Large top quark Yukawa coupling and horizontal symmetries

Andrija Rašin
High Energy Section, International Centre for Theoretical Physics, Strada Costiera 11, 34100 Trieste, Italy
(Received 8 May 1997; published 13 February 1998)

We consider the maximal U(3) horizontal scheme as a handle on fermion masses and mixings. In particular, we attempt to explain the large top quark Yukawa coupling and the masses and mixing in the two heaviest generations. A simple model is constructed by enlarging the matter content of the standard model with that of a $10+\overline{10}$ pair of SU(5). The third generation particles get their masses when U(3) is broken to U(2). The top quark mass is naturally of order one. Bottom and tau masses are suppressed because of a hierarchy in the effective Yukawa couplings and *not* from the hierarchy in the Higgs doublet vacuum expectation values. The hierarchy is a consequence of the fact that the particle spectrum contains an incomplete vectorlike generation and can come from hierarchies between scales of breaking of different grand unified groups. Hierarchies and mixings between the second and third generations are obtained by introducing a single parameter ϵ' representing the breaking $U(2)\rightarrow U(1)$. As a consequence, we show that the successful (and previously obtained) relations $V_{cb}\approx m_s/m_b\approx\sqrt{m_c/m_t}$ easily follow from our scheme. [S0556-2821(98)01403-9]

PACS number(s): 12.15.Ff, 12.10.Dm, 12.60.Jv, 14.65.Ha

I. INTRODUCTION

Despite all its successes, the standard model (SM) has many unexplained features. Most of them are connected to the fermionic sector, such as the puzzling pattern of masses and mixings or the fact that quarks and leptons seem to neatly fit into three identical generations. The situation is best summarized by the fact that of the 19 arbitrary parameters in the SM, 13 reside in the fermionic sector. Thus, it seems that the search for a way beyond the SM will go through the reduction of arbitrary parameters in the fermionic sector.

A promising approach to explain some of the SM features is that of using the flavor symmetry of gauge interactions of the fermions, which is the $U(3)^5$ global symmetry of rotations with each U(3) belonging to one of the five charged fermion sectors of the SM (q,u^c,d^c,l,e^c). This flavor symmetry is broken by various degrees by the arbitrary Yukawa couplings of the SM. The idea essentially amounts to building an extension of the SM that is invariant under a certain subgroup of the maximal flavor symmetry with Yukawa couplings generated only when this horizontal symmetry gets broken. The SM is then the effective theory with Yukawa couplings carrying information on the broken horizontal symmetry. An example of this approach is the Froggatt-Nielsen (FN) mechanism [1,2], in which the Yukawa couplings get generated from higher dimensional operators when new scalars ϕ, *flavons*, get their vacuum expectation values (VEVs) and break the horizontal symmetries. The higher dimensional operators itself get generated by integrating out some extra matter or scalar fields with mass M (for example see later Figs. 4 and 5).

Since the top quark mass is of order weak scale [3], its corresponding Yukawa coupling is of order unity. On the other hand, if the VEVs of the Higgs doublets in the theory are comparable, b and τ couplings are much smaller than 1. In this paper we build a model that incorporates a large top and small bottom and tau Yukawa couplings.

How does one include the large top quark Yukawa coupling in a horizontal symmetry model? In theories based on Abelian symmetries or SU(2), it is usually assumed that the top quark does not transform under the horizontal symmetry considered, noting that it must come from a maximally broken SU(3).

In the maximal horizontal group SU(3) it does not make sense to say that such a large number comes from a higher dimensional operator which is suppressed by some inverse powers of some high scale M. Rather it means the horizontal symmetry in the top quark sector is broken maximally; i.e., the VEV of the ϕ is of the order of M. This means that if the horizontal symmetries were operative once at some high scale, either the third generation would have some large, unsuppressed mixing to some extra matter (unlike the other lighter generations) or the SM Higgs doublet is maximally mixed with some new scalars. In this paper we present a model which explores the first possibility.

We consider the full global U(3) symmetry in a manner similar to the U(2) case of Refs. [4,5]. U(2) [or SU(2)] horizontal symmetry has received a lot of interest lately as a natural solution to the supersymmetry (SUSY) flavor problem, forcing the squarks of the first two families to be approximately degenerate [6]. Thus, we will consider supersymmetric theories although we focus on conclusions in the fermionic sector (we discuss the scalar sector briefly at the end). The different hierarchies between m_c/m_t and m_s/m_b are also easily explained, as well as V_{cb}. The large top quark mass is explained by the addition of an extra $10+\overline{10}$ of SU(5).[1] The theory can also explain the smallness of the bottom and tau lepton masses without any suppression of Higgs doublet VEVs. It is easy to accommodate also the first generation in this scheme, but we chose to avoid doing so in this paper for clarity of the argument and reasons we discuss later.

[1] A similar field content in the context of supersymmetry was also recently proposed by Berezhiani in [7].

Motivated by grand unification and more predictivity, we consider the same U(3) acting on all charged sectors [rather than the maximal $U(3)^5$]. Thus the scale M could be some scale of order 10^{16} GeV or so, although we will comment on how low phenomenologically such a scale can be.

The feature of large mixings of the top quark with extra matter was explored in several papers. For example, in a supersymmetric Pati-Salam model there is an extra gaugino with charge $+2/3$ which can effectively play the role of an extra vectorlike quark singlet, and the large top quark mass can be related to the scale of SUSY breaking which is of the order of the weak scale [8,9]. Many other papers explore the possibility of having an extra vectorlike singlet up quark [10]. The issue of large top quark Yukawa coupling and large mixing in an inverse hierarchy scheme was discussed in Ref. [11]. A pseudo-Goldstone approach for the Higgs doublets where the top quark mixes with extra vectorlike matter can be found in Ref. [12].

The first attempt at building supersymmetric theories with non-Abelian horizontal symmetries was done by Berezhiani *et al.* in [13]. Later attempts include those listed in [14]. Cosmological consequences of a global SU(3) family symmetry broken at a grand unified theory (GUT) scale were studied in [15].

