Implications of 4 texture zeros mass matrices for neutrino anomalies

P. S. Gill* and Manmohan Gupta
Centre of Advanced Study, Department of Physics, Panjab University, Chandigarh-160014, India

(Received 2 May 1997; revised manuscript received 14 October 1997; published 19 February 1998)

Phenomenological 4 texture zeros mass matrices, successful in accommodating the CKM phenomenology, are used to simultaneously explain the three neutrino anomalies: the solar neutrino problem (SNP), the atmospheric neutrino problem (ANP), and the LSND anomaly. When the SNP is resolved through vacuum oscillations, we obtain a solution implying large mixing. In case the SNP is resolved through the MSW mechanism, the neutrino masses follow a ''natural'' hierarchy.

[S0556-2821(98)01007-8]

PACS number(s): 12.15.Ff, 14.60.Pq, 96.60.Jw

Neutrino physics at present has three anomalies usually referred to as the solar neutrino problem (SNP) [1–6], the atmospheric neutrino problem (ANP) [7–10] and the ''oscillations'' $\overline{\nu_\mu} \rightarrow \overline{\nu_e}$ supposedly observed at the Liquid Scintillator Neutrino Detector (the LSND anomaly) [11,12]. Even in the absence of direct evidence, neutrino flavor mixing is a widely recognized explanation for the above mentioned anomalies. To generate flavor mixing, the neutrinos have to be massive which would have cosmological implications [13] as well. This motivates one to consider a specific form of neutrino mass matrices. In this context, texture specific mass matrices have been considered with a fair degree of success [14].

Our purpose in this paper is to examine the implications of a particular kind of 4 texture zeros phenomenological mass matrices [15] which have shown a good deal of success in accommodating the Cabibbo-Kobayashi-Maskawa (CKM) phenomenology [16]. In particular, quark mass matrices of the form

$$M_i = \begin{pmatrix} 0 & A_i & 0 \\ A_i & D_i & B_i \\ 0 & B_i & C_i \end{pmatrix}, \quad [\mathrm{i=u,d}] \qquad (1)$$

not only accommodate $m_t = 175 \pm 15$ GeV [17] but are also able to fit the data [18], for example $|V_{cb}| = 0.040 \pm 0.003$, $R_{ub} = |V_{ub}/V_{cb}| = 0.08 \pm 0.02$ and $|V_{td}| = 0.0064 - 0.0124$.

Further, it has also been shown that such mass matrices could be generated from grand unified theories (GUTs) [19,20] as well as that these are ''natural'' in the sense of Peccei and Wang [21]. Therefore it becomes interesting to examine their implications for the three neutrino anomalies discussed above.

Before we go into our attempts to study the implications of the 4 texture zeros mass matrices for the three neutrino anomalies, we would like to present data in a manner which would facilitate discussion as well as comparison with our predictions. In the case of SNP, what is observed is the flux of the electron neutrino (ν_e) received on the earth's surface. In Table I, we have presented the solar neutrino data from four experiments, with the predictions for these from the two standard solar models: Bahcall and Pinsonneault (BP) [5] and Turck-Chieze and Lopes (TCL) [6]. In the table, the deviation of the values of R_ϕ from unity is the solar neutrino problem (SNP). Invoking neutrino oscillations to resolve SNP, the data is usually parametrized in terms of two flavors. The second flavor into which ν_e oscillates is muon neutrino (ν_μ) and the survival probability is given as

$$P_{ee} = 1 - P_{e\mu}, \qquad (2)$$

where $P_{e\mu}$, the transition probability, is given by

$$P_{e\mu} = \sin^2 2\theta \sin^2\left(\frac{1.267\Delta m^2 L}{E}\right), \qquad (3)$$

with θ representing the mixing angle, $L(=ct)$ the distance from the sun to the earth in meters, $\Delta m^2 = m_1^2 - m_2^2$ in eV^2 and E is the neutrino energy in MeV.

