Proof of factorization for diffractive hard scattering

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A proof is given that hard-scattering factorization is valid for deep-inelastic processes which are diffractive or which have some other condition imposed on the final state in the target fragmentation region. [S0556-2821(98)00507-4]

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I. INTRODUCTION

In this paper, I show how to prove hard-scattering factorization for diffractive deep-inelastic processes, and certain related processes. This is an important topic because it is known [1–4] that factorization fails for hard processes in diffractive hadron-hadron scattering[1] (such as the diffractive Drell-Yan process). Moreover, the violation of factorization appears to be confirmed by experiment [6–9]. So we must determine those diffractive processes, if any, for which factorization is actually predicted by QCD.

The precise form of the factorization property that I prove has been stated by Kunszt and Stirling [10], and by Berera and Soper [11,12], as a full QCD generalization of the Ingelman-Schlein model [13], but shorn of the Regge hypotheses. It is the same as factorization for inclusive hard processes, except that parton densities are replaced by diffractive parton densities. We can say that Ingelman-Schlein [13] factorization is hard-scattering factorization, such as is proved in the present paper, together with Regge factorization for the Pomeron exchange.

I will prove the theorem not only for diffractive deep-inelastic processes, but for any deep-inelastic process where a requirement is imposed on the final state in the target fragmentation region. Any requirement that is fixed relative to the beam is allowed: e.g., that there be detected particle(s) of particular kind(s) carrying some particular fraction of the beam's momentum and carrying some given transverse momentum. Hence the proof applies to the fracture function formalism of Trentadue and Veneziano [14], for deep-inelastic processes.[2] Factorization for diffractive scattering is a special case [15] of fracture function factorization.

Furthermore, it is possible to discuss any of the normal hard scattering processes which are lepton induced: in addition to the deep-inelastic cross section itself, the proof applies, for example, to the case where jets of large transverse momentum are detected and where particular particles in the ``current fragmentation region'' are detected.

The proof in the present paper justifies, from fundamental principles, the analysis [16,17] of diffractive deep-inelastic processes in terms of parton densities in the Pomeron. Note that the only real use of the Pomeron in these analyses is as a label for a particular power law for the x_{P} dependence of diffractive cross sections, with the exponent actually being a free power. Indeed, the QCD analysis by H1 [16], which has two phenomenological power laws, is also covered by the theorem proved in this paper. However, I will not at all address the separate and important question of whether *Regge* factorization is also valid. Regge factorization relates, for example, the power of x_{P} measured in diffractive deep-inelastic scattering to the power of s measured in hadron-hadron elastic scattering.

Berera and Soper [12] provided arguments that hard-scattering factorization should be true in diffractive lepton-induced processes, and the present paper completes the proof. The bulk of the proof follows the usual methods [5,18] for proving factorization, and, as pointed out by Berera and Soper [12], the only new element that is needed is a proper treatment of the soft-gluon cancellation for the processes in question. The essential point of the present paper is to show that there exists a contour deformation that permits the methods of Collins and Sterman [18] to be used.

II. FACTORIZATION, PARTON DENSITIES

In this section, I will review the factorization theorem that is to be proved.

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[1]Note that this knowledge predates QCD. Within the context of pre-QCD parton-model ideas it was shown that there are factorization-breaking terms [1] in both the diffractive and nondiffractive parts of the Drell-Yan process, and that these terms cancel [2,3] in the inclusive cross section, which is the sum of the diffractive and nondiffractive parts. This result forms part of the proof of factorization for inclusive hard processes in QCD [5].

[2]Note that since factorization fails for diffractive hard processes in hadron-hadron scattering, it follows that the fracture function formalism also fails in hadron-hadron scattering. The proof given by Trentadue and Veneziano does not treat the soft exchanges which break factorization in hadron-hadron scattering.

