Spin-orbit inversion of excited heavy quark mesons

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The mesonic heavy quark spin multiplets with $s_\ell^{\pi_\ell}=\frac{1}{2}^+$ and $\frac{3}{2}^+$ are expected to be the lowest-lying excitations above the pseudoscalar and vector ground states with $s_\ell^{\pi_\ell}=\frac{1}{2}^-$. I show that for charm and bottom these multiplets are probably inverted, with the 2^+ and 1^+ states with $s_\ell^{\pi_\ell}=\frac{3}{2}^+$ about 150 MeV *below* the 1^+ and 0^+ states with $s_\ell^{\pi_\ell}=\frac{1}{2}^+$. If verified, such an inversion would both support the expectation that confinement has no dynamical spin-dependence and indicate that heavy- and light-quark systems may be characterized by the same effective low-energy degrees of freedom. As an important by-product, this work establishes the dynamics of the strange quark as a critical link between heavy- and light-quark hadrons, justifying efforts toward a much more complete experimental and theoretical understanding of strange mesons and baryons and of strange quarkonia. [S0556-2821(98)00505-0]

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I. INTRODUCTION

Heavy quark symmetry [1,2] places very strong constraints on the spectroscopy [3] (including masses and decay widths) of hadrons containing a single heavy quark Q. In particular, in the limit $m_Q\to\infty$, the spectrum of such hadrons is required to consist of degenerate spin doublets with $J^P=(s_\ell\pm\frac{1}{2})^{\pi_\ell}$ built on "brown muck" states with light quantum numbers $s_\ell^{\pi_\ell}$ [4]. In this limit it is further required that the two states of an $s_\ell^{\pi_\ell}$ spin multiplet have the same total strong interaction width, with the relative strengths of their four couplings to any $s'^{\pi'_\ell}_\ell$ spin multiplet also determined by Clebsch-Gordan coefficients. Heavy quark symmetry also dictates how these limiting relations are broken by matrix elements of operators in a $1/m_Q$ expansion.

One of the things that heavy quark symmetry cannot do is predict the spectrum of the "brown muck," *i.e.*, the positions of the $s_\ell^{\pi_\ell}$ multiplets, since these are determined by the dynamics of strong QCD. In this case, however, the simplicity of these systems provides us with a powerful probe of this poorly understood dynamics: these hadrons are the closest analog we have in strongly interacting systems to the hydrogenic problem in QED. (For example, in the constituent quark model, the dynamics of mesons with a single heavy quark is that of a single constituent spin-$\frac{1}{2}$ (anti-)particle orbiting the origin.) As Bjorken has suggested [5], such systems offer unique opportunities to study the "brown muck" one chunk at a time.

II. A MODEL FOR THE LOW-LYING EXCITATIONS

In the constituent quark model, the "brown muck" of a meson containing a heavy quark Q is just a single constituent antiquark $\bar{q}$ interacting with Q *via* strong chromoelectric and chromomagnetic fields. This picture is also obtained in the large N_c limit of QCD. In the real world, this valence quark picture is modified by "unquenching," *i.e.*, by the effects of light quark-antiquark loops.

If both Q and $\bar{q}$ were heavy, *i.e.*, if both m_Q and m_q were large compared to Λ_{QCD}, then this system could be rigorously described in QCD by a nonrelativistic Schrödinger equation with a generic two-body effective potential of the form

$$V=V_c+V_{ss}+V_{so}, \tag{1}$$

where the three potentials are the central (c), spin-spin (ss), and spin-orbit (so) potentials, respectively. Using a double heavy quark expansion, it follows that (to leading order in $1/m_Q^2$, $1/m_Q m_q$, and $1/m_q^2$) these potentials have the most general forms

$$V_c(\vec{r})=V_{c0}(r), \tag{2}$$

$$V_{ss}(\vec{r})=\frac{1}{m_Q m_q}\left\{V_{hf}(r)\vec{S}_Q\cdot\vec{S}_{\bar{q}}+V_{ten}(r)\left(\frac{3\vec{S}_Q\cdot\vec{r}\vec{S}_{\bar{q}}\cdot\vec{r}}{r^2}-\vec{S}_Q\cdot\vec{S}_{\bar{q}}\right)\right\}, \tag{3}$$

and

$$V_{so}(\vec{r})=V_1(r)\vec{L}\cdot\left[\frac{\vec{S}_Q}{m_Q^2}+\frac{\vec{S}_{\bar{q}}}{m_q^2}\right]+\frac{V_2(r)}{m_Q m_q}\vec{L}\cdot(\vec{S}_Q+\vec{S}_{\bar{q}}), \tag{4}$$

where V_{c0}, V_{hf}, V_{ten}, V_1, and V_2 represent the leading *mass-independent* pieces of what are, respectively, the low-energy static interquark potential, the hyperfine interaction, the tensor interaction, the one-body-type spin orbit interaction and the two-body-type spin orbit interaction. In quark models in which the interquark forces arise from flux tube formation plus one-gluon exchange, V_{c0} contains the static Coulomb and linear potentials, V_{hf}, V_{ten}, and V_2 arise from the Breit-Fermi reduction of one gluon exchange, and V_1 contains both color-magnetic effects from one gluon exchange and the effects of Thomas precession in V_{c0}. In a more general case (*e.g.*, if quark loops are taken into account), the *interpretation* of these potentials changes, but this generic expansion does not: it relies only on the validity of

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an adiabatic approximation to the interquark potentials which freezes out all but the assumed heavy Q and $\bar{q}$ degrees of freedom [6].

The standard assumption of the nonrelativistic valence quark model is that this same Schrödinger equation, which *would* be valid for the low-lying states of sufficiently heavy quarks Q and $\bar{q}$, may be extrapolated to constituent quark masses of the order of Λ_{QCD}. I will adopt this model for the following discussion, noting that it meets the minimum requirements of respecting the constraints of heavy quark symmetry for $m_Q \gg m_q$ as well as the constraints of light quark flavor symmetry when $m_Q = m_q$.

III. CALCULATIONS

A. The mass matrix

We are primarily interested here in the low-lying positive parity excitations of a heavy meson system, which we assume may be described in a first approximation (*e.g.* as part of a $1/N_c$ expansion) by the $s_\ell^{\pi_\ell} = \frac{3}{2}^+$ and $\frac{1}{2}^+$ multiplets associated with an $L=1$ excitation of the $Q\bar{q}$ relative coordinate in the nonrelativistic valence quark approximation. We therefore introduce the excited state basis [3]

$$|^{3/2}E_2\rangle = |^3P_2\rangle \tag{5}$$

$$|^{3/2}E_1\rangle = \sqrt{2/3}|^1P_1\rangle + \sqrt{1/3}|^3P_1\rangle \tag{6}$$

$$|^{1/2}E_1\rangle = \sqrt{1/3}|^1P_1\rangle - \sqrt{2/3}|^3P_1\rangle \tag{7}$$

$$|^{1/2}E_0\rangle = |^3P_0\rangle \tag{8}$$

where the notation is $|^{s_\ell}E_J\rangle$ and $|^{2S+1}L_J\rangle$ with $\vec{S} = \vec{S}_Q + \vec{S}_{\bar{q}}$ the total quark spin and $\vec{L}$ the $Q\bar{q}$ relative angular momentum. In the $|^{s_\ell}E_J\rangle$ basis, the matrix elements of $V_{ss}+V_{so}$ are

$$\delta m_2 = \frac{5h-2t+20o_2}{20m_Q m_q} + \frac{o_1}{2}\left(\frac{1}{m_Q^2}+\frac{1}{m_q^2}\right) \tag{9}$$

$$\delta m_1 = \begin{bmatrix} \frac{-5h+2t-4o_2}{12m_Q m_q} + \frac{o_1}{6}\left(\frac{-5}{m_Q^2}+\frac{3}{m_q^2}\right) & -\frac{\sqrt{2}(h+t/2-o_2)}{3m_Q m_q} + \frac{\sqrt{2}o_1}{3m_Q^2} \\ -\frac{\sqrt{2}(h+t/2-o_2)}{3m_Q m_q} + \frac{\sqrt{2}o_1}{3m_Q^2} & \frac{-h+4t-8o_2}{12m_Q m_q} + \frac{o_1}{3}\left(\frac{1}{m_Q^2}-\frac{3}{m_q^2}\right) \end{bmatrix} \tag{10}$$

$$\delta m_0 = \frac{h-4t-8o_2}{4m_Q m_q} - o_1\left(\frac{1}{m_Q^2}+\frac{1}{m_q^2}\right) \tag{11}$$

in an obvious notation where h, t, o_1 and o_2 are the expectation values of $V_{hf}(r)$, $V_{ten}(r)$ and the spin-orbit potentials $V_1(r)$ and $V_2(r)$, respectively, as defined by Eqs. (3) and (4).

