Regarding the enigmas of *P*-wave meson spectroscopy

L. Burakovsky* and T. Goldman†
Theoretical Division, MS B285, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

(Received 19 February 1997; published 10 February 1998)

The mass spectrum of P-wave mesons is considered in a nonrelativistic constituent quark model. The results show the common mass degeneracy of the isovector and isodoublet states of the scalar and tensor meson nonets, $m(a_0)\cong m(a_2)$, $m(K_0^*)\cong m(K_2^*)$, and do not exclude the possibility of a similar degeneracy of the same states of the axial-vector and pseudovector nonets. Current experimental hadronic and τ-decay data suggest, however, a different scenario leading to the a_1 meson mass $\simeq$1190 MeV and the K_{1A}-K_{1B} mixing angle $\simeq(37\pm3)°$. Possible $s\bar{s}$ states of the four nonets are also discussed. [S0556-2821(98)00307-5]

PACS number(s): 12.39.Jh, 12.39.Pn, 12.40.Yx, 14.40.Cs

I. INTRODUCTION

The existence of a gluon self-coupling in QCD suggests that, in addition to the conventional $q\bar{q}$ states, there may be non-$q\bar{q}$ mesons: bound states including gluons (gluonia and glueballs, and $q\bar{q}g$ hybrids) and multiquark states [1]. Since the theoretical guidance on the properties of unusual states is often contradictory, models that agree in the $q\bar{q}$ sector differ in their predictions about new states. Among the naively expected signatures for gluonium are (i) no place in $q\bar{q}$ nonet, (ii) flavor-singlet coupling, (iii) enhanced production in gluon-rich channels such as $J/\psi(1S)$ decay, (iv) reduced $\gamma\gamma$ coupling, and (v) exotic quantum numbers not allowed for $q\bar{q}$ (in some cases). Points (iii) and (iv) can be summarized by the Chanowitz S parameter [2]

$$S=\frac{\Gamma(J/\psi(1S)\rightarrow\gamma X)}{S_P(J/\psi(1S)\rightarrow\gamma X)}\,\frac{S_P(X\rightarrow\gamma\gamma)}{\Gamma(X\rightarrow\gamma\gamma)},$$

where S_P stands for phase space. S is expected to be larger for gluonium than for $q\bar{q}$ states. Of course, mixing effects and other dynamical effects such as form factors can obscure these simple signatures. Even if the mixing is large, however, simply counting the number of observed states remains a clear signal for non-exotic non-$q\bar{q}$ states. Exotic quantum number states $(0^{--},0^{+-},1^{-+},2^{+-},\ldots)$ would be the best signatures for non-$q\bar{q}$ states. It should be also emphasized that no state has yet unambiguously been identified as gluonium, or as a multiquark state, or as a hybrid.

In this paper we shall discuss P-wave meson states, the interpretation of which as members of conventional quark model $q\bar{q}$ nonets encounters difficulties. We shall be concerned with the scalar, axial-vector, pseudovector, and tensor meson nonets which have the following $q\bar{q}$ quark model assignments, according to the most recent Particle Data Group book [3]:

(1) $1\,^1P_1$ pseudovector meson nonet, $J^{PC}=1^{+-}$, $b_1(1235),h_1(1170),h_1'(1380),K_{1B}$ [1]
(2) $1\,^3P_0$ scalar meson nonet, $J^{PC}=0^{++}$, $a_0(?),f_0(?),f_0'(?),K_0^*(1430)$
(3) $1\,^3P_1$ axial-vector meson nonet, $J^{PC}=1^{++}$, $a_1(1260),f_1(1285),f_1'(1510),K_{1A}$ [1]
(4) $1\,^3P_2$ tensor meson nonet, $J^{PC}=2^{++}$, $a_2(1320),f_2(1270),f_2'(1525),K_2^*(1430)$, and briefly mention the problems associated with these four nonets.