As stated above, the factorization theorem for diffractive hard processes has the same form as for inclusive processes. For example, for diffractive deep-inelastic scattering $e + p \rightarrow e' + X + p'$, we have[3]

$$F_2^{(D)}(x_{\rm bj}, Q, x_{\mathbb{P}}, t) = \sum_i C_{2i} \otimes f_i^D + \text{nonleading power of } Q. \tag{1}$$

Here, $x_{\rm bj}$ and Q are the usual deep-inelastic variables, $x_{\mathbb{P}} = 1 - q \cdot p' / q \cdot p$ is the fractional loss of longitudinal momentum by the diffracted proton,[4] and $t = (p - p')^2$ is the invariant momentum transfer from the diffracted proton, while $\otimes$ signifies a convolution of the hard-scattering coefficient C_{2i} with the diffractive parton density f_i^D. The factorization theorem applies when Q is made large while $x_{\rm bj}$, $x_{\mathbb{P}}$, and t are held fixed. It asserts not only that an expansion of the form of Eq. (1) is true, but also that

> C_{2i} is the *same* hard scattering coefficient as in ordinary (inclusive) deep-inelastic scattering (DIS), with i being a label for parton flavor (gluon, u-quark, etc.).
>
> The diffractive parton densities f_i^D are those defined by Berera and Soper [12], as suitable "cut matrix elements" of the same operators that define ordinary parton densities.
>
> They therefore obey exactly the same DGLAP evolution equations as ordinary parton densities.

Generalizations of the theorem that are covered by the proof in this paper are of two kinds:

> The requirement that there be a diffracted proton p' in the final state may be replaced by any other requirement in the "target fragmentation region" that is fixed relative to the initial hadron. For example, p' may be a neutron, or it may be replaced by a two-pion state of some invariant mass that has a fraction $1 - x_{\mathbb{P}}$ of the longitudinal momentum of p and that has some given value of t. (Longitudinal momentum must be interpreted in the sense of the appropriate light-cone momentum, so that the definition of the parton densities is invariant under longitudinal boosts.)
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> Any other standard hard process may be considered. Then the coefficient C_{2i} is replaced by the appropriate
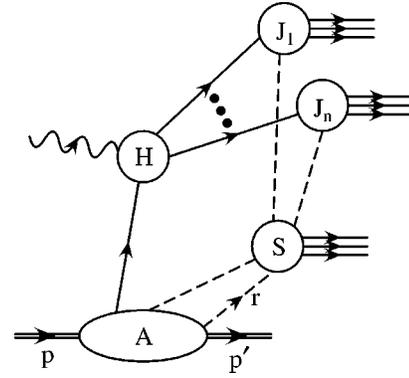
FIG. 1. Leading regions for diffractive DIS.

coefficient for the process, times fragmentation functions if necessary. Thus the theorem applies to the longitudinal structure function F_L^D, and to differential cross sections for jet production in the "current fragmentation region."

The first generalization implies that the theorem applies at all $x_{\mathbb{P}}$ away from zero, and not just to the diffractive region of small $x_{\mathbb{P}}$. This justifies the analysis [16] given by H1, who analyzed F_2^D in terms of two powers of $x_{\mathbb{P}}$, both a leading diffractive power, and a nonleading power. It also justifies the fracture function formalism of Trentadue and Veneziano [14], but only for deep-inelastic processes. Note that Trentadue and Veneziano define their cross sections to be integrated over the transverse momentum of the final-state hadron p'. This complicates the formalism: Whereas the diffractive parton densities without the integral over transverse momentum obey standard Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations, the corresponding equations for fracture functions in [14] are more complicated, since the outgoing particle p' may be at large transverse momentum and thus be associated with the hard scattering. The theorem proved here does not need the integral over the transverse momentum of p'.

III. PERTURBATIVE PROOF

As was explained by Berera and Soper [12], the proof of factorization for diffractive hard processes is the same as for inclusive hard processes [5], except for the treatment of the cancellation of soft exchanges.

A. Regions

The leading regions of Feynman graphs for amplitudes for diffractive deep-inelastic scattering may be represented as in Fig. 1, the analysis [19] being independent of the diffractive requirement. There is a subgraph A consisting of lines collinear to p and p'. One parton[5] from A is incident on the hard subgraph H, consisting of lines of virtuality of order Q^2 connected to the virtual photon. From H are produced one or more lines that go into jet subgraphs, $J_1, \ldots$. There may be

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[3]For the purposes of this paper, I define F_2^D to be the value of F_2 computed from those events containing a final-state proton p' with the specified kinematics. So the use of the word "diffractive" to describe the process is not really correct. Our definition is the one used by the H1 experiment [16], and it contrasts with the definition used by the ZEUS experiment [17], which subtracts the nondiffractive contribution. Of course, given the "diffractive" F_2^D defined here, one can extract the leading power at small $x_{\mathbb{P}}$, which, at least for our present purposes, is the definition of the truly diffractive part. Factorization for the complete F_2^D, as defined here, implies factorization for the purely diffractive part, with the diffractive parton densities f_i^D being replaced by their diffractive components.