We start in Sec. II with the masses of the third generation. We show how the top quark Yukawa coupling can be generated from the breaking of U(3) and still be of order one, while the bottom and tau Yukawa couplings are suppressed *without* a hierarchy in the VEVs of the standard Higgs doublets. The masses of the second generation fermions, discussed in Sec. III, are generated in a manner somewhat similar to Ref. [5] and come from the breaking of the remaining U(2) symmetry down to U(1). Section IV is reserved for the discussion of the origin of the nonrenormalizable terms and the generated ratio m_b/m_t. We conclude with some final thoughts in Sec. V.

II. MASS OF THE THIRD GENERATION FERMIONS

In order to explain the large top quark Yukawa coupling within a FN scheme we must add some extra matter fields. There exist strong limits on extra matter, such as SM-like generations, from electroweak precision measurements [16]. However, extra vectorlike matter is almost not constrained. Furthermore, gauge coupling unification is not spoiled if matter is added in $5+\overline{5}$ or $10+\overline{10}$s.[2] It is interesting that string compactification can give three generations and extra vectorlike matter with a SM invariant mass which is not necessarily at the Planck scale [18,19]. Since we will discuss the grand unification of such a theory, we will assume masses of the order GUT scale, although we will comment later on how low can such a scale be.

We add to the three generations of the SM (q_a, u^c_a, d^c_a, l_a, e^c_a, $a=1,2,3$) vectorlike matter with the content of $10+\overline{10}$ of SU(5):

$$Q_1 U_1^c E_1^c,$$

$$Q_2^c U_2 E_2. \tag{1}$$

The index a denotes generations and goes from 1 to 3. Notice that the $SU(2)_L$ *doublet* Q_2^c carries fields with the same electric charges as the $SU(2)_L$ *singlets* u^c_a and d^c_a, and that the $SU(2)_L$ *singlet* U_2 carries the same electric charge as the up quark fields in the $SU(2)_L$ *doublets* q_a. Also, the $SU(2)_L$ *singlet* E_2 carries the same electric charge as the charged field in the $SU(2)_L$ *doublets* l_a. We will call ''mismatch'' this wrong pairing of fields from different SU(2) multiplets with the same electric charge. As is well known and we show later, this will make the quark mixing matrix non-unitary.

A. Mass of the top quark

We assume that the horizontal group is maximal, i.e., U(3), under which the three generations transform as $\overline{3}$, while the extra vectorlike matter is neutral. In addition, we assume that there is an extra SM singlet flavon field ϕ^a which transforms as a $\overline{3}$ under U(3), and the upper index denotes a charge opposite to the charge of a field with a lower index. We denote the gauge invariant mass of the extra vectorlike matter by M.

Thus the most general mass terms come from

$$\phi^a Q_2^c q_a + \phi^a u_a^c U_2 + \phi^a e_a^c E_2 + M Q_2^c Q_1 + M U_1^c U_2 + M E_1^c E_2 + H_2 U_1^c Q_1 + H_1 Q_2^c U_2, \tag{2}$$

from which we obtain the mass matrices for up quarks, down quarks, and charged leptons,

$$L_{M_u} = (u_a^c U_1^c U_2^c) \begin{pmatrix} \mathbf{0} & \mathbf{0} & \phi^a \\ \mathbf{0} & H_2^0 & M \\ \phi^a & M & H_1^0 \end{pmatrix} \begin{pmatrix} u_a \\ U_1 \\ U_2 \end{pmatrix} + \text{H.c.}, \tag{3}$$

$$L_{M_d} = (d_a^c D_2^c) \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \phi^a & M \end{pmatrix} \begin{pmatrix} d_a \\ D_1 \end{pmatrix} + \text{H.c.}, \tag{4}$$

$$L_{M_e} = (e_a^c E_1^c) \begin{pmatrix} \mathbf{0} & \phi^a \\ \mathbf{0} & M \end{pmatrix} \begin{pmatrix} e_a \\ E_2 \end{pmatrix} + \text{H.c.}, \tag{5}$$

where H_1 and H_2 are the Higgs doublets. The index a runs from 1 to 3 so that the up type mass matrix is a 5×5, while the down and lepton matrices are 4×4. Boldfaced zeros denote the appropriate matrix, vector, or column with all elements equal to zero. Also, notice that, for example, U_2^c from the doublet Q_2^c is grouped with the singlets, reflecting the mismatch. For simplicity, we assumed that the up quark mass matrix is symmetric.[3]

The VEV of ϕ can always be rotated so that only one component obtains a VEV, say, ϕ^3. Thus, $\langle \phi^3 \rangle$ breaks the

[2]Perturbativity, however, constrains the number of such extra pairs [17].

[3]For example, the gauge invariant mass terms $U_1^c U_2$ and $U_2^c U_1$ do not have to be exactly equal. However, this does not qualitatively change our results.

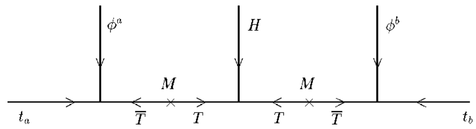

FIG. 1. Mechanism for generating the top quark mass.

rU(3) symmetry down to U(2). If we diagonalize the up quark mass matrix (3), we get the top quark with mass

$$m_t = v_2 \frac{\langle \phi^3 \rangle^2}{\langle \phi^3 \rangle^2 + M^2}, \tag{6}$$

while b and τ remain massless,

$$m_b = m_\tau = 0. \tag{7}$$

Equation (6) holds *regardless* of the values of $\langle \phi^3 \rangle$ and M, as long as they are both larger than v_1 and v_2. In particular, we take $\langle \phi^3 \rangle$ and M of the same order, in order for the top quark mass to be of the order weak scale [20]. In addition, there are four heavy states with mass $\sqrt{\langle \phi^3 \rangle^2 + M^2}$ (two in the up sector, one in down, and one in lepton sector).

Let us discuss the possibility that $\langle \phi \rangle$ (scale of maximal horizontal symmetry breaking) is of order M (mass of the vectorlike pair). In fact, there is *a priori* nothing that can stop it from doing so: The scalar potential involving ϕ will have all dimensionful parameters at scale M. Only subgroups that are preserved after the breaking of the largest group may get a smaller VEV than the natural scale M at some later stage (possibly from radiative corrections).