In the heavy quark limit, the expectation values h, t, o_1 and o_2 must be *independent of* m_Q. In our quark model, this constraint is realized since the internal wave function of the $Q\bar{q}$ system becomes independent of m_Q as $m_Q \to \infty$, and the potentials V_i themselves, as defined in Eqs. (3) and (4), are mass-independent. In the full heavy quark limit, the mass matrices of Eqs. (9)–(11) are diagonal with

$$m_{3/2} \equiv m(^{3/2}E_2) = m(^{3/2}E_1) = m_{si_0} + \frac{o_1}{2m_q^2} \tag{12}$$

and

$$m_{1/2} \equiv m(^{1/2}E_1) = m(^{1/2}E_0) = m_{si_0} - \frac{o_1}{m_q^2}, \tag{13}$$

as required [3]. Here m_{si_0} is the expectation value, common to all four states, of the spin-independent parts of the Hamiltonian as $m_Q \to \infty$. The main new result of this paper will be to show that o_1 is probably large and *negative*.

That spin-orbit multiplets of $Q\bar{d}$ systems might be inverted was suggested long ago by Schnitzer [7] in the context of the nonrelativistic quark model. In this work we incorporate much of the same physics in the rigorous framework of heavy quark symmetry, update the determination of unknown hadronic matrix elements, and consider the effects of $q\bar{q}$ loops on the extraction of these matrix elements. While differing from Ref. [7] in many details, our qualitative conclusions are remarkably similar.

B. The extension to light mesons

Our predictions for heavy meson systems are based on an analysis of the observed P-wave $Q\bar{q}$ states with $Q=s$ and u. Since the expectation values of h, t, o_1 and o_2 are independent of m_Q only as $m_Q \to \infty$, one might expect to lose any connection between these matrix elements (and in particular o_1 responsible for the mass splitting shown in Eqs. (12) and (13)) and the properties of systems with $Q=s$ and u. However, there is compelling phenomenological evidence that this connection is not lost. Figure 1 shows the evolution of the spectra of $Q\bar{d}$ systems for $Q=b$, c, s and u. These spectra and associated data strongly suggest that these systems all have the same low-energy effective degrees of free-

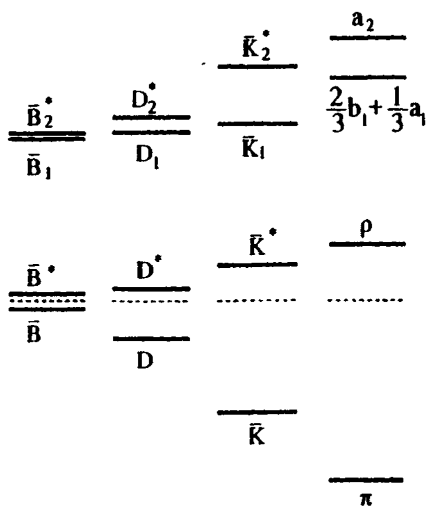

FIG. 1. The evolution of $Q\bar{d}$ spectra from $Q=b$ to $Q=u$; $\bar{B}_2^*$ and B_1, which are not yet separately identified, are shown here with their measured center-of-gravity and *predicted* splitting.

dom, and that the breaking of heavy quark symmetry is a smooth function of m_Q from $m_Q\to\infty$ to $m_Q=m_u$. In particular, heavy quark symmetry predicts that:

(1) the splittings of the centers-of-gravity of the $s_{\ell}^{\pi_{\ell}}=\frac{1}{2}^-$ and $\frac{3}{2}^+$ multiplets will be constant up to $1/m_Q$ corrections, as observed,

(2) the $1^- - 0^-$ splittings of the $s_{\ell}^{\pi_{\ell}}=\frac{1}{2}^-$ multiplets will open up like $1/m_Q$ (as observed with $\rho-\pi$: $\bar{K}^*-\bar{K}$: D^*-D: $\bar{B}^*-\bar{B}\simeq 1$: m_u/m_s:m_u/m_c:m_u/m_b for canonical [8] constituent quark masses of $m_u=0.33$ GeV, $m_s=0.55$ GeV, $m_c=1.82$ GeV and $m_b=5.20$ GeV),

(3) the $2^+ - 1^+$ splittings of the $s_{\ell}^{\pi_{\ell}}=\frac{3}{2}^+$ multiplets will also open up like $1/m_Q$ (as observed for the $D_2^*-D_1$ and $K_2^*(1420)-K_1(1280)$ splittings; in addition, as we will see below, even the apparently anomalous $a_2-(\frac{2}{3}b_1+\frac{1}{3}a_1)$ mass difference, involving the linear combination of the b_1 and a_1 masses that corresponds to the unmixed ${}^{3/2}E_1$ state, fits into the heavy quark symmetry pattern once the ${}^{3/2}E_1-{}^{1/2}E_1$ mixing required in the SU(3) limit is considered),

(4) the phase-space-corrected pion emission decay widths of the 2^+ and 1^+ members of the $s_{\ell}^{\pi_{\ell}}=\frac{3}{2}^+$ multiplets to their ground state $s_{\ell}^{\pi_{\ell}}=\frac{1}{2}^-$ multiplets should be equal (as observed; in addition, each of the D_2^* and K_2^* have approximately the predicted ratio of their amplitudes to the $[1^-\pi]_D$ and $[0^-\pi]_D$ final states), and

(5) the decay amplitudes of a given $s_{\ell}^{\pi_{\ell}}$ multiplet should be independent of m_Q (as observed: for example, the phase-space corrected decay amplitudes of D_2^* to $D^*\pi$ and of K_2^* to $K^*\pi$ are approximately equal).

While the persistence of these heavy quark symmetry predictions down to the light quark mass scale might be unexpected, the dynamics of these systems within the quark model provides some understanding of its resilience. We first note that the solutions [8] of the $V_{c0}+V_{hf}$ problem (with a regulated hyperfine interaction) give wave functions for the $u\bar{d}$, $s\bar{d}$, $c\bar{d}$ and $b\bar{d}$ systems with radii in the ratios of 1:1.07:1.10:1.05 for the 0^- states, 1:1.10:1.27:1.33 for the 1^- states, and 1:1.07:1.18:1.25 for all of the P-wave states. While not negligible, these variations from heavy quark symmetry are relatively modest. One also learns from the quark model calculations that the matrix elements which govern the strength of $1/m_Q$ effects, namely h,t,o_1,o_2, and the expectation value of p^2, are themselves of the scale of m_d so that while the perturbations they represent may not be accurately computed by first order perturbation theory, they will not be wildly misrepresented.

The hypothesis that heavy quark symmetry might be smoothly extended to light quarks has been adopted by a number of authors. For applications to semileptonic decays, see Refs. [9] in addition to Ref. [8]. For spectroscopic calculations closely related to ours see Refs. [10,11].

C. Unquenching the quark model

The greatest threat to the assumed smoothness of the extrapolation in m_Q from heavy to light masses is probably *not* in the m_Q dependence of the Breit-Fermi reduction of the confinement-plus-one-gluon-exchange interaction, but rather in the m_Q dependence of the effects of light quark-antiquark loops (a $1/N_c$ effect).

In the former case, m_Q dependence arises from the expansion of the Q currents in the gluonic potentials generated by the ‘‘brown muck.’’ While a nonrelativistic expansion of these currents may be crude, the constituent quark masses are after all free parameters which one may expect to be able to hide many of the defects of the nonrelativistic expansion. Assuming a smooth behavior of the effects of ‘‘unquenching the quark model’’ is potentially a much more precarious proposition. To understand why this is the case, we briefly review some recent work in this area [12], and then calculate loop-induced mass shifts of the $L=1$ mesons.

1. A review

We begin by addressing the origin of the valence approximation itself. A form of this approximation emerges from the large N_c limit [13] in the sense that diagrams in which only valence quark lines propagate through hadronic two-point functions dominate as $N_c\to\infty$. While suggestive, this dominance does not seem to correspond to the usual valence approximation since the Z-graph pieces of such diagrams will produce a $q\bar{q}$ sea. Consider, however, the Dirac equation for a single light quark interacting with a static color source. This equation represents the sum of a set of Feynman graphs which also include Z graphs, but the effects of those graphs is captured in the lower components of the single-particle Dirac spinor. That is, such Z graphs correspond to relativistic corrections to the quark model. That such corrections are important in the quark model has been known for a long time [14]. For us the important point is that while they have quantitative effects on quark model predictions like g_A, they do not qualitatively change the single-particle nature of the spectrum of the quark of our example, nor would they qualitatively change the spectrum of $q\bar{q}$ or qqq systems. In particular, a $Q\bar{d}$ spectrum would retain its valence ‘‘quarkonium’’ character with respect to the degrees of freedom it displays and their J^P quantum numbers. Of course, relativistic effects will in general induce interactions which can dramatically affect the level structure *within* this quarkonium spectrum, *e.g.*, by creating large spin-dependent splittings comparable to orbital splittings. See Fig. 1.