[4]Of course, the proton may be replaced by any other hadronic state, e.g., a nucleus.

[5]Plus arbitrarily many gluons with scalar polarizations [20], if we are in a covariant gauge. These gluons are a gauge artifact.
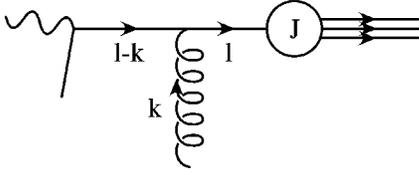
FIG. 2. Soft gluon attaching to jet.

a soft subgraph S (not necessarily connected) consisting of low momentum lines (in the Breit frame); it is joined by *gluon* lines to the ''jet subgraphs'' A and J_i. Some lines from S may go into the final state.

It is important that it is only necessary to consider regions where the contours of the integrations over loop momenta are trapped. To define a unique contribution from each region of the form of Fig. 1, subtractions should be devised to avoid double counting from the different regions that contribute for a single graph. This issue is the same as for nondiffractive scattering, so we do not need to treat it here.

To analyze the process quantitatively, we use light-cone coordinates[6] in the Breit frame so that

$$q^\mu = \left(-\frac{Q}{\sqrt{2}}, \frac{Q}{\sqrt{2}}, \mathbf{0}_T \right),$$

$$p^\mu = \left(\frac{Q}{x_{\mathrm{bj}}\sqrt{2}}, \frac{m^2 x_{\mathrm{bj}}}{Q\sqrt{2}}, \mathbf{0}_T \right), \quad (2)$$

$$p'^\mu = \left(\frac{(1-x_\mathbb{P})Q}{x_{\mathrm{bj}}\sqrt{2}}, \frac{(m^2+\mathrm{p}_T^2)x_{\mathrm{bj}}}{Q\sqrt{2}(1-x_\mathbb{P})}, \mathrm{p}_T \right).$$

B. Single soft gluon attaching to jet

By definition a soft momentum k^μ is one all of whose components are much less than Q in the Breit frame: $|k^\mu| \ll Q$.

As a first example, which is readily generalized, let the hard scattering be the Born graph and let a soft gluon of momentum k^μ attach to the outgoing quark (Fig. 2). We will show that after a suitable approximation in the jet subgraph, a Ward identity can be applied to factor out the soft attachment. The relevant factor in the jet subgraph is

$$J^\mu(l,k) = \frac{1}{(l-k)^2 - m^2 + i\epsilon}\Gamma^\mu, \quad (3)$$

where m is the quark mass, and Γ^μ is the vertex which couples the gluon to the jet subgraph, together with the attached numerator factors. The jet momentum l^μ is $(0, Q/\sqrt{2}, \mathbf{0}_T)$, plus terms that are smaller by a power of Q. The largest component of Γ^μ is Γ^- (by a power of Q), so it is a good approximation to replace Γ^μ by $\Gamma^- n_J^\mu$, where $(n_J^+, n_J^-, \mathbf{n}_{TJ}) = (0,1,\mathbf{0}_T)$.

Suppose first that all components of k^μ are comparable in size. Then it is a good approximation to replace

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[6]$V^\mu = (V^+, V^-, V_T)$, where $V^\pm = (V^0 \pm V^z)/\sqrt{2}$.
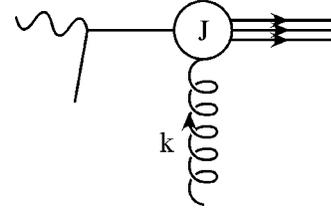
FIG. 3. Soft gluon attaching anywhere to jet subgraph.

$(l-k)^2 - m^2 + i\epsilon$ by $l^2 - m^2 - 2l^- k^+ + i\epsilon$, that is, to replace k^μ by its $+$ component everywhere in J. Thus

$$J^\mu(l,k) = J^-(l,(k^+,0,\mathbf{0}_T))n_J^\mu + \text{power correction}$$

$$= k^+ J^-(l,(k^+,0,\mathbf{0}_T)) \frac{n_J^\mu}{k^+} + \text{power correction}.$$

$$(4)$$

The $k^+ J^-$ factor is of a form to which a Ward identity can be applied: a Green function of the gluon field contracted with the gluon's momentum. If Eq. (4) is correct, then we can apply the argument used by Collins and Sterman in the proof of factorization for inclusive e^+e^- annihilation [18], and factorization would be true for our process also.

