Electroweak symmetry restoration at high temperature in a four-generation fermion condensate scheme

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We research the gap equation at finite temperature in a four-generation fermion condensate scheme and find that the critical temperature T_c for electroweak symmetry restoration in this scheme may be lower than in the top-quark condensate scheme. The critical chemical potentials of relevant fermions in the zero-temperature limit will submit to an elliptic equation. In the zero chemical potential case, we obtain $T_c \simeq 257$ GeV and prove that the dynamical fermion masses near T_c have a $(T_c^2 - T^2)^{1/2}$ behavior with an additional factor dependent on the temperature T and momentum cutoff Λ. This indicates that, in spite of the extra factor, the symmetry restoration is a second-order phase transition in this scheme. [S0556-2821(98)01405-2]

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I. INTRODUCTION

It has been known that the top-quark condensate scheme [1] of electroweak symmetry breaking at zero-temperature may have a version of the four-generation fermion extension in which the fine-tuning problem of the coupling constant could be greatly alleviated owing to the enormous descent of the compositeness momentum scale [2]. Therefore, just as research on the top-quark condensate scheme at finite temperature [3], it is certainly interesting to research the four-generation fermion condensate scheme at finite temperature from both field theory itself and cosmological implications [4]. Generally, in a dynamically symmetry breaking model of the Nambu–Jona-Lasinio (NJL) form [5], as is discussed here, restoration of symmetry could happen at high temperature. In this paper we will research this problem by means of the gap equation of the dynamical fermion masses which are related to the order parameter responsible for electroweak symmetry breaking. Without loss of essentiality of the model, the whole discussions will be conducted in the fermionic bubble graph approximation and the color interactions will be completely neglected.

II. GAP EQUATION IN N-GENERATION SCHEME

In an n-generation fermion condensate scheme one assumes that electroweak symmetry breaking is induced by effective four-fermion interactions of the n generations of fermions at some high momentum scale Λ [6]. At zero-temperature, the neutral scalar sector of the effective four-fermion Lagrangian

$$\mathcal{L}_{4F}^{N_s} = \frac{1}{4}\sum_{Q,Q'} g_{Q'Q'}^{1/2} g_{QQ}^{1/2} (\overline{Q'}Q')(\overline{Q}Q),$$

$$Q', Q = U_\alpha, D_\alpha \quad (\alpha = 1, \ldots, n) \tag{2.1}$$

will generate the dynamical mass of the Q fermion

$$m_Q(0) = -\frac{1}{2} g_{QQ}^{1/2} \sum_{Q'} g_{Q'Q'}^{1/2} \langle \overline{Q'}Q' \rangle_0, \tag{2.2}$$

where $\langle \overline{Q'}Q' \rangle_0$ represents the vacuum expectation value of $\overline{Q'}Q'$ (the Q'-fermion condensates), if the combined condensates $\Sigma_{Q'} g_{Q'Q'}^{1/2} \langle \overline{Q'}Q' \rangle_0 \neq 0$. From Eq. (2.2) we can obtain the relations

$$m_Q(0)/m_{Q'}(0) = g_{QQ}^{1/2}/g_{Q'Q'}^{1/2},$$

$$Q, Q' = U_\alpha, D_\alpha \quad (\alpha = 1, \ldots, n) \tag{2.3}$$

and the gap equation

$$1 = \sum_Q g_{QQ} I_Q^0, \tag{2.4}$$

where

$$I_Q^0 = -\frac{1}{2m_Q(0)} \langle \overline{Q}Q \rangle_0 = \frac{d_Q(R)}{2m_Q(0)} \int \frac{d^4k}{(2\pi)^4} \mathrm{tr} \frac{i}{\not{k} - m_Q(0) + i\varepsilon} \tag{2.5}$$

and $d_Q(R)$ denotes the dimension of the color $SU_c(3)$ representation R.