The above equation describes an oscillation with amplitude equal to $\sin^2 2\theta$ and the oscillation length λ is given by

$$\lambda = 2.48\left(\frac{E}{\Delta m^2}\right). \qquad (4)$$

It needs to be mentioned that if oscillation length λ is much shorter than the size of the neutrino source or that of the detector (or both) the periodic term in Eq. (3) averages to 1/2 and the oscillation probability becomes

TABLE I. Results for solar neutrino flux in SNU (1 SNU $= 10^{-36}$ capture/atom/sec).

Experiment	Data	BPSSM	TCLSSM	$R_\phi = \frac{\text{Data}}{\text{BPSSM}}$
Homestake [1]	2.55 ± 2.0	8.0 ± 1.0	6.4	0.27 ± 0.03
Kamiokande [2]	2.89 ± 0.42	5.7 ± 2.4	4.4	0.44 ± 0.06
Gallex [3]	79 ± 12	132 ± 21	123	0.56 ± 0.06
Sage [4]	79 ± 19	132 ± 21	123	0.50 ± 0.09

*Permanent address: Sri Guru Gobind Singh College, Chandigarh, India.

TABLE II. Atmospheric neutrino data showing double ratio of fluxes for ν_μ and ν_e.

Experiment	Energy	$R_D = \frac{\phi(\mu/e)_{\rm obs}}{\phi(\mu/e)_{MC}}$
KAM II [7]	sub-GeV	0.60 ± 0.08
KAM II [7]	multi-GeV	0.57 ± 0.11
IMB [8]	sub-GeV	0.54 ± 0.09
NUSEX [9]	sub-GeV	0.99 ± 0.40
SOUDAN-2 [10]	sub-GeV	0.69 ± 0.21

$$P_{e\mu} = \frac{1}{2}\sin^2 2\theta. \tag{5}$$

The data, presented in Table I, can be fitted with the following range of parameters:

$$\Delta m^2 = 6.0 \times 10^{-11} \ \ \mathrm{eV}^2 \tag{6}$$

and

$$\sin^2 2\theta = 0.96. \tag{7}$$

The above values actually correspond to the range

$$5 \times 10^{-11} \ \ \mathrm{eV}^2 < \Delta m^2 < 10^{-10} \ \ \mathrm{eV}^2 \tag{8}$$

and

$$0.7 \leqslant \sin^2 2\theta \leqslant 1.0 \tag{9}$$

as emphasized by Bahcall and Krastev recently [22].

A more popular and natural solution is the Mikhayev-Smirnov-Wolfenstein (MSW) oscillation mechanism [23] based on $\nu_e \rightarrow \nu_x$ conversion in the solar interior. The required range of Δm^2 and $\sin^2 2\theta$ corresponds to the two solutions [22,24]

$$\Delta m_\odot^2 \sim 10^{-5} \ \ \mathrm{eV}^2, \quad \sin^2 2\theta \approx 10^{-2} \quad (\text{small angle}), \tag{10}$$

$$\Delta m_\odot^2 \sim 10^{-5} \ \ \mathrm{eV}^2, \quad \sin^2 2\theta \approx 1 \quad (\text{large angle}). \tag{11}$$

The second neutrino anomaly is concerning cosmic ray neutrinos originating from the pion and subsequent muon decays. The depletion of ν_μ flux relative to ν_e flux has been observed in several experiments [7–10]. The results of these experiments are summarized in Table II. In the third column of Table II, we present R_D, the ratio of the ratios of observed to Monte Carlo predictions of ν_μ and ν_e fluxes. The deviation of R_D from the expected value of 1 is the atmospheric neutrino problem (ANP). The data (presented in Table II) clearly shows the depletion of atmospheric ν_μ flavor by a factor of ~ 2. This can be summarized in terms of two flavor mixing [25] as

$$\Delta m_{\mu x}^2 \sim 10^{-2} \ \ \mathrm{eV}^2, \quad \sin^2 2\theta_{\mu x} \sim 1, \tag{12}$$

where x can be e or τ.

