Corrections to oblique parameters induced by anomalous vector boson couplings

J. J. van der Bij and Boris Kastening
Albert-Ludwigs-Universität Freiburg, Fakultät für Physik, Hermann-Herder-Straße 3, D-79104 Freiburg, Germany
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We study quadratically divergent radiative corrections to the oblique parameters at CERN LEP1 induced by nonstandard vector boson self-couplings. We work in the Stückelberg formalism and regulate the divergences through a gauge-invariant higher derivative scheme. Using consistency arguments together with the data we find a limit on the anomalous magnetic moment $\Delta\kappa$ of the W boson, $|\Delta\kappa|\lesssim 0.26$. [S0556-2821(98)04205-2]

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I. INTRODUCTION

With the running of the CERN e^+e^- collider LEP-200 and with results from the Fermilab Tevatron the self-interactions of the vector bosons are nowadays being measured directly. Within the standard model the vector boson self-interactions are fully determined by the gauge structure of the theory. Deviations from the standard model can be parametrized by a set of operators describing so-called anomalous couplings and experiment can put a limit on the coefficient of these operators. However the presence of anomalous gauge boson self-couplings will violate the renormalizability of the theory. As a consequence one can generate divergent contributions to quantities at lower energies than the two vector boson threshold. When one uses a cutoff procedure one can estimate the induced effects and use low-energy data to put limits on the assumed anomalous couplings. As the data at low energy have become very precise since LEP-100, strong limits can be found. Indeed in a recent discussion it was argued that the LEP-100 data can obviate the LEP-200 data, with the exception of so-called blind directions in coupling constant space. These blind directions correspond to operators that do not have direct effects in propagators and can therefore only be seen after insertion inside a loop, indirectly generating propagator effects. In the original articles [1,2] on these induced effects quadratically and quartically divergent contributions were found, leading to relatively severe restrictions. These results were criticized in [3,4], where it was argued that the quadratic divergences would be gauge-dependent and nonphysical, so one should use dimensional regularization as a cutoff, which gives logarithmic divergence and weak constraints. In a more recent calculation [5], dimensional regularization in d dimensions was used to determine the quadratic divergences as poles in $(d-2)$. In [6] the divergences were regularized by using the Higgs field as a regulator. An analysis based on the philosophy of [3,4] is presented in [7]. Both calculations [5,6] confirm the original calculations as having quadratic divergent contributions, as is consistent with power counting in chiral perturbation theory. However also here the situation is not fully satisfactory, as only one cutoff scale is assumed to be present. In reality however, there are different cutoff scales present. This is most easily seen from the vector boson propagators, which consist of longitudinal and transversal parts, which could have different form factors. Indeed one would expect the longitudinal part to have structure at a relatively low scale, as this part describes effects coming from the Goldstone boson part of the theory, dependent on the mechanism of spontaneous symmetry breaking, where strong interactions might be present. In order to clarify the situation we therefore perform in this paper a calculation of induced low energy effects from anomalous effects using a higher derivative regulator. More precisely, we describe vector boson physics without a Higgs boson as a gauged nonlinear sigma-model. The anomalous couplings are then given by higher dimensional operators. This is the Stückelberg formalism and is closely related to chiral perturbation theory. This has the advantage that the whole calculation can be performed in a gauge-invariant way. The quadratic and higher divergences are regulated via covariant higher derivative terms; the remaining logarithmic ones via dimensional regularization.

We limit the anomalous couplings to terms that have no CP violation, as we know CP violation to be very small. Furthermore we limit the discussion to terms that correspond to dimension four operators in the unitary gauge. Within the standard model there is an extra custodial $\mathrm{SU}(2)_R$ symmetry in the limit of vanishing hypercharge, which has as a consequence that the ρ-parameter deviates from unity only through hypercharge couplings. This symmetry has to be protected at least to some level also in the anomalous couplings and we will focus mostly on the operators where the custodial symmetry is only violated through a minimal coupling to hypercharge.

We have assumed, that the only relevant gauge bosons are those of the $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ gauge group and that new physics does not couple directly to light fermions. Therefore any contribution of new physics below the vector-boson pair threshold can only come from vacuum polarization corrections to gauge boson propagators [8,9]. For most of the available low-energy, Z and W observables it is possible to parametrize these corrections by the six so-called oblique parameters S, T, U, V, W, X [8,9]. These parameters are therefore well suited to compare experiment with our calculation and we will use a recent analysis in this terminology.

In Sec. II, we will outline the model we use to describe the electroweak sector of the standard model, give the various anomalous couplings and describe our regularization procedure. In Sec. III, we give our results for the oblique parameters. In Sec. IV we investigate the contribution of our regularization procedure to the oblique parameters and study

the consistency of the method. In Sec. V, we analyze our results with respect to experimental data.

II. THE MODEL

Since the origin of electroweak symmetry breaking is unknown, we do not assume the existence of a Higgs field, but describe the breaking using the Stückelberg formalism [10,11]. That is, we write the spontaneously broken $SU(2)_L \times U(1)_Y$ theory as a gauged nonlinear sigma model.

We need the following definitions. Let

$$\boldsymbol{W}_{\mu\nu} = \tfrac{1}{2}\tau_a W^a_{\mu\nu} = \partial_\mu \boldsymbol{W}_\nu - \partial_\nu \boldsymbol{W}_\mu + ig[\boldsymbol{W}_\mu, \boldsymbol{W}_\nu] \tag{1}$$

and

$$\boldsymbol{B}_{\mu\nu} = \tfrac{1}{2}\tau_3 B_{\mu\nu} = \partial_\mu \boldsymbol{B}_\nu - \partial_\nu \boldsymbol{B}_\mu \tag{2}$$

be the $SU(2)_L$ and $U(1)_Y$ field strengths. Let U be an SU(2) valued field that describes the longitudinal degrees of freedom of the vector fields and let U transform as

$$U \to U_L U U_Y \tag{3}$$

under $SU(2)_L \times U(1)_Y$ gauge transformations with $U_L = \exp(-i/2\, g \vec{\Theta}_L \cdot \vec{\tau})$ and $U_Y = \exp(-i/2\, g' \Theta_Y \tau_3)$, where g' is the hypercharge coupling. Define auxiliary quantities

$$\boldsymbol{T} = U \tau_3 U^\dagger \tag{4}$$

and

$$\boldsymbol{V}_\mu = -\frac{i}{g}(D_\mu U) U^\dagger \tag{5}$$

with

$$D_\mu U = \partial_\mu U + ig \boldsymbol{W}_\mu U + ig' U \boldsymbol{B}_\mu . \tag{6}$$

Under $SU(2)_L \times U(1)_Y$ gauge transformations, they transform as $\boldsymbol{T} \to U_L \boldsymbol{T} U_L^\dagger$ and $\boldsymbol{V}_\mu \to U_L \boldsymbol{V}_\mu U_L^\dagger$.

Electroweak theory without fermions and without the Higgs scalar is then described by the Lagrange density

$$\mathcal{L}_{EW} = -\tfrac{1}{2}\,\mathrm{Tr}(\boldsymbol{W}_{\mu\nu}\boldsymbol{W}^{\mu\nu}) - \tfrac{1}{2}\,\mathrm{Tr}(\boldsymbol{B}_{\mu\nu}\boldsymbol{B}^{\mu\nu}) + \frac{g^2 v^2}{4}\,\mathrm{Tr}(\boldsymbol{V}_\mu \boldsymbol{V}^\mu), \tag{7}$$

where v replaces the vacuum expectation value of the Higgs field.

In this formalism, the CP conserving anomalous three and four vector boson couplings that are of dimension four in unitary gauge ($U = \mathbf{1}$) are described by the following set of gauge-invariant operators:

$$\mathcal{L}_1 = -i\,\mathrm{Tr}(\boldsymbol{W}^{\mu\nu}[\boldsymbol{V}_\mu, \boldsymbol{V}_\nu]), \tag{8}$$

$$\mathcal{L}_2 = -\frac{i}{2} B^{\mu\nu}\mathrm{Tr}(\boldsymbol{T}[\boldsymbol{V}_\mu, \boldsymbol{V}_\nu]), \tag{9}$$

$$\mathcal{L}_3 = -\frac{i}{2}\,\mathrm{Tr}(\boldsymbol{T}\boldsymbol{W}^{\mu\nu})\mathrm{Tr}(\boldsymbol{T}[\boldsymbol{V}_\mu, \boldsymbol{V}_\nu]), \tag{10}$$

$$\mathcal{L}_4 = (\,\mathrm{Tr}[\boldsymbol{V}_\mu \boldsymbol{V}_\nu])^2, \tag{11}$$

$$\mathcal{L}_5 = (\mathrm{Tr}[\boldsymbol{V}^\mu \boldsymbol{V}_\mu])^2, \tag{12}$$

$$\mathcal{L}_6 = \mathrm{Tr}(\boldsymbol{V}^\mu \boldsymbol{V}^\nu)\ \mathrm{Tr}(\boldsymbol{T}\boldsymbol{V}_\mu)\mathrm{Tr}(\boldsymbol{T}\boldsymbol{V}_\nu), \tag{13}$$

$$\mathcal{L}_7 = \mathrm{Tr}(\boldsymbol{V}^\mu \boldsymbol{V}_\mu)(\mathrm{Tr}[\boldsymbol{T}\boldsymbol{V}_\nu])^2, \tag{14}$$

$$\mathcal{L}_8 = (\mathrm{Tr}[\boldsymbol{T}\boldsymbol{V}_\mu])^2(\mathrm{Tr}[\boldsymbol{T}\boldsymbol{V}_\nu])^2, \tag{15}$$

which we introduce by adding

$$\mathcal{L}_{ano} = \sum_{i=1}^{8} g_i \mathcal{L}_i \tag{16}$$

to $\mathcal{L}_{EW}$. In our treatment, the aforementioned approximate custodial $SU(2)_R$ symmetry is realized by $U \to U U_R$ with $U_R \in \mathrm{SU}(2)$. Among the operators (8)–(15), only $\mathcal{L}_1$, $\mathcal{L}_4$ and $\mathcal{L}_5$ conserve this custodial symmetry in the limit of vanishing hypercharge coupling. At the same time, the absence of the other operators leads to a cancellation of quartic divergences in oblique electroweak parameters [2], as will be seen below. In other words, $\mathcal{L}_1$, $\mathcal{L}_4$ and $\mathcal{L}_5$ correspond to the so-called blind directions in coupling constant space which do not receive the severe constraints that the presence of quartic divergences would impose [12]. We will therefore assume that the custodial symmetry is respected by the anomalous couplings and thus restrict our analysis with respect to experimental results to these three operators.

Since higher than logarithmic divergences are set to zero by dimensional regularization, we have to parametrize them using a different method. We will apply the method of higher covariant derivatives [13]. In the version used here, it leaves only logarithmic divergences in the anomalous contribution to the oblique parameters in Landau gauge. These remaining divergences are then regulated dimensionally. Specifically, we add to the theory

$$\mathcal{L}_{hc,tr} = \frac{1}{2\Lambda_W^2}\,\mathrm{Tr}[(D_\alpha \boldsymbol{W}_{\mu\nu})(D^\alpha \boldsymbol{W}^{\mu\nu})] + \frac{1}{2\Lambda_B^2}\,\mathrm{Tr}[(\partial_\alpha \boldsymbol{B}_{\mu\nu}) \times (\partial^\alpha \boldsymbol{B}^{\mu\nu})] \tag{17}$$

for the transverse degrees of freedom of the gauge fields and

$$\mathcal{L}_{hc,lg} = -\frac{g^2 v^2}{4\Lambda_V^2}\,\mathrm{Tr}[(D^\alpha \boldsymbol{V}^\mu)(D_\alpha \boldsymbol{V}_\mu)] \tag{18}$$

for the longitudinal ones, where the Λ_X parametrize the quadratic divergences and are expected to represent the scales where new physics comes in. The covariant derivatives in (17) and (18) are defined by

$$D_\alpha \boldsymbol{W}_{\mu\nu} = \partial_\alpha \boldsymbol{W}_{\mu\nu} + ig[\boldsymbol{W}_\alpha, \boldsymbol{W}_{\mu\nu}], \tag{19}$$

$$D_\alpha \boldsymbol{V}_\mu = \partial_\alpha \boldsymbol{V}_\mu + ig[\boldsymbol{W}_\alpha, \boldsymbol{V}_\mu]. \tag{20}$$

As a variant of $\mathcal{L}_{hc,lg}$, one can use e.g.

$$\mathcal{L}'_{hc,lg}=-\frac{v^2}{4\Lambda_V^2}\,\mathrm{Tr}[(D^\alpha D^\beta U)(D_\alpha D_\beta U)^\dagger]$$

$$=-\frac{g^2v^2}{4\Lambda_V^2}\Bigg\{\mathrm{Tr}[(D^\alpha \boldsymbol{V}^\mu)(D_\alpha \boldsymbol{V}_\mu)]+\frac{g}{2}\mathcal{L}_1+\frac{g'}{2}\mathcal{L}_2+g^2\mathcal{L}_4-\frac{g^2}{2}\mathcal{L}_5\Bigg\} \tag{21}$$

instead, which is closer to a natural regularization in the linear model. The quartic divergences are invariant under this change due to the additional suppression factor g^2v^2/Λ_V^2. Once we impose absence of quartic divergences by setting $g_2=g_3=g_6=g_7=g_8=0$, it is easily seen that now the quadratic divergences are invariant under the change in regularization.

We remark here that a reasonable assumption of the dynamics would make Λ_V the smallest, being related to the Goldstone sector of the theory. Also, one would expect Λ_B to be very large, as it is hard to imagine a fundamental dynamics, where strong interactions would start in the Abelian sector of the theory. The presence of the approximate custodial symmetry tells us that terms with explicit T or $B_{\mu\nu}$ should be heavily suppressed. We finally note that the signs in front of $\Lambda_V^2,\Lambda_W^2,\Lambda_B^2$ are not determined *a priori*. The method of gauge fixing we use is outlined in Appendix B.

Finally, our conventions lead to the following definitions of the usual gauge fields:

$$\begin{pmatrix}Z_\mu\\A_\mu\end{pmatrix}=\begin{pmatrix}c & s\\ -s & c\end{pmatrix}\begin{pmatrix}W_\mu^3\\B_\mu\end{pmatrix}, \tag{22}$$

$$W_\mu^\pm=\frac{1}{\sqrt{2}}(W_\mu^1\mp iW_\mu^2), \tag{23}$$

where we have used the abbreviations $c=\cos\Theta_W$, $s=\sin\Theta_W$ and where the weak mixing angle is defined by $\tan\Theta_W=g'/g$.

III. OBLIQUE PARAMETERS

In models where new physics comes in at scales much larger than the electroweak scale, it is usually assumed that an expansion of the vacuum polarizations linear in k^2 is sufficiently accurate to parametrize the new physics effects at the electroweak scale. Accordingly, a description of new physics effects in terms of three parameters S, T, U is appropriate [8]. In our description this assumption is explicitly violated as can be seen from the structure of the k^2 and k^4 terms in the vacuum polarizations given in Appendix A 2 by (A5)–(A8). We therefore need all six parameters S, T, U, V, W, X used when observables at the scales 0, m_Z^2, m_W^2 are taken into account and the above assumption is not valid [9].