1. Scalar meson nonet

The spectrum of the scalar meson nonet is a long-standing problem of light meson spectroscopy, since the number of resonances found in the region of 1–2 GeV exceeds the number of states that conventional quark models can accommodate, in both the isoscalar and isovector channels [3]. While extra states could be interpreted alternatively as $K\bar{K}$ molecules, glueballs, multi-quark states or hybrids, the correct $q\bar{q}$ assignment for the scalar nonet yet remains unknown.

2. Axial-vector meson nonet

(1) One of the uncertainties related to the axial-vector nonet is the still undefined properties of its $I=1$ member, the $a_1(1260)$ meson. This meson has a huge width of $\sim$400

*Email address: BURAKOV@PION.LANL.GOV
†Email address: GOLDMAN@T5.LANL.GOV

[1]The K_{1A} and K_{1B} are nearly 45^o mixed states of the $K_1(1270)$ and $K_1(1400)$ [3], their masses is therefore $\simeq$1340 MeV.

MeV, due to strong coupling to a dominant decay channel $a_1 \to \rho\pi$, which makes the determination of its mass rather difficult. The value currently adopted by the Particle Data Group (PDG) is $m(a_1)=1230\pm 40$ MeV [3].

(2) The $q\bar{q}$ model predicts a nonet that includes two isoscalar $1\ {}^3P_1$ states with masses below $\sim$ 1.6 GeV, but three ''good'' 1^{++} objects are known, the $f_1(1285), f_1(1420)$, and $f_1(1510)$, one more than expected, and therefore, one of the three must be a non-$q\bar{q}$ meson.

3. Pseudovector meson nonet

Experimental information on the h_1 and h_1' mesons is rather restricted, and while the $h_1(1170)$ may be considered as firmly established, the $h_1'(1380)$ still needs confirmation [3].

4. Tensor meson nonet

Here, as in case of the scalar nonet, the number of states in the isoscalar channel exceeds that allowed by conventional quark model, although the two $1\ {}^3P_2$ $q\bar{q}$ states are likely the well-known $f_2(1270)$ and $f_2'(1525)$ currently adopted by the PDG.

5. Let us also discuss the $I=1/2$ $1\ {}^3P_1$ and $1\ {}^1P_1$ mesons, $K_1(1270)$ and $K_1(1400)$, with masses 1273 ± 7 MeV and 1402 ± 7 MeV, respectively [3]. It has been known that their decay satisfies a dynamical selection rule:

$$\Gamma(K_1(1270)\to K\rho) \gg \Gamma(K_1(1270)\to K^*\pi),$$

$$\Gamma(K_1(1400)\to K^*\pi) \gg \Gamma(K_1(1400)\to K\rho),$$

which prompted experimentalists to suspect large mixing (with a mixing angle close to 45°) between the $I=1/2$ members of the axial-vector and pseudovector nonets, K_{1A} and K_{1B}, respectively, leading to the physical K_1 and K_1' states [4]. Numerical values for the K_{1A}-K_{1B} mixing angle indicated in the literature lie in the range $\sim$33°–45° [5–10], consistent with 33.6°–56.4° that we obtained in Ref. [11].

Since the experimentally established isodoublet states of the scalar and tensor meson nonets, K_0^* and K_2^*, are mass degenerate, 1429 ± 6 MeV and 1429 MeV, respectively [3], and different models (like those considered in Refs. [12–14]) lead to the $q\bar{q}$ assignment for the scalar nonet which includes both the $a_0(1320)$ and $f_0(1525)$ mesons which are mass degenerate with the corresponding tensor mesons $a_2(1320)$ and $f_2'(1525)$, the question naturally suggests itself as to whether the scalar and tensor nonets are intrinsically mass degenerate[2] [15]. Similar questions may be asked regarding the mass degeneracy of the axial-vector and pseudovector nonets in the $I=1$ and $I=1/2$ channels. If this mass degeneracy of two pairs of nonets, $({}^3P_0-{}^3P_2)$ and $({}^3P_1-{}^1P_1)$, is actually the case, it should be reproduced in a simple phenomenological model of QCD, e.g., in a nonrelativistic constituent quark model. The purpose of this work is to apply the latter model for P-wave meson spectroscopy in order to establish whether mass degeneracy of the two pairs of nonets discussed above actually occurs.

