Chaotic behavior of multiparticle production in pp collisions at 400 GeV/c

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The entropy indices are calculated from the data of 400 GeV/c pp collisions. The entropy indices are increased with decreasing average multiplicities of the final states. The multiparticle production in pp collisions is chaotic. [S0556-2821(98)00705-X]

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I. INTRODUCTION

How to measure the chaoticity of multiparticle production in high energy collisions is a complicated problem. For classical nonlinear dynamics, the Lyapunov exponent λ can be defined to characterize divergent distances between trajectories; no such conventional description of chaotic behavior of QCD branching processes exists. Recently, Cao and Hwa have investigated the possible signatures of chaoticity of multiparticle production in high energy collisions [1–3]. They have defined a new entropy index μ_q which can be used as an adequate parameter to measure chaotic behavior in branching processes. The entropy indices also provide an efficient way to distinguish whether the initial parton in QCD branching processes is a quark or a gluon [3]. In order to extract the entropy indices, one should study the phase space (for example, the pseudorapidity η space) and the event space simultaneously. For each event one can calculate the factorial moments in η space, event by event; the values of factorial moments fluctuate greatly. The entropy index μ_q describes the degree of such a fluctuation from event to event; i.e., the entropy index is a simple quantity that can characterize the degree of fluctuations of both the parton multiplicity and the spatial pattern of the final states. It is a quantity that can be measured experimentally with ease.

According to Monte Carlo simulations, the authors of [1] were able to show that perturbative QCD branching exhibits chaotic behavior, whereas a model lacking the characteristics of QCD does not. It is found that the entropy index is very large when the parton which initiates the branching process is a quark, especially in the case of a lower average multiplicity of the final states [3].

In this paper the entropy indices are calculated from the experimental data of multiparticle production in pp collisions at 400 GeV/c in order to learn the chaoticity of multiparticle production in pp collisions and to see how the entropy index looks in reactions not dominated by QCD parton showering.

II. ENTROPY AND ENTROPY INDEX IN THE EVENT SPACE

The details of the method can be found in [2]; here, we describe it briefly. The pseudorapidity space is divided into M bins and calculated the normalized factorial moments for each event according to the following formula:

$$F_q^e(M)=f_q^e(M)/[f_1^e(M)]^q \tag{1}$$

and

$$f_q^e(M)=M^{-1}\sum_{j=1}^{M} n_j(n_j-1)\cdots(n_j-q+1), \tag{2}$$

where n_j is the number of particles in the jth bin for event e. For different events, the fluctuation of $F_q^e(M)$ may be very large. In order to measure the degree of this fluctuation, a new normalized moment was defined in [1],

$$C_{p,q}(M)=\langle F_q^p(M)\rangle/\langle F_q^1(M)\rangle^p \tag{3}$$

and

$$\langle F_q^p(M)\rangle=\frac{1}{N}\sum_{e=1}^{N}[F_q^e(M)]^p, \tag{4}$$

where N is the number of events and p is a positive real number. If $C_{p,q}(M)$ has a power law behavior in M, i.e.,

$$C_{p,q}(M)\sim M^{\phi_q(p)}, \tag{5}$$

then a new entropy index is defined by [1]

$$\mu_q=\frac{d}{dp}\,\phi_q(p)|_{p=1} \tag{6}$$

and a new entropy in the event space is defined as

$$S_q=\sum_{e=1}^{N} P_q^e\,\ln(P_q^e), \tag{7}$$

where

$$P_q^e=F_q^e(M)\Big/\sum_{e=1}^{N}F_q^e(M). \tag{8}$$

The relationship between the entropy and entropy index is

$$S_q=\ln(NM^{-\mu_q}). \tag{9}$$

It can be seen from the above formula that a small μ_q corresponds to a large entropy, which means no chaotic behav-