The grand unified origin of terms in Eq. (2) is straightforward. We consider a Froggatt-Nielsen [1,2] theory with the flavor group U(3) and unification group SU(5). Ordinary matter is in $t_a(10) + f_a(\overline{5})$, extra FN vector matter is in $T(10) + \overline{T}(\overline{10})$, and Higgs fields are in $H(5)$ and $\overline{H}(\overline{5})$, where the transformation properties under SU(5) are spelled out in the brackets. Flavons ϕ^a are SU(5) singlets. The most general renormalizable interactions are

$$\phi^a \overline{T} t_a + M T \overline{T} + TTH + \overline{TTH}. \tag{8}$$

On integrating out heavy states, there is a single diagram, given in Fig. 1, which generates the top quark mass as in Eq. (6). We conclude that the top quark is heavy because at the scale of U(3) breaking the only FN fields transform as 10.

B. Mass of the bottom and tau

Masses of lighter fermions may be generated in a way similar to the U(2) case [5]. We use a flavon field ϕ^{ab} which is symmetric in flavor indices [a $\overline{6}$ of U(3)] and which can generate some higher order operators of the form

$$\frac{\phi^{ab}}{M_H}[u_a^c q_b H_2 + d_a^c q_b H_1 + e_a^c l_b H_1]. \tag{9}$$

The crucial point is that the mass M_H has no reason to be of the same order as M. Mass M is the SU(5) invariant mass of the $10 + \overline{10}$ pair, while M_H can, for example, come from the SO(10) invariant mass, and thus can be higher by several orders of magnitude. This will be discussed in more detail in Sec. IV. From now on, we will assume

$$\epsilon \equiv M/M_H \approx 10^{-2}. \tag{10}$$

The field ϕ^{ab} may be a new field added to the theory or, more economically, an effective field made out of the product of two fundamentals $\phi^a \phi^b$. Later we will give an explicit SU(5) realization of the model where we discuss possible ways of generating the hierarchy (10).

U(3) is also broken by the VEV of ϕ^{33} and we expect it to be of the same order as $\langle \phi^3 \rangle \approx M$. This then modifies the Yukawa matrices in Eqs. (3)–(5) in the 3,3 entry,

$$L_{M_u} = (u_i^c u_3^c U_1^c U_2^c) \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\langle \phi^{33} \rangle}{M_H} v_2 & 0 & \langle \phi^3 \rangle \\ \mathbf{0} & 0 & v_2 & M \\ \mathbf{0} & \langle \phi^3 \rangle & M & v_1 \end{pmatrix} \begin{pmatrix} u_i \\ u_3 \\ U_1 \\ U_2 \end{pmatrix} + \text{H.c.}, \tag{11}$$

$$L_{M_d} = (d_i^c d_3^c D_2^c) \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\langle \phi^{33} \rangle}{M_H} v_1 & 0 \\ \mathbf{0} & \langle \phi^3 \rangle & M \end{pmatrix} \begin{pmatrix} d_i \\ d_3 \\ D_1 \end{pmatrix} + \text{H.c.}, \tag{12}$$

$$L_{M_e} = (e_i^c e_3^c E_1^c) \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\langle \phi^{33} \rangle}{M_H} v_1 & \langle \phi^3 \rangle \\ \mathbf{0} & 0 & M \end{pmatrix} \begin{pmatrix} e_i \\ e_3 \\ E_2 \end{pmatrix} + \text{H.c.}, \tag{13}$$

where $i = 1,2$. Diagonalizing we see that the top quark mass stays almost unchanged. However, bottom and tau masses are generated and they are of order

$$m_b \approx m_\tau \approx \frac{\langle \phi^{33} \rangle}{M_H} v_1 \approx \frac{M}{M_H} v_1 \equiv \epsilon v_1. \tag{14}$$

This realization of the heaviest generation masses is different than [7], where the top-quark–bottom-quark splitting was left to be explained as usual [either a large ratio of the Higgs doublet VEVs (large $\tan\beta$) or a large ratio of Yukawa couplings put in by hand].

The terms in Eq. (9) can be generated in SU(5) if we add the nonrenormalizable operators suppressed by the high scale M_H:

$$\frac{\phi^{ab}}{M_H}(t_a t_b H + t_a \overline{f_b} \overline{H}), \tag{15}$$

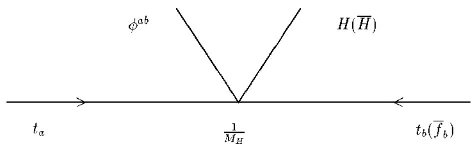

FIG. 2. A mechanism for generating bottom and tau masses. This mechanism can also be used to generate lighter generation masses.

as shown in the diagram of Fig. 2. Instead of using the symmetric flavon field ϕ^{ab} to generate the bottom and tau masses, a more economical way is to use only the fundamental fields ϕ^a. In this case we introduce nonrenormalizable operators of the form

$$\frac{1}{M_H}(T\overline{f}_a\phi^a\overline{H}+Tt_a\phi^a H), \tag{16}$$

which generate nonzero entries in the up, down and lepton mass matrices from the diagrams shown in Fig. 3.

Before going on to generate masses of lighter generations, let us discuss the diagonalization of the sector involving the third family and the extra vectorlike fields and the ensuing quark mixing matrix. It is obvious from the above equations that the rotation to get the top and bottom quark mass involves the same rotation on the left fields so that the Kobayashi-Maskawa (KM) matrix element V_{33} will be close to one. Let us do this in more detail. The up quark mass matrix is diagonalized by the following rotations both on the left and on the right:

$$V_L^u = V_R^u \approx \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{M}{\sqrt{M^2+\langle\phi^3\rangle^2}} & \frac{\langle\phi^3\rangle}{\sqrt{M^2+\langle\phi^3\rangle^2}} & 0 \\ \mathbf{0} & -\frac{\langle\phi^3\rangle}{\sqrt{M^2+\langle\phi^3\rangle^2}} & \frac{M}{\sqrt{M^2+\langle\phi^3\rangle^2}} & 0 \\ \mathbf{0} & 0 & 0 & 1 \end{pmatrix}$$
$$\times \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 1 & 0 & O\left(\frac{v_2}{M}\right) \\ \mathbf{0} & 0 & 1 & 0 \\ \mathbf{0} & O\left(\frac{v_2}{M}\right) & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 1 & 0 & 0 \\ \mathbf{0} & 0 & c & s \\ \mathbf{0} & 0 & -s & c \end{pmatrix}, \tag{17}$$