The LSND experiment at Los Alamos National Lab has brought the first direct though controversial evidence for the nonzero neutrino masses and mixing angles and is the ''third neutrino problem.'' The experiment might have observed the neutrino oscillations $\overline{\nu_\mu} \rightarrow \overline{\nu_e}$ with two flavor parameters:

$$\Delta m_{e\mu}^2 \geqslant 0.3 \ \ \mathrm{eV}^2 \tag{13}$$

and

$$\sin^2 2\theta = 10^{-3} - 4 \times 10^{-2}. \tag{14}$$

The result may be marginally compatible with the earlier E776 experiment [26] at BNL and KARMEN experiment [27] at Rutherford Appleton Lab.

The LSND conclusions for the candidate events of $\overline{\nu_\mu} \rightarrow \overline{\nu_e}$ oscillation when combined with other experiments [26–28], translate [29] the constraints (13) and (14) to

$$0.3 \ \ \mathrm{eV}^2 \leqslant \Delta m_{\mathrm{LSND}}^2 \leqslant 2.3 \ \ \mathrm{eV}^2, \tag{15}$$

$$\Delta m^2 = 0.2 - 100 \ \ \mathrm{eV}^2, \tag{16}$$

and

$$\sin^2 2\theta \cong 10^{-3} - 4 \times 10^{-2}. \tag{17}$$

A few observations of the resolution of three neutrino anomalies in terms of two flavor oscillations are in order. To provide a simultaneous solution to these we need at least three independent mass differences [29,30], which requires the introduction of the fourth neutrino. The fourth neutrino in the present scenario must be sterile [SU(2) singlet]. The other observation is regarding the fact that data has been analyzed keeping in mind oscillations between two flavors whereas if we have to invoke mixing between three flavors simultaneously the data should also be analyzed accordingly.

Several attempts have been made in the three flavor oscillation scenario to reconcile the data pertaining to the neutrino anomalies [22,31,32]. It is hoped that the three flavors would be richer in phenomenology due to the fact that deficit or absence of a particular species could be due to the oscillation of that to the other two flavors. In this context, Cardall and Fuller [32] have observed that a solution to three anomalies could be obtained wherein one mass scale dominance (OMSD) [33] is presumed. Apart from the solution mentioned with OMSD, solutions within three flavor mixing are possible when we have degenerate neutrinos [34] or there is an inverted hierarchy [35].

An important point in the case of three neutrinos to be noted pertains to the possibility that if oscillations become fast or $\Delta m^2 L/E$ becomes large for the case of solar neutrinos, two flavor oscillations predict P_{ee} to be 1/2 whereas in the three flavor case this number can be pushed to 1/3 [36] which could be in agreement with data from all experiments except Homestake.

To begin with, we consider lepton mass matrices which are written in analogy with the quark mass matrices as given by Eq. (1). At present we do not bother about the origin of such mass matrices. In fact these could represent the left-handed Dirac neutrinos with right-handed neutrinos being SU(2) singlets or there could be an ansatz for neutrino mass matrices in the sense of Babu and Shafi [37]. Further unlike quark mass matrices where the CKM mixing matrix requires the elements of any of the mass matrices to satisfy the hierarchy, $|A| \ll |B| \approx D < C$, no such restriction is imposed on the neutrino mass matrices. Therefore, M_ν is written as

$$M_\nu = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & D_\nu & B_\nu \\ 0 & B_\nu & C_\nu \end{pmatrix}, \tag{18}$$

where $A_\nu = |A_\nu e^{i\alpha}$ and $B_\nu = |B_\nu| e^{i\beta}$. The mass matrix M_ν can be expressed as

$$M_\nu = P_\nu \overline{M}_\nu P_\nu^\dagger, \tag{19}$$

where $\overline{M}_\nu$ is the real mass matrix

$$\overline{M}_\nu = \begin{pmatrix} 0 & |A_\nu| & 0 \\ |A_\nu| & D_\nu & |B_\nu| \\ 0 & |B_\nu| & C \end{pmatrix} \tag{20}$$