Given the valence approximation, it is still surprising that quark model spectroscopy survives ''unquenching.'' Consider two resonances which are separated by a mass gap δm in the narrow resonance approximation. In general we would expect that departures from the narrow resonance approximation, which produce resonance widths Γ, ought also to produce mass shifts Δm of order Γ. Yet even though a typical hadronic mass spectrum is characterized by mass gaps δm of order 500 MeV, and typical hadronic widths are of order 250 MeV, this does not seem to happen. In the flux tube model [15], the quark potential model arises from an adiabatic approximation to the gluonic degrees of freedom embodied in the flux tube. At short distances where perturbation theory applies, the effect of N_f types of light $q\bar{q}$ pairs is (in lowest order) to shift the coefficient of the Coulombic potential from $\alpha_s^{(0)}(Q^2)=12\pi/33\ln(Q^2/\Lambda_0^2)$ to $\alpha_s^{(N_f)}(Q^2)=12\pi/(33-2N_f)\ln(Q^2/\Lambda_{N_f}^2)$. The net effect of such pairs is thus to produce a *new* effective short distance $Q\bar{Q}$ potential. Similarly, when pairs bubble up in the flux tube (*i.e.*, when the flux tube breaks to create a $Q\bar{q}$ plus $q\bar{Q}$ system and then ''heals'' back to $Q\bar{Q}$), their net effect is to cause a shift $\Delta E_{N_f}(r)$ in the ground state gluonic energy which in turn produces a new long-range effective $Q\bar{Q}$ potential. It has indeed been shown [12] that the net long-distance effect of the bubbles is to create a new string tension b_{N_f} (*i.e.*, that the potential remains linear). Since this string tension is to be associated with the observed string tension, after renormalization *pair creation has no effect on the long-distance structure of the quark model in the adiabatic approximation*. The net effect of mass shifts from pair creation is thus much smaller than one would naively expect from the typical width Γ: such shifts can only arise from nonadiabatic effects. For heavy quarkonium, these shifts can in turn be associated with states which are strongly coupled to nearby thresholds. It should be emphasized that it was necessary to sum over very large towers of $Q\bar{q}$ plus $q\bar{Q}$ intermediate states to see that the spectrum was only weakly perturbed (after unquenching and renormalization). In particular, no simple truncation of the set of meson loop graphs can reproduce such results.

The final puzzle of hadronic dynamics which we must address before ''unquenching'' is the success of the Okubo-Zweig-Iizuka (OZI) rule [16]. A generic OZI-violating amplitude A_{OZI} can also be shown to vanish like $1/N_c$. However, there are several unsatisfactory features of this ''solution'' to the origin of the OZI rule [17]. Consider ω-ϕ mixing as an example. This mixing receives contributions from both true ''hairpin graphs'' and from the virtual hadronic loop process $\omega\rightarrow K\bar{K}\rightarrow\phi$, both steps of which are OZI allowed, and each of which scales with N_c like $\Gamma^{1/2}\sim N_c^{-1/2}$. The large N_c result that this OZI-violating amplitude behaves like N_c^{-1} is thus not peculiar to large N_c: it just arises from ''unitarity'' in the sense that the real and imaginary parts of a generic hadronic loop diagram will have the same dependence on N_c. The usual interpretation of the OZI rule in this case—that ''double hairpin graphs'' are dramatically suppressed—is untenable in the light of these OZI-allowed loop diagrams. They expose the deficiency of the large N_c argument since $A_{OZI}\sim\Gamma$ is *not* a good representation of the OZI rule. (Continuing to use ω-ϕ mixing as an example, we note that $m_\omega-m_\phi$ is numerically comparable to a typical hadronic width, so the large N_c result would predict an ω-ϕ mixing angle of order unity in contrast to the observed pattern of very weak mixing which implies that $A_{OZI}\ll\Gamma\ll m$.) Unquenching the quark model thus endangers the naive quark model's agreement with the OZI rule. It has been shown [12] how this disaster is naturally averted in the flux tube model through a ''miraculous'' set of cancellations between mesonic loop diagrams consisting of apparently unrelated sets of mesons (*e.g.*, the $K\bar{K}$, $K\bar{K}^*+K^*\bar{K}$, and $K^*\bar{K}^*$ loops tend to strongly cancel against loops containing a K or K^* plus one of the four strange mesons of the $L=1$ meson nonets). Of course the ''miracle'' occurs for a good reason. In the flux tube model, where pair creation occurs in the $^3P_0$ state, the overlapping double hairpin graphs which correspond to OZI-violating loop diagrams cannot contribute in a closure-plus-spectator approximation since the 0^{++} quantum numbers of the produced (or annihilated) pair do not match those of the initial and final state for any established nonet. In fact [12] this approximation gives zero OZI violation in all but the (still obscure) 0^{++} nonet. In addition, corrections to the closure-plus-spectator approximation are small, so that the observed hierarchy $A_{OZI}\ll\Gamma$ is reproduced. It must be emphasized once again that such cancellations require the summation of a very large set of meson loop diagrams with cancellations between what are apparently unrelated sets of intermediate states, and that no truncated low energy hadronic effective theory could reproduce such behavior.

2. Overview of quark loop effects

With this background in mind, we first consider the breaking of heavy quark symmetry by quark loops in the ''safe'' region $m_Q\gg\Lambda_{QCD}$. In this region the masses and coupling constants of all particles contributing to a given light-quark-induced set of hadronic loop diagrams will be near their heavy quark limit, each hadronic loop generated by the quark loop will have an amplitude which may be expanded in $1/m_Q$ as

$$a_i=a_{i0}+a_{i1}\left(\frac{1}{m_Q}\right)+\cdots\ , \tag{14}$$

and a smooth $1/m_Q$ expansion is guaranteed. Heavy quark symmetry will therefore be displayed by the loop-shifted spectra. However, as $m_Q\rightarrow m_s$ and m_u, two potentially dangerous effects arise which could destroy this smooth expansion:

(1) For $m_Q=m_s$, and *a fortiori* for $m_Q=m_u$, symmetry breaking in the ground state masses is so large that the $1/m_Q$ expansion might simply fail.

(2) For $m_Q=m_s$ or m_u, when the light quark loop is an s or u loop, respectively, new coherent processes arise which can change the amplitudes a_i to new functions.

Having raised these potentially serious objections to a smooth $1/m_Q$ extrapolation, let us immediately see why their effects are unlikely to be dramatic. We begin with the second difficulty. In the adiabatic approximation, all flavors of quark loops contribute to the renormalized Coulomb and linear potentials: $\alpha_s=\alpha_s^{(N_f)}$ and $b=b^{(N_f)}$. Thus the effect of the new amplitudes a_i can only be to reshuffle strength from one threshold to another with no net effect. Consider the concrete example of the mass shifts experienced by scalar mesons. We will focus on three states: the generic heavy quark meson ${}^{1/2}E_0$ (with quark content $Q\bar{d}$) and the two non-flavor-singlet light systems $\bar{K}_0^*$ and a_0 with quark contents $s\bar{d}$ and $u\bar{d}$, respectively. Most of their total mass shifts will be subsumed into the adiabatic potentials associated with $\alpha_s^{(N_f)}$ and $b^{(N_f)}$, but they will also experience nonadiabatic shifts associated with nearby thresholds. In particular, let us examine the mass shifts they experience due to their couplings to the lightest channels arising from pseudoscalar-pseudoscalar loop diagrams induced by $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ quark loops. These diagrams correspond for the generic heavy scalar to ${}^{1/2}E_0\to P\pi\to{}^{1/2}E_0$, ${}^{1/2}E_0\to P\eta\to{}^{1/2}E_0$, ${}^{1/2}E_0\to P\eta'\to{}^{1/2}E_0$, and ${}^{1/2}E_0\to P_s\bar{K}\to{}^{1/2}E_0$ (P denotes the $Q\bar{d}$ or $Q\bar{u}$ pseudoscalar; P_s is the $Q\bar{s}$ pseudoscalar), for the $s\bar{d}$ scalar to $\bar{K}_0^*\to\bar{K}\pi\to\bar{K}_0^*$, $\bar{K}_0^*\to\bar{K}\eta\to\bar{K}_0^*$, and $\bar{K}_0^*\to\bar{K}\eta'\to\bar{K}_0^*$, and for the $u\bar{d}$ scalar to $a_0\to\eta\pi\to a_0$, $a_0\to\eta'\pi\to a_0$, and $a_0\to K\bar{K}\to a_0$. Despite the fact that there are discrepancies between the strengths and even numbers of these thresholds, we know that in the SU(3) limit, $\delta m_{a_0}=\delta m_{\bar{K}_0^*}$, and in the limit $m_s\gg\Lambda_{QCD}$ by heavy quark symmetry $\delta m_{K_0^*}=\delta m_{{}^{1/2}E_0}$.