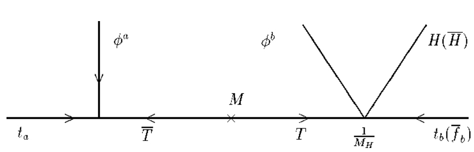

FIG. 3. Another mechanism for generating bottom and tau masses.

while the down and charged lepton mass matrices are diagonalized by

$$V_L^d = V_R^e \approx \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{M}{\sqrt{M^2+\langle\phi^3\rangle^2}} & \frac{\langle\phi^3\rangle}{\sqrt{M^2+\langle\phi^3\rangle^2}} \\ \mathbf{0} & -\frac{\langle\phi^3\rangle}{\sqrt{M^2+\langle\phi^3\rangle^2}} & \frac{M}{\sqrt{M^2+\langle\phi^3\rangle^2}} \end{pmatrix}, \tag{18}$$

up to corrections of order v^2/M^2 or $\epsilon v/M$. The last matrix on the right-hand side of Eq. (17) is the rotation in the heavy-sector and is unimportant for our discussion.[4] From the above discussion we see that the left-handed and right-handed top quarks, left-handed bottom and right-handed taus are actually maximally mixed states (for $\langle\phi\rangle \approx M$) of the third flavored generation and the extra matter in 10 of SU(5).

In the Appendix we show that the quark mixing matrix is in fact a 10×8 matrix with two 5×4 blocks. The upper block contains the light quark mixings in the 3×3 sector which can be identified with the KM matrix $\Sigma_{k=1,4}(V_L^{u\dagger})_{ik}(V_L^d)_{kj}$ $(i,j=1,2,3)$. We see at this level, from Eqs. (17) and (18), that the KM matrix is equal to unity. Departure of elements from those of the unit matrix is of the order v^2/M^2 or $\epsilon v/M$ which is negligible for M of order GUT or Planck scale.

In the lepton sector the situation is similar. The right-handed rotation defines a combination of e_3^c and E_1^c as the right-handed component of the tau lepton.

A note on scalar masses. It is interesting that the right-handed down squarks and left-handed sleptons remain approximately degenerate even though the U(3) symmetry is broken [7]. This is because the rotations on these fields are suppressed by ϵ. We mention the consequences of this towards the end of the paper.

III. MASS OF THE SECOND GENERATION FERMIONS

Second generation masses are generated when the remaining U(2) symmetry breaks down to U(1) (which keeps the first generation massless). We can obtain this breaking economically from the same symmetric flavon field ϕ^{ab} (or an additional symmetric field ϕ'^{ab}), when it gets VEVs of the same order in the (2,2), (2,3), and (3,2) entries. We parametrize this breaking by a parameter

$$\epsilon' \equiv \frac{\langle\phi^{22}\rangle}{M_H} \approx \frac{\langle\phi^{23}\rangle}{M_H} = \frac{\langle\phi^{32}\rangle}{M_H}. \tag{19}$$

The structure of the fermion mass matrices in the weak eigenstate basis is

[4] $s=1/\sqrt{2}$ for symmetric up matrix, and $s=O(v/M)$ otherwise.

$$L_{M_u}=(u_1^c u_2^c u_3^c U_1^c U_2^c)\times\begin{pmatrix}0&0&0&0&0\\0&\epsilon' v_2&\epsilon' v_2&0&0\\0&\epsilon' v_2&\epsilon v_2&0&\langle\phi^3\rangle\\0&0&0&v_2&M\\0&0&\langle\phi^3\rangle&M&v_1\end{pmatrix}\begin{pmatrix}u_1\\u_2\\u_3\\U_1\\U_2\end{pmatrix}+\text{H.c.},\tag{20}$$

$$L_{M_d}=(d_1^c d_2^c d_3^c D_2^c)\begin{pmatrix}0&0&0&0\\0&\epsilon' v_1&\epsilon' v_1&0\\0&\epsilon' v_1&\epsilon v_1&0\\0&0&\langle\phi^3\rangle&M\end{pmatrix}\begin{pmatrix}d_1\\d_2\\d_3\\D_1\end{pmatrix}+\text{H.c.},\tag{21}$$

$$L_{M_e}=(e_1^c e_2^c e_3^c E_1^c)\begin{pmatrix}0&0&0&0\\0&\epsilon' v_1&\epsilon' v_1&0\\0&\epsilon' v_1&\epsilon v_1&\langle\phi^3\rangle\\0&0&0&M\end{pmatrix}\begin{pmatrix}e_1\\e_2\\e_3\\E_2\end{pmatrix}+\text{H.c.}\tag{22}$$

Diagonalizing the third generation+heavy sector as in the previous section (with $\langle\phi^3\rangle\approx M$) will not change much the structure of the second generation sector:[5]

$$L_{M_u}=(u_1^c u_2^c u_{M3}^c u_{M4}^c u_{M5}^c)\times\begin{pmatrix}0&0&0&0&0\\0&\epsilon' v_2&\epsilon' v_2&(\epsilon' v_2)&(\epsilon' v_2)\\0&\epsilon' v_2&v_2&0&0\\0&(\epsilon' v_2)&0&M&0\\0&(\epsilon' v_2)&0&0&M\end{pmatrix}\begin{pmatrix}u_1\\u_2\\u_{3M}\\u_{4M}\\u_{5M}\end{pmatrix}+\text{H.c.},\tag{23}$$

$$L_{M_d}=(d_1^c d_2^c d_{M3}^c d_{M4}^c)\begin{pmatrix}0&0&0&0\\0&\epsilon' v_1&\epsilon' v_1&0\\0&\epsilon' v_1&\epsilon v_1&0\\0&(\epsilon' v_1)&0&M\end{pmatrix}\begin{pmatrix}d_1\\d_2\\d_{3M}\\d_{4M}\end{pmatrix}+\text{H.c.},\tag{24}$$

$$L_{M_e}=(e_1^c e_2^c e_{3M}^c e_{4M}^c)\begin{pmatrix}0&0&0&0\\0&\epsilon' v_1&\epsilon' v_1&(\epsilon' v_1)\\0&\epsilon' v_1&\epsilon v_1&0\\0&0&0&M\end{pmatrix}\begin{pmatrix}e_1\\e_2\\e_{3M}\\e_{4M}\end{pmatrix}+\text{H.c.},\tag{25}$$

where we denoted only the order of magnitude of relevant entries, and the index M denotes the approximate mass eigenstates. We can neglect the bracketed terms (which represent mixings between the light and heavy fields) since they yield only order $\epsilon' v/M$ mixings, without significantly changing mass eigenvalues.