The six oblique parameters are computed from the $\Pi_{XY}^g(k^2)$ part of the nonstandard model contribution to the vacuum polarizations,

$$\Pi_{XY}^{\mu\nu}(k^2)=\Pi_{XY}^g(k^2)g^{\mu\nu}+\Pi_{XY}^k(k^2)\frac{k^\mu k^\nu}{k^2}, \tag{24}$$

with $XY=WW,ZZ,ZA,AA$. Their definitions are, according to [9] (except that our conventions lead to a different sign of s),

$$\alpha S=4s^2c^2\left[\frac{\Pi_{ZZ}^g(m_z^2)-\Pi_{ZZ}^g(0)}{m_z^2}+\frac{c^2-s^2}{sc}\Pi_{ZA}^{g\prime}(0)-\Pi_{AA}^{g\prime}(0)\right], \tag{25}$$

$$\alpha T=\frac{\Pi_{WW}^g(0)}{m_w^2}-\frac{\Pi_{ZZ}^g(0)}{m_z^2}, \tag{26}$$

$$\alpha U=4s^2\left[\frac{\Pi_{WW}^g(m_w^2)-\Pi_{WW}^g(0)}{m_w^2}-c^2\frac{\Pi_{ZZ}^g(m_z^2)-\Pi_{ZZ}^g(0)}{m_z^2}+2sc\Pi_{ZA}^{g\prime}(0)-s^2\Pi_{AA}^{g\prime}(0)\right], \tag{27}$$

$$\alpha V=\Pi_{ZZ}^{g\prime}(m_z^2)-\frac{\Pi_{ZZ}^g(m_z^2)-\Pi_{ZZ}^g(0)}{m_z^2}, \tag{28}$$

$$\alpha W=\Pi_{WW}^{g\prime}(m_w^2)-\frac{\Pi_{WW}^g(m_w^2)-\Pi_{WW}^g(0)}{m_w^2}, \tag{29}$$

$$\alpha X=sc\left[\frac{\Pi_{ZA}^g(m_z^2)}{m_z^2}-\Pi_{ZA}^{g\prime}(0)\right]. \tag{30}$$

These combinations are well-suited for comparison with experimental data. In particular, the W parameter only appears in the rather poorly measured W width and can therefore be dropped from the analysis.

To present the results of our calculation, a modification of the S and U parameters is useful. Let us define

$$\hat{S}=S-4s^2c^2V, \tag{31}$$

$$\hat{U}=U+4s^2c^2V-4s^2W. \tag{32}$$

In this way, T is getting contributions only from k-independent terms, $\hat{S}$ and $\hat{U}$ only from k^2 terms and V, W, X only from k^4 terms (higher powers of k are absent in our treatment of the quadratically divergent terms). The relevance of this is that the k^4 terms of the various vacuum polarizations are essentially identical, while our predictive power for the k^2 terms and the k-independent terms hinges on additional assumptions, as will be seen below.

Including all anomalous couplings (8)–(15), we have computed the quartically divergent contributions to the $\Pi_{XY}^g(k^2)$. The results can be found in Appendix A 1. These contributions are k-independent. A look at our definitions of the oblique parameters (25)–(30) shows that only T, representing the correction to the ρ parameter, depends on k-independent parts of vacuum polarizations and therefore only it can be quartically divergent. We get

$$\alpha T = g_2^2 \frac{\Lambda_V^2 \Lambda_B^2}{m_{W0}^4}\left(-\frac{3}{4\epsilon} - \frac{5}{8} + \frac{3}{4}\frac{\Lambda_V^2 \ln\frac{\Lambda_V^2}{\mu^2} - \Lambda_B^2 \ln\frac{\Lambda_B^2}{\mu^2}}{\Lambda_V^2 - \Lambda_B^2}\right) + (2g_1 g_3 + g_3^2)\frac{\Lambda_V^2 \Lambda_W^2}{m_{W0}^4}\left(-\frac{3}{4\epsilon} - \frac{5}{8} + \frac{3}{4}\frac{\Lambda_V^2 \ln\frac{\Lambda_V^2}{\mu^2} - \Lambda_W^2 \ln\frac{\Lambda_W^2}{\mu^2}}{\Lambda_V^2 - \Lambda_W^2}\right) + g_6 \frac{\Lambda_V^4}{m_{W0}^4}\left(-\frac{13}{4\epsilon} - \frac{31}{8} + \frac{13}{4}\ln\frac{\Lambda_V^2}{\mu^2}\right) + g_7 \frac{\Lambda_V^4}{m_{W0}^4}\left(-\frac{4}{\epsilon} - \frac{9}{2} + 4\ln\frac{\Lambda_V^2}{\mu^2}\right) + g_8 \frac{\Lambda_V^4}{m_{W0}^4}\left(-\frac{3}{\epsilon} - \frac{7}{2} + 3\ln\frac{\Lambda_V^2}{\mu^2}\right) + \mathcal{O}(\Lambda^2). \tag{33}$$

Here, ϵ is defined by $d = 4 - 2\epsilon$, where d is the dimension of spacetime. From the presence of these quartic divergences we have therefore severe constraints on the quartic vector boson couplings. This is in agreement with [14], but in contrast to [15], who however use dimensional regularization and therefore find only a logarithmic divergence. Evidently, absence of the custodial symmetry breaking couplings g_2, g_3, g_6, g_7, g_8 leads to a cancellation of the quartic divergencies in T. In the further analysis we will therefore only keep the anomalous couplings g_1, g_4, g_5 nonzero. This is consistent with the dynamical principle from [2], that the breaking of the custodial symmetry should be only through the minimal coupling to hypercharge.

Our results for the $\Pi_{XY}^g(k^2)$ are given in Appendix A 2. From them we get

$$\alpha \hat{S} = -s^2 \frac{g_1^2}{(4\pi)^2}\left(\frac{20\Lambda_V^2\Lambda_W^4}{3m_{W0}^2(\Lambda_V^2 - \Lambda_W^2)^2} + \frac{2\Lambda_V^2\Lambda_W^2(14\Lambda_V^4 - 33\Lambda_V^2\Lambda_W^2 + 9\Lambda_W^4)}{3m_{W0}^2(\Lambda_V^2 - \Lambda_W^2)^3}\ln\frac{\Lambda_V^2}{\Lambda_W^2}\right) - 2s^2\frac{\Lambda_V^2}{m_{W0}^2}\frac{g_1 g}{(4\pi)^2}\left(\frac{1}{\epsilon} + 1 - \ln\frac{\Lambda_V^2}{\mu^2}\right), \tag{34}$$

$$\alpha T = -\frac{3s^2}{4c^2}\frac{g_1^2}{(4\pi)^2}\left(\frac{\Lambda_B^2\Lambda_W^2}{m_{W0}^2(\Lambda_B^2 - \Lambda_W^2)}\ln\frac{\Lambda_B^2}{\Lambda_W^2}\right) + \frac{s^2\Lambda_B^2}{c^2 m_{W0}^2}\left[\frac{g_4}{(4\pi)^2}\left(\frac{15}{4\epsilon} + \frac{13}{8} - \frac{15}{4}\ln\frac{\Lambda_B^2}{\mu^2}\right) + \frac{g_5}{(4\pi)^2}\left(\frac{3}{2\epsilon} + \frac{5}{4} - \frac{3}{2}\ln\frac{\Lambda_B^2}{\mu^2}\right)\right], \tag{35}$$

$$\alpha \hat{U} = \frac{2s^4}{c^2}\frac{g_1^2}{(4\pi)^2}\frac{\Lambda_V^2\Lambda_B^2}{m_{W0}^2(\Lambda_V^2 - \Lambda_B^2)}\ln\frac{\Lambda_V^2}{\Lambda_B^2}, \tag{36}$$

$$\alpha V = -\frac{\Lambda_V^2}{4m_{W0}^2}\frac{g_1^2}{(4\pi)^2}, \tag{37}$$

$$\alpha W = -\frac{\Lambda_V^2}{4m_{W0}^2}\frac{g_1^2}{(4\pi)^2}, \tag{38}$$

$$\alpha X = \frac{s^2\Lambda_V^2}{4m_{W0}^2}\frac{g_1^2}{(4\pi)^2} \tag{39}$$

for the quadratically divergent contributions to the oblique parameters. The $1/\epsilon$ terms represent logarithmic divergences that are left even after the quadratic divergences are parametrized by the scales Λ_X and that do not cancel between the vacuum polarizations in the oblique parameters. Our interpretation is that the $1/\epsilon$ terms are replacing numerical coefficients whose values depend on the details of what happens at the scale where new physics comes in.

IV. CONSISTENCY OF THE METHOD

The results that we derived above cannot be compared directly with experiment without some further considerations. The reason for this is that the oblique corrections receive also contributions from the regulator terms themselves and these contributions should be consistent with the terms calculated from the radiative corrections.

The tree-level contribution to the $\Pi_{XY}^g(k^2)$ can be read off the quadratic part of the Lagrange density (C8) and is

$$\Pi_{AA}^g(k^2) = \left(\frac{s^2}{\Lambda_W^2} + \frac{c^2}{\Lambda_B^2}\right)k^4, \tag{40}$$

$$\Pi_{ZA}^g(k^2) = sc\left(\frac{1}{\Lambda_B^2} - \frac{1}{\Lambda_W^2}\right)k^4, \tag{41}$$

$$\Pi_{ZZ}^g(k^2) = \left(\frac{c^2}{\Lambda_W^2} + \frac{s^2}{\Lambda_B^2}\right)k^4 - \frac{m_{Z0}^2}{\Lambda_V^2}k^2, \tag{42}$$

$$\Pi_{WW}^g(k^2) = \frac{1}{\Lambda_W^2}k^4 - \frac{m_{W0}^2}{\Lambda_V^2}k^2. \tag{43}$$

The corresponding contributions to the oblique parameters are

$$\alpha S = 4s^2\left(\frac{c^2}{\Lambda_W^2} + \frac{s^2}{\Lambda_B^2} - \frac{1}{\Lambda_V^2}\right)m_{W0}^2, \tag{44}$$

$$\alpha T = 0, \tag{45}$$

$$\alpha U = 4s^4\left(\frac{1}{\Lambda_W^2} - \frac{1}{\Lambda_B^2}\right)m_{W0}^2, \tag{46}$$

$$\alpha V = \left(\frac{c^2}{\Lambda_W^2} + \frac{s^2}{\Lambda_B^2}\right)\frac{m_{W0}^2}{c^2}, \tag{47}$$

$$\alpha W=\frac{m_{W0}^2}{\Lambda_W^2}, \tag{48}$$

$$\alpha X=s^2\left(\frac{1}{\Lambda_B^2}-\frac{1}{\Lambda_W^2}\right)m_{W0}^2. \tag{49}$$

We observe that Λ_V enters only the S parameter.

These tree-level contributions should be compared with the loop corrections to check whether no inconsistency arises. The philosophy we adopt here is the following. The structure for the vector boson propagators, parametrized by Λ_B, Λ_W, Λ_V is generated by the self-interactions among the vector bosons, as parametrized by g_1. Therefore the tree-level and the loop-corrections should be of similar size. Whereas S, T, U depend on the details of the interactions, V, W, X are given by a universal contribution. We therefore impose the conditions $V_{tree}=V_{loop}$, $W_{tree}=W_{loop}$, $X_{tree}=X_{loop}$. This leads to the following result:

$$1/\Lambda_B^2=0, \tag{50}$$

$$\frac{m_{W0}^2}{\Lambda_W^2}=-\frac{1}{4}\frac{g_1^2}{(4\pi)^2}\frac{\Lambda_V^2}{m_{W0}^2}. \tag{51}$$

After imposing these conditions, consistency further demands that the radiative corrections (34)–(36) should be of the same order of magnitude as the tree level relations (44)–(46). We see that this is indeed the case. The relations (50), (51) have an interesting physical interpretation. The fact that $\Lambda_B \gg \Lambda_W^2, \Lambda_V^2$ means that the hypercharge field, being a simple Abelian field, contains no structure. Furthermore it is seen that the cutoff Λ_W is only an indirect effect being generated by g_1, connected with the interactions in the Goldstone boson sector. Note the opposite signs for Λ_W^2 and Λ_V^2. These relations were already qualitatively expected in Sec. II. Given these relations, one can now make a comparison with experiment.

V. EXPERIMENTAL BOUNDS

We use the following experimental constraints for oblique parameters, which were provided to us by Takeuchi. They describe the deviation from standard model expectations for $m_t=175$ GeV, $m_H=300$ GeV, $m_Z=91.18630$ GeV, $\alpha^{-1}=128.9$, $\alpha_S(m_Z)=0.123$:

$$\begin{aligned} S&=-1.0\pm 1.5,\\ T&=-0.57\pm 0.80,\\ U&=0.07\pm 0.82,\\ V&=0.49\pm 0.82,\\ X&=0.22\pm 0.51, \end{aligned} \tag{52}$$

with the correlation matrix

	S	T	U	V	X
S	1	0.79	0.54	−0.77	−0.95
T	0.79	1	−0.05	−0.98	−0.56
U	0.54	−0.05	1	0.05	−0.76
V	−0.77	−0.98	0.05	1	0.55
X	−0.95	−0.56	−0.76	0.55	1

(53)

Although there is no Higgs particle in our model, the dependence of the oblique parameters on the Higgs mass is very weak and we can utilize the data above. We will now use these data to put bounds on Λ_V and Λ_W. We will have to consider two cases, depending on the sign of Λ_V^2.

A. The case $\Lambda_V^2>0$, $\Lambda_W^2<0$

In the comparison with experiment, we will now use the relations (50), (51) and give limits on Λ_W and Λ_V from the formulas (44)–(49). One might wonder whether it would not be more appropriate to use formulas (34)–(39), but here the comparison is complicated due to the arbitrariness involved by the undetermined coefficients. The procedure we take gives the most conservative, i.e. the least restrictive limits. In order to facilitate the discussion, we change in this subsection the notation $\Lambda_W^2 \to -\Lambda_W^2$. We also define an auxiliary $\Lambda_{\rm eff}^2=g_1^2\Lambda_V^2$. We will use the data on U, V, X to put a limit on Λ_W. Subsequently, we use the information on S to put a limit on Λ_V.

Using U, V and X, we get from (53) the statistically independent combinations

$$U-0.74V+2.0X=-0.14\pm 0.28, \tag{54}$$

$$U-0.59V-0.72X=-0.4\pm 1.3, \tag{55}$$

$$U+1.6V+0.087X=0.9\pm 1.6, \tag{56}$$

which, using (46), (47), (49), (50), (51), translate into

$$\Lambda_{\rm eff}^2=(0.4\pm 1.3)m_{W0}^2, \tag{57}$$

giving at 95% confidence level

$$\Lambda_{\rm eff}^2<2.5m_{W0}^2. \tag{58}$$

Using (51)and $m_{W0}=80.26$ GeV, this can be written as

$$\Lambda_W>1.3\text{TeV}. \tag{59}$$

Subsequently, using (44), (50) and the data on S, we get at 95% confidence level

$$\left(\frac{c^2}{\Lambda_W^2(\text{TeV})}+\frac{1}{\Lambda_V^2(\text{TeV})}\right)<4.2. \tag{60}$$

This can be written as

$$\Lambda_V>0.49\ \text{TeV}. \tag{61}$$

When we express the results in terms of the anomalous magnetic moment of the vector boson $\Delta\kappa=g_1/g$ we get the following equation

$$|\Delta\kappa| = \frac{0.25}{\Lambda_W(\mathrm{TeV})\Lambda_V(\mathrm{TeV})} \lesssim 0.26. \tag{62}$$

To arrive at the numerical bound, we took the linear combination (55), together with the bound on S and their statistical correlation, made a confidence level contour plot and determined the value of $\Delta\kappa$, where its line in the plot is tangential to the ellipse bounded by 1.64σ lines. Since we assume $\Lambda_V^2 > 0$, $\Lambda_W^2 < 0$, this gives an at least 95% confidence level bound on $\Delta\kappa$ for this case. This is a conservative procedure since it ignores some region in the plot out side the 1.64σ ellipse that would also give smaller $|\Delta\kappa|$. Although (55) among the three independent linear combinations (54)–(56) gives the weakest bounds on Λ_{eff}^2 and Λ_W, its strong anticorrelation with S causes it to give in combination with the limit on S the best limit on $\Delta\kappa$. This is true also for the case considered next.

We notice that the careful separation of longitudinal and transversal structure functions allows us to put a limit on $\Delta\kappa$ independent of assumptions on the size of the cutoff. This is in contrast with other methods, where an arbitrary estimate of the size of the cutoff is made, typically of the order of a TeV.