and

$$P_\nu = \mathrm{diag}(1, e^{-i\alpha}, e^{-i(\alpha+\beta)}). \tag{21}$$

The real matrix $\overline{M}_\nu$ can be diagonalized exactly by the orthogonal transformations, for example,

$$\overline{M}_\nu = O_\nu^T M_\nu^{\mathrm{diag}} O_\nu, \tag{22}$$

where $M_\nu^{\mathrm{diag}} = (m_1, -m_2, m_3)$, m_i being the neutrino mass eigenstates. In a similar manner, replacing ν by l we can get the diagonalizing transformation for the charged lepton sector. The detailed form of the diagonalizing transformation is given in the Appendix.

The mixing matrix V_ν in the leptonic sector is defined as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{23}$$

where ν_e, ν_μ, ν_τ are the three flavor eigenstates whereas ν_1, ν_2, ν_3 are the three mass eigenstates. The matrix V_ν is given by

$$V_\nu = O_\nu^\dagger P_{\nu l} O_l, \tag{24}$$

where $O_{\nu,l}$ are the diagonalizing transformations and the phase matrix $P_{\nu l} = P_\nu^\dagger P_l$.

In the absence of any observation of CP violation in the leptonic sector as well as for the sake of simplicity, we ignore the phase in V_ν, so that

$$V_\nu = O_\nu^T O_l. \tag{25}$$

The parametrization of V_ν in terms of CKM like angles is presented for the sake of ready reference, for example [18],

$$V_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}. \tag{26}$$

To find the survival (disappearance) probabilities from the mixing matrix, first we sketch the details pertaining to survival (disappearance) probabilities in terms of three flavors as well as the corresponding oscillations.

The time evoluation of a massive neutrino with momentum $\vec{p}$ produced in a state ν_α at time t=0 is given by

$$\nu_\alpha(t) = e^{i\vec{p}\cdot\vec{r}} \sum_{i=1}^{3} V_{\alpha i} \nu_i, \tag{27}$$

where $E_i = (p^2 + m_i^2)^{1/2}$. If neutrinos are nondegenerate the three terms in Eq. (27) get out of phase, i.e., a state ν_α acquires a component ν_β $(\beta \neq \alpha)$. The general transition and survival probabilities P_{ij} for 3 generations [38] are

$$\begin{aligned} P_{ij} = \sum_{k=1}^{3} (V_{ik}V_{jk})^2 &+ 2|V_{i1}V_{i2}V_{j1}V_{j2}| \cos\left(\frac{2.53\Delta_{21}^2 L}{E}\right) \\ &+ 2|V_{i1}V_{i3}V_{j1}V_{j3}| \cos\left(\frac{2.53\Delta_{31}^2 L}{E}\right) \\ &+ 2|V_{i3}V_{i2}V_{j3}V_{j2}| \cos\left(\frac{2.53\Delta_{32}^2 L}{E}\right), \end{aligned} \tag{28}$$

where E (in MeV) is the energy, L (in m) is the distance between sun and earth and $\Delta m_{ij}^2 = m_i^2 - m_j^2$ (in eV^2). The survival probability of ν_e in solar neutrino flux, P_{ee}, is approximately equal to the ratio R_D whereas $P_{\mu\mu}$ is measured in the experiments as the double ratio,

$$R_D = \frac{\phi\left(\frac{\nu_\mu}{\nu_e}\right)_{obs}}{\phi\left(\frac{\nu_\mu}{\nu_e}\right)_{MC}} = \frac{P_{\mu\mu} + P_{\mu e}/R}{P_{ee} + RP_{e\mu}}, \tag{29}$$

where R=2.45 is the ratio of the number of ν_e's to ν_μ's in the atmosphere.