To derive Eq. (4), we assumed that all components of k^μ are comparable, so that the largest term in $k^2 - 2J \cdot k$ is $-2J^- k^+$. The argument fails if k^+ is too small compared to the other components of k. Exactly the same problem had to be overcome in the proofs of factorization for inclusive e^+e^- annihilation [18] and for the Drell-Yan cross section [5], etc.

Now, in the dangerous region $|k^+ k^-| \ll k_T^2$,[7] so that the only nearby pole in k^+ is the explicit pole in Eq. (3). We may therefore deform the k^+ integration contour away from the pole and out of the dangerous region. This is exactly the same argument used for e^+e^- annihilation by Collins and Sterman [18]. We must interpret the n_J^μ/k^+ factor in Eq. (4) as $n_J^\mu/(k^+ - i\epsilon)$, so that the pole at $k^+=0$ does not interfere with the contour deformation.

The soft approximation Eq. (4) therefore applies over the whole of the soft region for k, on the deformed contour.

Exactly the same contour deformation and the same approximation can be applied to all attachments of the soft gluon to a final-state jet subgraph, Fig. 3. The reason is that [18], just as in e^+e^- annihilation, all interactions of soft gluons with the jet are in the final state relative to the hard scattering. Because the direction of the contour deformation is the same in all cases, Ward identities[8] can be applied con-

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[7]This region was called the Coulomb region in [18]. Note that if k^+ is smaller than the other components of k but $|k^+ k^-|$ is comparable with k_T^2, then $|k^-/k^+| \gg 1$, so that k is collinear to J rather than being in the soft region, which is our present concern.

[8]To implement the Ward identities correctly, account must be taken of graphs where the hard scattering is coupled to the jet subgraph J by extra gluons of scalar polarization as well as the explicitly written quark line. This part of the argument is identical to the same part of the argument for inclusive scattering, and so we do not need to go into the details.
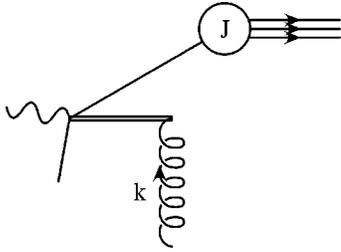
FIG. 4. Result of summing over all graphs of the form of Fig. 3.

sistently to factor the soft gluon out of the jet subgraph. The result is shown in Fig. 4, where the double line represents an eikonalized quark propagator, $1/(k^+ - i\epsilon)$.

C. General soft gluon attachments to jet subgraphs

The argument in the previous section, III B immediately generalizes, exactly as in the proof [18] of factorization for inclusive e^+e^- annihilation, to any attachments of the soft subgraph to any of the final-state jets subgraphs in Fig. 1. Provided that we can also apply the argument to soft-gluon attachments to the A subgraph, a sum over real and virtual emission of soft gluons can be used, just as in e^+e^- annihilation, to cancel the complete soft gluon factors. The cancellation only concerns a kinematic region unaffected by the diffractive requirement on the final state.

As explained out by Berera and Soper [12], the desired factorization theorem immediately follows.

D. Single soft attachment to A

However, we cannot apply the same argument to the attachment of a soft gluon to the A subgraph, since this subgraph contains both initial- and final-state interactions. The graph of a typical leading region, Fig. 1 illustrates this. We have labeled one of the soft gluons attaching to A by its momentum r. The appropriate soft approximation is

$$A^\mu(r,p,\ldots)=r^- A^+((0,r^-,\mathbf{0}_T),p,\ldots)\,\frac{n_A^\mu}{r^-}$$

$$+\text{power correction}, \qquad (5)$$

where $n_A^\mu=(1,0,\mathbf{0}_T)$. This approximation is valid only if r^- is not too small. The obvious generalization of the argument in Sec. III B would have us deform r^- away from the poles of denominators in A to avoid the region where the approximation fails. Precisely because A contains both initial- and final-state interactions, there are nearby poles in both the upper and lower half-planes, and we cannot deform the r^- contour to where the soft approximation is valid.