The generalization to the case of finite temperature means to replace the vacuum expectation value $\langle \overline{Q}Q \rangle_0$ by the thermal expectation value $\langle \overline{Q}Q \rangle_T$. Thus the dynamical mass $m_Q(T, \mu_Q)$ of the Q-fermion at finite temperature T and finite chemical potential μ_Q can be expressed by

$$m_Q(T, \mu_Q) = -\frac{1}{2} g_{QQ}^{1/2} \sum_{Q'} g_{Q'Q'}^{1/2} \langle \overline{Q'}Q' \rangle_T. \tag{2.6}$$

Equation (2.6) will lead to a relation similar to Eq. (2.3), i.e.,

$$m_Q(T, \mu_Q)/m_{Q'}(T, \mu_{Q'}) = g_{QQ}^{1/2}/g_{Q'Q'}^{1/2} \tag{2.7}$$

and the gap equation at finite temperature

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$$1=\sum_Q g_{QQ}I_Q, \tag{2.8}$$

where

$$I_Q=2d_Q(R)\Bigg\{\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\varepsilon}-\int\frac{d^3k}{(2\pi)^3 2\omega_Q}\bigg\{\frac{1}{\exp[\beta(\omega_Q-\mu_Q)]+1}+\frac{1}{\exp[\beta(\omega_Q+\mu_Q)]+1}\bigg\}\Bigg\},$$

$$\omega_Q=(\mathbf{k}^2+m_Q^2)^{1/2}, \tag{2.9}$$

which is obtained from Eq. (2.5) by the substitutions [7]

$$m_Q(0)\to m_Q(T,\mu_Q)\equiv m_Q, \tag{2.10}$$

$$\frac{i}{\not k-m_Q(0)+i\varepsilon}\to(\not k+m_Q)\left[\frac{i}{k^2-m_Q^2+i\varepsilon}-2\pi\delta(k^2-m_Q^2)\sin^2(k^0,\mu_Q)\right], \tag{2.11}$$

where

$$\sin^2(k^0,\mu_Q)=\frac{\theta(k^0)}{\exp[\beta(k^0-\mu_Q)]+1}+\frac{\theta(-k^0)}{\exp[\beta(-k^0+\mu_Q)]+1}, \tag{2.12}$$

with $\beta=1/T$. In the above derivation we have used the real-time formalism of the thermal field theory which is identical to the imaginary-time formalism in the present simple problem [3]. When $|\mathbf{k}|\to\infty$, only the first integral in Eq. (2.9) is divergent. This allows us to take the momentum cutoff merely in the first integral and, as a good approximation, extend the integral limit of $|\mathbf{k}|$ toward infinite in the second one [3]. After the wick rotation and the angular integration, the gap equation (2.8) will take the form

$$1=\sum_Q g_{QQ}\frac{d_Q(R)\Lambda^2}{8\pi^2}\left\{1-\frac{m_Q^2}{\Lambda^2}\ln\left(\frac{\Lambda^2}{m_Q^2}+1\right)-8\frac{T^2}{\Lambda^2}[I_3(y_Q,-r_Q)+I_3(y_Q,r_Q)]\right\}, \tag{2.13}$$

where Λ is the four-dimension Euclidean momentum cutoff and

$$I_3(y_Q,\mp r_Q)=\frac{1}{2}\int_0^\infty dx\frac{x^2}{(x^2+y_Q^2)^{1/2}}\frac{1}{\exp[(x^2+y_Q^2)^{1/2}\mp r_Q]+1} \tag{2.14}$$

with

$$y_Q=\beta m_Q,\quad r_Q=\beta\mu_Q. \tag{2.15}$$

When $T\to 0$, Eq. (2.13) will be reduced to the gap equation at zero-temperature [6]:

$$1=\sum_Q g_{QQ}\frac{d_Q(R)\Lambda^2}{8\pi^2}\left[1-\frac{m_Q^2(0)}{\Lambda^2}\ln\left(\frac{\Lambda^2}{m_Q^2(0)}+1\right)\right]. \tag{2.16}$$

III. CRITICAL TEMPERATURE AND CHEMICAL POTENTIALS

Combining Eq. (2.13) with Eq. (2.16), we obtain

$$\sum_Q g_{QQ}d_Q(R)\{m_Q^2(0)\ln[\Lambda^2/m_Q^2(0)+1]-m_Q^2\ln[\Lambda^2/m_Q^2+1]-8T^2[I_3(y_Q,-r_Q)+I_3(y_Q,r_Q)]\}=0. \tag{3.1}$$