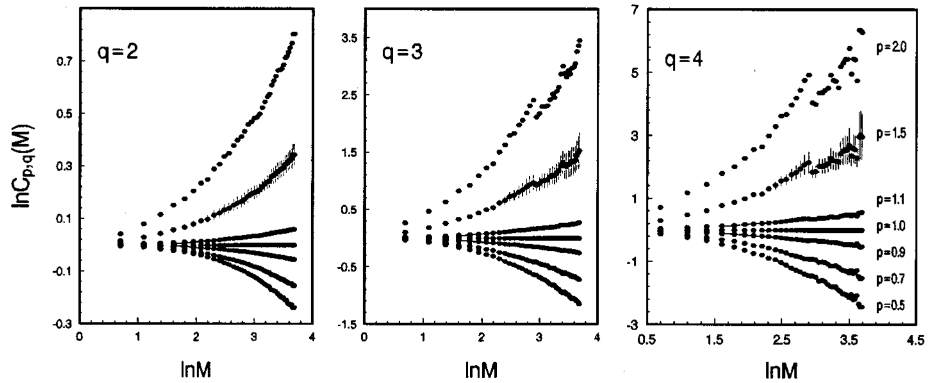

FIG. 1. ln $C_{p,q}(M)$ versus ln M for an $N_{ev}\geqslant 4$ event sample. The error bars are drawn only for one curve; others are dropped.

ior of particle production in branching processes. In order to decrease the entropy, the μ_q must be large, and so a large μ_q means chaotic behavior.

III. EXPERIMENTAL DATA

In the present investigation, the angular distributions for charged particles produced in pp collisions at 400 GeV/c were measured by using the Lexan Bubble Chamber (LEBC) films offered by the CERN NA27 Collaboration. The details of the NA27 experiment, which was performed using the rapid cycling hydrogen bubble chamber LEBC in association with the European Hybrid spectrometer at the CERN Super Proton Synchrotron (SPS), was described elsewhere [4]. The diameter of the bubble was 17 μm. The density of the bubbles was 80 cm^{-1}. Without a magnetic field, the tracks were linear and the pictures were clear, which helped to measure the angular distributions of the tracks accurately. These films were measured on a semiautomatic scanning table with magnification 17. In order to get the angular distribution in the c.m. system (c.m.s.), a visual ionization scan was performed. Details of the measurement are given in [5]. A total of 3730 nonsingle-diffractive events ($n_{ch}\geqslant 4$) were measured. The accuracy in pseudorapidity in the region of interest ($-2\leqslant\eta\leqslant 2$) is of the order of 0.1 pseudorapidity units.

IV. RESULTS

The single-particle density distribution in pseudorapidity space is nonflat. In order to reduce the effect of a nonflat particle density distribution $\rho(\eta)$, one can use a new variable $X(\eta)$, which was defined by [6,7]

$$X(\eta)=\int_{\eta_1}^{\eta}\rho(\eta')d\eta' \bigg/ \int_{\eta_1}^{\eta_2}\rho(\eta')d\eta', \qquad (10)$$

where η_1 and η_2 are the extreme points in the distribution $\rho(\eta)$. In $X(\eta)$ space the single-particle density is uniformly distributed from 0 to 1. Then we divided the region [0-1] into M bins and calculated the moments $C_{p,q}(M)$ for $q=2,3,4$, $p=0.5,0.7,0.9,1.0,1.1,1.5,2.0$, and $M=1,2,...,40$, respectively. The ln $C_{p,q}(M)$ versus ln M are plotted in Fig. 1. The linear fits to the experimental data in the region of $M=5-25$ have been performed for $q=2,3,4$ and $p=0.9,1.1$, respectively. The fit parameters are given in Table I. From these parameters we can extract the entropy indices which are plotted in Fig. 2(a). It can be seen from Fig. 2(a) that the entropy indices are quite large. Comparing with the theoretical predication in [3], the experimental data are very close to the Monte Carlo–simulated values of a quark jet at a fixed coupling constant $\alpha_s=0.05$ [see Fig. 2(b)]. However, it is significantly greater than the simulated values of a gluon jet at the same coupling constant. It looks like the original parton that initiates the parton shower in 400 GeV/c pp collisions is a quark.