The following remarks are due here. The most widely used potential models are the relativized model of Godfrey and Isgur [8] for the $q\bar{q}$ mesons, and Capstick and Isgur [16] for the qqq baryons. But first, these models differ from the nonrelativistic quark potential model only in relatively minor ways, such as the use of $H_{kin}=\sqrt{m_1^2+\mathbf{p}_1^2}+\sqrt{m_2^2+\mathbf{p}_2^2}$ in place of that given in Eq. (1) below, the retention of the m/E factors in the matrix elements, and the introduction of coordinate smearing in the singular terms such as $\delta(\mathbf{r})$, which makes them more difficult to deal with than the nonrelativistic models. Second, the nonrelativistic models themselves are very successful in the description of, at least, meson spectroscopy, even for the lightest S-wave states, as we demonstrate explicitly below, where they might seem not to work at all. Physical reasons for this success of the nonrelativistic models are analyzed by Lucha *et al.* in Ref. [17]. Finally, more recent analysis by the same authors [18] reveals that ''contrary to one's physical intuition, a relativistic treatment of bound states in a potential model provides *no improvement at all* compared to the corresponding nonrelativistic description.'' Thus, all of the above completely justifies our choice of the nonrelativistic constituent quark model.

II. NONRELATIVISTIC CONSTITUENT QUARK MODEL

In the constituent quark model conventional mesons are bound states of a spin 1/2 quark and spin 1/2 antiquark bound by a phenomenological potential which has some basis in QCD [17]. The quark and antiquark spins combine to give a total spin 0 or 1, which is coupled to the orbital angular momentum L. This leads to meson parity and charge conjugation given by $P=(-1)^{L+1}$ and $C=(-1)^{L+S}$, respectively. One typically assumes that the $q\bar{q}$ wave function is a solution of a nonrelativistic Schrödinger equation with the generalized Breit-Fermi Hamiltonian, H_{BF},

$$H_{BF}\psi_n(\mathbf{r})\equiv[H_{kin}+V(\mathbf{p},\mathbf{r})]\psi_n(\mathbf{r})=E_n\psi_n(\mathbf{r}), \qquad (1)$$

where $H_{kin}=m_1+m_2+\mathbf{p}^2/2\mu-(1/m_1^3+1/m_2^3)\mathbf{p}^4/8$, $\mu=m_1m_2/(m_1+m_2)$, m_1 and m_2 are the constituent quark masses, and to first order in $(v/c)^2=\mathbf{p}^2c^2/E^2\simeq\mathbf{p}^2/m^2c^2$, $V(\mathbf{p},\mathbf{r})$ reduces to the standard nonrelativistic result,

$$V(\mathbf{p},\mathbf{r})\simeq V(r)+V_{SS}+V_{LS}+V_T, \qquad (2)$$

with $V(r)=V_V(r)+V_S(r)$ being the confining potential which consists of a vector and a scalar contribution, and V_{SS}, V_{LS}, and V_T the spin-spin, spin-orbit, and tensor terms, respectively, given by [17]

[2]In the scenario suggested in Refs. [12,13], due to instanton effects, the mass of the f_0' meson is shifted down to $\sim$1 GeV, as compared to the mass $\approx$1275 MeV of its tensor ''partner'' f_2.

$$V_{SS}=\frac{2}{3m_1m_2}\mathbf{s}_1\cdot\mathbf{s}_2\triangle V_V(r), \tag{3}$$

$$V_{LS}=\frac{1}{4m_1^2m_2^2}\frac{1}{r}\left(\{[(m_1+m_2)^2+2m_1m_2]\mathbf{L}\cdot\mathbf{S}_+ +(m_2^2-m_1^2)\mathbf{L}\cdot\mathbf{S}_-\}\frac{dV_V(r)}{dr}-[(m_1^2+m_2^2)\mathbf{L}\cdot\mathbf{S}_+ +(m_2^2-m_1^2)\mathbf{L}\cdot\mathbf{S}_-]\frac{dV_S(r)}{dr}\right), \tag{4}$$

$$V_T=\frac{1}{12m_1m_2}\left(\frac{1}{r}\frac{dV_V(r)}{dr}-\frac{d^2V_V(r)}{dr^2}\right)S_{12}. \tag{5}$$