B. The case $\Lambda_V^2 < 0$, $\Lambda_W^2 > 0$

The analysis in this case proceeds exactly analogous to the previous case. Only here we change the notation to $\Lambda_V^2 \to -\Lambda_V^2$. Following the same steps as before, we now find

$$\Lambda_{\mathrm{eff}}^2 < 1.8 m_{W0}^2, \tag{63}$$

$$\Lambda_W > 1.5 \ \mathrm{TeV}, \tag{64}$$

$$\Lambda_V > 0.74 \ \mathrm{TeV}, \tag{65}$$

$$|\Delta\kappa| = \frac{0.25}{\Lambda_W(\mathrm{TeV})\Lambda_V(\mathrm{TeV})} \lesssim 0.08. \tag{66}$$

When combining (62) and (66), we have in principle to take into account that we do not know which case is realized. Since (66) is significantly more stringent than (62), the case $\Lambda_V^2 < 0$, $\Lambda_W^2 > 0$ with $|\Delta\kappa| > 0.26$ has negligible probability and the bound (62) gives a 95% confidence level overall bound.

C. Anomalous contribution to the photon structure function

Here we relate our results to two works dealing with the changes to the photon structure function induced by new physics.

To make contact with an earlier paper by one of the authors [14] we use again the identity $\Delta\kappa = g_1/g$. Besides this we identify Λ there with Λ_V in the present article. Translating the limit found there,

$$|\Delta\kappa(\Lambda/m_{W0})| \lesssim 33 \tag{67}$$

gives

$$|\Lambda_{\mathrm{eff}}| \lesssim 21 m_{W0} \tag{68}$$

and we see that our bounds improve more than an order of magnitude on this.

Measurements of the running of α can be used to put limits on Λ_{eff}. In [16] bounds at the 95% confidence level on the effective scale where new physics comes in were given as

$$\Lambda_- > 702 \ \mathrm{GeV}, \tag{69}$$

$$\Lambda_+ > 535 \ \mathrm{GeV}. \tag{70}$$

Identifying Λ_- or Λ_+ with Λ_{expt} and Λ_V with Λ in the relation

$$\Lambda_{\mathrm{expt}} = \frac{8\pi m_{W0}^2}{e\Lambda\Delta\kappa} \tag{71}$$

from [14] gives limits

$$\Lambda_{\mathrm{eff}} < 6.0 m_{W0}, \tag{72}$$

$$\Lambda_{\mathrm{eff}} < 7.9 m_{W0}, \tag{73}$$

which are considerably weaker than our bounds.

D. Relation to direct searches

The only gauge-boson self-coupling parameter being measured directly that can be compared to our results is $\Delta\kappa_\gamma$ in the phenomenological Lagrange density [17,18]

$$\begin{aligned}\mathcal{L} = &-igc[\Delta g_1^Z Z^\mu (W^-_{\mu\nu} W^{+\nu} - W^+_{\mu\nu} W^{-\nu}) + \Delta\kappa_Z W^+_\mu W^-_\nu Z^{\mu\nu}] \\ &+ igs\Delta\kappa_\gamma W^+_\mu W^-_\nu F^{\mu\nu} - \frac{i\lambda_Z}{m_W^2} Z^\nu_\mu W^{+\rho}_\nu W^{-\mu}_\rho \\ &- \frac{i\lambda_\gamma}{m_W^2} F^\nu_\mu W^{+\rho}_\nu W^{-\mu}_\rho,\end{aligned} \tag{74}$$

where $F^{\mu\nu}$ is the electromagnetic field strength. The relations to our triple gauge boson couplings are

$$g_1 = c^2 g \Delta g_1^Z, \tag{75}$$

$$g_2 = csg(\Delta\kappa_Z - \Delta\kappa_\gamma), \tag{76}$$

$$g_3 = -c^2 g \Delta g_1^Z + c^2 g \Delta\kappa_Z + s^2 g \Delta\kappa_\gamma, \tag{77}$$

$$\lambda_Z = \lambda_\gamma = 0. \tag{78}$$

Custodial symmetry for $g' \to 0$ requires $g_2 = g_3 = 0$, leading to

$$\Delta\kappa \equiv \Delta\kappa_\gamma = \Delta\kappa_Z = c^2 \Delta g_1^Z, \tag{79}$$

and thus

$$\Delta\kappa = g_1/g. \tag{80}$$

Another popular set of parameters is

$$\alpha_{w\phi} = c^2 \Delta g_1^Z, \tag{81}$$

$$\alpha_w = \lambda_\gamma, \tag{82}$$

$$\alpha_{b\phi}=\Delta\kappa_\gamma-c^2\Delta g_1^Z, \tag{83}$$

together with the constraints

$$c^2\Delta g_1^Z=c^2\Delta\kappa_Z+s^2\Delta\kappa_\gamma, \tag{84}$$

$$\lambda_Z=\lambda_\gamma=0. \tag{85}$$

While from (84) already follows $g_3=0$, the demand that also $g_2=\lambda_Z=\lambda_\gamma=0$ yields

$$\alpha_{w\phi}=\Delta\kappa=g_1/g, \tag{86}$$

$$\alpha_w=\alpha_{b\phi}=0. \tag{87}$$

The best available Fermilab bound combined from several Tevatron runs is compiled by the D0 Collaboration and reads [19]

$$-0.33<\Delta\kappa<0.45 \tag{88}$$

at 95% confidence level. This bound assumes that $\Delta g_1^Z=0$. As can be inferred from Fig. 3d in [19], our assumption that $\Delta\kappa=c^2\Delta g_1^Z$ leads to a bound that is roughly twice as stringent. However, we note that this limit assumes a cutoff of 1.5 TeV in the analysis. This maybe too optimistic, as we have seen that the longitudinal cutoff could be smaller. If we assume that one can take $\Lambda_V>1.5$ TeV and use the results from U, V, X, we would find $\Delta\kappa<0.13$. Therefore the Fermilab data appear to be on the verge of being competitive now.

The best limit from CERN experiments so far is provided by the LEP2 Collaboration ALEPH from combined hadronically and semileptonically decaying W^+W^- pairs and reads [20]

$$-0.62(0.14)<\alpha_{w\phi}<0.41(0.12) \tag{89}$$

at 95% confidence level, where the numbers in parentheses give systematic uncertainties.

We conclude therefore that at present the best limit on $\Delta\kappa$ still comes from the high precision LEP-100 data. However LEP-200 is already competitive and should be able to improve the limits [17]. The situation at Fermilab is somewhat less clear, as the limits depend on the assumed form factors. An analysis of the Fermilab data in terms of our cutoff propagators with Λ_B, Λ_W, Λ_V should be useful in order to clarify the situation. This is in particular important, in order to determine the ultimate precision on the anomalous couplings that can be reached after the upgrade of the Tevatron.

E. Comparison with other methods

Finally we make a comparison with other results in the literature.

In [6] the quadratic divergences are regulated by introducing the Higgs particle in the Lagrangian. The anomalous couplings are in this model generated through spontaneous symmetry breaking from higher dimension operators coupling vector boson operators with the Higgs sector. This regulates some of the quadratic divergences, but others still have to be treated by other means, i.e. as poles in $(d-2)$ in dimensional regularization. This way two cut-offs appear, m_H and Λ. This method should qualitatively give the same results as our method with the replacements $m_H\to\Lambda_V$ and $\Lambda\to\Lambda_{B,W}$. Unfortunately Ref. [6] calculated only the terms which are linear in the anomalous couplings, which are less divergent, so we can only compare the $g_1 g$ term in the S parameter. This term is actually of the expected form. Moreover it is found in [6] that the higher divergences are physical. The contribution to T from g_2 found in [6] is of a higher degree in the cutoff than the contribution from g_1. This supports the arguments concerning the breaking of the $SU_R(2)$ invariance. A numerical comparison is impossible, given the fact that quantities with different cutoff dependence were calculated. It should be interesting to compare the results for V,W,X with the scheme of [6].

In Ref. [5] the quadratic divergences were regulated by replacing poles in $(d-2)$ by Λ^2. This should roughly correspond with our results for $\Lambda_W=\Lambda_V$. Translated in our notation Ref. [5] finds $-0.013<\Delta\kappa<0.033$ for a cutoff of 3 TeV. If we use our formula (62) we find $|\Delta\kappa|<0.028$. So there is at least a qualitative agreement.

In [7] quadratic divergences are not considered, as dimensional regularization is used. In the case only g_1 is considered it is found in our notation $-0.07<\Delta\kappa<0.05$ for a cutoff of 2 TeV. If we use our formula (62) we find $|\Delta\kappa|<0.06$. This agreement is accidental, as the regularization methods are quite different. In [7] the logarithmically divergent terms containing one power of the anomalous coupling are studied, whereas we consider the more divergent terms containing two anomalous couplings. This difference becomes clearer, when one considers the contributions from the four-point vertices g_4 and g_5. Both we and Ref. [7] find that the corrections appear in the combination $5g_4+2g_5$, thereby confirming the previous results from Ref. [2]. Translated in our notation Ref. [7] quotes a limit of $-0.15<5g_4+2g_5<0.14$, for a cutoff of 2 TeV. Ignoring the logarithmic enhancement of the correction, but keeping the quadratic part we find the stronger limit, $-0.066<(5g_4+2g_5)\Lambda_B^2(\mathrm{TeV})<0.026$. The difference is clearly due to the different treatment of the quadratic divergences. As there are however more terms contributing to T, one should be careful in the interpretation of this limit.

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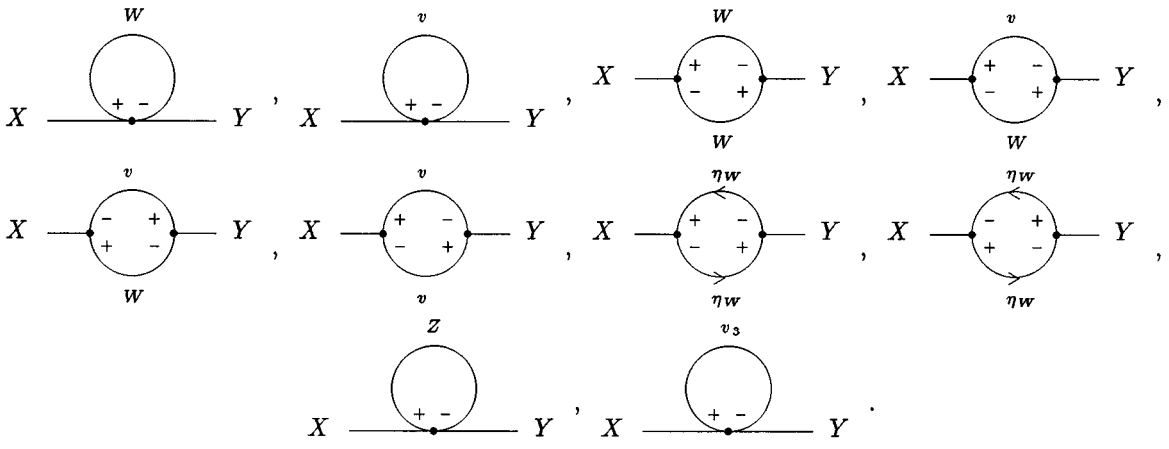
APPENDIX A: RESULTS

Here we present our results for the vacuum polarizations. Only the $\Pi_{XY}^g(k^2)$ are needed, since the contribution of the $\Pi_{XY}^k(k^2)$ part is suppressed in experimentally accessible observables by the smallness of the involved fermion masses.

When evaluating integrals, we assume that $\xi\ll m_{W0}^2/\Lambda_V^2, m_{Z0}^2/\Lambda_V^2$. If this is not the case, the more than logarithmic diver-

TABLE I. One-loop diagrams contributing to $\Pi_{XY}(k^2)$, where $XY=AA,ZA,ZZ$. The last two diagrams exist only for $XY=ZZ$.



gences in one-loop graphs are not limited to vacuum polarization corrections for terms containing both anomalous and gauge couplings.

Tables I and II show the one-loop vacuum polarization diagrams that can be constructed from the Feynman rules given in Appendix C. The integrals needed for their evaluation can be found in Appendix D.

1. $\Pi^g_{XY}(k^2)$: Quartically divergent terms

The quartically divergent contributions to the vacuum polarizations terms when all of the couplings (8)–(15) are present are given by

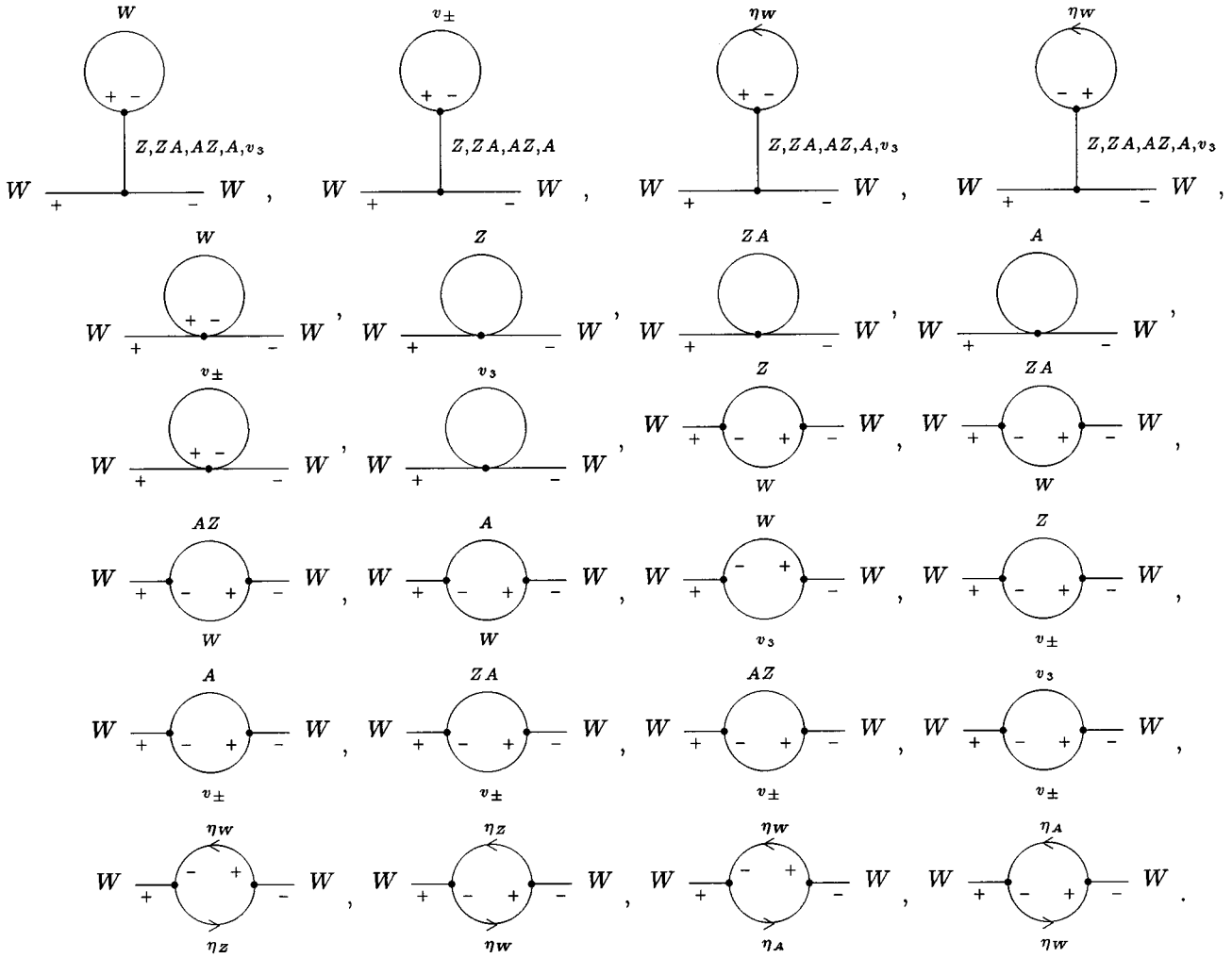
$$(4\pi)^2\Pi^g_{AA}(k^2)=\mathcal{O}(\Lambda^2), \tag{A1}$$