The oscillatory functions in Eq. (28) get averaged out when arguments of these, e.g., $2.53\Delta m_{ij}^2 L$, are large. In particular, for the case solar neutrinos propagating in vacuum, the survival probability P_{ee}, when $\Delta m_{ij}^2 L$ is large, becomes

$$P_{ee} = \sum_{k=1}^{3} |V_{ek}|^4 + |V_{e1}|^2|V_{e2}|^2 + |V_{e1}|^2|V_{e3}|^2 + |V_{e2}|^2|V_{e3}|^2, \tag{30}$$

whereas, for ν_μ,

$$\begin{aligned} P_{\mu\mu} = \sum_{k=1}^{3} |V_{\mu k}|^4 &+ |V_{\mu 1}|^2|V_{\mu 2}|^2 + |V_{\mu 1}|^2|V_{\mu 3}|^2 \\ &+ |V_{\mu 2}|^2|V_{\mu 3}|^2. \end{aligned} \tag{31}$$

The transition probability of $\overline{\nu_e} \to \overline{\nu_\mu}$ in the LSND experiment from Eq. (28) becomes

TABLE III. Solutions for degenerate neutrinos for $m_3=2.5$ eV.

$R_{l3}=\frac{D_l}{m_{l3}}$	$R_{\nu 3}=\frac{D_\nu}{m_{\nu 3}}$	m_2 (eV)	m_1 (eV)	P_{ee}	$P_{\mu\mu}$	$P_{e\mu}\times 10^{-2}$
0.2	0.46	1.30	0.3	0.43	0.51	0.34
	0.44	1.31	0.4	0.41	0.50	0.33
	0.26	1.33	0.3	0.43	0.56	0.54
	0.17	1.36	0.3	0.49	0.60	0.53
	0.10	1.39	0.3	0.54	0.53	0.15
0.4	0.41	1.30	0.4	0.35	0.48	0.29
	0.35	1.33	0.3	0.52	0.52	0.53
	0.25	1.35	0.4	0.42	0.52	0.54
	0.10	1.35	0.3	0.56	0.59	0.55
0.6	0.35	1.30	0.3	0.42	0.44	0.54
	0.30	1.30	0.3	0.46	0.49	0.52
	0.20	1.35	0.4	0.49	0.46	0.17
	0.12	1.35	0.3	0.56	0.50	0.15
	0.10	1.36	0.3	0.58	0.50	0.54
0.8	0.34	1.30	0.3	0.48	0.40	0.52
	0.31	1.32	0.3	0.49	0.40	0.53

$$P_{e\mu}(\mathrm{LSND})=\sum_{k=1}^{3}|V_{ek}|^2|V_{\mu k}|^2 +2|V_{e1}V_{e2}V_{\mu 1}V_{\mu 2}|\cos\left(2.53\frac{\Delta m_{12}^2 L}{E}\right) +2|V_{e1}V_{e3}V_{\mu 1}V_{\mu 3}|\cos\left(2.53\frac{\Delta m_{31}^2 L}{E}\right) +2|V_{e2}V_{e3}V_{\mu 2}V_{\mu 3}|\cos\left(2.53\frac{\Delta m_{32}^2 L}{E}\right). \tag{32}$$

Using Eq. (28), we have reproduced required probabilities for different ranges of neutrino masses. In Table III, we have presented the results of our calculations when SNP is resolved through vacuum oscillations. The solution has been obtained through ''rapid'' oscillations; therefore the mass scale required is quite different from the usual vacuum oscillation solution. A typical solution with corresponding mixing matrix is given as follows:

$$m_3=2.5\ \text{eV},\quad m_2=1.3\ \text{eV},\quad m_1=0.3\ \text{eV} \tag{33}$$

and

$$V_\nu=\begin{pmatrix}0.74 & 0.65 & 0.18\\ 0.59 & 0.75 & 0.31\\ 0.34 & 0.12 & 0.95\end{pmatrix}. \tag{34}$$