Here $\mathbf{S}_+\equiv\mathbf{s}_1+\mathbf{s}_2$, $\mathbf{S}_-\equiv\mathbf{s}_1-\mathbf{s}_2$, and

$$S_{12}\equiv3\left(\frac{(\mathbf{s}_1\cdot\mathbf{r})(\mathbf{s}_2\cdot\mathbf{r})}{r^2}-\frac{1}{3}\mathbf{s}_1\cdot\mathbf{s}_2\right). \tag{6}$$

For constituents with spin $s_1=s_2=1/2$, S_{12} may be rewritten in the form

$$S_{12}=2\left(3\frac{(\mathbf{S}\cdot\mathbf{r})^2}{r^2}-\mathbf{S}^2\right), \quad \mathbf{S}=\mathbf{S}_+\equiv\mathbf{s}_1+\mathbf{s}_2. \tag{7}$$

Since $(m_1+m_2)^2+2m_1m_2=6m_1m_2+(m_2-m_1)^2$, $m_1^2+m_2^2=2m_1m_2+(m_2-m_1)^2$, the expression for V_{LS}, Eq. (4), may be rewritten as follows:

$$V_{LS}=\frac{1}{2m_1m_2}\frac{1}{r}\left[\left(3\frac{dV_V(r)}{dr}-\frac{dV_S(r)}{dr}\right)+\frac{(m_2-m_1)^2}{2m_1m_2}\left(\frac{dV_V(r)}{dr}-\frac{dV_S(r)}{dr}\right)\right]\mathbf{L}\cdot\mathbf{S}_+ +\frac{m_2^2-m_1^2}{4m_1^2m_2^2}\frac{1}{r}\left(\frac{dV_V(r)}{dr}-\frac{dV_S(r)}{dr}\right)\mathbf{L}\cdot\mathbf{S}_- \equiv V_{LS}^+ + V_{LS}^-. \tag{8}$$

Since two terms corresponding to the derivatives of the potentials with respect to r are of the same order of magnitude, the above expression for V_{LS}^+ may be rewritten as

$$V_{LS}^+=\frac{1}{2m_1m_2}\frac{1}{r}\left(3\frac{dV_V(r)}{dr}-\frac{dV_S(r)}{dr}\right)\mathbf{L}\cdot\mathbf{S}\left[1+\frac{(m_2-m_1)^2}{2m_1m_2}O(1)\right]. \tag{9}$$

A. *S*-wave spectroscopy

Let us first apply the Breit-Fermi Hamiltonian to the *S*-wave which consists of the two, ${}^1S_0$ $J^{PC}=0^{-+}$ pseudoscalar and ${}^3S_1$ 1^{--} vector, meson nonets. We shall consider only the $I=1$ and $I=1/2$ mesons which are pure $n\bar{n}$ and $(n\bar{s},s\bar{n})$ states, respectively. Since the expectation values of the spin-orbit and tensor terms vanish for $L=0$ or $S=0$ states [17], the mass of a $q\bar{q}$ state with $L=0$ is given by

$$M=m_1+m_2+E+\Lambda\frac{\mathbf{s}_1\cdot\mathbf{s}_2}{m_1m_2}, \tag{10}$$

where E is the nonrelativistic binding energy. As shown below, the sum of just the constituent quark masses and the quark-quark hyperfine interaction term describes the masses of the *S*-wave mesons extremely well. Moreover, Eq. (10) with no E is consistent with the empirical mass squared splitting $\triangle M^2 \equiv M^2(^3S_1) - M^2(^1S_0) \approx$ const for all the corresponding mesons composed by the n,s, and c quarks, aside from charmonia. Physically, these observations mean that E is small compared to m_1 and m_2 or approximately constant over all of these states, and so may be absorbed in the definition of the latter. For the higher L nonets, E decreases with the increasing quark masses, according to the Feynman-Hellmann theorem; it therefore may be absorbed into the constituent quark mass defined for every L. We shall dwell on this point in more detail in the following section.