$$(4\pi)^2\Pi^g_{ZA}(k^2)=\mathcal{O}(\Lambda^2), \tag{A2}$$

$$\begin{aligned}(4\pi)^2\Pi^g_{ZZ}(k^2)=&g_1^2\frac{\Lambda_V^2\Lambda_W^2}{c^2m_{W0}^2}\left(-\frac{3}{2\epsilon}-\frac{5}{4}+\frac{3}{2}\frac{\Lambda_V^2\ln\frac{\Lambda_V^2}{\mu^2}-\Lambda_W^2\ln\frac{\Lambda_W^2}{\mu^2}}{\Lambda_V^2-\Lambda_W^2}\right)+g_4\frac{\Lambda_V^4}{c^2m_{W0}^2}\left(\frac{2}{\epsilon}+\frac{5}{2}-2\ln\frac{\Lambda_V^2}{\mu^2}\right)+g_5\frac{\Lambda_V^4}{c^2m_{W0}^2}\left(\frac{7}{2\epsilon}+\frac{15}{4}\right.\\&\left.-\frac{7}{2}\ln\frac{\Lambda_V^2}{\mu^2}\right)+g_6\frac{\Lambda_V^4}{c^2m_{W0}^2}\left(\frac{7}{2\epsilon}+\frac{17}{4}-\frac{7}{2}\ln\frac{\Lambda_V^2}{\mu^2}\right)+g_7\frac{\Lambda_V^4}{c^2m_{W0}^2}\left(\frac{5}{\epsilon}+\frac{11}{2}-5\ln\frac{\Lambda_V^2}{\mu^2}\right)+g_8\frac{\Lambda_V^4}{c^2m_{W0}^2}\left(\frac{3}{\epsilon}+\frac{7}{2}-3\ln\frac{\Lambda_V^2}{\mu^2}\right)\\&+\mathcal{O}(\Lambda^2),\end{aligned} \tag{A3}$$

$$\begin{aligned}(4\pi)^2\Pi^g_{WW}(k^2)=&\left(g_1^2+g_1g_3+\frac{g_3^2}{2}\right)\frac{\Lambda_V^2\Lambda_W^2}{m_{W0}^2}\left(-\frac{3}{2\epsilon}-\frac{5}{4}+\frac{3}{2}\frac{\Lambda_V^2\ln\frac{\Lambda_V^2}{\mu^2}-\Lambda_W^2\ln\frac{\Lambda_W^2}{\mu^2}}{\Lambda_V^2-\Lambda_W^2}\right)+g_2^2\frac{\Lambda_V^2\Lambda_B^2}{m_{W0}^2}\left(-\frac{3}{4\epsilon}-\frac{5}{8}\right.\\&\left.+\frac{3}{4}\frac{\Lambda_V^2\ln\frac{\Lambda_V^2}{\mu^2}-\Lambda_B^2\ln\frac{\Lambda_B^2}{\mu^2}}{\Lambda_V^2-\Lambda_B^2}\right)+g_4\frac{\Lambda_V^4}{m_{W0}^2}\left(\frac{2}{\epsilon}+\frac{5}{2}-2\ln\frac{\Lambda_V^2}{\mu^2}\right)+g_5\frac{\Lambda_V^4}{m_{W0}^2}\left(\frac{7}{2\epsilon}+\frac{15}{4}-\frac{7}{2}\ln\frac{\Lambda_V^2}{\mu^2}\right)+g_6\frac{\Lambda_V^4}{m_{W0}^2}\left(\frac{1}{4\epsilon}+\frac{3}{8}\right.\\&\left.-\frac{1}{4}\ln\frac{\Lambda_V^2}{\mu^2}\right)+g_7\frac{\Lambda_V^4}{m_{W0}^2}\left(\frac{1}{\epsilon}+1-\ln\frac{\Lambda_V^2}{\mu^2}\right)+\mathcal{O}(\Lambda^2).\end{aligned} \tag{A4}$$

TABLE II. One-loop diagrams contributing to $\Pi_{WW}(k^2)$. The tadpole graphs in the first line turn out to vanish.



2. $\Pi^g_{XY}(k^2)$ for $g_2=g_3=g_6=g_7=g_8=0$

Here we display the quartically and quadratically divergent parts of the vacuum polarizations for the case when the anomalous couplings preserve the custodial $SU(2)_R$ symmetry in the limit of vanishing hypercharge coupling, i.e. when $g_2=g_3=g_6=g_7=g_8=0$.

Our results for the at least quadratically divergent contributions to the $\Pi^g_{XY}(k^2)$ are

$$(4\pi)^2\Pi^g_{AA}(k^2)=s^2\left\{g_1^2\frac{\Lambda_V^2\Lambda_W^2}{\Lambda_V^2-\Lambda_W^2}\ln\frac{\Lambda_V^2}{\Lambda_W^2}\left(\frac{k^2}{m_{W0}^2}\right)+g_1g2\Lambda_V^2\left(\frac{1}{\epsilon}+1-\ln\frac{\Lambda_V^2}{\overline{\mu^2}}\right)\left(\frac{k^2}{m_{W0}^2}\right)-\frac{1}{4}g_1^2\Lambda_V^2\left(\frac{k^2}{m_{W0}^2}\right)^2\right\}+\mathcal{O}(\Lambda^0), \quad \text{(A5)}$$

$$(4\pi)^2\Pi^g_{ZA}(k^2)=\frac{s}{c}\left\{g_1^2\Lambda_V^2\Lambda_W^2\left[-\frac{1}{\Lambda_V^2-\Lambda_W^2}+\left(\frac{\Lambda_V^2+\Lambda_W^2}{2(\Lambda_V^2-\Lambda_W^2)^2}+\frac{s^2}{\Lambda_V^2-\Lambda_W^2}\right)\ln\frac{\Lambda_V^2}{\Lambda_W^2}\right]\left(\frac{k^2}{m_{W0}^2}\right)-g_1g\frac{3-4s^2}{2}\Lambda_V^2\left(\frac{1}{\epsilon}+1\right.\right.$$
$$\left.\left.-\ln\frac{\Lambda_V^2}{\overline{\mu^2}}\right)\left(\frac{k^2}{m_{W0}^2}\right)+\frac{c^2}{4}g_1^2\Lambda_V^2\left(\frac{k^2}{m_{W0}^2}\right)^2\right\}+\mathcal{O}(\Lambda^0), \quad \text{(A6)}$$

$$(4\pi)^2\Pi^g_{ZZ}(k^2)=g_1^2\frac{\Lambda_V^2\Lambda_W^2}{c^2m_{W0}^2}\left(-\frac{3}{2\epsilon}-\frac{5}{4}+\frac{3}{2}\frac{\Lambda_V^2\ln\frac{\Lambda_V^2}{\overline{\mu^2}}-\Lambda_W^2\ln\frac{\Lambda_W^2}{\overline{\mu^2}}}{\Lambda_V^2-\Lambda_W^2}\right)+g_4\frac{\Lambda_V^4}{c^2m_{W0}^2}\left(\frac{2}{\epsilon}+\frac{5}{2}-2\ln\frac{\Lambda_V^2}{\overline{\mu^2}}\right)+g_5\frac{\Lambda_V^4}{c^2m_{W0}^2}\left(\frac{7}{2\epsilon}+\frac{15}{4}\right.$$

$$-\frac{7}{2}\ln\frac{\Lambda_V^2}{\overline{\mu^2}}\Big)-g_1^2\frac{3}{2c^2}\Lambda_W^2-\frac{1}{c^2}g_1g\Lambda_W^2\Big(\frac{3}{\epsilon}+\frac{5}{2}-3\ln\frac{\Lambda_W^2}{\overline{\mu^2}}\Big)-\frac{1}{c^2}g^2\Lambda_V^2\Big(\frac{1}{\epsilon}+\frac{3}{4}-\ln\frac{\Lambda_V^2}{\overline{\mu^2}}\Big)-g_4\frac{s^2\Lambda_B^2}{c^4}\Big(\frac{9}{2\epsilon}+\frac{9}{4}$$
$$-\frac{9}{2}\ln\frac{\Lambda_B^2}{\overline{\mu^2}}\Big)-g_4\frac{\Lambda_W^2}{c^2}\Big(\frac{6}{\epsilon}+\frac{7}{2}-6\ln\frac{\Lambda_W^2}{\overline{\mu^2}}\Big)-g_5\frac{s^2\Lambda_B^2}{c^4}\Big(\frac{9}{2\epsilon}+\frac{9}{4}-\frac{9}{2}\ln\frac{\Lambda_B^2}{\overline{\mu^2}}\Big)-g_5\frac{\Lambda_W^2}{c^2}\Big(\frac{21}{2\epsilon}+\frac{17}{4}-\frac{21}{2}\ln\frac{\Lambda_W^2}{\overline{\mu^2}}\Big)$$
$$+\frac{1}{c^2}g_1^2\Lambda_V^2\Lambda_W^2\Big[\frac{3\Lambda_V^2-8\Lambda_W^2}{3(\Lambda_V^2-\Lambda_W^2)^2}-\frac{2s^2}{\Lambda_V^2-\Lambda_W^2}+\Big(\frac{-17\Lambda_V^4+33\Lambda_V^2\Lambda_W^2-6\Lambda_W^4}{6(\Lambda_V^2-\Lambda_W^2)^3}+\frac{(\Lambda_V^2+\Lambda_W^2)s^2}{(\Lambda_V^2-\Lambda_W^2)^2}+\frac{s^4}{\Lambda_V^2-\Lambda_W^2}\Big)\ln\frac{\Lambda_V^2}{\Lambda_W^2}\Big]$$
$$\times\Big(\frac{k^2}{m_{W0}^2}\Big)+g_1g\Lambda_V^2(1-2s^2)\Big(\frac{1}{\epsilon}+1-\ln\frac{\Lambda_V^2}{\overline{\mu^2}}\Big)\Big(\frac{k^2}{m_{W0}^2}\Big)-\frac{c^2}{4}g_1^2\Lambda_V^2\Big(\frac{k^2}{m_{W0}^2}\Big)^2+\mathcal{O}(\Lambda^0), \tag{A7}$$

$$(4\pi)^2\Pi_{WW}^g(k^2)=g_1^2\frac{\Lambda_V^2\Lambda_W^2}{m_{W0}^2}\Big(-\frac{3}{2\epsilon}-\frac{5}{4}+\frac{3}{2}\frac{\Lambda_V^2\ln\frac{\Lambda_V^2}{\overline{\mu^2}}-\Lambda_W^2\ln\frac{\Lambda_W^2}{\overline{\mu^2}}}{\Lambda_V^2-\Lambda_W^2}\Big)+g_4\frac{\Lambda_V^4}{m_{W0}^2}\Big(\frac{2}{\epsilon}+\frac{5}{2}-2\ln\frac{\Lambda_V^2}{\overline{\mu^2}}\Big)+g_5\frac{\Lambda_V^4}{m_{W0}^2}\Big(\frac{7}{2\epsilon}+\frac{15}{4}-\frac{7}{2}\ln\frac{\Lambda_V^2}{\overline{\mu^2}}\Big)$$
$$-g_1^2\Lambda_W^2\Big(\frac{3}{2}+\frac{3s^2}{4c^2}\frac{\Lambda_B^2}{\Lambda_B^2-\Lambda_W^2}\ln\frac{\Lambda_B^2}{\Lambda_W^2}\Big)-g_1g\Lambda_W^2\Big(\frac{3}{\epsilon}+\frac{5}{2}-3\ln\frac{\Lambda_W^2}{\overline{\mu^2}}\Big)-g^2\Lambda_V^2\Big(\frac{1}{\epsilon}+\frac{3}{4}-\ln\frac{\Lambda_V^2}{\overline{\mu^2}}\Big)-g_4\frac{s^2\Lambda_B^2}{c^2}\Big(\frac{3}{4\epsilon}+\frac{5}{8}$$
$$-\frac{3}{4}\ln\frac{\Lambda_B^2}{\overline{\mu^2}}\Big)-g_4\Lambda_W^2\Big(\frac{6}{\epsilon}+\frac{7}{2}-6\ln\frac{\Lambda_W^2}{\overline{\mu^2}}\Big)-g_5\frac{s^2\Lambda_B^2}{c^2}\Big(\frac{3}{\epsilon}+1-3\ln\frac{\Lambda_B^2}{\overline{\mu^2}}\Big)-g_5\Lambda_W^2\Big(\frac{21}{2\epsilon}+\frac{17}{4}-\frac{21}{2}\ln\frac{\Lambda_W^2}{\overline{\mu^2}}\Big)$$
$$+g_1^2\Big(\frac{s^2}{2c^2}\frac{\Lambda_V^2\Lambda_B^2}{\Lambda_V^2-\Lambda_B^2}\ln\frac{\Lambda_V^2}{\Lambda_B^2}+\frac{1}{3}\Lambda_V^2\Lambda_W^2\frac{3\Lambda_V^2-8\Lambda_W^2}{(\Lambda_V^2-\Lambda_W^2)^2}-\frac{1}{6}\Lambda_V^2\Lambda_W^2\frac{17\Lambda_V^4-33\Lambda_V^2\Lambda_W^2+6\Lambda_W^4}{(\Lambda_V^2-\Lambda_W^2)^3}\ln\frac{\Lambda_V^2}{\Lambda_W^2}\Big)\Big(\frac{k^2}{m_{W0}^2}\Big)$$
$$+g_1g\Lambda_V^2\Big(\frac{1}{\epsilon}+1-\ln\frac{\Lambda_V^2}{\overline{\mu^2}}\Big)\Big(\frac{k^2}{m_{W0}^2}\Big)-\frac{1}{4}g_1^2\Lambda_V^2\Big(\frac{k^2}{m_{W0}^2}\Big)^2+\mathcal{O}(\Lambda^0). \tag{A8}$$

APPENDIX B: GAUGE FIXING

To fix the gauge we introduce a variant of the class of R_ξ gauges suitable to cancel the quadratic mixing terms between would-be Goldstone bosons and longitudinal gauge bosons in the presence of the higher covariant derivative terms.

Specificially, we use the gauge fixing term

$$\mathcal{L}_{\rm gf}=-\frac{1}{2\xi}F_{Wa}^2-\frac{1}{2\xi}F_B^2 \tag{B1}$$

with

$$F_{Wa}=\partial_\mu W_a^\mu-\tfrac{1}{2}\xi g v^2(1+\Lambda_V^{-2}\partial^2)u_a \tag{B2}$$

and

$$F_B=\partial_\mu B^\mu-\tfrac{1}{2}\xi g' v^2(1+\Lambda_V^{-2}\partial^2)u_3, \tag{B3}$$

where the u_a are defined by writing $U=\exp(iu_a\tau_a)$. The necessary ghost terms are given by

$$\mathcal{L}_{gh}=-(\overline{\eta}_{Wa},\overline{\eta}_B)\begin{pmatrix}\delta_{ab}\partial_\mu D^\mu+\xi\Big(\frac{gv}{2}\Big)^2(1+\Lambda_V^{-2}\partial^2)(\delta_{ab}-\epsilon_{abc}u_c) & \xi\frac{gg'v^2}{4}(1+\Lambda_V^{-2}\partial^2)(\delta_{a3}+\epsilon_{a3c}u_c)\\ \xi\frac{gg'v^2}{4}(1+\Lambda_V^{-2}\partial^2)(\delta_{3b}-\epsilon_{3bc}u_c) & \partial^2+\xi\Big(\frac{g'v}{2}\Big)^2(1+\Lambda_V^{-2}\partial^2)\end{pmatrix}\begin{pmatrix}\eta_{Wb}\\ \eta_B\end{pmatrix}$$
$$+\mathcal{O}(u^2)\,\overline{\eta}\,\eta \tag{B4}$$

with

$$D^\mu \eta_{Wa} = \partial^\mu \eta_{Wa} + g \epsilon_{abc} \eta_{Wb} W_c^\mu . \tag{B5}$$

Due to the relative simplicity of our gauge fixing terms, the absence of quadratically divergent integrals in the oblique parameters becomes manifest only in Landau gauge, i.e. $\xi = 0$ [13].