In Table III, we have considered some of those solutions which are able to reproduce the ranges of probabilities,

$$P_{ee}=0.18-0.77,\quad P_{\mu\mu}=0.4-0.8,$$

and

$$P_{e\mu}=(0.15-0.55)\times 10^{-2}, \tag{35}$$

required to resolve the SNP and ANP as well as accommodate the LSND experiment. As is evident from the table, these ranges can be accommodated for almost degenerate neutrino masses in the eV range [34]. The mixing implied by these masses is quite large, which is in agreement with the recent analysis by Pakvasa and Acker [38]. It is perhaps desirable to mention that we have been able to obtain an overall agreement with the data except for the Homestake experiment, where our solution just touches the higher end of the data. However, if we take into account the consequences of a recent measurement of the reaction $\gamma+{}^8B\rightarrow{}^7Be+p$, which suggests lowering of the cross section for the inverse reaction ${}^7Be+p\rightarrow{}^8B+\gamma$ at energies relevant to the solar core, a better agreement with our calculations even for the Homestake data can be obtained [38,39].

If we invoke the MSW [23] solutions to the solar neutrino problem, then the comparison with data is not that straightforward. A detailed analysis in this regard without assuming a specific mass matrix in the three neutrino oscillations scenario has been carried out by several authors [22,24,29–32]. For our purpose we make use of the analysis of by Cardall and Fuller [32]. These authors have shown that SNP, ANP, and LSND data can be simultaneously fitted provided

$$\Delta m_{12}^2=7\times 10^{-6}\ \text{eV}^2,\quad \Delta m_{13}^2\sim\Delta m_{23}^2\sim 0.3\ \text{eV}^2$$
$$\sin^2\theta_{12}\sim 2\times 10^{-3},\quad \sin^2\theta_{13}\sim 10^{-2},\quad \sin^2\theta_{23}\sim 0.5. \tag{36}$$

To realize a Cardall and Fuller scenario invoking the MSW effect for the solar neutrino problem, we have tried to reproduce the above angles through natural hierarchy for neutrino masses. In Table IV we have presented some such solutions which approximately reproduce the values given by Eq. (35). A typical solution is of the form

$$m_3=0.5\ \text{eV},\quad m_2=9.6\times 10^{-2}\ \text{eV},\quad m_1=5.1\times 10^{-5}\ \text{eV}, \tag{37}$$

$$\sin^2\theta_{12}=(6.3)\times 10^{-3}, \tag{38}$$

$$\sin^2\theta_{13}=(1.2)\times 10^{-2}, \tag{39}$$

$$\sin^2\theta_{23}=0.53. \tag{40}$$

A few comments are in order. The two solutions were obtained for SNP, ANP, and LSND with the particular set of mass matrices wherein an important role is played by the 22 elements D_i. Similar elements for quark mass matrices also play a crucial role in accommodating the CKM phenomena [16]. This clearly brings out the crucial role of the 4 texture zeros mass matrices considered here. It is perhaps desirable to mention that attempts have already been made to generate such mass matrices [19,20]. Also, it may be of interest to mention that we have explored the case when a vacuum solution for SNP is reproduced with mass scale $\Delta m_{ij}^2\sim 10^{-10}$ eV^2. In this situation it becomes extremely difficult to obtain a fit to ANP and LSND data even with full variation of the parameter D_ν; however, a solution can be obtained if we

TABLE IV. Solutions with hierarchical neutrino masses. $R_3 = D_\nu / m_3$, m_3 is in units of eV, m_2 in 10^{-2} eV, m_1 in 10^{-5} eV, $S_{12}^2 = \sin^2 2\theta_{12} \times 10^{-3}$, $S_{13}^2 = \sin^2 2\theta_{13} \times 10^{-2}$, and $S_{23}^2 = \sin^2 2\theta_{23}$.