Note that the obtained structure of mass matrices for the second and third generations is similar to the one of the U(2) model in Ref. [5].

Now, we see immediately that the following relations approximately hold:

$$\frac{m_c}{m_t}\approx\epsilon',\tag{26}$$

$$\frac{m_s}{m_b}\approx\frac{m_\mu}{m_\tau}\approx\frac{\epsilon'}{\epsilon}.\tag{27}$$

We see that for $\epsilon'\approx\epsilon^2$ we get good agreement with experiment. Moreover, we get the successful relations [5,22]

$$V_{cb}\approx\frac{m_s}{m_b}\approx\sqrt{\frac{m_c}{m_t}}.\tag{28}$$

These relations will also approximately hold in the SU(5) grand unified theory, with the relevant contributions coming from diagrams as the one shown in Fig. 2. Although not the main aim of this paper, one can make the relations more precise. We assume that the VEVs of ϕ^{22} and ϕ^{23} are of order $\epsilon' M_H$ since they break U(2) down to U(1). At first sight, the precise relations among the four observables m_c, m_s, m_μ, and V_{cb} can always be fixed to fit the experimental values by the four unknown numbers of order one, $\langle\phi^{22}\rangle/(\epsilon' M)$, $\langle\phi^{23}\rangle/(\epsilon' M)$, and the numbers of order one in front of the two nonrenormalizable operators in Eq. (15). However, if the flavon field ϕ^{ab} is a SU(5) singlet, then we have the relation $m_\mu=m_s$ at the GUT scale. However, notice that if ϕ^{ab} that contributes to the (2,2) entry is such a multiplet that $\phi^{ab}\overline{H}$ is a 45 of SU(5), then a more successful relation emerges at the GUT scale $m_\mu=3m_s$ [23]. For example, this can be achieved with ϕ^{ab} in 24 or 75 of SU(5). However, notice that this then forbids the up quark mass entries (and thus m_c), since the 45 is in the antisymmetric part of 10×10, thus prompting the use of more complicated representations. For example, if one wants the ϕ^{ab} to lie in the 24, one can also construct $\phi^{ab}\overline{H}$ as a 5 of SU(5), with some additional vector states (see next section).

Although somewhat beyond the scope of this paper, it is possible to extend this analysis to explain the masses of the first generation, using, for example, an antisymmetric representation of U(3). However, we chose not do so in this paper for clarity. Anyway the values of the lightest generation masses are still to some degree undetermined because of at least two reasons. Planck scale physics can alter the values of the lightest fermions through higher dimensional operators. Also the issue of whether the mass of the lightest quark is zero or not is far from being settled [24,25].

IV. ORIGIN OF THE NONRENORMALIZABLE OPERATORS AND THE HIERARCHY $M\ll M_H$

The nonrenormalizable operators with ϕ^{ab} introduced in Sec. III can be generated in two ways from the heavy scale

[5]Compact formulas for block diagonalizing such matrices can be found in the Appendix of [21].

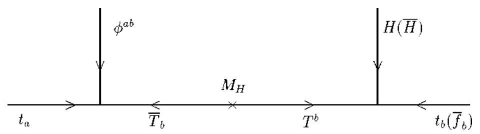

FIG. 4. A mechanism for generating the higher dimensional operators in Eq. (15).

[1,2,26]. One is the usual Froggatt-Nielsen way in which there are heavy fields on the matter line. For example, we can exchange a pair of $T^a(10)+\overline{T}_a(\overline{10})$ with mass M_H as shown in Fig. 4. Another way [26] is where there are some heavy fields on the line of the fields which get a VEV, as shown in Fig. 5 [here the ϕ^{ab} transforms as 24 under SU(5)]. The choice of 45 in Fig. 5 is convenient to generate the desired relation $m_\mu = 3m_s$ at the GUT scale.

Before going on one needs to explain the ratio of masses M/M_H. One can explain easily such a ratio by a discrete symmetry softly broken by the M mass term and the VEVs of field ϕ^a. However, there is also a deeper understanding for such a ratio as we now explain. As advertised before, it is entirely possible that the origin lies in the different scales of breaking SU(5) and SO(10). Notice that M is the SU(5) invariant mass of $T\overline{T}$, while M_H is the SU(5) invariant mass of $T^a\overline{T}_a$. However, we could have added also FN fields in the $F^a(5)$ and $\overline{F}_a(\overline{5})$ representations, also with mass M_H, which would generate new higher dimensional operators, but with no new contribution to the order of masses that did not exist before. Thus, we can imagine that T_a and $\overline{F}_a$ come from a 16_a of SO(10), and $\overline{T}_a$ and F_a from a $\overline{16_a}$ of SO(10), so that 16_a and $\overline{16_a}$ can be combined to have an SO(10) invariant mass M_H. On the other hand, new fields $F+\overline{F}$ are forbidden to have mass of order M because they would force the Yukawa couplings of the bottom and tau to be of order one, in contrast to our assumption that the Higgs doublet VEVs are of the order weak scale. Thus the mass M is an SU(5) invariant mass only of the pair $T+\overline{T}$, while $F+\overline{F}$ mass remains at the higher scale M_H.