It is instructive to see how this happens in detail. In the flux tube-breaking model the couplings of ${}^{1/2}E_0$ to $P\pi$, $P\eta$, $P\eta'$, and P_sK are proportional to $1:\sqrt{1/3}\cos\phi_P:\sqrt{1/3}\sin\phi_P:\sqrt{2/3}$; those of $\bar{K}_0^*$ to $\bar{K}\pi$, $\bar{K}\eta$, $\bar{K}\eta'$ to $1:\sqrt{1/3}\cos\phi_P-\sqrt{2/3}\sin\phi_P:\sqrt{1/3}\sin\phi_P+\sqrt{2/3}\cos\phi_P$; and those of a_0 to $\eta\pi$, $\eta'\pi$, and $K\bar{K}$ to $\sqrt{4/3}\cos\phi_P:\sqrt{4/3}\sin\phi_P:-\sqrt{2/3}$. [These ratios are all quoted using the angle ϕ_i which describes the deviation from ideal mixing: $\phi_i=\theta_{ideal}-\theta_i$ with $\theta_{ideal}\simeq 35.3°$ and θ_i the ordinary SU(3) mixing angle so that, *e.g.*, $\eta=\sqrt{1/2}(u\bar{u}+d\bar{d})\cos\phi_P-s\bar{s}\sin\phi_P$ and $\eta'=\sqrt{1/2}(u\bar{u}+d\bar{d})\sin\phi_P+s\bar{s}\cos\phi_P$. If $\theta\simeq-10°$, then one would be close to the case of ''perfect'' pseudoscalar mixing [18] where $\phi_P\simeq 45°$. In what follows, for simplicity we quote numerical results for the two cases $\phi_P=45°$ and $\phi_V=0$ since our qualitative conclusions are independent of this angle.) One immediately sees that if the intrinsic coupling strengths S of each state to these loop diagrams are the same (as supported by the discussion of the previous subsection), and if all thresholds were equidistant from the state being shifted, that each of ${}^{1/2}E_0$, $\bar{K}_0^*$, and a_0 would experience a mass shift proportional to $2|S|^2$. From this example we see that the ''second potentially dangerous effect'' of coherent new amplitudes a_i is actually controlled by the same physics as the first: nonsmooth behavior in m_Q can occur if important low-lying thresholds are split in such a way that they affect a state discontinuously as a function of m_Q.

TABLE I. S-wave decay amplitudes (in units of S) of the $s_{\ell}^{\pi_\ell}=\frac{3}{2}^+$ and $\frac{1}{2}^+$ excited state heavy quark spin multiplets to the $s_{\ell}^{\pi_\ell}=\frac{1}{2}^-$ ground state heavy quark spin multiplet and a light pseudoscalar or vector. Decays to $P\pi$, $V\pi$, $P\rho$, and $V\rho$ (where P and V are the 0^- and 1^- $Q\bar{d}$ ground states) are shown explicitly. Decays to other light ground states may be obtained using the flavor ratios $(P,V)(\pi,\rho):(P,V)(\eta,\omega):(P,V)(\eta',\phi):(P_s,V_s)(\bar{K},\bar{K}^*)=1:\sqrt{1/3}\cos\phi_i:\sqrt{1/3}\sin\phi_i:\sqrt{2/3}$, where P_s and V_s are the $Q\bar{s}$ ground states and ϕ_i is the light flavor multiplet mixing angle defined in the text.

Excited state	$P\pi$	$V\pi$	$P\rho$	$V\rho$
${}^{3/2}E_2$	0	0	0	$-\frac{2}{\sqrt{3}}$
${}^{3/2}E_1$	0	0	$-\frac{2\sqrt{2}}{3}$	$\frac{2}{3}$
${}^{1/2}E_1$	0	-1	$\frac{1}{3}$	$\frac{\sqrt{2}}{3}$
${}^{1/2}E_0$	1	0	0	$\frac{1}{\sqrt{3}}$

The danger from such discontinuities is real, and an examination of their potential impact on our analysis will be the focus of the remainder of this subsection. We begin by remarking that the thresholds which are likely to produce nonsmooth behavior in $1/m_Q$ are those associated with S-wave channels: such channels have a cusp discontinuity right at threshold, while for higher partial waves the coupling strengths are shifted to higher masses and smoothed out. Table I gives the low-lying S-wave amplitudes of the four $Q\bar{d}$ states of the $s_{\ell}^{\pi_\ell}=\frac{3}{2}^+$ and $\frac{1}{2}^+$ heavy quark spin multiplets to the $s_{\ell}^{\pi_\ell}=\frac{1}{2}^-$ ground states and a ground state light quark pseudoscalar or vector meson. The amplitudes for the analogous $Q=s$ states are given in Table II, while those for the $Q=u$ states a_2, b_1, a_1, and a_0 are quoted in Table III.

These tabulated decay amplitudes A have been defined so that for a kinematically allowed decay the corresponding partial width $\Gamma\equiv|A(q^2)|^2q$ where q is the center-of-mass three-momentum of the decay. In the ${}^3P_0$ approximation to the flux tube model we use, and in a set of harmonic oscillator variational wave functions, the common amplitude S for our generic decay $E\to(P,V)(\pi,\rho)$ is proportional to [19]

$$A_Q(q^2)=A_Q(0)\left[1-\frac{q^2}{4\beta_Q^2}(1-\xi_Q)(1+\xi_Q)\right] \quad (15)$$

with

$$A_Q(0)\equiv\frac{8\gamma_0\pi^{3/4}}{9\beta_Q^{1/2}}\left[\frac{\beta_Q}{\beta_E}\right]^{5/2}\left[\frac{\beta_Q^2}{\beta_{P,V}\beta_{\pi,\rho}}\right]^{3/2}F_Q(q^2), \quad (16)$$

$$\beta_Q^{-2}\equiv\frac{1}{3}(\beta_E^{-2}+\beta_{P,V}^{-2}+\beta_{\pi,\rho}^{-2}), \quad (17)$$

TABLE II. The S-wave decay amplitudes (in units of S) of the ${}^{3/2}K_2$, ${}^{3/2}K_1$, ${}^{1/2}K_1$, and ${}^{1/2}K_0$ mesons analogous to those of Table I. Amplitudes to the mixed pairs of states (η,η') and (ω,ϕ) are given in the table in the format $(A_\eta, A_{\eta'})$ and (A_ω, A_ϕ), respectively.

State	$K\pi$	$(K\eta, K\eta')$
${}^{3/2}K_2$	-	-
${}^{3/2}K_1$	-	-
${}^{1/2}K_1$	-	-
${}^{1/2}K_0$	$+1$	$\left(\frac{(\cos\phi_P-\sqrt{2}\sin\phi_P)}{\sqrt{3}}, \frac{(\sin\phi_P+\sqrt{2}\cos\phi_P)}{\sqrt{3}}\right)$

State	$K^*\pi$	$(K^*\eta, K^*\eta')$	$K\rho$	$(K\omega, K\phi)$
${}^{3/2}K_2$	-	-	-	-
${}^{3/2}K_1$	0	$\left(+\frac{4\sin\phi_P}{3\sqrt{3}}, -\frac{4\cos\phi_P}{3\sqrt{3}}\right)$	$-\frac{2\sqrt{2}}{3}$	$-\left(\frac{2\sqrt{2}\cos\phi_V}{3\sqrt{3}}, \frac{2\sqrt{2}\sin\phi_V}{3\sqrt{3}}\right)$
${}^{1/2}K_1$	-1	$-\left(\frac{3\cos\phi_P+\sqrt{2}\sin\phi_P}{3\sqrt{3}}, \frac{3\sin\phi_P-\sqrt{2}\cos\phi_P}{3\sqrt{3}}\right)$	$\frac{1}{3}$	$\left(\frac{\cos\phi_V+3\sqrt{2}\sin\phi_V}{3\sqrt{3}}, \frac{\sin\phi_V-3\sqrt{2}\cos\phi_V}{3\sqrt{3}}\right)$
${}^{1/2}K_0$	-	-	-	-

State	$K^*\rho$	$(K^*\omega, K^*\phi)$
${}^{3/2}K_2$	$-\frac{2}{\sqrt{3}}$	$-\left(\frac{2(\cos\phi_V-\sqrt{2}\sin\phi_V)}{3}, \frac{2(\sin\phi_V+\sqrt{2}\cos\phi_V)}{3}\right)$
${}^{3/2}K_1$	$\frac{2}{3}$	$\left(-\frac{2(\sqrt{2}\sin\phi_V+\cos\phi_V)}{3\sqrt{3}}, +\frac{2(\sqrt{2}\cos\phi_V-\sin\phi_V)}{3\sqrt{3}}\right)$
${}^{1/2}K_1$	$\frac{\sqrt{2}}{3}$	$\left(-\frac{\sqrt{2}(\sqrt{2}\sin\phi_V+\cos\phi_V)}{3\sqrt{3}}, +\frac{\sqrt{2}(\sqrt{2}\cos\phi_V-\sin\phi_V)}{3\sqrt{3}}\right)$
${}^{1/2}K_0$	$\frac{1}{\sqrt{3}}$	$\left(\frac{(\cos\phi_V-\sqrt{2}\sin\phi_V)}{3}, \frac{(\sin\phi_V+\sqrt{2}\cos\phi_V)}{3}\right)$

$$\xi_Q \equiv \frac{\beta_Q^2}{3\beta_E^2}\left(1-\Delta_Q\,\frac{\beta_E^2}{\beta_{P,V}^2}\right), \tag{18}$$

$$F_Q(q^2)\equiv \exp\left(-\frac{q^2\beta_Q^2}{12}\left[\frac{\beta_{\pi,\rho}^2(1+\Delta_Q^2)+\beta_{P,V}^2+\beta_E^2\Delta_Q^2}{2\beta_{\pi,\rho}^2\beta_{P,V}^2\beta_Q^2}\right]\right), \tag{19}$$

and

$$\Delta_Q=\frac{m_Q-m_d}{m_Q+m_d}, \tag{20}$$

with β_i the variationally determined [8] harmonic oscillator parameters for the state i, *i.e.*, $\psi_i \sim (\text{polynomial})_i \exp[-\frac{1}{2}\beta_i^2 r^2]$.