$R_{\nu 3}$	R_{l3}	m_3	m_2	m_1	S_{12}^2	S_{13}^2	S_{23}^2
0.8	0.05	0.5	9.1	9.1	6.2	0.94	0.53
				9.6	6.3	0.99	0.53
			9.6	3.6	5.9	0.82	0.53
				4.1	6.0	0.93	0.53
				4.6	6.2	1.04	0.53
				5.1	6.3	1.15	0.53
				5.6	6.4	1.26	0.53
				6.1	6.6	1.38	0.52
				6.6	6.7	1.49	0.52
	0.10	0.5	9.1	9.1	6.8	0.89	0.46
				9.6	6.9	0.95	0.46
			9.6	4.1	6.6	0.88	0.46
				4.6	6.8	0.98	0.46
				5.1	6.9	1.09	0.46
				5.6	7.1	1.19	0.46
				6.1	7.3	1.30	0.46
				6.6	7.5	1.40	0.45
0.85	0.10	0.5	6.6	9.1	6.9	0.91	0.51
				9.6	7.0	0.96	0.51
			7.1	4.1	6.6	0.89	0.51
				4.6	6.8	1.00	0.51
				5.1	7.0	1.10	0.51
				5.6	7.2	1.21	0.51
				6.1	7.4	1.31	0.51
				6.6	7.6	1.42	0.51

resort to fine tuning. The two solutions obtained here involve neutrino mass of eV range, also making these relevant for the relation [40]

$$\sum_{k=1}^{3} |m_{\nu_i}| \approx 7 \quad \text{eV}, \tag{41}$$

implied by the requirement that neutrinos provide the hot dark matter of the universe.

In conclusion, we would like to emphasize that phenomenological 4 texture zeros mass matrices not only accommodate the CKM phenomenology in the quark sector [16], but also are able to reconcile the three neutrino anomalies in the leptonic sector. In particular, we find when SNP is resolved through vaccum oscillations we obtain a solution suggesting large neutrino mixing and almost degenerate neutrino masses; whereas for the MSW solution we find a solution with natural hierarchy of neutrino masses.

ACKNOWLEDGMENTS

The authors gratefully acknowledge a few useful discussions with Professor M. P. Khanna. P.S.G. would like to thank the chairman, Department of Physics, for providing facilities to work in the department as well as the principal of his college for his kind cooperation.

APPENDIX

The diagonalizing matrix, O_ν used in the text is given below:

$$O_{\nu 11} = \sqrt{\frac{m_2 m_3 (m_3 - m_2 - D_\nu)}{C_\nu (m_3 - m_1)(m_2 + m_1)}},$$

$$O_{\nu 12} = -\sqrt{\frac{m_1 m_3 (m_3 + m_1 - D_\nu)}{C_\nu (m_3 + m_2)(m_3 - m_1)}},$$

$$O_{\nu 13} = \sqrt{\frac{m_1 m_3 (m_2 - m_1 + D_\nu)}{C_\nu (m_3 + m_1)(m_3 - m_1)}},$$

$$O_{\nu 21} = \sqrt{\frac{m_1 (m_3 - m_2 - D_\nu)}{(m_3 - m_1)(m_2 + m_1)}},$$

$$O_{\nu 22} = \sqrt{\frac{m_2 (m_3 + m_1 - D_\nu)}{(m_3 + m_2)(m_2 + m_1)}},$$

$$O_{\nu 23} = \sqrt{\frac{m_3 (m_2 - m_1 + D_\nu)}{(m_3 + m_2)(m_3 - m_1)}},$$

$$O_{\nu 31} = -\sqrt{\frac{m_1 (m_3 + m_1 - D_\nu)(m_2 - m_1 + D_\nu)}{C_\nu (m_3 - m_1)(m_2 + m_1)}},$$

$$O_{\nu 32} = -\sqrt{\frac{m_2 (m_3 - m_2 - D_\nu)(m_2 - m_1 + D_\nu)}{C_\nu (m_3 + m_2)(m_2 + m_1)}},$$

$$O_{\nu 33} = \sqrt{\frac{m_3 (m_3 - m_2 - D_\nu)(m_3 + m_1 - D_\nu)}{C_\nu (m_3 + m - 2)(m_3 - m_1)}}.$$

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