Instead, we appeal to a deformation of the other longitudinal momentum component r^+. The simplest case is the exchange of a single soft gluon, Fig. 5. We already know that to obtain the soft approximation where this gluon attaches to the *final-state* jet subgraph, we must deform the r^+ contour away from the (final-state) poles in the jet subgraph. The limits to this deformation are when the contour reaches the pole in the putative soft gluon propagator at $r^+=r_T^2/2r^-$ or one of the poles in the A subgraph at $r^+\sim Q$: Fig. 6. In either case the contour is deformed to a region where
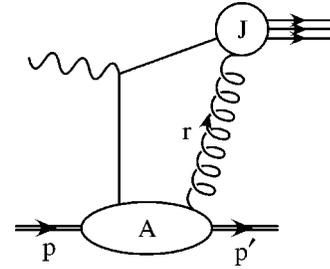
FIG. 5. Soft gluon exchange between proton subgraph and jet subgraph.

$|r^+|\gg|r^-|$, which is not part of the soft region. This is sufficient to show that there is no pinch in the soft region at small r^-. Hence we can use the soft approximation at both the A and J ends of the soft gluon.

Notice that it is not necessary to specify the sign of an $i\epsilon$ for the $1/r^-$ factor in Eq. (5). Once subtractions are made to define the soft factor unambiguously, to remove the collinear contributions, our proof implies that the soft factor is zero at $r^-=0$, and thus the $1/r^-$ pole is cancelled.

E. General soft attachment to A

For the most general case of soft gluons attaching to the subgraph A, we refer back to Fig. 1. To get the desired result we must show that if r^- is very small, then we can deform the r^+ contour to another region. This is a bit tricky, since the deformation may be restricted by poles in other parts of the soft subgraph, and these give restrictions that are more severe than those imposed by the poles in the jet subgraphs.

The first point to notice is that by hypothesis we start in a part of the soft region where $r_T\gg r^-$, the part where Eq. (5) fails. This implies that the pole of the propagator for the line r does not restrict the deformation.

Moreover, the r^+ contour is *not* trapped by the A subgraph. So any pinch would arise from a pinch by other soft lines or by jet lines. It would occur only in a situation like Fig. 7, where we suppose that the lines l_1+r and l_2-r both have positive $-$ components of momenta. Moreover, l_1^+ and l_2^+ must not be much larger than r^+ and l_1^- and l_2^- must not be so small that the lines are in the Coulomb region.

But if we do have such a pinch, then we can reroute the momentum as in Fig. 8, unless the left-hand line l_1+l_2 is an external momentum.

So we now have a prescription for avoiding a pinch, if it is possible at all. This is to start at the top end of the line r, and to route r *back* against the flow of $-$ momentum, as in Fig. 8. If by this procedure we do not arrive at the bottom end of the line r, then we arrive at one of the two incoming lines, either the proton or the virtual photon. In either case
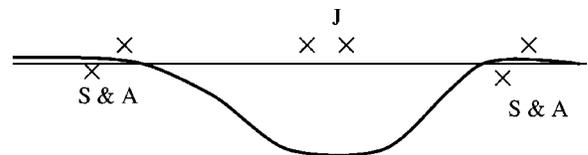
FIG. 6. Contour deformation for gluon momentum r^+ in Fig. 5. The poles are labeled by A, S or J according to the subgraph that causes them.
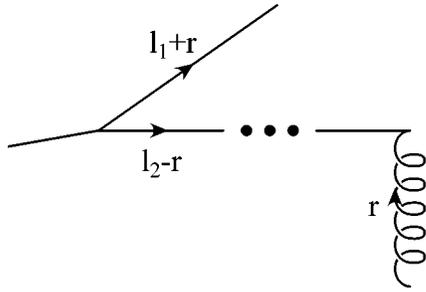
FIG. 7. A situation giving a pinch of r^+.

we can finish the construction of the route for r by taking it on lines in the A subgraph. Since by definition these have large $+$ momenta, while r^+ is small, none of these lines contribute to a possible pinch of r^+.

This completes the proof that the contour of integration over loop momenta is not trapped in a region where the soft approximation Eq. (5) fails for the attachment of a soft gluon to the A subgraph.

IV. NONPERTURBATIVE FINAL-STATE INTERACTIONS

The above proof of factorization relies strictly on the power counting obtained in perturbation theory. We now show that nonperturbative soft effects do not affect the proof, at least in the context of normal models, such as those appropriate to the soft Pomeron physics treated in Refs. [2,3] for the case of the Drell-Yan process.

