M_B in the expression for $\Gamma(B\to K^*\gamma)$ and its subsequent cancellation. In order to reduce such possible uncertainty, we prefer here to push back exactly the same factor E_{K*}/M_B available in $|f_1(0)|^2$ and $|f_2(0)|^2$ into the corresponding quark level integrals under the same loose binding approximation with which it was brought out from the energy argument of the delta function. Such a procedure has already been adopted in our earlier work on radiative transitions of light and heavy mesons in Ref. [24]. Hence $\sqrt{E_{K*}/M_B}=\sqrt{(E_{p_b+k}+E_{p_d})/(E_{p_b}+E_{p_d})}$, when brought into the quark level integrals in Eqs. (27) and (28), the expression for the transition form factor $f_1(0)$ got modified to

$$f_1(0)=\frac{1}{\sqrt{N(0)N(\vec{k})}}\int\frac{d\vec{p}_b G_B(\vec{p}_b,-\vec{p}_b)G_{K*}(\vec{p}_b+\vec{k},-\vec{p}_b)Q(\vec{p}_b,\vec{k})\sqrt{(E_{p_b+k}+E_{p_d}}}{\sqrt{4E_{p_b}E_{p_b+k}(E_{p_b}+E_{p_d})}}. \tag{30}$$

We also prefer here to take the three-momentum-squared $(|\vec{k}|^2=|\vec{q}|^2=E_\gamma^2)$ of the final K^* meson to be fixed at the quark level with $\overline{E}_\gamma=(m_b^2-m_s^2)/2m_b$ wherever it appears inside the quark level integrals for $f_1(0)$ and $f_2(0)$. Since the most dominant mechanism for $B\to K^*\gamma$ is assumed to be obtained from the quark subprocess $b\to s\gamma$, it is a good approximation to say that the s quark is recoiled with the momentum $\vec{k}$. This lends credence to such a preference for the value of $|\vec{k}|=E_\gamma$ to be fixed at the quark level. However, the value of E_γ appearing elsewhere in the overall meson normalization factor $N(\vec{k})$ belonging to the K^* meson is fixed at the meson level with $E_\gamma=(M_B^2-M_{K*}^2)/2M_B$. In view of these considerations along with Eq. (28), the expression for the decay width $\Gamma(B\to K^*\gamma)$ is obtained finally in terms of the single form factor $f_1(0)$ in the familiar form

$$\Gamma(B\to K^*\gamma)=\frac{\alpha G_F^2 m_b^2}{32\pi^4}|\mathcal{V}_{cb}\mathcal{V}_{cs}^*|^2|C_7(m_b)|^2 \times M_B^3\left(1-\frac{M_{K*}^2}{M_B^2}\right)^3|f_1(0)|^2. \tag{31}$$

V. RESULTS AND DISCUSSION

In this section we evaluate the form factor and branching ratio for different exclusive rare radiative decays such as $B^0\to K^{*0}\gamma$, $B^\pm\to K^{\pm *}\gamma$, $B\to\rho\gamma$, $B_s\to\phi\gamma$, and $B_s\to K^*\gamma$ using Eqs. (30) and (31). The calculation primarily involves the potential parameters (a,V_0) and quark masses m_q as the input parameters. From earlier applications of the present model in the meson as well as the baryon sector [21–29], the potential parameters are taken as

$$(a;V_0)\equiv(0.017166\ \mathrm{GeV}^3;\ -0.1375\ \mathrm{GeV}). \tag{32}$$

With this choice of potential parameters and two separate sets of quark masses, the present model has generated the ground-state hyperfine mass splittings of the light $(\rho,\pi;K^*,K)$ [22] as well as heavy $(D^*,D;B^*,B)$ mesons [27] in good agreement with the experiment by appropriately taking into account the corrections due to the one-gluon exchange at short distance, the quark-pion-like interaction and the spurious center of mass motion. The quark masses as per Refs. [21–29] referred to as the parameter set.1 determine the quark binding energy E_q, which, in fact, plays the role of

TABLE II. Predictions for the rare radiative decay form factor $f_1(0)$ in comparison with relativistic quark model [20], light cone sum rule [12], and hybrid sum rule [13] results.

Physical process	Our result Set.1 (Set.2)	Ref. [20]	Ref. [12]	Ref. [13]
$B^{\pm}\to K^{\pm *}\gamma$	0.406 (0.384)	-	-	-
$B^0\to K^{0*}\gamma$	0.407 (0.384)	0.32 ± 0.03	0.32 ± 0.05	$0.308\pm 0.013\pm 0.036\pm 0.006$
$B\to\rho\gamma$	0.252 (0.229)	0.26 ± 0.03	0.24 ± 0.04	$0.27\pm 0.011\pm 0.032$
$B_s\to\phi\gamma$	0.345 (0.315)	0.27 ± 0.03	0.29 ± 0.05	-
$B_s\to K^*\gamma$	0.212 (0.185)	0.23 ± 0.02	0.20 ± 0.04	-

the effective constituent quark mass. The quark masses as per Refs. [23, 24, 26, 27] referred to as the parameter set.2 are those in the current quark mass limit. Both these sets of quark masses, their corresponding binding energies and other model quantities, are given in Table I. Since theoretical uncertainty in the perturbative approach of determining the meson masses in the model [22,27] cannot be overlooked, we would prefer to use the observed meson masses for the participating mesons, whenever they appear in the calculation instead of using the model masses [22,27]. The relevant CKM parameters required for all the processes considered here are taken from Ref. [31] as

$$\mathcal{V}_{bc}=0.041;\quad \mathcal{V}_{cs}=1.01;\quad \mathcal{V}_{cd}=0.224. \tag{33}$$