As indicated in [3], for the QCD parton shower, the parton multiplicities are low, when the fixed coupling constant α_s is small and especially so for the Q jet. The fluctuation is very large when the average multiplicity is lower. It causes a larger μ_q. This is coincident with the experimental results of multiplicity production in pp collisions at 400 GeV/c. Be-

TABLE I. The fit parameters obtained by linear fit ($y=ax+b$) to the experimental data ($N_{ev}\geqslant 4$ event sample).

q	p	a	b
2	0.9	-0.0197 ± 0.0009	0.0284 ± 0.0018
2	1.1	0.0209 ± 0.0009	-0.0289 ± 0.0018
3	0.9	-0.0897 ± 0.0022	0.1101 ± 0.0046
3	1.1	0.0923 ± 0.0020	-0.1064 ± 0.0042
4	0.9	-0.1743 ± 0.0049	0.1788 ± 0.0104
4	1.1	0.1760 ± 0.0059	-0.1659 ± 0.0127

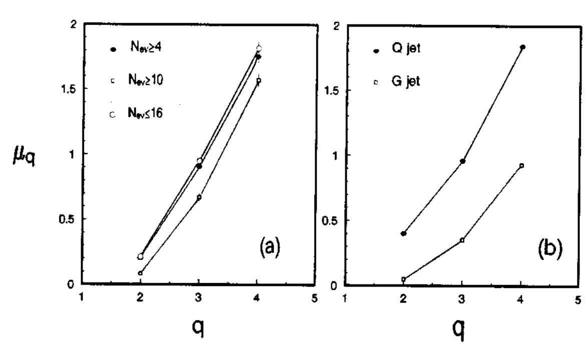

FIG. 2. μ_q versus q for different average multiplicity event samples. (a) The experimental results. (b) The Monte Carlo–simulated results obtained by Cao and Hwa [3].