One of the key points that enabled us to use the soft approximation, Eq. (4), was that in finite order perturbation theory the only soft subgraphs that give a leading power are those which attach to the collinear subgraphs purely by gluon lines. Any such soft gluon joins two vertices with momenta of very different rapidities, so that the vertex Γ^μ in Eq. (4) can be replaced by $\Gamma^- n_J^\mu$.

We know that there must be nonperturbative final-state interactions that perform hadronization, and that these interactions give a distribution of particles with several per unit rapidity. These interactions can be represented by graphs like Fig. 1 except that the soft attachments to the jets are not purely gluons joining vertices of very different rapidities. In a perturbative model of this situation, to get a contribution that does not fall off as a power of Q, the rapidities carried by lines in the graph must cover the whole range from the rapidity of A to the rapidity of J, without large gaps. This implies that the order of the graph must be at least of order the available rapidity range, i.e., the order of the relevant
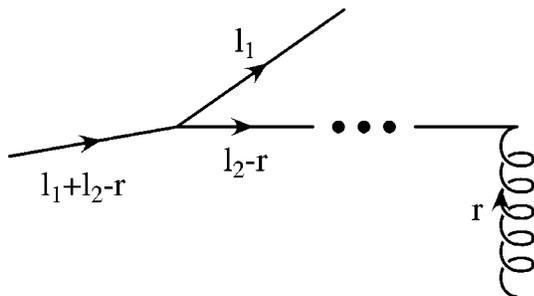
graphs increases at least as fast as $\ln Q$ at large Q.

Luckily, the second part of the argument leading to the soft approximation still applies, that is, the contour deformation. In general, when the momentum transfer s^μ across the subgraph S is associated the nonperturbative hadronization interactions, we expect s^μ to have components of order $(\Lambda/Q^2, \Lambda/Q^2, \Lambda)$. Once we deform s^+ to values of order Λ or bigger, as is the result of our argument, the jet lines in which s^μ flows become off-shell by order ΛQ. We now have a perturbative region where we can use the usual power-counting rules.

This argument is very similar to arguments used before the advent of QCD to prove that parton model formulas are valid. See, for example, Refs. [2,3,21]. In those arguments it was assumed that the result of contour deformations such as we perform is that the contours can be taken to infinity with a zero result—the assumption of soft behavior of vertices. In QCD we cannot take the contours to infinity, but instead we take the contours from the original region to one that we can treat either purely perturbatively or with the aid of Ward identities.

V. CONCLUSIONS

We have proved the factorization theorem for the general class of diffractive deep-inelastic processes, and generalizations including those to which the fracture function formalism of Trentadue and Veneziano [14] applies. The proof includes a treatment of nonperturbative effects at the level of Refs. [2,3,21].

Given the results of Refs. [2,3] on the Drell-Yan process, we must not expect the theorem to be applicable to hadron-hadron collisions. Absorptive corrections should reduce diffractive hard-scattering cross sections below the expectations given by the factorization formula on the basis of deep-inelastic data. Furthermore, the "coherent Pomeron mechanism" of [4,11,22] may exist. It is only when one of the initiating particles is a lepton that the proof of factorization is valid.

The proof would appear to apply also to *direct* photoproduction of jets, etc., because the initiating particle of the hard scattering is a lepton. However, the proof does not apply to *resolved* photoproduction processes, since these are in effect hadron-hadron processes. The lack of an absolutely unambiguous separation between direct and resolved photoproduction will presumably limit the accuracy of the application of the factorization formula to direct diffractive photoproduction.

Note added. After completion of this paper, a paper by Grazzini, Trentadue, and Veneziano [23] appeared, in which the concept of an "extended fracture function" is defined, with the aid of the cut-vertex formalism of Mueller [24]. Extended fracture functions are exactly the same as the diffractive parton densities I define in this paper; they are fracture functions without the integral over the transverse momentum of the detected final-state hadron. Grazzini *et al.* give a brief proof of factorization in the case of $(\phi^3)_6$ theory. This theory is simpler than QCD since soft exchanges are power suppressed. Given this fact, the proofs and results in the paper of Grazzini *et al.* are completely compatible with those in the present paper.

FIG. 8. Rerouting r in this way avoids the pinch given by Fig. 7.

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