Since $g_{QQ}>0$ [6], a physically reasonable solution of Eq. (3.1) is

$$m_Q^2(0)\ln[\Lambda^2/m_Q^2(0)+1]=m_Q^2\ln[\Lambda^2/m_Q^2+1]+8T^2[I_3(y_Q,-r_Q)+I_3(y_Q,r_Q)],\quad \text{for each } Q. \tag{3.2}$$

We see from Eq. (3.2) that since $I_3(y_Q,\mp r_Q)$ are positive-definite, as T increases m_Q will decrease and finally could become zero. At this point, symmetry restoraton will be attained. Now multiplying $d_Q(R)$ to each term of Eq. (3.2) and doing the sum of Q we will obtain

$$\sum_Q d_Q(R)m_Q^2(0)\ln[\Lambda^2/m_Q^2(0)+1]=\sum_Q d_Q(R)\{m_Q^2\ln[\Lambda^2/m_Q^2+1]+8T^2[I_3(y_Q,-r_Q)+I_3(y_Q,r_Q)]\}. \tag{3.3}$$

Equation (3.3) can be regarded as an equivalent equation to the gap equation (2.13) and is applicable to the case of n generations of fermions. In the following we will limit ourselves to the four-generation fermion condensate scheme where the heavy fermions relevant to the fermion condensates are only the top-quark and a mass-degenerate fourth generation Dirac U-fermions. Considering the basic relation which defines the vacuum expectation value v of the effective Higgs scalar field in this scheme [2],

$$\frac{4\sqrt{2}\pi^2}{G_F}=8\pi^2v^2=\sum_{Q=U,t} f_Q d_Q(R) m_Q^2(0)\ln[\Lambda^2/m_Q^2(0)+1], \tag{3.4}$$

where G_F is the Fermi constant and f_Q is the flavor number of the Q-fermion with $f_U=2$ and $f_t=1$, we may change Eq. (3.3) into

$$8\pi^2v^2=\sum_{Q=U,t} f_Q d_Q(R)\{m_Q^2\ln[\Lambda^2/m_Q^2+1] + 8T^2[I_3(y_Q,-r_Q)+I_3(y_Q,r_Q)]\}. \tag{3.5}$$

We may assume that when some critical temperature T_c is arrived, the combined condensates $\Sigma_Q g_{QQ}^{1/2}\langle\overline{Q}Q\rangle_T=0$; then based on Eq. (2.6) all the m_Q will become zeros. Therefore, the equaton which determines the critical temperature T_c and the critical chemical potentials μ_{Uc} and μ_{tc} may be obtained by taking $m_U\to 0$, $m_t\to 0$ (i.e., $y_U\to 0$, $y_t\to 0$) in Eq. (3.5) and has the form

$$\pi^2v^2=T^2\sum_{Q=U,t} f_Q d_Q(R)[I_3(0,-r_Q)+I_3(0,r_Q)]. \tag{3.6}$$

By means of the formula [8]

$$I_3(0,\mp r_Q)=\frac{1}{2}\int_0^\infty dx\frac{x}{\exp(x\mp r_Q)+1} =\frac{1}{2}\exp(\pm r_Q)\Phi[-\exp(\pm r_Q),2,1] \tag{3.7}$$

expressed by the Lerch's transcendent function $\Phi[z,s,a]=\Sigma_{k=0}^\infty z^k/(k+a)^s$ Eq. (3.6) can be changed into

$$2\pi^2v^2=T^2\sum_{Q=U,t} f_Q d_Q(R)\{\exp(\beta\mu_Q)\Phi[-\exp(\beta\mu_Q),2,1] +\exp(-\beta\mu_Q)\Phi[-\exp(-\beta\mu_Q),2,1]\} \tag{3.8}$$

or

$$2\pi^2v^2=\sum_{Q=U,t} f_Q d_Q(R)\mu_Q^2\left\{\frac{1}{r_Q^2}\int_0^\infty dx\left[\frac{x}{\exp(x-r_Q)+1} + \frac{x}{\exp(x+r_Q)+1}\right]\right\}. \tag{3.9}$$