APPENDIX C: FEYNMAN RULES

Since the Feynman rules in higher covariant derivative regularization have an unfamiliar appearance, we give here all rules in our version of R_ξ gauge explicitly.

To avoid confusion with the momentum s appearing in the four-vertices, write now $s_\theta = \sin\Theta_W$ and then also $c_\theta = \cos\Theta_W$, $t_\theta = \tan\Theta_W$. Additionally to (22) and (23), we need the following field redefinitions:

$$(\overline{\eta_Z}, \overline{\eta_A}) = (\overline{\eta_{W3}}, \overline{\eta_B}) \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix}, \tag{C1}$$

$$\begin{pmatrix} \eta_Z \\ \eta_A \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \eta_{W3} \\ \eta_B \end{pmatrix}, \tag{C2}$$

$$v_\pm = \frac{v}{\sqrt{2}} (u_1 \mp i u_2), \tag{C3}$$

$$v_3 = v u_3 , \tag{C4}$$

$$\overline{\eta_{W\pm}} = \frac{1}{\sqrt{2}} (\overline{\eta_{W1}} \mp i \overline{\eta_{W2}}), \tag{C5}$$

$$\eta_{W\pm} = \frac{1}{\sqrt{2}} (\eta_{W1} \mp i \eta_{W2}). \tag{C6}$$

Define also

$$m_{W0} \equiv \frac{gv}{2}, \quad m_{Z0} \equiv \frac{gv}{2c_\theta}. \tag{C7}$$

The quadratic part of the Lagrangian extracted from (7), (17), (18), (B1), (B4) reads in terms of the redefined fields

$$\begin{aligned} \mathcal{L}_2 = {} & W_\mu^+ \left\{ [\Lambda_W^{-2} (\partial^2)^2 + (1 + \Lambda_V^{-2} m_{W0}^2) \partial^2 + m_{W0}^2] \left(g^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\partial^2} \right) + \left[\left(\frac{1}{\xi} + \Lambda_V^{-2} m_{W0}^2 \right) \partial^2 + m_{W0}^2 \right] \frac{\partial^\mu \partial^\nu}{\partial^2} \right\} W_\nu^- + \tfrac{1}{2} (Z_\mu, A_\mu) \\ & \times \left[\mathcal{D}_{ZA}^{tr} \left(g^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\partial^2} \right) + \mathcal{D}_{ZA}^{lg} \frac{\partial^\mu \partial^\nu}{\partial^2} \right] \begin{pmatrix} Z_\nu \\ A_\nu \end{pmatrix} - v_+ [(1 + \Lambda_V^{-2} \partial^2) \partial^2 + \xi m_{W0}^2 (1 + \Lambda_V^{-2} \partial^2)^2] v_- - \tfrac{1}{2} v_3 [(1 + \Lambda_V^{-2} \partial^2) \partial^2 \\ & + \xi m_{Z0}^2 (1 + \Lambda_V^{-2} \partial^2)^2] v_3 - \overline{\eta_{W+}} [(1 + \xi \Lambda_V^{-2} m_{W0}^2) \partial^2 + \xi m_{W0}^2] \eta_{W-} - \overline{\eta_{W-}} [(1 + \xi \Lambda_V^{-2} m_{W0}^2) \partial^2 + \xi m_{W0}^2] \eta_{W+} \\ & - \overline{\eta_Z} [(1 + \xi \Lambda_V^{-2} m_{Z0}^2) \partial^2 + \xi m_{Z0}^2] \eta_Z - \overline{\eta_A} \partial^2 \eta_A \end{aligned} \tag{C8}$$

with

$$\mathcal{D}_{ZA}^{tr} = \begin{pmatrix} \left(\frac{c_\theta^2}{\Lambda_W^2} + \frac{s_\theta^2}{\Lambda_B^2} \right) (\partial^2)^2 + \left(1 + \frac{m_{Z0}^2}{\Lambda_V^2} \right) \partial^2 + m_{Z0}^2 & \left(\frac{1}{\Lambda_B^2} - \frac{1}{\Lambda_W^2} \right) s_\theta c_\theta (\partial^2)^2 \\ \left(\frac{1}{\Lambda_B^2} - \frac{1}{\Lambda_W^2} \right) s_\theta c_\theta (\partial^2)^2 & \left(\frac{s_\theta^2}{\Lambda_W^2} + \frac{c_\theta^2}{\Lambda_B^2} \right) (\partial^2)^2 + \partial^2 \end{pmatrix} \tag{C9}$$

and

$$\mathcal{D}^{lg}_{ZA}=\begin{pmatrix} \left(\frac{1}{\xi}+\Lambda_V^{-2}m^2_{Z0}\right)\partial^2+m^2_{Z0} & 0 \\ 0 & \frac{1}{\xi}\partial^2 \end{pmatrix}. \tag{C10}$$

1. Propagators

Some of the propagators have an unusual form caused by the higher covariant derivative terms. However, they can be decomposed into combinations of standard propagator terms with modified masses and normalization factors as indicated below:

$$\begin{aligned}\Delta^W_{\mu\nu}(k)&=-i\left[\frac{-\Lambda_W^2}{(k^2-m^2_{W<})(k^2-m^2_{W>})}(g_{\mu\nu}-k_\mu k_\nu/k^2)+\frac{Z^{lg}_W\xi k_\mu k_\nu/k^2}{k^2-m^2_{Wlg}}\right]\\&=-i\left[Z^{tr}_W\left(\frac{1}{k^2-m^2_{W<}}-\frac{1}{k^2-m^2_{W>}}\right)(g_{\mu\nu}-k_\mu k_\nu/k^2)+\frac{Z^{lg}_W\xi k_\mu k_\nu/k^2}{k^2-m^2_{Wlg}}\right],\end{aligned} \tag{C11}$$

$$\begin{aligned}\Delta^Z_{\mu\nu}(k)&=-i\left[\frac{\Lambda_W^2\Lambda_B^2-\Lambda_Z^2k^2}{(k^2-m^2_{Z<})(k^2-m^2_{Z>})(k^2-m^2_{A>})}(g_{\mu\nu}-k_\mu k_\nu/k^2)+\frac{Z^{lg}_Z\xi k_\mu k_\nu/k^2}{k^2-m^2_{Zlg}}\right]\\&=-i\left[\left(\frac{Z^{ZZ}_{Z<}}{k^2-m^2_{Z<}}+\frac{Z^{ZZ}_{Z>}}{k^2-m^2_{Z>}}+\frac{Z^{ZZ}_{A>}}{k^2-m^2_{A>}}\right)(g_{\mu\nu}-k_\mu k_\nu/k^2)+\frac{Z^{lg}_Z\xi k_\mu k_\nu/k^2}{k^2-m^2_{Zlg}}\right],\end{aligned} \tag{C12}$$

$$\begin{aligned}\Delta^{ZA}_{\mu\nu}(k)=\Delta^{AZ}_{\mu\nu}(k)&=-i\frac{(\Lambda_W^2-\Lambda_B^2)k^2s_\theta c_\theta}{(k^2-m^2_{Z<})(k^2-m^2_{Z>})(k^2-m^2_{A>})}(g_{\mu\nu}-k_\mu k_\nu/k^2)\\&=-i\left(\frac{Z^{ZA}_{Z<}}{k^2-m^2_{Z<}}+\frac{Z^{ZA}_{Z>}}{k^2-m^2_{Z>}}+\frac{Z^{ZA}_{A>}}{k^2-m^2_{A>}}\right)(g_{\mu\nu}-k_\mu k_\nu/k^2),\end{aligned} \tag{C13}$$

$$\begin{aligned}\Delta^A_{\mu\nu}(k)&=-i\left[\frac{\Lambda_W^2\Lambda_B^2[(1+\Lambda_V^{-2}m^2_{Z0})k^2-m^2_{Z0}]-\Lambda_A^2(k^2)^2}{k^2(k^2-m^2_{Z<})(k^2-m^2_{Z>})(k^2-m^2_{A>})}(g_{\mu\nu}-k_\mu k_\nu/k^2)+\frac{\xi k_\mu k_\nu/k^2}{k^2}\right]\\&=-i\left[\left(\frac{1}{k^2}+\frac{Z^{AA}_{Z<}}{k^2-m^2_{Z<}}+\frac{Z^{AA}_{Z>}}{k^2-m^2_{Z>}}+\frac{Z^{AA}_{A>}}{k^2-m^2_{A>}}\right)(g_{\mu\nu}-k_\mu k_\nu/k^2)+\frac{\xi k_\mu k_\nu/k^2}{k^2}\right],\end{aligned} \tag{C14}$$

$$\Delta^{v\pm}(k^2)=i\frac{-Z^{lg}_W\Lambda_V^2}{(k^2-m^2_{Wlg})(k^2-\Lambda_V^2)}=i\left(\frac{1}{k^2-m^2_{Wlg}}-\frac{1}{k^2-\Lambda_V^2}\right), \tag{C15}$$

$$\Delta^{v3}(k^2)=i\frac{-Z^{lg}_Z\Lambda_V^2}{(k^2-m^2_{Zlg})(k^2-\Lambda_V^2)}=i\left(\frac{1}{k^2-m^2_{Zlg}}-\frac{1}{k^2-\Lambda_V^2}\right), \tag{C16}$$

$$\Delta^{\eta_W}(k^2)=\frac{iZ^{lg}_W}{k^2-m^2_{Wlg}}, \tag{C17}$$

$$\Delta^{\eta_Z}(k^2)=\frac{iZ^{lg}_Z}{k^2-m^2_{Zlg}}, \tag{C18}$$

$$\Delta^{\eta_A}(k^2)=\frac{i}{k^2} \tag{C19}$$

with

$$Z_W^{tr}=\frac{\Lambda_W^2}{m_{W>}^2-m_{W<}^2}=\frac{1}{\sqrt{(1+\Lambda_V^{-2}m_{W0}^2)^2-4\Lambda_W^{-2}m_{W0}^2}}=1+\mathcal{O}(\Lambda^{-2}), \tag{C20}$$

$$Z_W^{lg}=\frac{1}{1+\xi\Lambda_V^{-2}m_{W0}^2}=1+\mathcal{O}(\Lambda^{-2}), \tag{C21}$$

$$Z_{Z<}^{ZZ}=\frac{\Lambda_B^2\Lambda_W^2-\Lambda_Z^2m_{Z<}^2}{(m_{Z<}^2-m_{Z>}^2)(m_{Z<}^2-m_{A>}^2)}=1+\mathcal{O}(\Lambda^{-2}), \tag{C22}$$

$$Z_{Z>}^{ZZ}=\frac{\Lambda_B^2\Lambda_W^2-\Lambda_Z^2m_{Z>}^2}{(m_{Z>}^2-m_{Z<}^2)(m_{Z>}^2-m_{A>}^2)}=-c_\theta^2+\mathcal{O}(\Lambda^{-2}), \tag{C23}$$

$$Z_{A>}^{ZZ}=\frac{\Lambda_B^2\Lambda_W^2-\Lambda_Z^2m_{A>}^2}{(m_{A>}^2-m_{Z<}^2)(m_{A>}^2-m_{Z>}^2)}=-s_\theta^2+\mathcal{O}(\Lambda^{-2}), \tag{C24}$$

$$Z_{Z<}^{ZA}=\frac{(\Lambda_W^2-\Lambda_B^2)m_{Z<}^2s_\theta c_\theta}{(m_{Z<}^2-m_{Z>}^2)(m_{Z<}^2-m_{A>}^2)}=\mathcal{O}(\Lambda^{-2}), \tag{C25}$$

$$Z_{Z>}^{ZA}=\frac{(\Lambda_W^2-\Lambda_B^2)m_{Z>}^2s_\theta c_\theta}{(m_{Z>}^2-m_{Z<}^2)(m_{Z>}^2-m_{A>}^2)}=s_\theta c_\theta+\mathcal{O}(\Lambda^{-2}), \tag{C26}$$

$$Z_{A>}^{ZA}=\frac{(\Lambda_W^2-\Lambda_B^2)m_{A>}^2s_\theta c_\theta}{(m_{A>}^2-m_{Z<}^2)(m_{A>}^2-m_{Z>}^2)}=-s_\theta c_\theta+\mathcal{O}(\Lambda^{-2}), \tag{C27}$$

$$Z_{Z<}^{AA}=\frac{\Lambda_W^2\Lambda_B^2-m_{Z>}^2m_{A>}^2(1-\Lambda_V^{-2}m_{Z<}^2)-\Lambda_A^2m_{Z<}^2}{(m_{Z<}^2-m_{Z>}^2)(m_{Z<}^2-m_{A>}^2)}=\mathcal{O}(\Lambda^{-4}), \tag{C28}$$

$$Z_{Z>}^{AA}=\frac{\Lambda_W^2\Lambda_B^2-m_{Z<}^2m_{A>}^2(1-\Lambda_V^{-2}m_{Z>}^2)-\Lambda_A^2m_{Z>}^2}{(m_{Z>}^2-m_{Z<}^2)(m_{Z>}^2-m_{A>}^2)}=-s_\theta^2+\mathcal{O}(\Lambda^{-2}), \tag{C29}$$

$$Z_{A>}^{AA}=\frac{\Lambda_W^2\Lambda_B^2-m_{Z<}^2m_{Z>}^2(1-\Lambda_V^{-2}m_{A>}^2)-\Lambda_A^2m_{A>}^2}{(m_{A>}^2-m_{Z<}^2)(m_{A>}^2-m_{Z>}^2)}=-c_\theta^2+\mathcal{O}(\Lambda^{-2}), \tag{C30}$$

$$Z_Z^{lg}=\frac{1}{1+\xi\Lambda_V^{-2}m_{Z0}^2}=1+\mathcal{O}(\Lambda^{-2}), \tag{C31}$$

$$m_{W_<^>}^2=\tfrac{1}{2}\Lambda_W^2[(1+\Lambda_V^{-2}m_{W0}^2)\pm\sqrt{(1+\Lambda_V^{-2}m_{W0}^2)^2-4\Lambda_W^{-2}m_{W0}^2}]=\begin{Bmatrix}\Lambda_W^2\\ m_{W0}^2\end{Bmatrix}\times(1+\mathcal{O}(\Lambda^{-2})), \tag{C32}$$

$$m_{Wlg}^2=\frac{\xi m_{W0}^2}{1+\xi\Lambda_V^{-2}m_{W0}^2}, \tag{C33}$$

$$m_{Zlg}^2=\frac{\xi m_{Z0}^2}{1+\xi\Lambda_V^{-2}m_{Z0}^2}, \tag{C34}$$

$$\Lambda_Z^2=\Lambda_W^2 c_\theta^2+\Lambda_B^2 s_\theta^2, \tag{C35}$$

$$\Lambda_A^2=\Lambda_W^2 s_\theta^2+\Lambda_B^2 c_\theta^2, \tag{C36}$$

and where $m_{Z<}^2$, $m_{Z>}^2$, $m_{A>}^2$ are determined by

$$(k^2-m_{Z<}^2)(k^2-m_{Z>}^2)(k^2-m_{A>}^2)=(k^2)^3-[\Lambda_W^2+\Lambda_B^2+\Lambda_V^{-2}\Lambda_Z^2 m_{Z0}^2](k^2)^2+[\Lambda_W^2\Lambda_B^2(1+\Lambda_V^{-2}m_{Z0}^2)+\Lambda_Z^2 m_{Z0}^2]k^2-\Lambda_W^2\Lambda_B^2 m_{Z0}^2 \tag{C37}$$

i.e.

$$\Lambda_W^2+\Lambda_B^2+\Lambda_V^{-2}\Lambda_Z^2 m_{Z0}^2=m_{Z<}^2+m_{Z>}^2+m_{A>}^2, \tag{C38}$$

$$\Lambda_W^2\Lambda_B^2+\Lambda_Z^2 m_{Z0}^2+\Lambda_V^{-2}\Lambda_W^2\Lambda_B^2 m_{Z0}^2=m_{Z<}^2 m_{Z>}^2+m_{Z<}^2 m_{A>}^2+m_{Z>}^2 m_{A>}^2, \tag{C39}$$

$$\Lambda_W^2\Lambda_B^2 m_{Z0}^2=m_{Z<}^2 m_{Z>}^2 m_{A>}^2 \tag{C40}$$

with

$$m_{Z<}^2=m_{Z0}^2(1+\mathcal{O}(\Lambda^{-2})), \tag{C41}$$

$$m_{Z>}^2=\Lambda_W^2(1+\mathcal{O}(\Lambda^{-2})), \tag{C42}$$

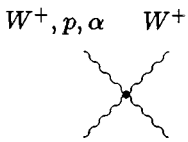
$$m_{A>}^2=\Lambda_B^2(1+\mathcal{O}(\Lambda^{-2})). \tag{C43}$$

The masses and renormalization constants have to be evaluated to higher order than explicitly given here.