3. Loop-induced mass shifts

With the couplings of Tables I–III in hand, we are prepared to explicitly examine the formally $1/N_c$ effects of quark loops. We begin by reminding ourselves that these effects appear at many levels in heavy quark systems:

(1) As shown in Refs. [12], nonadiabatic effects are expected to shift the renormalized radial and orbital (principal quantum numbers n and ℓ) spectral splittings of the valence quark model by of order 20% (100 out of a typical 500 MeV). This ''brown muck'' shift affects the overall splitting between the centers-of-gravity of the $s_\ell^{\pi_\ell}=\frac{1}{2}^-$ and $s_\ell^{\pi_\ell}=\frac{3}{2}^+$ states of Fig. 1.

(2) As also shown in Refs. [12], nonadiabatic effects on spin-dependent splittings (hyperfine and spin-orbit splittings) are expected to be smaller, but still significant: unless protected by some symmetry [*e.g.*, heavy quark symmetry or SU(6)] such splittings are also vulnerable to ''random'' 20% shifts (20 out of a typical 100 MeV). This ''brown muck'' effect is related to the light SU(6) flavor-spin splitting *between* the $s_\ell^{\pi_\ell}=\frac{3}{2}^+$ and $s_\ell^{\pi_\ell}=\frac{1}{2}^+$ excited states. It is central to our discussion and will be examined carefully below.

(3) While the ''brown muck'' physics of $q\bar{q}$ loops cannot break heavy quark symmetry as $m_Q\to\infty$, it can contribute to

TABLE III. The S-wave decay amplitudes (in units of S) of the a_2, b_1, a_1, and a_0 mesons, analogous to those of Table I. Amplitudes to the mixed pairs of states (η,η') and (ω,ϕ) are given in the table in the format $(A_\eta,A_{\eta'})$ and (A_ω,A_ϕ), respectively.

Excited state	$\pi(\eta,\eta')$	$K\overline{K}$
a_2	-	-
b_1	-	-
a_1	-	-
a_0	$\frac{2(\cos\phi_P,\sin\phi_P)}{\sqrt{3}}$	$-\sqrt{2/3}$

Excited state	$\pi\rho$	$\pi(\omega,\phi)$	$\rho(\eta,\eta')$	$K^*\overline{K}+\overline{K}^*K$
a_2	-	-	-	-
b_1	0	$-\frac{2(\cos\phi_V,\sin\phi_V)}{3}$	$-\frac{2(\cos\phi_P,\sin\phi_P)}{3}$	$-\frac{2}{3}$
a_1	$\frac{4}{3}$	0	0	$\frac{2\sqrt{2}}{3}$
a_0	-	-	-	-

Excited state	$\rho\rho$	$\rho(\omega,\phi)$	$K^*\overline{K}^*$
a_2	0	$-\frac{4(\cos\phi_V,\sin\phi_V)}{3}$	$-\frac{2\sqrt{2}}{3}$
b_1	$\frac{2\sqrt{2}}{3}$	0	$\frac{2}{3}$
a_1	0	0	0
a_0	0	$\frac{2(\cos\phi_V,\sin\phi_V)}{3}$	$\frac{\sqrt{2}}{3}$

the matrix elements determining the coefficients of the $1/m_Q$ expansion. Examining the effects of hadronic loops on the smoothness of the $1/m_Q$ expansion in passing from $m_Q \to m_b \to m_c \to m_s \to m_u$ *within* the excited $s_\ell^{\pi_\ell}=\frac{3}{2}^+$ and $s_\ell^{\pi_\ell}=\frac{1}{2}^+$ states is one of the main goals of this subsection.

(4) Finally, loop effects cannot destroy the smoothness of the extrapolation from m_s to m_u since this smoothness is simply a consequence of ordinary light SU(3) symmetry. Nevertheless, this extrapolation provides the link required to connect the underlying physics of light quark systems through the ''quasi-heavy'' s quark to $m_Q=\infty$.

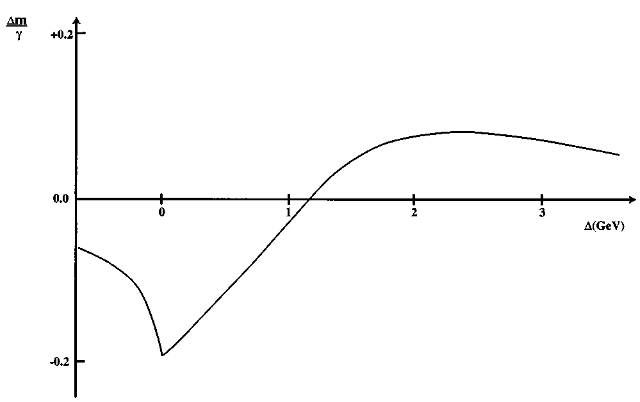

FIG. 2. A typical S-wave mass shift and its dependence on $\Delta \equiv m^{valence}-m^{threshold}$; shown is $\Delta m/\gamma$ for $K_0^* \to K\pi \to K_0^*$ for the simple width function $\Gamma=\gamma q e^{-q^2}$ with q in GeV.

Figure 2 sets the stage for this discussion by showing a typical mass shift of one of our S-wave-coupled states as a function of its intrinsic coupling strength and of the position of the zeroth-order valence quark state relative to the S-wave channel in question. The details of this curve depend on the kinematics and model dynamics of this example, but the qualitative features are universal. If $m^{valence}$ is more than a GeV below threshold it will experience a small negative mass shift; as $m^{valence}$ approaches $m^{threshold}$ the negative shift grows rapidly. It then quickly decreases in magnitude above threshold with a cusp singularity, and finally becomes relatively small and positive about a GeV above threshold as $m^{valence}$ is pushed up more by the low invariant mass continuum than the high invariant mass continuum [the effect of which is suppressed by the hadronic form factors $F_Q(Q^2)$ of Eq. (18) associated with the vertices of the loop diagrams].

We begin by discussing the smoothness of the heavy quark symmetry expansion in a given $s_\ell^{\pi_\ell}$ multiplet. Consider first the case $s_\ell^{\pi_\ell}=\frac{3}{2}^+$ where the S-wave thresholds are of the $P\rho$-type ($P\rho$, $P\omega$, $P\phi$, and P_sK^*) and $V\rho$-type ($V\rho$, $V\omega$, $V\phi$, and V_sK^*). Since we know from the $c\overline{d}$ and $b\overline{d}$ systems that the asymptotic splitting between the $Q\overline{d}$ $s_\ell^{\pi_\ell}=\frac{3}{2}^+$ multiplet and its $s_\ell^{\pi_\ell}=1/2^-$ ground state is about 450 MeV, we know that the *asymptotic* positions of the four (degenerate) $P\rho$- and $V\rho$-type thresholds are approximately 320, 330, 570, and 590 MeV above the degenerate ${}^{3/2}E_2$ and ${}^{3/2}E_1$ states. In the exact heavy quark symmetry limit, the

TABLE IV. Estimated loop contributions to the spectroscopic parameters, fit values of those parameters, and the deduced valence contributions to them (in MeV). These estimates are based on an S-wave width coefficient (see Fig. 2) of $\gamma=0.8$ deduced from $b_1 \to \omega\pi$.

	h/m_d^2	t/m_d^2	o_1/m_d^2	o_2/m_d^2
Loops	+50	−70	+40	+15
Fit	+90	−40	−120	+160
Valence	+40	+30	−160	+145

coupling strength of ${}^{3/2}E_2$ and ${}^{3/2}E_1$ at each of these thresholds must be identical: *e.g.*, from Table I, $\Delta m({}^{3/2}E_2 \to V\rho \to {}^{3/2}E_2) \propto \frac{4}{3}|S|^2$ while $\Delta m({}^{3/2}E_1 \to P\rho \to {}^{3/2}E_1) + \Delta m({}^{3/2}E_1 \to V\rho \to {}^{3/2}E_1) \propto \frac{8}{9}|S|^2 + \frac{4}{9}|S|^2 = \frac{4}{3}|S|^2$. In the $c\bar{d}$ and $b\bar{d}$ sectors two effects can break this equality: (1) the bare wave functions, and therefore the intrinsic couplings, of ${}^{3/2}E_2$ and ${}^{3/2}E_1$ can differ at order $1/m_Q$, and (2) the threshold positions will no longer coincide (*e.g.*, the $D_2^* \to D^*\rho$, $D_1 \to D\rho$ and $D_1 \to D^*\rho$ threshold splittings are 320, 220, and 360 MeV and not all at their asymptotic value of 320 MeV). Within our model it may be shown that the former effect is negligible. However, since these channels have strengths of $\frac{4}{3}$, $\frac{8}{9}$, and $\frac{4}{9}$, respectively, the splitting of their thresholds can and does make a substantial contribution to the $D_2^* - D_1$ splitting. At its calculated value of about 10 MeV, this contribution is not negligible compared to the observed $D_2^* - D_1$ splitting of 40 MeV, and we can conclude that the coefficient of the $1/m_Q$ expansion of this splitting has a non-negligible $1/N_c$ correction from hadronic loop mass shifts.