In the special case with $\mu_U=\mu_t=0$ we may obtain from Eq. (3.8) that

$$T_c^2=12v^2/[2d_U(R)+d_t(R)]. \tag{3.10}$$

If the Higgs vacuum expectation value $v=246$ GeV is taken, then we get, from Eq. (3.10),

$$T_c=\begin{cases}257\text{ GeV} & \text{if } d_U(R)=4 \text{ and } d_t(R)=3,\\ 492\text{ GeV} & \text{if } d_U(R)=0 \text{ and } d_t(R)=3.\end{cases} \tag{3.11}$$

The results show that the value of T_c in the four-generation fermion condensate scheme $[d_U(R)=4]$ is only almost one half of the one in the top-quark condensate scheme $[d_U(R)=0]$. Hence, inclusion of the fourth generation of fermions in the scheme will be able to make the temperature of electroweak symmetry restoration go down evidently. The same conclusion is true for the case with nonzero chemical potentials. This can be seen from Eq. (3.8). Since the functions $\Phi[-\exp(\beta\mu_Q),2,1](Q=U,t)$ are positive-definite by Eq. (3.7), for the left-handed side of Eq. (3.8) being kept to be the constant $2\pi^2v^2$, the addition of the terms with $d_U(R)$ will necessarily cause the descent of T_c.

On the other hand, in the special case when $T\to 0$ or equivalently, $r_Q=\mu_Q/T\to\infty$ if μ_Q is finite, we have the limits

$$\lim_{r_Q\to\infty}\frac{1}{r_Q^2}\int_0^\infty dx\frac{x}{\exp(x\mp r_Q)+1}=\begin{cases}\frac{1}{2}\\ 0\end{cases}. \tag{3.12}$$

As a result, Eq. (3.9) will give

$$4\pi^2v^2=2d_U(R)\mu_{Uc}^2+d_t(R)\mu_{tc}^2. \tag{3.13}$$

Hence, when $T=0$ the critical chemical potentials μ_{Uc} and μ_{tc} will satisfy an elliptic equaton. Similar to variation of the critical temperature as scheme, we can see from Eq. (3.13) that the critical chemical potential μ_{Qc} of some given Q-fermions in the four-generation scheme is always less than the corresponding one in the scheme in which only the Q-fermions are included. This conclusion is also valid in the case of $T\neq 0$ if we note that when T is fixed, the two terms in the right-handed side of Eq. (3.9) are respectively positive-definite and monotonically increasing function of μ_U and μ_t.

IV. CRITICAL BEHAVIOR OF FERMION MASSES NEAR T_c

We will only consider the case of zero chemical potentials. Equation (3.5) is thus reduced to

$$8\pi^2v^2=\sum_{Q=U,t} f_Q d_Q(R)[m_Q^2\ln(\Lambda^2/m_Q^2+1) + 16T^2I_3(y_Q,0)]. \tag{4.1}$$

By the standard technique of high temperature expansion of thermal integrals [9,10], we find

$$I_3(y_Q,0)=(\pi^2/24)+(y_Q^2/8)\{\ln(y_Q/\pi)+\gamma-1/2 +O[(y_Q/2\pi)^2]\},\quad Q=U,t, \tag{4.2}$$

where $\gamma=0.5772$ is the Euler constant and we have used the fact that at the temperature $T\lesssim T_c$, both m_U and m_t are small so that the terms of the orders $O[(y_Q/2\pi)^2]=O[(m_Q/2\pi T)^2]$ and above could be neglected. Considering the definition of y_Q in Eq. (2.15) and $\Lambda/m_Q\gg 1(Q=U,t)$, we get, from Eq. (4.1) and the equation (3.10) of T_c when $\mu_U=\mu_t=0$,

$$[2d_U(R)m_U^2+d_t(R)m_t^2][\ln(\Lambda/T\pi)+\gamma-1/2] =(\pi^2/3)[2d_U(R)+d_t(R)](T_c^2-T^2). \quad (4.3)$$