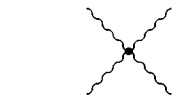
2. Vertices

All momenta are outgoing. Only vertices needed for one-loop gauge propagator corrections are displayed.

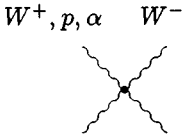
a. Four-vertices



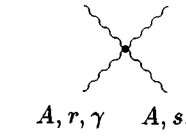
$$
\begin{aligned}
= i\Big\{ & g_{\alpha\beta}g_{\gamma\delta}\Big[2(2gg_1+2gg_3+g_4) \\
& \quad +g^2\left(2+4\Lambda_V^{-2}m_{w_0}^2+\Lambda_W^{-2}[4(p\cdot q)+(p+q)\cdot(r+s)+4(r\cdot s)]\right)\Big] \\
& -g_{\alpha\gamma}g_{\beta\delta}\Big[(2gg_1+2gg_3-g_4-2g_5) \\
& \quad +g^2\left(1+2\Lambda_V^{-2}m_{w_0}^2+\Lambda_W^{-2}[(p\cdot q)+2(p\cdot r)+2(q\cdot s)+(r\cdot s)]\right)\Big] \\
& -g_{\alpha\delta}g_{\beta\gamma}\Big[(2gg_1+2gg_3-g_4-2g_5) \\
& \quad +g^2\left(1+2\Lambda_V^{-2}m_{w_0}^2+\Lambda_W^{-2}[(p\cdot q)+2(p\cdot s)+2(q\cdot r)+(r\cdot s)]\right)\Big] \\
& -g^2\Lambda_W^{-2}\Big[\ g_{\alpha\beta}(2(p-q)_\gamma(p-q)_\delta-(p+q)_\gamma r_\delta-s_\gamma(p+q)_\delta+2s_\gamma r_\delta) \\
& \quad +g_{\gamma\delta}(2(r-s)_\alpha(r-s)_\beta-(r+s)_\alpha p_\beta-q_\alpha(r+s)_\beta+2p_\beta q_\alpha) \\
& \quad +g_{\alpha\gamma}(2p_\beta r_\delta-p_\beta p_\delta-r_\beta r_\delta+p_\beta q_\delta-s_\beta(q-r)_\delta) \\
& \quad +g_{\alpha\delta}(2p_\beta s_\gamma-p_\beta p_\gamma-s_\beta s_\gamma+p_\beta q_\gamma-r_\beta(q-s)_\gamma) \\
& \quad +g_{\beta\gamma}(2q_\alpha r_\delta-q_\alpha q_\delta-r_\alpha r_\delta+q_\alpha p_\delta-s_\alpha(p-r)_\delta) \\
& \quad +g_{\beta\delta}(2q_\alpha s_\gamma-q_\alpha q_\gamma-s_\alpha s_\gamma+q_\alpha p_\gamma-r_\alpha(p-s)_\gamma)\Big]\Big\}
\end{aligned}
\tag{C44}
$$



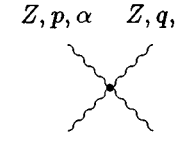
$$
\begin{aligned}
= -i\Big\{ & g_{\alpha\beta}g_{\gamma\delta}\Big[4gg_1-2(g_5+g_7)c_\theta^{-2} \\
& \quad +g^2\Big(2c_\theta^2+2\Lambda_V^{-2}m_{w_0}^2(c_\theta^2+c_\theta^{-2}) \\
& \quad +\Lambda_W^{-2}c_\theta^2\left(4(p\cdot q)+(p+q)\cdot(r+s)+4(r\cdot s)\right)\Big)\Big] \\
& -g_{\alpha\gamma}g_{\beta\delta}\Big[2gg_1+(g_4+g_6)c_\theta^{-2} \\
& \quad +g^2\Big(c_\theta^2+2\Lambda_V^{-2}m_{w_0}^2+\Lambda_W^{-2}c_\theta^2((p\cdot q)+2(p\cdot r)+2(q\cdot s)+(r\cdot s))\Big)\Big] \\
& -g_{\alpha\delta}g_{\beta\gamma}\Big[2gg_1+(g_4+g_6)c_\theta^{-2} \\
& \quad +g^2\Big(c_\theta^2+2\Lambda_V^{-2}m_{w_0}^2+\Lambda_W^{-2}c_\theta^2((p\cdot q)+2(p\cdot s)+2(q\cdot r)+(r\cdot s))\Big)\Big] \\
& -g^2\Lambda_W^{-2}c_\theta^2\Big[\ g_{\alpha\beta}(2(p-q)_\gamma(p-q)_\delta-(p+q)_\gamma r_\delta-s_\gamma(p+q)_\delta+2s_\gamma r_\delta) \\
& \quad +g_{\gamma\delta}(2(r-s)_\alpha(r-s)_\beta-(r+s)_\alpha p_\beta-q_\alpha(r+s)_\beta+2p_\beta q_\alpha) \\
& \quad +g_{\alpha\gamma}(2p_\beta r_\delta-p_\beta p_\delta-r_\beta r_\delta+p_\beta q_\delta-s_\beta(q-r)_\delta) \\
& \quad +g_{\alpha\delta}(2p_\beta s_\gamma-p_\beta p_\gamma-s_\beta s_\gamma+p_\beta q_\gamma-r_\beta(q-s)_\gamma) \\
& \quad +g_{\beta\gamma}(2q_\alpha r_\delta-q_\alpha q_\delta-r_\alpha r_\delta+q_\alpha p_\delta-s_\alpha(p-r)_\delta) \\
& \quad +g_{\beta\delta}(2q_\alpha s_\gamma-q_\alpha q_\gamma-s_\alpha s_\gamma+q_\alpha p_\gamma-r_\alpha(p-s)_\gamma)\Big]\Big\}
\end{aligned}
\tag{C45}
$$



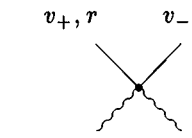
$$
\begin{aligned}
&= it_\theta \Big\{ g_{\alpha\beta}g_{\gamma\delta}\Big[2gg_1 + g^2c_\theta^2\Big(2 + 2\Lambda_V^{-2}m_{w_0}^2 + \Lambda_W^{-2}[4(p\cdot q) + (p+q)\cdot(r+s) + 4(r\cdot s)]\Big] \\
&\quad - g_{\alpha\gamma}g_{\beta\delta}\Big[gg_1 + g^2\Big(c_\theta^2 + \Lambda_V^{-2}m_{w_0}^2 + \Lambda_W^{-2}c_\theta^4[(p\cdot q)+2(p\cdot r)+2(q\cdot s)+(r\cdot s)]\Big)\Big] \\
&\quad - g_{\alpha\delta}g_{\beta\gamma}\Big[gg_1 + g^2\Big(c_\theta^2 + \Lambda_V^{-2}m_{w_0}^2 + \Lambda_W^{-2}c_\theta^4[(p\cdot q)+2(p\cdot s)+2(q\cdot r)+(r\cdot s)]\Big)\Big] \\
&\quad - g^2\Lambda_W^{-2}c_\theta^2\Big[\ g_{\alpha\beta}(2(p-q)_\gamma(p-q)_\delta - (p+q)_\gamma r_\delta - s_\gamma(p+q)_\delta + 2s_\gamma r_\delta) \\
&\qquad + g_{\gamma\delta}(2(r-s)_\alpha(r-s)_\beta - (r+s)_\alpha p_\beta - q_\alpha(r+s)_\beta + 2p_\beta q_\alpha) \\
&\qquad + g_{\alpha\gamma}(2p_\beta r_\delta - p_\beta p_\delta - r_\beta r_\delta + p_\beta q_\delta - s_\beta(q-r)_\delta) \\
&\qquad + g_{\alpha\delta}(2p_\beta s_\gamma - p_\beta p_\gamma - s_\beta s_\gamma + p_\beta q_\gamma - r_\beta(q-s)_\gamma) \\
&\qquad + g_{\beta\gamma}(2q_\alpha r_\delta - q_\alpha q_\delta - r_\alpha r_\delta + q_\alpha p_\delta - s_\alpha(p-r)_\delta) \\
&\qquad + g_{\beta\delta}(2q_\alpha s_\gamma - q_\alpha q_\gamma - s_\alpha s_\gamma + q_\alpha p_\gamma - r_\alpha(p-s)_\gamma)\Big]\Big\}
\end{aligned}
\tag{C46}
$$



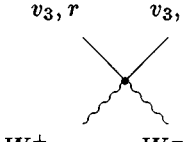
$$
\begin{aligned}
&= -ig^2s_\theta^2\Big\{\ g_{\alpha\beta}g_{\gamma\delta}\Big(2 + 2\Lambda_V^{-2}m_{w_0}^2 + \Lambda_W^{-2}[4(p\cdot q) + (p+q)\cdot(r+s) + 4(r\cdot s)]\Big) \\
&\quad - g_{\alpha\gamma}g_{\beta\delta}\Big(1 + \Lambda_W^{-2}c_\theta^2[(p\cdot q) + 2(p\cdot r) + 2(q\cdot s) + (r\cdot s)]\Big) \\
&\quad - g_{\alpha\delta}g_{\beta\gamma}\Big(1 + \Lambda_W^{-2}c_\theta^2[(p\cdot q) + 2(p\cdot s) + 2(q\cdot r) + (r\cdot s)]\Big) \\
&\quad - \Lambda_W^{-2}\Big[\ g_{\alpha\beta}(2(p-q)_\gamma(p-q)_\delta - (p+q)_\gamma r_\delta - s_\gamma(p+q)_\delta + 2s_\gamma r_\delta) \\
&\qquad + g_{\gamma\delta}(2(r-s)_\alpha(r-s)_\beta - (r+s)_\alpha p_\beta - q_\alpha(r+s)_\beta + 2p_\beta q_\alpha) \\
&\qquad + g_{\alpha\gamma}(2p_\beta r_\delta - p_\beta p_\delta - r_\beta r_\delta + p_\beta q_\delta - s_\beta(q-r)_\delta) \\
&\qquad + g_{\alpha\delta}(2p_\beta s_\gamma - p_\beta p_\gamma - s_\beta s_\gamma + p_\beta q_\gamma - r_\beta(q-s)_\gamma) \\
&\qquad + g_{\beta\gamma}(2q_\alpha r_\delta - q_\alpha q_\delta - r_\alpha r_\delta + q_\alpha p_\delta - s_\alpha(p-r)_\delta) \\
&\qquad + g_{\beta\delta}(2q_\alpha s_\gamma - q_\alpha q_\gamma - s_\alpha s_\gamma + q_\alpha p_\gamma - r_\alpha(p-s)_\gamma)\Big]\Big\}
\end{aligned}
\tag{C47}
$$



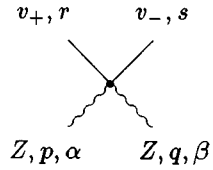
$$
= 2ic_\theta^{-4}(g_4 + g_5 + 2g_6 + 2g_7 + 2g_8)(g_{\alpha\beta}g_{\gamma\delta} + g_{\alpha\gamma}g_{\beta\delta} + g_{\alpha\delta}g_{\beta\gamma}) \tag{C48}
$$



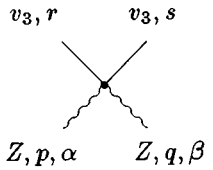
$$
\begin{aligned}
&= -\tfrac{i}{2}\Big\{ g_{\alpha\beta}\Big[m_{w_0}^{-2}\Big(gg_1(p-q)\cdot(r-s) + 2gg_3(p\cdot r + q\cdot s) + 2(g_4+2g_5)(r\cdot s)\Big) - 2g^2\Lambda_V^{-2}(r\cdot s)\Big] \\
&\quad + (gg_1m_{w_0}^{-2} + g^2\Lambda_V^{-2})[2(r_\alpha s_\beta - s_\alpha r_\beta) + q_\alpha(r-s)_\beta - (r-s)_\alpha p_\beta] \\
&\quad + 2gg_3m_{w_0}^{-2}(r_\alpha s_\beta - s_\alpha r_\beta - r_\alpha p_\beta - q_\alpha s_\beta) + 2g_4(2r_\alpha s_\beta + s_\alpha r_\beta) + g_5 4 s_\alpha r_\beta\Big\}
\end{aligned}
\tag{C49}
$$



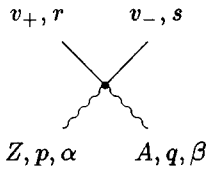
$$
= 2i\left[g^2\Lambda_V^{-2} - (g_5 + g_7)\right](r\cdot s)g_{\alpha\beta} - im_{w_0}^{-2}(g_4 + g_6)(r_\alpha s_\beta + s_\alpha r_\beta) \tag{C50}
$$



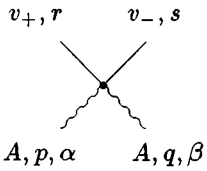
$$
\begin{aligned}
= -i\Big\{ & g_{\alpha\beta}\Big[2g^2 s_\theta^2\Big(1+\Lambda_V^{-2}(p\cdot q)\Big) - 2c_\theta^{-2}\Big(g^2\Lambda_V^{-2}(c_\theta^4+s_\theta^4) - (g_5+g_7)m_{w_0}^{-2}\Big)(r\cdot s) \\
& \quad -\Big((gg_1 s_\theta^2 - gg_2 s_\theta c_\theta - gg_3 c_\theta^2)m_{w_0}^{-2} + g^2\Lambda_V^{-2}s_\theta^2\Big)(p+q)^2\Big] \\
& -2g^2\Lambda_V^{-2}s_\theta^2(r-s)_\alpha(r-s)_\beta \\
& -\Big((gg_1 s_\theta^2 - gg_2 s_\theta c_\theta - gg_3 c_\theta^2)m_{w_0}^{-2} - g^2\Lambda_V^{-2}s_\theta^2\Big)[(r+s)_\alpha p_\beta + q_\alpha(r+s)_\beta] \\
& +(g_4+g_6)c_\theta^{-2}m_{w_0}^{-2}(r_\alpha s_\beta + s_\alpha r_\beta)\Big\}
\end{aligned}
\tag{C51}
$$



$$
= -2ic_\theta^{-2}m_{w_0}^{-2}(g_4+g_5+2g_6+2g_7+2g_8)(r\cdot s g_{\alpha\beta} + r_\alpha s_\beta + r_\beta s_\alpha)
\tag{C52}
$$