Since $\mathbf{s}_1 \cdot \mathbf{s}_2 = -3/4$ for spin-0 mesons and $+1/4$ for spin-1 mesons, one has the four relations [in the following, π stands for the mass of the π meson, etc., and n and s for the masses of the non-strange and strange quarks, respectively, unless otherwise specified, and we assume SU(2) flavor symmetry, $m_u = m_d = n$],

$$\pi = 2n - \frac{3}{4}\frac{\Lambda}{n^2}, \tag{11}$$

$$\rho = 2n + \frac{1}{4}\frac{\Lambda}{n^2}, \tag{12}$$

$$K = n + s - \frac{3}{4}\frac{\Lambda}{ns}, \tag{13}$$

$$K^* = n + s + \frac{1}{4}\frac{\Lambda}{ns}. \tag{14}$$

It then follows from these relations that

$$n = \frac{\pi + 3\rho}{8}, \tag{15}$$

$$s = \frac{2K + 6K^* - \pi - 3\rho}{8}, \tag{16}$$

$$\frac{\Lambda}{n^2} = \frac{\rho - \pi}{2}, \tag{17}$$

$$\frac{\Lambda}{ns} = \frac{K^* - K}{2}. \tag{18}$$

By expressing the ratio n/s in two different ways, directly from Eqs. (15),(16) and dividing Eqs. (18) by (17), one obtains the relation

$$\frac{n}{s} = \frac{\pi + 3\rho}{2K + 6K^* - \pi - 3\rho} = \frac{K^* - K}{\rho - \pi}. \tag{19}$$

For the physical values of π,ρ,K, and K^* (in MeV), 138, 769, 495, and 892, respectively, the above relation reads $0.629 = 0.627$, i.e., the result is consistent within the accuracy provided by the assumption of exact SU(2) flavor symmetry. The values of n,s, and K provided by (15)–(18) are $n = 306$ MeV, $s = 487$ MeV, $\Lambda = 0.0592$ GeV^3 $= (390$ MeV$)^3$. The values of the meson masses, as calculated from (11)–(14), are (in MeV) $\pi = 137.8$, $\rho = 770.0$, $K = 495.0$, $K^* = 892.3$. The relation (10) may also be applied successfully to the $^3S_1$ $I = 0$ mesons too, assuming that they are pure $n\bar{n}$ and $s\bar{s}$ states. In this case, as follows from Eq. (12), $\omega = \rho = 770$ MeV, and $\phi = 2s + \Lambda/(4s^2) = 1036$ MeV. Both numbers are within 1.5% of the physical values 782 and 1019 MeV, respectively.

Let us note that, although Eq. (11) contains no information on chiral symmetry, one may deal with the chiral limit $\pi = 0$ by the introduction of the so called ''dynamical'' quark mass [19], m_{dyn}, defined as the solution of $2m_{dyn} - 3\Lambda/(4m_{dyn}^2) = 0$. Although this does not restore chiral symmetry, it does incorporate the masslessness of the pion, in accord with common understanding of the latter as the Nambu-Goldstone boson of broken chiral symmetry, as well as calculating the chiral limit values of ρ and K^*, in agreement with other models [20].

III. *P*-WAVE SPECTROSCOPY

We now wish to apply the Breit-Fermi Hamiltonian to the P-wave mesons. By calculating the expectation values of different terms of the Hamiltonian defined in Eqs. (3),(7),(8), taking into account the corresponding matrix elements $\langle \mathbf{L}\cdot\mathbf{S}\rangle$ and S_{12} [17], one obtains the relations [10]

$$M(^3P_0) = M_0 + \frac{1}{4}\langle V_{SS}\rangle - 2\langle V_{LS}^+\rangle + \langle V_T\rangle,$$

$$M(^3P_2) = M_0 + \frac{1}{4}\langle V_{SS}\rangle + \langle V_{LS}^+\rangle + \frac{1}{10}\langle V_T\rangle,$$