It is indicated that the combination Eq. (2.3) and Eq. (2.7) will lead to

$$m_U/m_t=m_U(0)/m_t(0)\equiv\lambda, \quad (4.4)$$

i.e., the mass ratio m_U/m_t is temperature-independent. With this relation we may obtain from Eq. (4.3)

$$m_U^2=\lambda^2 m_t^2 =\frac{\pi^2}{3}\frac{[2d_U(R)+d_t(R)]}{[2d_U(R)+d_t(R)/\lambda^2]}\frac{(T_c^2-T^2)}{[\ln(\Lambda/T\pi)+\gamma-1/2]}. \quad (4.5)$$

In addition, from Eq. (2.6) we also have

$$\sum_Q g_{QQ}^{1/2}\langle\overline{Q}Q\rangle_T=-\frac{2m_Q}{g_{QQ}}=-2\frac{\left(\sum_Q m_Q^2\right)^{1/2}}{\left(\sum_Q g_{QQ}\right)^{1/2}} =-2\frac{(2m_U^2+m_t^2)^{1/2}}{(2g_{UU}+g_{tt})^{1/2}}. \quad (4.6)$$

Equations (4.5) and (4.6) show that at $T\lesssim T_c$, all the dynamical fermion masses $m_Q(Q=U,t)$ and the combined thermal expectation value $\Sigma_Q g_{QQ}^{1/2}\langle\overline{Q}Q\rangle_T$ as the order parameter responsible for symmetry breaking, have $(T_c^2-T^2)^{1/2}$ behavior with an additional factor dependent on $\ln(\Lambda/T\pi)$. This result is identical to the one obtained in the top-quark condensate scheme and different from the one obtained in the standard electroweak model with elementary Higgs scalar field [10] by the extra factor. However, based on a general argument, it may be shown that the extra factor does not change the fact that the symmetry restoration is a second-order phase transition. We first note that the momentum cutoff or the compositeness scale Λ is simply a fixed finite constant in a given model and the critical temperature $T_c\neq 0$, so the extra factor containing $\ln(\Lambda/T\pi)$ has no singularity. In addition, owing to Eq. (4.4) there is essentially only a single order parameter for symmetry breaking in this scheme, hence, consistent with Eqs. (4.5) and (4.6), we may generally write the order parameter by

$$\eta=f(T)(T_c^2-T^2)^{1/2}, \quad f(T)=b/[\ln(\Lambda/T\pi)+\gamma-1/2]^{1/2}, \quad (4.7)$$

where b is a positive constant. Then we may expand the thermodynamical potential $\Omega(T,\eta)\equiv\Omega(T,\mu=0,\eta)$ into power-series of η near the critical temperature (phase transition point) as [11,3]

$$\Omega(T,\eta)=\Omega_0(T)+a(T-T_c)\eta^2+[a/4T_cf^2(T_c)]\eta^4+\cdots, \quad (4.8)$$

where a is a positive constant. It is easy to verify that the thermodynamical potential of the form (4.8) will lead to the temperature behavior (4.7) of η and it is indeed able to describe the system considered. From Eq. (4.8) we can prove that the entropy of the system $S=-[\partial\Omega(T,\eta)/\partial T]_V$ is continuous but the special heat $c_v=T(\partial S/\partial T)_V$ is not and will reduce a finite quantity when T acrosses T_c increasely. This fact indicates that the symmetry restoration is indeed a second-order phase transition. The extra factor containing $\ln(\Lambda/T\pi)$ has no effect on such feature. Since the feature of the phase transition is determined essentially by the $(T_c^2-T^2)^{1/2}$ temperature behavior of the unique order parameter η in the scheme, it is conjectured that the above conclusion of the second-order symmetry retoring phase transition will be maintained when all the gauge interactions are opened. Therefore, it is conceivable that the thermodynamics of the four-generation fermion condensate scheme will be the same as the one of the four-generation standard electroweak model with elementary Higgs scalar field.

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