With Tables I–III, and the analogues of Fig. 2, the effects of loops on the $L=1$ spectrum of all $Q\bar{d}$ systems can be estimated. These estimates are not very reliable quantitatively: their overall magnitudes are quite sensitive to the assumed momentum dependence of the amplitudes, and even their dependence on m_Q is quite model dependent. However, our studies indicate that our estimates for mass shifts in a given channel can be expected to be good to within ± 10 MeV or a factor of 2 (whichever is larger), as can the dependence of a given shift on m_Q. Thus the estimates may be taken to be a reasonable *qualitative* guide to the effects of ''unquenching the quark model.'' Since for large m_Q the loop effects must obey heavy quark symmetry, most of these estimates may be encoded into a set of coefficients h^{loop}/m_d^2, t^{loop}/m_d^2, o_1^{loop}/m_d^2, and o_2^{loop}/m_d^2, which give the estimated loop contributions to these universal expansion coefficients. The results are displayed in Table IV.

Table IV also shows the ''best fit'' values of the spectroscopic parameters (to be discussed below) and the difference which we use to define an estimate of the valence contribution to these parameters. We see that, relative to the valence contributions, loop effects make small contributions to the spin-orbit parameters which are central to our discussion. Their contributions to hyperfine splittings are small compared to the S-wave hyperfine splitting parameter $h_{P,V}/m_d^2 \simeq 640$ MeV, but large compared to the P-wave parameter h/m_d^2. Since in the nonrelativistic limit $h^{valence}/m_d^2 = 0$ (see Sec. IV), this may be viewed as natural. The loop contribution to t/m_d^2 is, in contrast, surprisingly large compared to the valence estimate. As will be seen in Sec. IV, the latter is quite consistent with expectations. This is apparently an example where $1/N_c$ is not sufficient small to make the large-N_c expansion quantitatively useful.

While the description of the loop contributions to the spectrum of $Q\bar{d}$ systems by the universal set of parameters of Table IV is guaranteed for m_Q large, as discussed above its validity for $Q=s$ and u is dubious. Our estimates in fact show it working remarkably well (within 20%) for $Q=s$, but failing for $Q=u$ where $h_u^{loop}/m_d^2 \simeq -30$ MeV, $t_u^{loop}/m_d^2 \simeq -45$ MeV, and $o_{1u}^{loop}/m_d^2 + o_{2u}^{loop}/m_d^2 \simeq +30$ MeV (only the combination o_1+o_2 can be determined from $Q=u$; see below). These values, and especially h_u^{loop}/m_d^2, are inconsistent with the results shown in Table IV. The source of this failure of the heavy quark expansion is easily traced. As one passes from Q to b to c to s, the masses of the states ${}^{3/2}E_1$ and ${}^{1/2}E_1$ approach the $P\rho$ (and related) thresholds very smoothly from below, but for $Q=u$, where $P=\pi$, they jump above the cusp and a linear extrapolation fails dramatically. A similar effect occurs for the states ${}^{1/2}E_0$ with the $P\pi$ channel. As a result, while we may expect a smooth extrapolation to $Q=u$ of the valence properties of $u\bar{d}$ systems, we must carefully examine the effects of loops on these systems.

Before leaving our analysis of loop effects, we focus for a moment on the loop-induced splitting of the centers-of-gravity of the $s_\ell^{\pi_\ell} = \frac{3}{2}^+$ and $s_\ell^{\pi_\ell} = \frac{1}{2}^+$ multiplets, which is one of the main foci of this work. In a $Q\bar{D}$ system (where D is a hypothetical heavy-quark version of the d quark), in which the adiabatic approximation would be valid and where heavy quark symmetry would guarantee the spin independence of the loop contribution to the adiabatic potential, this splitting would vanish. An explicit example of this may be seen in Table I: if the P''π'', V''π'', P''ρ'', and V''ρ'' thresholds were all equal, as they would be for ''π'' and ''ρ'' being $D\bar{u}$ systems, then since the sums of the squares of the couplings of each of the four states to these four channels are equal, their loop-induced mass shifts would all be equal. The vanishing of the ${}^{3/2}E_2 - {}^{3/2}E_1$ and ${}^{1/2}E_1 - {}^{1/2}E_0$ splittings is just a result of heavy quark symmetry and does not require $m_D \to \infty$; the vanishing of the splitting between these two degenerate multiplets is a consequence of the adiabatic approximation. In contrast, since d is *not* a heavy quark, we expect, and our calculations provide, loop-induced violations of this degeneracy in $Q\bar{d}$ systems.

D. The data and some comments on it

Our fit is based on data [20] from the kaon ($s\bar{d}$) sector, where we use the masses of the two states $K_2^*(1430)$ and $K_1(1270)$ associated with the $s_\ell^{\pi_\ell} = \frac{3}{2}^+$ multiplet, and the two states $K_1(1400)$ and $K_0^*(1430)$ associated with the $s_\ell^{\pi_\ell} = \frac{1}{2}^+$ multiplet. In making these associations, we rely on analyses of the decay patterns of the $K_1(1270)$ and $K_1(1400)$ which show them to be quite near to being pure $s_\ell^{\pi_\ell} = \frac{3}{2}^+$ and $s_\ell^{\pi_\ell} = \frac{1}{2}^+$ states, respectively, with

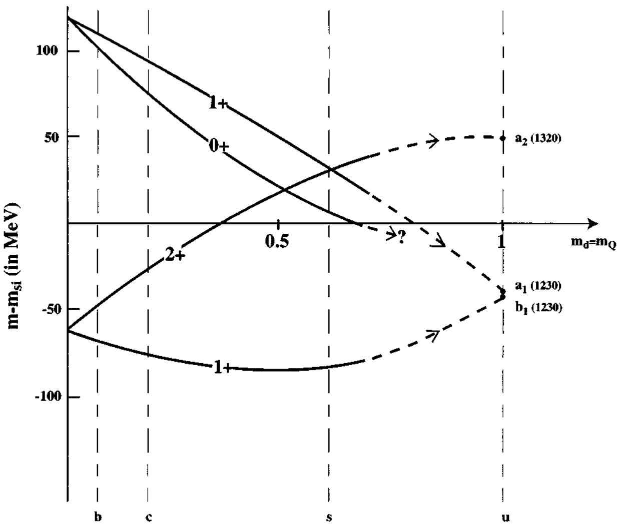

FIG. 3. The m_Q dependence of the P-wave spectra (solid curves). Vertical lines show the values of m_d/m_Q corresponding to each of $Q=u$, s, c and b. The dashed curves illustrate possible connections to the observed (and unobserved) $u\bar{d}$ states.

$$|K_1(1270)\rangle = \cos\,\phi_{K_1}|^{3/2}K_1\rangle + \sin\,\phi_{K_1}|^{1/2}K_1\rangle \quad (21)$$

$$|K_2(1400)\rangle = \sin\,\phi_{K_1}|^{3/2}K_1\rangle - \cos\,\phi_{K_1}|^{1/2}K_1\rangle \quad (22)$$

with a mixing angle $\phi_{K_1}\simeq -12\pm 7°$ [11,21].

Our fit also takes into account theoretical constraints from the $u\bar{d}$ light meson sector, where isospin symmetry guarantees that the mass eigenstates are states of definite charge conjugation versus states of definite $s_\ell^{\pi_\ell}$. Since tensor mixing (in this case between $^3P_2$ and $^3F_2$) is expected to be negligible, the 2^+ state of the $s_\ell^{\pi_\ell}=\frac{3}{2}^+$ multiplet can be identified with $a_2(1320)$ while the 0^+ of the $s_\ell^{\pi_\ell}=\frac{1}{2}^+$ multiplet would be uniquely associated with the $u\bar{d}$ state a_0. Unfortunately, the experimental status of this latter state is very murky. The two $J^P=1^+$ states $b_1(1235)$ and $a_1(1260)$ in this sector are particularly interesting. In this case the $s_\ell^{\pi_\ell}=\frac{3}{2}^+$ and $s_\ell^{\pi_\ell}=\frac{1}{2}^+$ *must* mix to precisely a mixing angle of $\cos^{-1}\sqrt{2/3}\simeq 35.3°$ to produce states of good charge conjugation:

$$|b_1\rangle = \sqrt{2/3}|^{3/2}E_1\rangle + \sqrt{1/3}|^{1/2}E_1\rangle \quad (23)$$

$$|a_1\rangle = \sqrt{1/3}|^{3/2}E_1\rangle - \sqrt{2/3}|^{1/2}E_1\rangle . \quad (24)$$

In this sector, indeed, the 1^+ mass matrix (10) collapses to the form

$$\bar{m} + \frac{(h+t/2-o_1-o_2)}{3m_d^2}\begin{pmatrix} -\frac{1}{2} & -\sqrt{2} \\ -\sqrt{2} & +\frac{1}{2} \end{pmatrix} \quad (25)$$

with

TABLE V. Comparison of the fit to experiment. The fit values of m_{si} for $Q=u$, s, c, and b are 1280, 1385, 2490, and 5765 MeV, respectively.