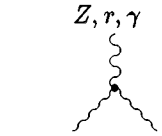


$$
\begin{aligned}
= -it_\theta\Big\{ & g_{\alpha\beta}\Big[g^2(c_\theta^2-s_\theta^2)(1+\Lambda_V^{-2}(p\cdot q)+2\Lambda_V^{-2}(r\cdot s)) \\
& \quad +(gg_1 m_{w_0}^{-2}+g^2\Lambda_V^{-2})(pc_\theta^2 - qs_\theta^2)\cdot(r+s) \\
& \quad +gg_2 m_{w_0}^{-2}(ps_\theta c_\theta - qc_\theta^3 s_\theta^{-1})\cdot(r+s) \\
& \quad -gg_3 m_{w_0}^{-2}c_\theta^2(r+s)^2\Big] \\
& -g^2\Lambda_V^{-2}(c_\theta^2-s_\theta^2)(r-s)_\alpha(r-s)_\beta \\
& -\Big((gg_1 c_\theta^2 + gg_2 s_\theta c_\theta + gg_3 c_\theta^2)m_{w_0}^{-2} + g^2\Lambda_V^{-2}s_\theta^2\Big)(r+s)_\alpha p_\beta \\
& +\Big((gg_1 s_\theta^2 + gg_2 c_\theta^3 s_\theta^{-1} - gg_3 c_\theta^2)m_{w_0}^{-2} + g^2\Lambda_V^{-2}c_\theta^2\Big)q_\alpha(r+s)_\beta\Big\}
\end{aligned}
\tag{C53}
$$

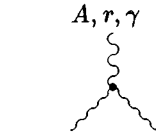


$$
\begin{aligned}
= i\Big\{ & g_{\alpha\beta}\Big[2g^2 s_\theta^2\left(1+\Lambda_V^{-2}[(p\cdot q)+2(r\cdot s)]\right) \\
& \quad -\Big((gg_1 s_\theta^2 - gg_2 s_\theta c_\theta + gg_3 s_\theta^2)m_{w_0}^{-2} + g^2 s_\theta^2\Lambda_V^{-2}\Big)(p+q)^2\Big] \\
& -2g^2 s_\theta^2\Lambda_V^{-2}(r-s)_\alpha(r-s)_\beta \\
& -\Big((gg_1 s_\theta^2 - gg_2 s_\theta c_\theta + gg_3 s_\theta^2)m_{w_0}^{-2} - g^2 s_\theta^2\Lambda_V^{-2}\Big)[(r+s)_\alpha p_\beta + q_\alpha(r+s)_\beta]\Big\}
\end{aligned}
\tag{C54}
$$

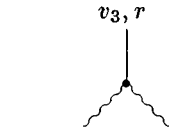
b. Three-vertices



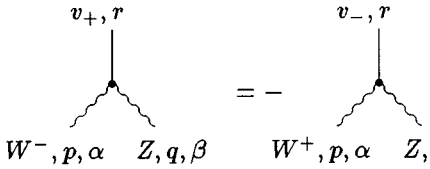
$$
\begin{aligned}
= -i\Big\{ \quad & g_{\alpha\beta}\Big[g(c_\theta + \Lambda_V^{-2} m_{w0}^2 c_\theta)(p-q)_\gamma + g_1 c_\theta^{-1}(p-q)_\gamma \\
& \qquad -g\Lambda_W^{-2} c_\theta \Big([(p-q)\cdot p] p_\gamma + [(p-q)\cdot q] q_\gamma \Big) \Big] \\
& + g_{\beta\gamma}\Big[g(c_\theta + \Lambda_V^{-2} m_{w0}^2 c_\theta^{-1})(q-r)_\alpha + g_1(c_\theta^{-1} q_\alpha - c_\theta r_\alpha) - (g_2 s_\theta + g_3 c_\theta) r_\alpha \\
& \qquad -g\Lambda_W^{-2} c_\theta \Big([(q-r)\cdot q] q_\alpha + [(q-r)\cdot r] r_\alpha \Big) \Big] \\
& + g_{\alpha\gamma}\Big[g(c_\theta + \Lambda_V^{-2} m_{w0}^2 c_\theta^{-1})(r-p)_\beta + g_1(c_\theta r_\beta - c_\theta^{-1} p_\beta) + (g_2 s_\theta + g_3 c_\theta) r_\beta \\
& \qquad -g\Lambda_W^{-2} c_\theta \Big([(r-p)\cdot r] r_\beta + [(r-p)\cdot p] p_\beta \Big) \Big] \\
& - g\Lambda_W^{-2} c_\theta [(p-q)_\gamma r_\alpha r_\beta + (q-r)_\alpha p_\beta p_\gamma + (r-p)_\beta q_\gamma q_\alpha] \Big\}
\end{aligned}
\tag{C55}
$$



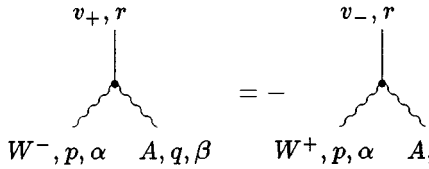
$$
\begin{aligned}
= is_\theta \Big\{ \quad & g_{\alpha\beta} g \Big[(1 + \Lambda_V^{-2} m_{w0}^2)(p-q)_\gamma - \Lambda_W^{-2} \Big([(p-q)\cdot p] p_\gamma + [(p-q)\cdot q] q_\gamma \Big) \Big] \\
& + g_{\beta\gamma}\Big[g(q-r)_\alpha - (g_1 - c_\theta s_\theta^{-1} g_2 + g_3) r_\alpha - g\Lambda_W^{-2} \Big([(q-r)\cdot q] q_\alpha + [(q-r)\cdot r] r_\alpha \Big) \Big] \\
& + g_{\alpha\gamma}\Big[g(r-p)_\beta + (g_1 - c_\theta s_\theta^{-1} g_2 + g_3) r_\beta - g\Lambda_W^{-2} \Big([(r-p)\cdot r] r_\beta + [(r-p)\cdot p] p_\beta \Big) \Big] \\
& - g\Lambda_W^{-2} [(q-r)_\alpha p_\beta p_\gamma + (r-p)_\beta q_\gamma q_\alpha + (p-q)_\gamma r_\alpha r_\beta] \Big\}
\end{aligned}
\tag{C56}
$$



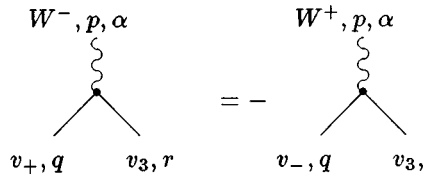
$$
= m_{w0}^{-1} \left[g_1 [(p-q)\cdot r] g_{\alpha\beta} - \left(g_1 + g\Lambda_V^{-2} m_{w0}^2 \right) (p_\beta r_\alpha - q_\alpha r_\beta) \right]
\tag{C57}
$$



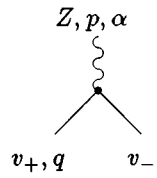
$$
\begin{aligned}
= m_{w0} \Big\{ & g_{\alpha\beta} \Big[g s_\theta^2 c_\theta^{-1} \left(1 - \Lambda_V^{-2} p^2 \right) + m_{w0}^{-2} g_1 c_\theta^{-1} (p\cdot r) - m_{w0}^{-2} \left(g_1 c_\theta + g_2 s_\theta + g_3 c_\theta \right) (q\cdot r) \Big] \\
& - p_\beta r_\alpha \left(g_1 m_{w0}^{-2} c_\theta^{-1} + g\Lambda_V^{-2} c_\theta \right) + q_\alpha r_\beta \left[m_{w0}^{-2} (g_1 c_\theta + g_2 s_\theta + g_3 c_\theta) + g\Lambda_V^{-2} c_\theta^{-1} \right] \\
& - r_\alpha r_\beta g \Lambda_V^{-2} s_\theta^2 c_\theta^{-1} \Big\}
\end{aligned}
\tag{C58}
$$



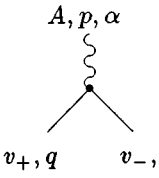
$$
\begin{aligned}
= m_{w0} \Big\{ & \left[g s_\theta \left(1 - \Lambda_V^{-2} p^2 \right) + m_{w0}^{-2} (g_1 s_\theta - g_2 c_\theta + g_3 s_\theta)(q\cdot r) \right] g_{\alpha\beta} \\
& + g s_\theta \Lambda_V^{-2} r_\alpha (p-r)_\beta + m_{w0}^{-2} (-g_1 s_\theta + g_2 c_\theta - g_3 s_\theta) q_\alpha r_\beta \Big\}
\end{aligned}
\tag{C59}
$$



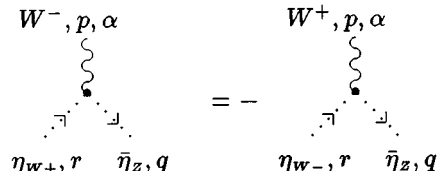
$$= -i\left\{\tfrac{1}{2}g\left[1-\Lambda_V^{-2}\left(q^2+r^2\right)\right](q-r)_\alpha - g_1 m_{w0}^{-2}[(p\cdot q)r_\alpha - (p\cdot r)q_\alpha]\right\} \tag{C60}$$



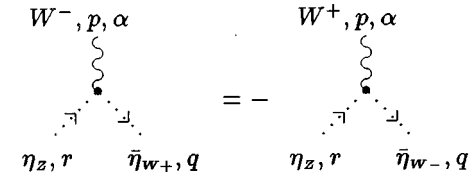
$$\begin{aligned} = -i\Big\{ & m_{w0}^{-2}(g_1 c_\theta + g_2 s_\theta + g_3 c_\theta)[(p\cdot q)r_\alpha - (p\cdot r)q_\alpha] \\ & -\tfrac{1}{2}g c_\theta^{-1}\left[(c_\theta^2 - s_\theta^2)\left(1+\Lambda_V^{-2}[(r-q)\cdot q]\right) + \Lambda_V^{-2}(p\cdot r)\right] q_\alpha \\ & +\tfrac{1}{2}g c_\theta^{-1}\left[(c_\theta^2 - s_\theta^2)\left(1+\Lambda_V^{-2}[(q-r)\cdot r]\right) + \Lambda_V^{-2}(p\cdot q)\right] r_\alpha \Big\} \end{aligned} \tag{C61}$$



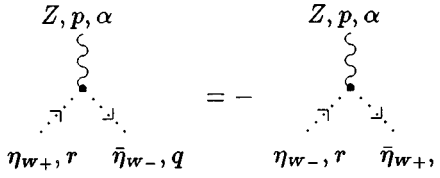
$$\begin{aligned} = i\Big\{ & m_{w0}^{-2}(g_1 s_\theta - g_2 c_\theta + g_3 s_\theta)[(p\cdot q)r_\alpha - (p\cdot r)q_\alpha] \\ & -g s_\theta\left[\left(1+\Lambda_V^{-2}[(r-q)\cdot q]\right) q_\alpha - \left(1+\Lambda_V^{-2}[(q-r)\cdot r]\right) r_\alpha\right] \Big\} \end{aligned} \tag{C62}$$



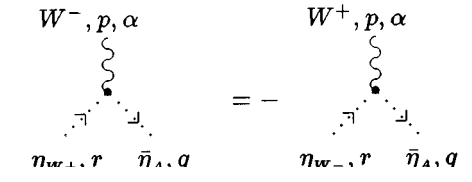
(C63)



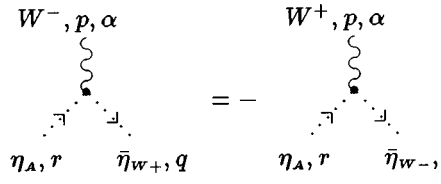
(C64)



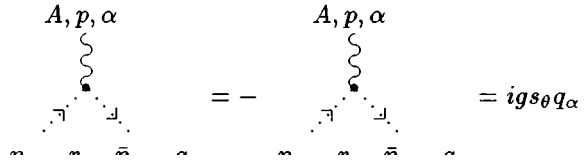
(C65)



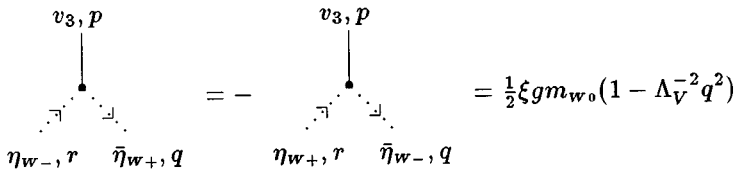
(C66)



(C67)



$$= - \qquad = igs_\theta q_\alpha \tag{C68}$$



$$= - \qquad = \tfrac{1}{2}\xi g m_{w0}(1-\Lambda_V^{-2}q^2) \tag{C69}$$

APPENDIX D: ONE-LOOP INTEGRALS

Define ϵ by

$$d=4-2\epsilon, \tag{D1}$$

where d is the spacetime dimension, and $\overline{\mu}$ by

$$\ln 4\pi\mu^2-\gamma_E=\ln\overline{\mu^2} \tag{D2}$$

and $\int_p$ by

$$\int_p=\int\frac{d^dp}{(2\pi)^d}. \tag{D3}$$

The only integrals we need are

$$I(m^2)\equiv\int_p\frac{1}{p^2-m^2}=\frac{im^2}{(4\pi)^2}\left(\frac{1}{\epsilon}+1-\ln\frac{m^2}{\overline{\mu^2}}\right)+\mathcal{O}(\epsilon) \tag{D4}$$

and

$$\begin{aligned}
I(k^2;m_a^2,m_b^2)&\equiv\int_p\frac{1}{[(p+k)^2-m_a^2+i\varepsilon](p^2-m_b^2+i\varepsilon)}\\
&=\int_0^1dx\int_p\frac{1}{[p^2+2xp\cdot k+xk^2-xm_a^2-(1-x)m_b^2+i\varepsilon]^2}\\
&=\frac{i\mu^{2\epsilon}\Gamma(\epsilon)}{(4\pi)^{2-\epsilon}}\int_0^1\frac{dx}{[-x(1-x)k^2+xm_a^2+(1-x)m_b^2-i\varepsilon]^\epsilon}\\
&=\frac{i}{(4\pi)^2}\left(\frac{1}{\epsilon}-\gamma_E\right)\left(1-\epsilon\int_0^1dx\ln\frac{-x(1-x)k^2+xm_a^2+(1-x)m_b^2-i\varepsilon}{4\pi\mu^2}\right)+\mathcal{O}(\epsilon)\\
&=\frac{i}{(4\pi)^2}\left(\frac{1}{\epsilon}-\int_0^1dx\ln\frac{-x(1-x)k^2+xm_a^2+(1-x)m_b^2-i\varepsilon}{\overline{\mu^2}}\right)+\mathcal{O}(\epsilon)\\
&=\frac{i}{(4\pi)^2}\left(\frac{1}{\epsilon}-\int_0^1dx\ln\frac{k^2(x-x_0)^2-D/(4k^2)-i\varepsilon}{\overline{\mu^2}}\right)+\mathcal{O}(\epsilon),
\end{aligned} \tag{D5}$$

where

$$x_0\equiv\frac{k^2+m_b^2-m_a^2}{2k^2} \tag{D6}$$

and

$$D \equiv k^4 + m_a^4 + m_b^4 - 2k^2 m_a^2 - 2k^2 m_b^2 - 2m_a^2 m_b^2 . \tag{D7}$$

We need to investigate here only the case where the argument of the logarithm is non-negative for $0 \leqslant x \leqslant 1$ and therefore $I(k^2; m_a^2, m_b^2)$ is purely imaginary. This is obviously the case for $D \leqslant 0$. For $D > 0$ this is the case if and only if $x_0 \leqslant 0$ or $x_0 \geqslant 1$, i.e. $k^2 \leqslant |m_a^2 - m_b^2|$.