$$M(a_1) = M_0 + \frac{1}{4}\langle V_{SS}\rangle - \langle V_{LS}^+\rangle - \frac{1}{2}\langle V_T\rangle,$$

$$M(b_1) = M_0 - \frac{3}{4}\langle V_{SS}\rangle,$$

$$\begin{pmatrix} M(K_1) \\ M(K_1') \end{pmatrix} = \begin{pmatrix} M_0 + \frac{1}{4}\langle V_{SS}\rangle - \langle V_{LS}^+\rangle - \frac{1}{2}\langle V_T\rangle & \sqrt{2}\langle V_{LS}^-\rangle \\ \sqrt{2}\langle V_{LS}^-\rangle & M_0 - \frac{3}{4}\langle V_{SS}\rangle \end{pmatrix} \begin{pmatrix} K_{1A} \\ K_{1B} \end{pmatrix},$$

where M_0 stands for the sum of the constituent quark masses in either case. The V_{LS}^- term acts only on the $I=1/2$ singlet and triplet states giving rise to the spin-orbit mixing between these states,[3] and is responsible for the physical masses of the K_1 and K_1'. Let us assume, for simplicity, that

$$\sqrt{2}\langle V_{LS}^-\rangle(K_{1B})\simeq-\sqrt{2}\langle V_{LS}^-\rangle(K_{1A})\equiv\Delta.$$

The masses of the K_{1A}, K_{1B} are then determined by relations similar to those for the a_1,b_1 above, and $K_1\simeq K_{1A}+\Delta$, $K_1'\simeq K_{1B}-\Delta$, or[4]

$$\Delta\simeq K_1-K_{1A}\simeq K_{1B}-K_1'. \tag{20}$$

We consider, therefore, the following formulas for the masses of all eight $I=1,1/2$ P-wave mesons, b_1,a_0,a_1,a_2, $K_{1B},K_0^*,K_{1A},K_2^*$:

$$M(^1P_1)=m_1+m_2-\frac{3}{4}\frac{a}{m_1m_2}, \tag{21}$$

$$M(^3P_0)=m_1+m_2+\frac{1}{4}\frac{a}{m_1m_2}-\frac{2b}{m_1m_2}+\frac{c}{m_1m_2}, \tag{22}$$

$$M(^3P_1)=m_1+m_2+\frac{1}{4}\frac{a}{m_1m_2}-\frac{b}{m_1m_2}-\frac{c}{2m_1m_2}, \tag{23}$$

$$M(^3P_2)=m_1+m_2+\frac{1}{4}\frac{a}{m_1m_2}+\frac{b}{m_1m_2}+\frac{c}{10m_1m_2}, \tag{24}$$

where a,b, and c are related to the matrix elements of V_{SS}, V_{LS}, and V_T [see Eqs. (3),(5),(9)], and assumed to be the same for all of the P-wave states. In the above expressions the nonrelativistic binding energies are absorbed in the constituent quark masses, as discussed above. The same constituent quark masses appear also in the denominators of the hyperfine interaction terms in Eqs. (5)–(8) above, similar to S-wave spectroscopy considered in a previous section. Since this is usually done only for the lowest S-wave states, we briefly review the precedent and argument for the generality of these forms.

It was shown in [21] that a pure scalar potential contributes to the effective constituent quark mass. Bag models suggest that the kinetic energy also contributes to the effective constituent quark mass in the case of no potential [22]. These results were generalized further by Cohen and Lipkin [23] who have shown that both the kinetic and potential energy are included in the effective mass parameter which appears also in the denominators of the hyperfine interaction terms in the case of a scalar confining potential. The analyses of experimental data suggest that the non-strange and strange quarks are mainly subject to scalar part of the confining potential (whereas charmed and bottom quarks are more dominantly affected by Coulomb-like vector part) [17]. Moreover, the generality of the arguments by Cohen and Lipkin [23] allows one to apply them to any partial wave. Therefore, the constituent quark masses can be defined for any partial wave, through relations of the form (21)–(24); in this case they vary with the energies of the corresponding mass levels. Such an energy dependence of the constituent quark masses was considered in Refs. [24,25]. Also, a QCD-based mechanism which generates dynamical quark mass growing with L in a Regge-like manner was considered by Simonov [26].