State	Predicted mass (MeV)	Observed mass (MeV) [21]	Comments
a_2	See Fig. 3	1320 ± 5	
b_1	See Fig. 3	1230 ± 10	
a_1	See Fig. 3	1230 ± 40	
a_0	See Fig. 3	-	Note 1
$\bar{K}_2^*$	1415	1430 ± 5	
$\bar{K}_1$	1300	1275 ± 10	
$\bar{K}_1$	1415	1400 ± 10	Note 2
$\bar{K}_0^*$	1395	1430 ± 10	Note 2
ϕ_{K_1}	$-3°$	$-12\pm 7°$	
D_2^*	2460	2460 ± 5	
D_1	2415	2420 ± 5	
D_1^*	2585	-	Note 2
D_0	2565	-	Note 2
ϕ_{D_1}	$-2°$		
$\bar{B}_2^*$	5715	-	See [23]
$\bar{B}_1$	5700	-	See [23]
$\bar{B}_1$	5875	-	
$\bar{B}_0^*$	5870	-	
ϕ_{B_1}	$-1°$		

Note 1: predicted [19] to be very broad, $\Gamma\sim 500$ MeV, and strongly coupled to the nearby S-wave thresholds $\eta'\pi$ and $K\bar{K}$; see Ref. [22].

Note 2: $\bar{K}_1$ and $\bar{K}_0^*$ predicted [19] to have $\Gamma\simeq 250$ MeV as observed, with $\bar{K}_0^*$ very strongly coupled to the nearby S-wave $\bar{K}\eta'$ threshold; from heavy quark symmetry, D_1 and D_0^* should also have $\Gamma\simeq 250$ MeV.

$$\bar{m} = \frac{1}{2}\,(m_{3/2_{E_1}} + m_{1/2_{E_1}}) = m_{si} - \frac{[h-t+2(o_1+o_2)]}{4m_d^2}, \quad (26)$$

where m_{si} is the expectation value of the spin-independent part of the Hamiltonian, leading to the eigenvalues in the whole P-wave sector of

$$m_{2^{++}} = m_{si} + \frac{1}{m_d^2}\left[\frac{1}{4}h - \frac{1}{10}t + (o_1+o_2)\right] \quad (27)$$

$$m_{1^{++}} = m_{si} + \frac{1}{m_d^2}\left[\frac{1}{4}h + \frac{1}{2}t - (o_1+o_2)\right] \quad (28)$$

$$m_{0^{++}} = m_{si} + \frac{1}{m_d^2}\left[\frac{1}{4}h - t - 2(o_1+o_2)\right] \quad (29)$$

$$m_{1^{+-}} = m_{si} - \frac{3h}{4m_d^2} \quad (30)$$

corresponding to the standard matrix elements of V_{hf}, V_{ten}, V_1, and V_2 in the ${}^{2S+1}L_J$ basis. Note that in the light meson sector only the combination o_1+o_2 enters.

The ''fit'' parameters of Table IV are based on just the $s\bar{d}$ data. On the basis of the observations made above on the smoothness of the extrapolation from $m_Q\to\infty$ to $m_Q=m_s$, we apply the $s\bar{d}$ parameters for all $m_Q>m_s$ to predict the excited spectra in the $c\bar{d}$, $b\bar{d}$, and $Q\bar{d}$ systems. The results are shown in Fig. 3 and compared to experiment in Table V. The overall fit seems satisfactory. Given that the loop contributions are not smoothly behaved in passing to $Q=u$, for the $u\bar{d}$ system we might consider adding our valence parameters to the computed $u\bar{d}$ loop parameters to obtain the predicted $u\bar{d}$ spectrum. On doing so we obtain $h_u/m_d^2\simeq-5\pm30$ MeV, $t_u/m_d^2\simeq0\pm30$ MeV, and $o_{1u}/m_d^2+o_{2u}/m_d^2\simeq0\pm30$ MeV, where we have shown explicitly the estimated theoretical errors arising from our loop calculation. From these parameters one can deduce little about the $u\bar{d}$ system except that all four states should be loosely clustered around their center-of-mass. Therefore, instead of using these parameters to make a prediction, we show in Fig. 3 the *experimental* values of the a_2, a_1, and b_1 masses to illustrate that they are consistent with an extrapolation from heavier quark systems. (To determine m_{si}, we assumed, in the absence of other information, that the a_0 lies at the center-of-gravity of the other states, *i.e.*, at 1270 MeV.) This exercise shows that the quantitative failure of the heavy quark expansion of loop mass shifts for $Q=u$ does not have very dramatic qualitative consequences.

As shown in Eqs. (12) and (13), the main conclusion of this paper concerning the inversion of the $s_\ell^{\pi_\ell}=\frac{3}{2}^+$ and $\frac{1}{2}^+$ spin multiplets depends on correctly determining o_1. We should therefore examine the power of the data to determine this matrix element. A good understanding of the situation can be obtained by noting from Eqs. (9)–(11) that, since the ${}^{3/2}K_1-{}^{1/2}K_1$ mixing angle is small,

$$m_{K_2^*}-m_{K_0^*}\simeq1.8\left[\left(1.1\,\frac{o_1}{m_d^2}+\frac{o_2}{m_d^2}\right)+0.3\,\frac{t}{m_d^2}\right]\simeq0\text{ MeV} \tag{31}$$

and

$$m_{{}^{3/2}K_1}-m_{{}^{1/2}K_1}\simeq\left[\left(1.1\,\frac{o_1}{m_d^2}+0.2\,\frac{o_2}{m_d^2}\right)-0.2\,\frac{h+t/2}{m_d^2}\right]\simeq-120\text{ MeV}. \tag{32}$$

The first of these splittings depends on a contribution close to $o_1/m_d^2+o_2/m_d^2$ as in the SU(3) limit [see Eqs. (27)–(30)]; the second displays a strong departure from the SU(3) limit. This radical departure from SU(3) is signalled experimentally by ϕ_{K_1} which, like ϕ_{B_1} and ϕ_{D_1}, is close to zero in contrast to the SU(3) limit where it must be 35.3°. It is these unique features of the $s\bar{d}$ system that allow us to separate o_1/m_d^2 and o_2/m_d^2. Using the facts that $0.3(t/m_d^2)$ and $0.2[(h+t/2)/m_d^2]$ are small, the results of Table IV for o_1/m_d^2 and o_2/m_d^2 may actually be checked directly from these two equations.

These conclusions may also be qualitatively cross-checked against the $u\bar{d}$ system. As just mentioned, the $u\bar{d}$ sector is sensitive to only h, t and o_1+o_2, so that it alone cannot determine o_1. Moreover, since $m_{0^{++}}$ is not known, this sector cannot even uniquely determine these three combinations of matrix elements. However, as $m_{0^{++}}$ varies in the ''reasonable'' range [22] from a maximum of 1300 MeV down to 1000 MeV, m_{si} varies from 1273 ± 14 MeV to 1248 ± 14 MeV, h_u/m_d^2 varies from 57 ± 17 MeV to 23 ± 17 MeV, t_u/m_d^2 varies from -64 ± 33 MeV to $+103\pm33$ MeV, and $(o_{1u}+o_{2u})/m_d^2$ varies from $+26\pm10$ MeV to $+76\pm10$ MeV. This analysis is thus consistent with our earlier discussion of this sector. Moreover, given that loop contributions to $(o_1+o_2)/m_d^2$ are relatively smooth functions of m_Q even as $Q\to u$, it confirms the small value of the combination $(o_1+o_2)/m_d^2$ of greatest interest here.

The most important prediction shown in Table V is that in the asymptotic case (to which the physical case $Q=b$ is very close) $m({}^{1/2}E_1)-m({}^{3/2}E_1)\simeq+180$ MeV, and that even in the $c\bar{d}$ system there will be a splitting between the ${}^{1/2}E$ and ${}^{3/2}E$ centers-of-gravity of $[\frac{3}{4}m({}^{1/2}E_1)+\frac{1}{4}m({}^{1/2}E_0)]-[\frac{5}{8}m({}^{3/2}E_2)+\frac{3}{8}m({}^{3/2}E_1)]\simeq-\frac{3}{2}(o_1/m_d^2+o_2/m_dm_c)\simeq+135$ MeV. In the next section we will show that this unexpected result has a very interesting interpretation.

IV. INTERPRETATION: THE FLUX-TUBE-PLUS-GLUON-EXCHANGE MODEL

The starting point for the preceeding discussion, Eqs. (2)–(4), is a general expression for the leading adiabatic potential between two heavy quarks. The key assumption of this paper—which could certainly fail—is that the characteristics of this general form can be extrapolated to light quark masses first in $\bar{q}$ and then in Q. Precisely because it is *not* obviously true, this prediction is an interesting test of one of the main assumptions of the valence quark model: that analogous degrees of freedom control the properties of the low-lying spectra of heavy-heavy, heavy-light and light-light systems and that the effective forces between these degrees of freedom evolve smoothly as a function of the constituent quark masses.

Having determined the valence contributions to the matrix elements h, t, o_1 and o_2 that arise in this context, we will now see that there is a very plausible interpretation of these matrix elements in the context of the flux-tube-plus-one-gluon-exchange constituent quark model. In this picture, the predicted spin-orbit inversion is a consequence of the Thomas precession of the light antiquark $\bar{q}$ in the linear confining potential generated by the heavy quark Q.