A similar way of understanding this is that the U(3) symmetry is effectively a product of two symmetries U(3)×U(3). Imagine an SO(10) generalization of what we were doing so far in SU(5), with 16, $\overline{16}$, 16_a, etc., with all mass scales at M_H. The only thing we need is a 45 with a VEV (order M_H) in the 1 direction of SU(5). We need one fine-tuning to make one pair of tens of SU(5) light (mass M), while all other FN fields remain heavy (mass M_H). This gives the effective SU(5) theory that we have. Now the U(3) breaking will affect only the states in t_a, and is thus an effective U(3) carried by the t_a. Breaking of the U(3) in the $\overline{f}_a$ sector is

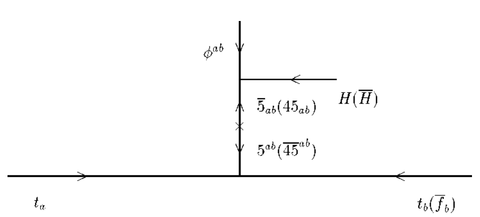

FIG. 5. Alternate mechanism for generating the higher dimensional operators in Eq. (15).

suppressed by $1/M_H$, and it looks like a separate U(3) on those states. [For a recent work with product of SU(3)'s as horizontal symmetries see [27].]

V. FINAL REMARKS AND CONCLUSIONS

One can ask how low can the mass of the extra vector matter fields be [28,29]. Suppose the mass matrices (20)–(22) were given without resorting to horizontal symmetries. Then, interestingly enough, existing experimental limits on V_{tb} or flavor changing Z decays would allow M to be as low as the weak scale, because all effects quickly decouple as M becomes large[6] (see the Appendix).

However, a much stronger limit on M comes from the fact that we are breaking global flavor symmetries spontaneously. The Goldstone bosons, known as *familons* [33,34], will actually produce too many flavor changing neutral currents (FCNCs), unless the scale is higher than about 2×10^{11} GeV [35]. The bound applies to the scale ϵM in the chain of scale of horizontal symmetry breaking $U(3)\stackrel{M}{\rightarrow}U(2)\stackrel{\epsilon M}{\rightarrow}U(1)\stackrel{v}{\rightarrow}$nothing. If we assume that M lies near a typical GUT scale, and that the lightest generation masses are generated by the last scale in the chain, v may reside actually very close to the lower bound [4]. Interestingly enough, the symmetry being broken at the lowest scale, U(1), has a color anomaly, so that we have an axion in the theory coming from the family symmetry [36]. Then there is also an upper bound on v as well coming from cosmology [37], $v<10^{12}$ GeV or so.

In this paper we envisioned the underlying theory to be a supersymmetric one, although all conclusions presented here concern the fermionic sector and are valid also in a nonsupersymmetric version. In SUSY, the U(3) symmetry acts on scalar partners as well. Here we will mention a few main points regarding the scalar masses and leave a more complete investigation to a future publication. It is interesting that because of the choice of representations of extra matter $(10+\overline{10})$ right-handed squarks and left-handed sleptons remain approximately degenerate in all three generations. This has some profound differences compared to the recent analysis based on U(2). In comparison to [4,5], we expect $\mu\rightarrow e\gamma$ and the electric dipole moment of the electron to be suppressed by ϵ^2 and the $K-\overline{K}$ mass difference by $(m_s/m_K)^2$. Recently, it has been pointed out that if there is more than one operator responsible for the same eigenvalue in the Yukawa matrix, the misalignment of A terms and Yukawa terms can actually produce the SUSY ϵ_K problem [38]. Our theory for lighter generations should essentially come from the operator containing ϕ^{ab} (and possibly from an antisymmetric flavon for the lightest generation) and is thus of the same form as the U(2) models which relax this problem. However, a more precise prediction of fermion masses and mixings may require more fields (as alluded in Sec. III). This then may require proportionality in order to avoid the problem [7].

[6]Present limits imply only $V_{tb}>0.05$ or so [30]. Also oblique corrections quickly dissapear as the $SU(2)_L$ invariant mass M becomes larger [31,32].

To summarize, in order for the Yukawa couplings of order one (top and/or bottom quarks) to find an explanation within the Froggat-Nielsen type of horizontal symmetry approach, it is necessary that the third generation particles mix maximally with some extra matter fields or that the Higgs doublet mixes maximally with extra scalars. In this paper we considered the first approach.

We have considered the maximal U(3) horizontal symmetry scheme, with the emphasis on the two heaviest generations and the large top quark Yukawa coupling. A simple scheme can be achieved with the extra matter in $10+\overline{10}$pair of SU(5). The third generation particles get their masses when U(3) is broken to U(2). The top quark mass is naturally of order one. Bottom and tau masses are suppressed because of a hierarchy in effective Yukawa couplings and *not* from the hierarchy in the Higgs doublet VEVs. The hierarchy of effective Yukawa couplings can come from hierarchies between scales of the breaking of different grand unified groups. Hierarchies and mixings between the second and third generations are obtained by introducing a single parameter ϵ' representing the breaking $\mathrm{U}(2)\to\mathrm{U}(1)$. As a consequence, we obtain the successful relations $V_{cb}\approx m_s/m_b \approx\sqrt{m_c/m_t}$.

ACKNOWLEDGMENTS

I am grateful to Lawrence Hall for many valuable comments and discussions and for constant encouragement. It is also a pleasure to thank Zurab Berezhiani, Gia Dvali, and Goran Senjanović for some important remarks, and to K.S. Babu, Biswajoy Brahmachari, Alejandra Melfo, Rabi Mohapatra, and Atsushi Yamada for many useful comments.

APPENDIX

In this Appendix we derive the quark mixing matrices for the case when vectorlike matter with the content of $10+\overline{10}$ of SU(5) are added to the three generations of the SM:

$$q_a u_a^c d_a^c l_a e_a^c,$$
$$Q_1 U_1^c E_1^c,$$
$$Q_2^c U_2 E_2. \tag{A1}$$

The quark mixing matrix is derived as follows (see also Ref. [39]). The charged weak current interaction is

$$L_W \sim [\overline{\mathbf{u}}^T\gamma^\mu \mathbf{d}+\overline{U_1}\gamma^\mu D_1+U_2^c\gamma^\mu \overline{D_2^c}]W_\mu^+. \tag{A2}$$

Mass eigenstates are related to the weak eigenstates by

$$\mathbf{u}_M=\mathbf{V}_L^{u\dagger}\begin{pmatrix}\mathbf{u}\\ U_1\\ U_2\end{pmatrix},$$

$$\mathbf{u}_M^{cT}=(\mathbf{u}^c U_1^c U_2^c)\mathbf{V}_R^u \tag{A3}$$

and

$$\mathbf{d}_M=\mathbf{V}_L^{d\dagger}\begin{pmatrix}\mathbf{d}\\ D_1\end{pmatrix},$$

$$\mathbf{d}_M^{cT}=(\mathbf{d}^c D_2^c)\mathbf{V}_R^d, \tag{A4}$$

where $V_{L,R}^u$ and $V_{L,R}^d$ are unitary 5×5 and 4×4 matrices, respectively.