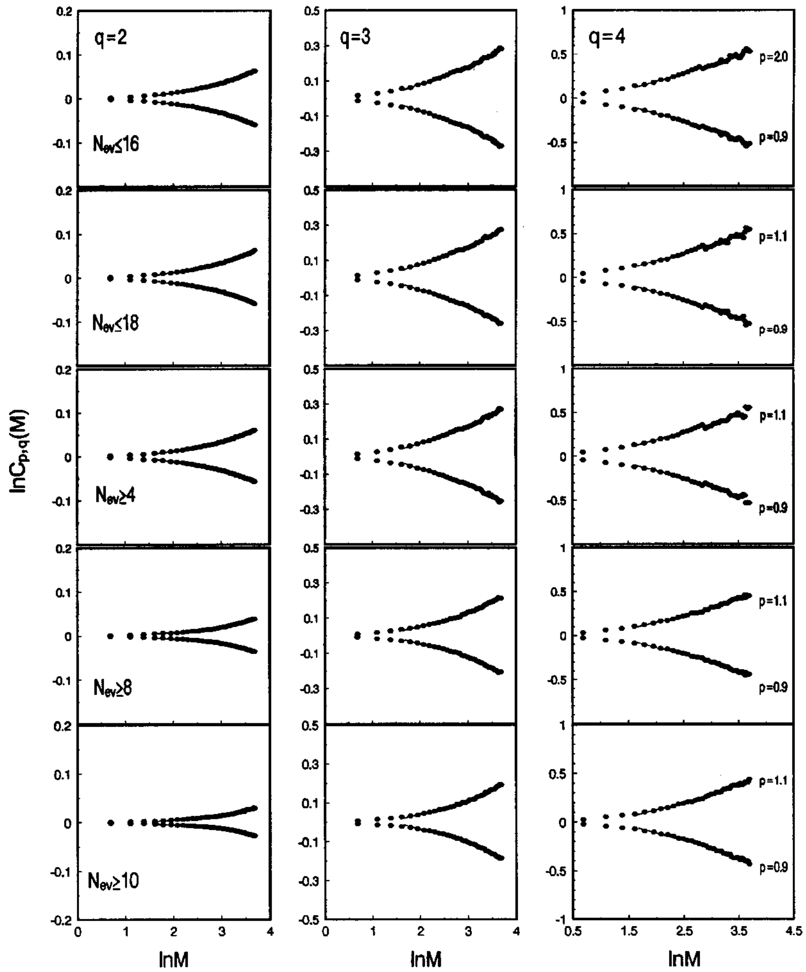

FIG. 3. $\ln C_{p,q}(M)$ versus $\ln M$ for different event samples.

cause the average multiplicity is lower, the average multiplicity $\overline{n}=10.27$ for an $N_{ev}\geq4$ event sample. It is reasonable to expect that the values of μ_q will be decreased when the average multiplicity is increased. This is because the gluon-initiated shower should be included and become important when the average multiplicity is increased. In order to confirm this, we have performed the same calculations for the following four multiplicity distributions: (1) $N_{ev}\geq10$, (2) $N_{ev}\geq8$, (3) $N_{ev}\leq18$, (4) $N_{ev}\leq16$. The event samples (1) and (2) correspond to the multiplicity distributions that are truncated at a lower tail; the event samples (3) and (4) correspond to the multiplicity distributions that are truncated at a higher tail. The $\ln C_{p,q}(M)$ versus $\ln M$ for different event samples are shown in Fig. 3. In order to make it clear, only the cases of $p=0.9$ and $p=1.1$ are shown in Fig. 3. It can be seen from Fig. 3 that the moment $C_{p,q}(M)$ becomes smaller when the average multiplicities become larger. The entropy indices obtained from above results and other relative parameters are listed in Table II. In Fig. 2(a) only a part of the entropy indices was plotted. It can be found from Fig. 2(a) and Table II that the entropy indices indeed become smaller when the average multiplicity becomes larger. These results are in agreement with the theoretical prediction in [3]. However, the experimental values are still significantly greater than the simulated values of the gluon jet. This means that the dominated original parton that initiates the parton shower in 400 GeV/c pp collisions may be a quark or the reactions may not be dominated by QCD parton showering.

TABLE II. Entropy indices and relative parameters.

Event samples	Event numbers	$\overline{n}$	μ_2	μ_3	μ_4
$N_{ev}\leq16$	3478	9.61	0.214 ± 0.009	0.951 ± 0.022	1.818 ± 0.052
$N_{ev}\leq18$	3612	9.92	0.206 ± 0.009	0.934 ± 0.023	1.787 ± 0.060
$N_{ev}\geq4$	3730	10.27	0.203 ± 0.009	0.909 ± 0.021	1.752 ± 0.054
$N_{ev}\geq8$	2864	11.74	0.119 ± 0.007	0.753 ± 0.030	1.627 ± 0.050
$N_{ev}\geq10$	2145	12.99	0.084 ± 0.005	0.670 ± 0.029	1.571 ± 0.052

V. CONCLUSION

In this paper, the entropy indices were calculated from the experimental data of 400 GeV/c pp collisions. Comparing with the theoretical prediction in [3], the experimental data are very close to the Monte Carlo–simulated values of a quark jet at a very small fixed coupling constant α_s. However, it is significantly greater than the simulated values of a gluon jet at the same coupling constant. It looks like the results show that the dominant original parton of the QCD branching process may be a quark for pp collisions at lower energy. The entropy indices are increased with decreasing average multiplicities of the final states. In the sense of the entropy index used, the multiparticle production in pp collisions at 400 GeV/c is chaotic.

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