1. $I(k^2; m_a^2, m_b^2)$ for $D \leqslant 0$

Now we can write

$$\begin{aligned}
I(k^2; m_a^2, m_b^2) &= \frac{i}{(4\pi)^2}\left(\frac{1}{\epsilon} - \ln\frac{k^2}{\overline{\mu^2}} - \int_{-x_0}^{1-x_0} dy \ln\left(y^2 + \frac{-D}{4k^4} \right) \right) + \mathcal{O}(\epsilon) \\
&= \frac{i}{(4\pi)^2}\left\{ \frac{1}{\epsilon} - \ln\frac{k^2}{\overline{\mu^2}} - \left[y \ln\left(y^2 + \frac{-D}{4k^4} \right) - 2y + 2\sqrt{\frac{-D}{4k^4}} \arctan\frac{y}{\sqrt{\frac{-D}{4k^4}}} \right]_{-x_0}^{1-x_0} \right\} + \mathcal{O}(\epsilon) \\
&= \frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon} + 2 - \ln\frac{k^2}{\overline{\mu^2}} - \frac{k^2 + m_a^2 - m_b^2}{2k^2} \ln\frac{m_a^2}{k^2} - \frac{k^2 + m_b^2 - m_a^2}{2k^2} \ln\frac{m_b^2}{k^2} - \frac{\sqrt{-D}}{k^2}\left(\arctan\frac{k^2 + m_a^2 - m_b^2}{\sqrt{-D}} \right.\right. \\
&\quad \left.\left. + \arctan\frac{k^2 + m_b^2 - m_a^2}{\sqrt{-D}} \right) \right] + \mathcal{O}(\epsilon) \\
&= \frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon} + 2 - \frac{k^2 + m_a^2 - m_b^2}{2k^2} \ln\frac{m_a^2}{\overline{\mu^2}} - \frac{k^2 + m_b^2 - m_a^2}{2k^2} \ln\frac{m_b^2}{\overline{\mu^2}} - \frac{\sqrt{-D}}{k^2}\left(\arctan\frac{k^2 + m_a^2 - m_b^2}{\sqrt{-D}} + \arctan\frac{k^2 + m_b^2 - m_a^2}{\sqrt{-D}} \right) \right] \\
&\quad + \mathcal{O}(\epsilon).
\end{aligned} \tag{D8}$$

2. $I(k^2; m_a^2, m_b^2)$ for $D \geqslant 0$ with $k^2 \leqslant |m_a^2 - m_b^2|$

Define

$$x_\pm \equiv \frac{k^2 + m_b^2 - m_a^2 \pm \sqrt{D}}{2k^2}, \tag{D9}$$

so that

$$1 - x_\pm = \frac{k^2 + m_a^2 - m_b^2 \mp \sqrt{D}}{2k^2}. \tag{D10}$$

Without loss of generality assume $m_a^2 \geqslant m_b^2$. Then $x_\pm \leqslant 0$ and $1 - x_\pm \geqslant 0$. We can write

$$\begin{aligned}
I(k^2; m_a^2, m_b^2) &= \frac{i}{(4\pi)^2}\left\{ \frac{1}{\epsilon} - \ln\frac{k^2}{\overline{\mu^2}} - \int_0^1 dx \ln[(x - x_+)(x - x_-)] \right\} + \mathcal{O}(\epsilon) \\
&= \frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon} - \ln\frac{k^2}{\overline{\mu^2}} - \int_0^1 dx \ln(x - x_+) - \int_0^1 dx \ln(x - x_-) \right] + \mathcal{O}(\epsilon) \\
&= \frac{i}{(4\pi)^2}\left\{ \frac{1}{\epsilon} - \ln\frac{k^2}{\overline{\mu^2}} - (1 - x_+)[\ln(1 - x_+) - 1] - x_+[\ln(-x_+) - 1] - (1 - x_-)[\ln(1 - x_-) - 1] - x_-[\ln(-x_-) \right. \\
&\quad \left. - 1] \right\} + \mathcal{O}(\epsilon)
\end{aligned}$$

$$=\frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon}+2-\ln\frac{k^2}{\overline{\mu^2}}-(1-x_+)\ln(1-x_+)-x_+\ln(-x_+)-(1-x_-)\ln(1-x_-)-x_-\ln(-x_-)\right]+\mathcal{O}(\epsilon). \quad \text{(D11)}$$

Now one can write either

$$I(k^2;m_a^2,m_b^2)=\frac{i}{(4\pi)^2}\left\{\frac{1}{\epsilon}+2-\ln\frac{k^2}{\overline{\mu^2}}-x_+\ln[(-x_+)(-x_-)]-(1-x_-)\ln[(1-x_+)(1-x_-)]+(x_+-x_-)\ln[(1-x_+)(-x_-)]\right\}$$
$$+\mathcal{O}(\epsilon)$$
$$=\frac{i}{(4\pi)^2}\left\{\frac{1}{\epsilon}+2-\frac{k^2-m_a^2+m_b^2+\sqrt{D}}{2k^2}\ln\frac{m_b^2}{\overline{\mu^2}}-\frac{k^2+m_a^2-m_b^2+\sqrt{D}}{2k^2}\ln\frac{m_a^2}{\overline{\mu^2}}+\frac{\sqrt{D}}{k^2}\ln\frac{m_a^2+m_b^2-k^2+\sqrt{D}}{2\overline{\mu^2}}\right\}+\mathcal{O}(\epsilon) \quad \text{(D12)}$$

or

$$I(k^2;m_a^2,m_b^2)=\frac{i}{(4\pi)^2}\left\{\frac{1}{\epsilon}+2-\ln\frac{k^2}{\overline{\mu^2}}-x_-\ln[(-x_-)(-x_+)]-(1-x_+)\ln[(1-x_-)(1-x_+)]+(x_--x_+)\ln[(1-x_-)(-x_+)]\right\}$$
$$+\mathcal{O}(\epsilon)=\frac{i}{(4\pi)^2}\left\{\frac{1}{\epsilon}+2-\frac{k^2-m_a^2+m_b^2-\sqrt{D}}{2k^2}\ln\frac{m_b^2}{\overline{\mu^2}}-\frac{k^2+m_a^2-m_b^2-\sqrt{D}}{2k^2}\ln\frac{m_a^2}{\overline{\mu^2}}-\frac{\sqrt{D}}{k^2}\ln\frac{m_a^2+m_b^2-k^2-\sqrt{D}}{2\overline{\mu^2}}\right\}$$
$$+\mathcal{O}(\epsilon) \quad \text{(D13)}$$

with

$$\sqrt{D}\equiv\sqrt{k^4+m_a^4+m_b^4-2m_a^2m_b^2-2k^2m_a^2-2k^2m_b^2}. \quad \text{(D14)}$$

Equations (D12) and (D13) are symmetric in m_a^2 and m_b^2 and therefore we can drop the restriction $m_a^2\geqslant m_b^2$.

In the following we will specialize to the cases that are needed for the evaluation of our one-loop diagrams.

3. $I(k^2;m^2,m^2)$

Only for $D=k^2(k^2-4m^2)\leqslant 0$, i.e. for $k^2\leqslant 4m^2$ we have purely imaginary $I(k^2;m^2,m^2)$. From (D8) we get

$$I(k^2;m^2,m^2)=\frac{i}{(4\pi)^2}\left(\frac{1}{\epsilon}+2-\ln\frac{m^2}{\overline{\mu^2}}-2\sqrt{\frac{4m^2}{k^2}-1}\arctan\frac{1}{\sqrt{\frac{4m^2}{k^2}-1}}\right)+\mathcal{O}(\epsilon). \quad \text{(D15)}$$

For $k^2\ll m^2$, we can expand in powers of k^2/m^2 to get

$$I(k^2;m^2,m^2)=\frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon}-\ln\frac{m^2}{\overline{\mu^2}}+\frac{1}{6}\left(\frac{k^2}{m^2}\right)+\frac{1}{60}\left(\frac{k^2}{m^2}\right)^2\right]+\mathcal{O}\left(\epsilon,\left(\frac{k^2}{m^2}\right)^3\right). \quad \text{(D16)}$$

4. $I(k^2;m^2,0)$

Now $D=|k^2-m^2|\geqslant 0$ and we need $k^2\leqslant m^2$ to have a purely imaginary $I(k^2;m^2,0)$. We get from (D12) and (D13)

$$I(k^2;0,m^2)=I(k^2;m^2,0)=\frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon}+2+\frac{m^2-k^2}{k^2}\ln\frac{m^2-k^2}{\overline{\mu^2}}-\frac{m^2}{k^2}\ln\frac{m^2}{\overline{\mu^2}}\right]+\mathcal{O}(\epsilon)$$
$$=\frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon}+2+\frac{m^2}{k^2}\ln\left(1-\frac{k^2}{m^2}\right)-\ln\frac{m^2-k^2}{\overline{\mu^2}}\right]+\mathcal{O}(\epsilon). \quad \text{(D17)}$$

For $k^2\ll m^2$, we can expand in powers of k^2/m^2 to get

$$I(k^2;0,m^2)=I(k^2;m^2,0)=\frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon}+1-\ln\frac{m^2}{\overline{\mu^2}}+\frac{1}{2}\left(\frac{k^2}{m^2}\right)+\frac{1}{6}\left(\frac{k^2}{m^2}\right)^2\right]+\mathcal{O}\left(\epsilon,\left(\frac{k^2}{m^2}\right)^3\right). \quad \text{(D18)}$$

5. $I(k^2;m_a^2,m_b^2)$ for $k^2,m_a^2 \ll m_b^2$

If $k^2,m_a^2 \ll m_b^2$, we can expand (D12) or (D13) in negative powers of m_b^2 to get

$$I(k^2;m_a^2,m_b^2)=\frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon}+1-\ln\frac{m_b^2}{\overline{\mu^2}}+\frac{\frac{1}{2}k^2+m_a^2\ln\frac{m_a^2}{m_b^2}}{m_b^2}+\frac{k^2\left(\frac{1}{6}k^2+\frac{3}{2}m_a^2\right)+m_a^2(k^2+m_a^2)\ln\frac{m_a^2}{m_b^2}}{m_b^4}\right]+\mathcal{O}(\epsilon,m_b^{-6}\ln m_b^2). \tag{D19}$$

6. $I(k^2;m_a^2,m_b^2)$ for $k^2 \ll m_a^2,m_b^2$

If $k^2 \ll m_a^2,m_b^2$, but the relative magnitude of k^2 and $|m_a^2-m_b^2|$ is unknown, it is not clear, which of (D8) on the one hand or (D12), (D13) on the other hand has to be used. Although they are connected by analytic continuation, here we will expand $I(k^2;m_a^2,m_b^2)$ in powers of k^2 to have an unambiguous result without having to worry about Riemann sheets.

Starting from the next-to-last line in (D5) we get

$$\begin{aligned}
I(k^2;m_a^2,m_b^2)&=\frac{i}{(4\pi)^2}\left(\frac{1}{\epsilon}-\int_0^1 dx\ln\frac{-x(1-x)k^2+xm_a^2+(1-x)m_b^2}{\overline{\mu^2}}\right)+\mathcal{O}(\epsilon)\\
&=\frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon}-\int_0^1 dx\ln\frac{xm_a^2+(1-x)m_b^2}{\overline{\mu^2}}-\int_0^1 dx\ln\left(1-\frac{x(1-x)k^2}{xm_a^2+(1-x)m_b^2}\right)\right]+\mathcal{O}(\epsilon)\\
&=\frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon}+1-\frac{m_a^2\ln\frac{m_a^2}{\overline{\mu^2}}-m_b^2\ln\frac{m_b^2}{\overline{\mu^2}}}{m_a^2-m_b^2}+\sum_{n=1}^{\infty}\frac{1}{n}\int_0^1 dx\left(\frac{x(1-x)k^2}{xm_a^2+(1-x)m_b^2}\right)^n\right]+\mathcal{O}(\epsilon).
\end{aligned} \tag{D20}$$

Expanding in k^2, we get

$$\begin{aligned}
I(k^2;m_a^2,m_b^2)&=\frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon}+1-\frac{m_a^2\ln\frac{m_a^2}{\overline{\mu^2}}-m_b^2\ln\frac{m_b^2}{\overline{\mu^2}}}{m_a^2-m_b^2}+k^2\int_0^1 dx\left(\frac{x(1-x)}{xm_a^2+(1-x)m_b^2}\right)+\frac{k^4}{2}\int_0^1 dx\left(\frac{x(1-x)}{xm_a^2+(1-x)m_b^2}\right)^2\right]+\mathcal{O}(k^6,\epsilon)\\
&=\frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon}+1-\frac{m_a^2\ln\frac{m_a^2}{\overline{\mu^2}}-m_b^2\ln\frac{m_b^2}{\overline{\mu^2}}}{m_a^2-m_b^2}+\left(\frac{m_a^2+m_b^2}{2(m_a^2-m_b^2)^2}-\frac{m_a^2m_b^2}{(m_a^2-m_b^2)^3}\ln\frac{m_a^2}{m_b^2}\right)k^2+\left(\frac{m_a^4+10m_a^2m_b^2+m_b^4}{6(m_a^2-m_b^2)^4}\right.\right.\\
&\quad\left.\left.-\frac{m_a^2m_b^2(m_a^2+m_b^2)}{(m_a^2-m_b^2)^5}\ln\frac{m_a^2}{m_b^2}\right)k^4\right]+\mathcal{O}(k^6,\epsilon).
\end{aligned} \tag{D21}$$

Note that (D20) tells us that subsequent powers of k^2 in (D21) are suppressed by negative powers of m_a^2 and m_b^2 and not just by their difference $m_a^2-m_b^2$, which might be small or even vanishing.

Indeed, setting $m_a^2=m^2+\delta m_a^2$, $m_b^2=m^2+\delta m_b^2$ with $k^2,\delta m_a^2,\delta m_b^2 \ll m^2$ and starting again from the next-to-last line in (D5) we get

$$\begin{aligned}
I(k^2;m^2+\delta m_a^2,m^2+\delta m_b^2)&=\frac{i}{(4\pi)^2}\left(\frac{1}{\epsilon}-\int_0^1 dx\ln\frac{m^2-x(1-x)k^2+x\,\delta m_a^2+(1-x)\,\delta m_b^2}{\overline{\mu^2}}\right)+\mathcal{O}(\epsilon)\\
&=\frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon}-\ln\frac{m^2}{\overline{\mu^2}}-\int_0^1 dx\ln\left(1-\frac{x(1-x)k^2-x\,\delta m_a^2-(1-x)\,\delta m_b^2}{m^2}\right)\right]+\mathcal{O}(\epsilon)\\
&=\frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon}-\ln\frac{m^2}{\overline{\mu^2}}+\sum_{n=1}^{\infty}\frac{1}{n}\int_0^1 dx\left(\frac{x(1-x)k^2-x\,\delta m_a^2-(1-x)\,\delta m_b^2}{m^2}\right)^n\right]+\mathcal{O}(\epsilon)
\end{aligned}$$

$$= \frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon} - \ln\frac{m^2}{\mu^2} + \frac{1}{6}\left(\frac{k^2}{m^2}\right) - \frac{1}{2}\left(\frac{\delta m_a^2}{m^2}\right) - \frac{1}{2}\left(\frac{\delta m_b^2}{m^2}\right) + \frac{1}{60}\left(\frac{k^2}{m^2}\right)^2 + \frac{1}{6}\left(\frac{\delta m_a^2}{m^2}\right)^2 + \frac{1}{6}\left(\frac{\delta m_b^2}{m^2}\right)^2 \right.$$
$$\left. - \frac{1}{12}\left(\frac{k^2}{m^2}\right)\left(\frac{\delta m_a^2}{m^2}\right) - \frac{1}{12}\left(\frac{k^2}{m^2}\right)\left(\frac{\delta m_b^2}{m^2}\right) + \frac{1}{6}\left(\frac{\delta m_a^2}{m^2}\right)\left(\frac{\delta m_b^2}{m^2}\right)\right] + \mathcal{O}(m^{-6}, \epsilon), \tag{D22}$$

which can also be obtained by expanding (D21).

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