The weak current interaction in the mass eigenstate basis is

$$L_W \sim [\overline{u_{Ma}}\gamma^\mu (V_L^{u\dagger})_{ai}(V_L^d)_{ib}d_{Mb} + u_{Ma}^c\gamma^\mu (V_R^{u\dagger})_{a5}(V_R^d)_{4b}\overline{d_{Mb}^c}]W_\mu^+, \tag{A5}$$

where $i=1,2,3,4$, $a=1,2,3,4,5$, and $b=1,2,3,4$.

From Eq. (A5) we can read off the quark mixing matrix which is now a 10×8 matrix and consists of two 5×4 blocks,

$$V_{a,b}=(V_L^{u\dagger})_{ai}(V_L^d)_{ib},$$

$$V_{5+a,5+b}=(V_R^{u\dagger})_{a5}(V_R^d)_{4b}, \tag{A6}$$

and is thus in general *not* unitary. The SM mixing matrix is in the upper 3×3 block of $V_{a,b}$. The approximate form of V for our model is discussed in the text.

Let us now turn to the neutral current:

$$L_Z \sim [\overline{\mathbf{u}}^T\gamma^\mu \mathbf{u}+\overline{U_1}\gamma^\mu U_1-U_2^c\gamma^\mu \overline{U_2^c}-\overline{\mathbf{d}}^T\gamma^\mu \mathbf{d}-\overline{D_1}\gamma^\mu D_1 + D_2^c\gamma^\mu \overline{D_2^c}-j_{em}^\mu]Z_\mu. \tag{A7}$$

Now let us go to the mass eigenstate basis. The electromagnetic part is flavor diagonal. However, the weak part has flavor changing pieces in the following terms:

$$L_Z^{FCNC}\sim \overline{u}_{Ma}^T(V_L^{u\dagger})_{ai}(V_L^u)_{ic}\gamma^\mu u_{Mc} - u_{Ma}^c(V_R^{u\dagger})_{a5}(V_R^u)_{5c}\gamma^\mu \overline{u_{Mc}^c} + d_{Mb}^c(V_R^{d\dagger})_{b4}(V_R^u)_{4d}\gamma^\mu \overline{d_{Md}^c}, \tag{A8}$$

where $i=1,2,3,4$, $a,c=1,2,3,4,5$, and $b,d=1,2,3,4$. There is no flavor changing part involving left-handed down quarks, reflecting the fact that there is no ‘‘mismatch’’ in that sector.

From Eqs. (17), (18) and (23), (24) we see that in our model the flavor changing Z interactions for example for the left-handed up quarks have the following order of magnitude:

$$(V_{uu}^{Z\text{-}FCNC})_{ac}\equiv(V_L^{u\dagger})_{ai}(V_L^u)_{ic}$$

$$
= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{\epsilon'}{\epsilon}\frac{v}{M} & \frac{\epsilon'}{\epsilon}\frac{v}{M} \\ 0 & 0 & 1 & \frac{v}{M} & \frac{v}{M} \\ 0 & \frac{\epsilon'}{\epsilon}\frac{v}{M} & \frac{v}{M} & 1 & 1 \\ 0 & \frac{\epsilon'}{\epsilon}\frac{v}{M} & \frac{v}{M} & 1 & 1 \end{pmatrix}. \tag{A9}
$$

These effects are, however, negligible for M near the GUT scale.

To summarize, there are several consequences of the ‘‘mismatch.’’

(i) The quark mixing matrix is no longer unitary (neither as a complete 10×8 matrix, neither in the two 5×4 blocks separately), unless the extra $\overline{10}$ is totally decoupled. In particular, it is not unitary in the 3×3 standard sector.

(ii) W_L couples also to the ‘‘right-handed’’ mass eigenstates $\mathbf{u}_M^c$ in the lower 5×5 block.

(iii) Couplings of fermions to the Z boson are flavor changing.

[1] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979).

[2] Z. Berezhiani, Phys. Lett. **129B**, 99 (1983); **150B**, 177 (1985); S. Dimopoulos, *ibid.* **129B**, 417 (1983).

[3] F. Abe *et al.*, Phys. Rev. Lett. **74**, 2626 (1995); S. Abachi *et al.*, *ibid.* **74**, 2632 (1995).

[4] R. Barbieri, G. Dvali, and L. J. Hall, Phys. Lett. B **377**, 76 (1996); R. Barbieri and L. J. Hall, Nuovo Cimento A **110**, 1 (1997).

[5] R. Barbieri *et al.*, Nucl. Phys. **B493**, 3 (1997); R. Barbieri, L. J. Hall, and A. Romanino, Phys. Lett. B **401**, 47 (1997).

[6] M. Dine, R. Leigh, and A. Kagan, Phys. Rev. D **48**, 4269 (1993).

[7] Z. Berezhiani, in *Proceedings of the Fourth International Conference on Supersymmetry, SUSY 96*, College Park, edited by R. N. Mohapatra and A. A. Rašin [Nucl. Phys. (Proc. Suppl.) **52A**, 153 (1997)]; (hep-ph/9607363); hep-ph/9609342.

[8] R. Barbieri and L. Hall, Nucl. Phys. **B319**, 1 (1989).

[9] K. S. Babu, B. S. Balakrishna, and R. N. Mohapatra, Phys. Lett. B **237**, 221 (1990).