We have taken $\tau_B\simeq 1.56\times 10^{-12}$ s and $\tau_{B_s}\simeq 1.61\times 10^{-12}$ s, which stand for the mean life of the decaying mesons B and B_s, respectively. The renormalized Wilson coefficient $C_7(m_b)$ used in the estimation of branching ratios is taken to be $|C_7(m_b)|=0.311477$ [32].

With the two sets of the input parameters already fixed from hadron spectroscopy, we perform here almost a parameter-free calculation. We first evaluate numerically the integral, defining the form factor $f_1(0)$ in Eq. (30). Our results for the form factor values given in Table II compare well with the recent calculations within the framework of the relativistic quark model [20], light cone QCD sum rule [12], and hybrid sum rule [13]. The present prediction for $B\to K^*\gamma$, in particular, shows $f_1(0)^{B\to K^*}=0.40(0.38)$ for parameter set.1 (set.2). This is found to be slightly on the higher side compared to the predictions of [12,13,20] and those of several other model calculations [15–17] yielding $f_1(0)^{B\to K^*}\simeq(0.25-0.31)$. In all other decay modes studied here, our results agree within errors [12,13,20]. It is observed that the large recoil effect involved in these processes, when considered appropriately through the factor $G_{K^*}(\vec{p}_b+\vec{k},-\vec{p}_b)$, provides a dominant exponential enhancement leading to these values for the form factors in Table II.

The uncertainty in the model predictions tends to be reduced in the form factor ratios. Our predictions on the ratios with respect to parameter set.1 (set.2) are as follows:

$$\frac{f_1(0)^{B\to\rho}}{f_1(0)^{B\to K^*}}=0.62(0.60),$$

$$\frac{f_1(0)^{B_s\to K^*}}{f_1(0)^{B_s\to\phi}}=0.61(0.59),$$

$$\frac{f_1(0)^{B_s\to K^*}}{f_1(0)^{B\to K^*}}=0.52(0.48). \tag{34}$$

These are comparable to the corresponding prediction of Ref. [12], which yielded (0.76 ± 0.06), (0.66 ± 0.09), and (0.60 ± 0.12), respectively.

With the calculated values of the form factors, the branching ratios for different channels are estimated using the expression in Eq. (31) as well as the mean life values of the appropriate decaying mesons. Our results for the branching ratio values are displayed in Table III. The branching ratios $B(B^0\to K^{0*}\gamma)$ and $B(B\pm\to K^{\pm *}\gamma)$ are found to be in reasonable agreement with the available data as well as with several theoretical predictions including those of [12–20]. Since there are no data as yet available for the branching ratios in case of the decays $B\to\rho\gamma$, $B_s\to\phi\gamma$, and $B_s\to K^*\gamma$, the model predictions need to be compared with other theoretical predictions available in the literature. In

TABLE III. Predictions for the branching ratio in comparison with the available data.

Branching ratio	Our result Set.1	Our result Set.2	Experiment [1,31]
$B^{\pm}\to K^{\pm *}\gamma$	6.65×10^{-5}	5.93×10^{-5}	$(5.7\pm 3.1\pm 1.1)\times 10^{-5}$
$B^0\to K^{0*}\gamma$	6.38×10^{-5}	5.69×10^{-5}	$(4.0\pm 1.7\pm 0.8)\times 10^{-5}$
$B\to\rho\gamma$	1.24×10^{-6}	1.02×10^{-6}	-
$B_s\to\phi\gamma$	4.87×10^{-5}	4.07×10^{-5}	-
$B_s\to K^*\gamma$	9.28×10^{-7}	7.09×10^{-7}	-

fact, there are only a few theoretical attempts made so far in this sector. Recently Singer predicted that in the relativistic quark model [33]

$$B(B_s \to \phi\gamma) \simeq (3.5 \pm 1.5) \times 10^{-5},$$

$$B(B_s \to K^*\gamma) \simeq 0.04 \;\; B(B_s \to \phi\gamma). \tag{35}$$

The heavy quark approach [18,34], using a monopole parametrization with $w_0 \simeq 1.1$, predicted

$$B(B_s \to \phi\gamma) = 3.56 \times 10^{-5}. \tag{36}$$

Our predicted values in Table III stand in good comparison with those values [18,33,34]. Finally the branching ratio $B(B \to \rho\gamma)$ predicted in this model is also comparable with that of Ref. [35].

In view of the consistency of our predictions with the large number of theoretical predictions as well as the experimental data, the present model provides a suitable alternative scheme to analyze the exclusive rare radiative B and B_s decays. Our predictions for the decays $B \to \rho\gamma$, $B_s \to \phi\gamma$, and $B_s \to K^*\gamma$ would certainly guide future experiments for a set of precise data in this sector.

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