In such quark models, as mentioned in Sec. II, the interquark forces arise from flux tube formation plus one-gluon exchange, and one can express the matrix elements h, t, o_1 and o_2 in terms of expectation values in the $Q\bar{d}\ {}^3P_2$ system as

$$\frac{h}{m_d^2}=\langle{}^3P_2|\,\frac{32\pi\alpha_s}{9m_d^2}\,\tilde{\delta}^3(\vec{r})|{}^3P_2\rangle \tag{33}$$

$$\frac{t}{m_d^2} = \langle {}^3P_2 | \frac{4\alpha_s}{3m_d^2 r^3} e^{-f_{mag} br^2} | {}^3P_2 \rangle \tag{34}$$

$$\frac{o_1}{m_d^2} = \langle {}^3P_2 | \frac{2\alpha_s}{3m_d^2 r^3} - \frac{b}{2m_d^2 r} | {}^3P_2 \rangle \tag{35}$$

and

$$\frac{o_2}{m_d^2} = \langle {}^3P_2 | \frac{4\alpha_s}{3m_d^2 r^3} | {}^3P_2 \rangle . \tag{36}$$

In these formulas, α_s is the strong fine structure constant (very appropriately named for this context), $\tilde{\delta}^3(\vec{r})$ is a delta-function smeared out by relativistic corrections and by the constituent quarks' nonzero effective sizes, and f_{mag}, of order unity, is a parameter characterizing the screening of the chromomagnetic field in the vacuum outside the flux tube.

Since $h=0$ for pointlike quarks in the nonrelativistic limit, to have a rough estimate of the size of h in systems with a light quark we must introduce a quark size. As $m_i \to \infty$, a quark's effective size would be characterized by its relativistic radius $1/m_i$. For $m_i \to 0$, this radius will freeze out at some constituent quark size $r_0 \ll 1$ fm. When two quarks interact *via* some intrinsic potential $V(r_{ji})$, we assume that this potential is smeared out into $\tilde{V}(r_{ji})$ given by

$$\tilde{V}(\vec{r}_{ji}) \equiv \int d^3\delta \rho_{ji}(\vec{\delta}) V(\vec{r}_{ji} + \vec{\delta}) \tag{37}$$

with

$$\rho_{ji}(\vec{\delta}) \equiv \int d^3\sigma \rho_j \left(\vec{\sigma} + \frac{1}{2}\vec{\delta} \right) \rho_i \left(\vec{\sigma} - \frac{1}{2}\vec{\delta} \right), \tag{38}$$

the convolution of the individual quark smearing functions. Thus in our Gaussian approximation

$$\rho_\alpha(x) = \left(\frac{3}{2\pi r_\alpha^2} \right)^{3/2} e^{-3x^2/2r_\alpha^2} \tag{39}$$

where for $\alpha = i$ or j, r_α is the effective radius of the quark and for $\alpha = ji$, $r_{ji}^2 = r_j^2 + r_i^2$.

With harmonic oscillator variational solutions [8] it immediately follows that

$$\frac{h}{m_d^2} = + \frac{64\alpha_s \beta_E^3}{27\sqrt{\pi} m_d^2} \frac{\beta_E^2 r_{ji}^2}{\left[1 + \frac{2}{3} \beta_E^2 r_{ji}^2 \right]^{5/2}} \tag{40}$$

$$\frac{t}{m_d^2} = + \frac{16\alpha_s \beta_E^3}{9\sqrt{\pi} m_d^2} \frac{1}{[1 + f_{mag} b/\beta_E^2]} \tag{41}$$

$$\frac{o_1}{m_d^2} = + \frac{8\alpha_s \beta_E^3}{9\sqrt{\pi} m_d^2} - \frac{2b\beta_E}{3\sqrt{\pi} m_d^2} \tag{42}$$

$$\frac{o_2}{m_d^2} = + \frac{16\alpha_s \beta_E^3}{9\sqrt{\pi} m_d^2} \tag{43}$$

and

$$\frac{h_{P,V}}{m_d^2} = + \frac{32\alpha_s \beta_{P,V}^3}{9\sqrt{\pi} m_d^2} \frac{1}{\left[1 + \frac{2}{3} \beta_{P,V}^2 r_{ji}^2 \right]^{3/2}}, \tag{44}$$

where β_E ($\beta_{P,V}$) is the Gaussian parameter characterizing the variational solution of the Coulomb-plus-linear problem for the excited state P-waves (ground state S-waves) in the $Q_i\bar{q}_j$ system. The new matrix element $h_{P,V}/m_d^2$ is the one responsible for the splitting of the ground state spin multiplet into its vector and pseudoscalar components; it will be helpful in the discussion which follows.

When evaluated (as appropriate to our fit) with the $s\bar{d}$ parameters of the first of Refs. [8] ($\alpha_s = 0.55$, $m_d = 0.33$ GeV, $b = 0.18$ GeV^2, $\beta_E = 0.30$ GeV, and $\beta_{P,V} = 0.42$ GeV), with $r_d \simeq r_s \simeq 0.15 \pm 0.15$ fm as given in Ref. [12], and with $f_{mag} = 1$, these formulas give $h_{P,V}/m_d^2 \simeq 620$ MeV (compared to the $\rho - \pi$ splitting of 630 MeV), $h/h_{P,V} = 0.03^{+0.04}_{-0.03}$ (Table IV would give 0.06), $t/m_d^2 = +45$ MeV (Table IV gives $+30$ MeV), $o_1/m_d^2 = -120$ MeV (Table IV gives -160 MeV) and $o_2/m_d^2 = +135$ MeV (Table IV gives $+145$ MeV). (As to be expected based on the discussion of Sec. III B, evaluation with $b\bar{d}$, $c\bar{d}$, and $u\bar{d}$ parameters gives similar results.) We note from (35) and (42) that the sign of o_1/m_d^2, critical to our main conclusions, depends on the relative strength of its Coulomb and confinement pieces: $o_1/m_d^2 \simeq +65$ MeV–185 MeV$\simeq -120$ MeV. Given the strong model-dependence of these two comparable terms, it is not surprising that most calculations fail to predict the spin-orbit inversion of $Q\bar{d}$ systems [24].

While this successful comparison with the predictions of the flux-tube-plus-gluon-exchange quark model is not a proof of its validity, it does show the remarkable ability of this model to describe the key features of not only the gross spectrum of $Q\bar{d}$ states, but also their ''fine structure.'' Within this context, our main result that $m_{1/2} - m_{3/2} \simeq +180$ MeV also has a simple but profound interpretation: as $m_Q \to \infty$, Thomas precession of the light $\bar{q}$ in the confinement potential br overwhelms the ordinary spin-orbit force familiar from atomic physics to invert spin-orbit multiplets [7,25]. If verified experimentally, this effect would lend strong support to the growing evidence that confinement is free of dynamical spin-dependent effects.

V. CONCLUSIONS

Based on the observed smooth evolution of the spectra of $Q\bar{d}$ systems from $Q=b$ to $Q=u$, we have extracted from the $s\bar{d}$ system estimates for the matrix elements which control the $1/m_Q$ expansion of heavy quark mesons. Checks on the validity of this approach are its correct prediction of the observed properties of the charm and bottom systems, and the comparison of its extrapolation to $Q=u$ with experiment.

The most striking result of our analysis is the conclusion that the $s_\ell^{\pi_\ell}=\frac{1}{2}^+$ states (with $J^P=0^+$ and 1^+) of the $c\bar{d}$ and $b\bar{d}$ sytems will lie *above* rather than below the $s_\ell^{\pi_\ell}=\frac{3}{2}^+$ states (with $J^P=1^+$ and 2^+). While contrary to conventional wisdom and intuition based on atomic and $Q\bar{Q}$ spectra, this inversion has a simple interpretation in the usual quark model: Thomas precession in the spin-independent linear confinement potential, a relativistic kinematic effect, has overwhelmed the usual atomic-like spin-orbit force from one-gluon exchange.

Although this would be an important conclusion, perhaps the most important ramification of the confirmation of this effect would be the support it would lend to the evidence discussed here that heavy- and light-quark systems may be characterized by the same low-energy degrees of freedom. Such an unexpected simplification of strong QCD would be an important step toward understanding the nature of confined hadronic systems. To complete this picture will require confirming the key role that the strange quark plays as the link between heavy- and light-quark systems *via* a much more complete experimental and theoretical understanding of strange mesons and baryons and of strange quarkonia.

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MeV, unless otherwise noted. The errors quoted are the quadrature of those of Ref. [20] and 5 MeV, a minimum error assigned to take into account errors associated with isospin breaking. The ''experimental'' mixing angle ϕ_{K_1} is taken from Ref. [11] with an error chosen to encompass model dependence. For other discussions of ϕ_{K_1}, see L. Burakovsky and T. Goldman, Phys. Rev. D **56**, R1368 (1997); M. Suzuki, *ibid.* **47**, 1252 (1993); **55**, 2840 (1997).

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[23] An effect consistent with several narrow states has been reported at a mass of about 5700 MeV. See [20] and references therein. We have used these observations to tentatively set our free parameter m_{si} for this system. Note that, as in each sector, our *predictions* are only for the splittings between the P-wave states, and not their center-of-gravity.

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