[10] B. S. Balakrishna, A. Kagan, and R. N. Mohapatra, Phys. Lett. B **205**, 345 (1988); E. Nardi, E. Roulet, and D. Tommasini, Nucl. Phys. **B386**, 239 (1992); B. Mukhopadhyaya and S. Nandi, Phys. Rev. D **46**, 5098 (1992); W.-S. Hou, Phys. Rev. Lett. **69**, 3587 (1992); E. Ma, Phys. Lett. B **322**, 363 (1994); V. Barger, M. S. Berger, and R. J. N. Phillips, Phys. Rev. D **52**, 1663 (1995); T. Morozumi *et al.*, Phys. Lett. B **410**, 233 (1997).

[11] Z. Berezhiani and R. Rattazzi, Nucl. Phys. **B407**, 249 (1993).

[12] R. Barbieri *et al.*, Nucl. Phys. **B432**, 49 (1994).

[13] Z. Berezhiani *et al.*, in *Proceedings of Quarks ’86*, Tbilisi, edited by A. Tavkhelidze *et al.* (INR Press, Moscow, 1987), p. 209.

[14] P. Pouliot and N. Seiberg, Phys. Lett. B **318**, 169 (1993); D. Kaplan and M. Schmaltz, Phys. Rev. D **48**, 4269 (1993); L. J. Hall and H. Murayama, Phys. Rev. Lett. **75**, 3985 (1995); P. H. Frampton and O. C. W. Kong, Phys. Rev. D **53**, 2293 (1996); Phys. Rev. Lett. **77**, 1699 (1996); P. H. Frampton and T. W. Kephart, Phys. Rev. D **51**, 1 (1995); A. Pomarol and D. Tommasini, Nucl. Phys. **B466**, 3 (1996); N. Arkani-Hamed, H. C. Cheng, and L. J. Hall, *ibid.* **B472**, 95 (1996); Phys. Rev. D **54**, 2242 (1996); K. S. Babu and S. M. Barr, Phys. Lett. B **387**, 87 (1996).

[15] M. Joyce and N. Turok, Nucl. Phys. **B416**, 389 (1994); C. Barnes and N. Turok, Phys. Rev. D **56**, 1989 (1997).

[16] See the review from J. Erler and P. Langacker, in *Particle Data Group*, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996); a recent review can be found in J. L. Rosner, hep-ph/9704331.

[17] T. Moroi, H. Murayama, and T. Yanagida, Phys. Rev. D **48**, 2995 (1993); B. Brahmachari, U. Sarkar, and K. Sridhar, Mod. Phys. Lett. A **8**, 3349 (1993).

[18] B. Greene *et al.*, Nucl. Phys. **B278**, 667 (1986); **B292**, 606 (1987).

[19] For a review of mirror matter see J. Maalampi and M. Roos, Phys. Rep. **186**, 53 (1990).

[20] For a similar conclusion in a different context see K. S. Babu and R. N. Mohapatra, Phys. Rev. D **41**, 1286 (1990); see also Ref. [11].

[21] K. S. Babu and S. M. Barr, Phys. Rev. D **56**, 2614 (1997).

[22] A similarly successful prediction $V_{cb}\approx m_s/m_b - m_c/m_t$ was obtained by Balakrishna, Kagan, and Mohapatra [10].

[23] H. Georgi and C. Jarlskog, Phys. Lett. **86B**, 297 (1979).

[24] H. Leutwyler, in *Proceedings of the 2nd IFT Workshop on Yukawa Couplings and the Origin of Mass*, Gainesville, 1994, edited by P. Ramond (International Press, Cambridge, 1996), p. 166.

[25] K. Choi, C. W. Kim, and W. K. Sze, Phys. Rev. Lett. **61**, 794 (1988); for a review, see T. Banks, Y. Nir, and N. Seiberg, in *Proceedings of the 2nd IFT Workshop on Yukawa Couplings and the Origin of Mass* [24], p. 26.

[26] L. Ibañez and G. G. Ross, Phys. Lett. B **332**, 100 (1994).

[27] A. Anselm and Z. Berezhiani, Nucl. Phys. **B484**, 97 (1997).

[28] M. Bowick and F. del Aguila, Phys. Lett. **119B**, 144 (1982); Nucl. Phys. **B224**, 107 (1983); P. Fishbane, S. Meshkov, and P. Ramond, Phys. Lett. **134B**, 81 (1984).

[29] For a discrete symmetry approach where the mass M is low, see G. Senjanović, F. Wilczek, and A. Zee, Phys. Lett. **141B**, 389 (1984).

[30] D. Gerdes, for CDF and D0 Collaborations, hep-ex/9609013.

[31] For precise expressions for oblique corrections from vectorlike quarks see L. Lavoura and J. P. Silva, Phys. Rev. D **47**, 2046 (1993).

[32] T. Inami, C. S. Lim, and A. Yamada, Mod. Phys. Lett. A **7**, 2789 (1992).

[33] D. Reiss, Phys. Lett. **115B**, 217 (1982); F. Wilczek, Phys. Rev. Lett. **49**, 1549 (1982); G. B. Gelmini, S. Nussinov, and T.

Yanagida, Nucl. Phys. **B219**, 31 (1983); D. Chang, P. Pal, and G. Senjanović, Phys. Lett. **153B**, 407 (1985).

[34] For a review, see J. E. Kim, Phys. Rep. **150**, 1 (1987).

[35] S. Adler *et al.*, Phys. Rev. Lett. **76**, 1421 (1996).

[36] R. Barbieri, D. V. Nanopoulos, and D. Wyler, Phys. Lett. **106B**, 303 (1981); A. Davidson and K. C. Wali, Phys. Rev. Lett. **48**, 11 (1982).

[37] J. Preskil, M. B. Wise, and F. Wilczek, Phys. Lett. **120B**, 127 (1983); L. F. Abbott and P. Sikivie, *ibid.* **120B**, 133 (1983); M. Dine and W. Fischler, *ibid.* **120B**, 137 (1983).

[38] Y. Nir and R. Rattazzi, Phys. Lett. B **382**, 363 (1996); Y. Grossmann, Y. Nir, and R. Rattazzi, hep-ph/9701231.

[39] P. Langacker and D. London, Phys. Rev. D **38**, 886 (1988).