The correction to V_{LS}^+ in formula (9), due to the difference in the masses of the n and s quarks, is ignored. Indeed, these masses, as calculated from Eqs. (21)–(24), are

$$n=\frac{3b_1+a_0+3a_1+5a_2}{24}, \tag{25}$$

$$s=\frac{6K_{1B}+2K_0^*+6K_{1A}+10K_2^*-3b_1-a_0-3a_1-5a_2}{24}. \tag{26}$$

With the physical values of the meson masses (in GeV), $a_1\simeq b_1\cong1.23$, $a_0\simeq a_2\cong1.32$, $K_{1A}\simeq K_{1B}\cong1.34$, $K_0^*\simeq K_2^*\cong1.43$, the above relations give

$$n\simeq640\ \text{MeV},\quad s\simeq740\ \text{MeV}, \tag{27}$$

so that the above-mentioned correction, according to (9), is $\sim100^2/(2\times640\times740)\simeq1\%$, i.e., comparable to isospin breaking on the scale considered here, and so completely negligible. It follows from Eqs. (21)–(24) that

$$\frac{9a}{m_1m_2}=M(^3P_0)+3M(^3P_1)+5M(^3P_2)-9M(^1P_1), \tag{28}$$

$$\frac{12b}{m_1m_2}=5M(^3P_2)-3M(^3P_1)-2M(^3P_0), \tag{29}$$

$$\frac{18c}{5m_1m_2}=2M(^3P_0)+M(^3P_2)-3M(^3P_1). \tag{30}$$

By expressing the ratio n/s in four different ways, viz., directly from Eqs. (25),(26) and dividing the expressions (28)–(30) for the $I=1/2$ and $I=1$ mesons by each other, similarly to the case of the S-wave mesons considered above, one obtains the following three relations:

[3]The spin-orbit $^3P_1$-$^1P_1$ mixing is a property of the model we are considering; the possibility that another mechanism is responsible for this mixing, such as mixing via common decay channels [7] should not be ruled out, but is not included here.

[4]Actually, as follows from Eq. (38) below,

$$\frac{K_1-K_{1A}}{K_{1B}-K_1'}=\frac{K_1'+K_{1B}}{K_1+K_{1A}}\simeq\frac{2K_{1B}}{2K_{1A}}\simeq1,$$

since the deviations K_1-K_{1A}, $K_{1B}-K_1'$ are small compared to K_{1A}, K_{1B}, and the mixing angle is $\sim45°$.

$$\frac{3a_1+3b_1+a_0+5a_2}{6K_{1A}+6K_{1B}+2K_0^*+10K_2^*-3a_1-3b_1-a_0-5a_2}=\frac{K_0^*+3K_{1A}+5K_2^*-9K_{1B}}{a_0+3a_1+5a_2-9b_1}, \tag{31}$$

$$\frac{K_0^*+3K_{1A}+5K_2^*-9K_{1B}}{a_0+3a_1+5a_2-9b_1}=\frac{5K_2^*-3K_{1A}-2K_0^*}{5a_2-3a_1-2a_0}, \tag{32}$$

$$\frac{5K_2^*-3K_{1A}-2K_0^*}{5a_2-3a_1-2a_0}=\frac{2K_0^*+K_2^*-3K_{1A}}{2a_0+a_2-3a_1}. \tag{33}$$

First consider Eq. (33) which may be rewritten, by a simple algebra, as

$$(K_2^*-K_0^*)(a_2-a_1)=(K_2^*-K_{1A})(a_2-a_0). \tag{34}$$

Since $K_2^*\cong K_0^*\approx 1430$ MeV, it then follows from Eq. (34) that either $K_2^*\cong K_0^*\cong K_{1A}$, or $a_0\cong a_2$. The first possibility should be discarded as unphysical, since it leads, through relations (29),(30) applied to the $I=1/2$ mesons, to $b=c=0$, which would in turn, from the same relations for the $I=1$ mesons, imply $a_0\cong a_1\cong a_2$, in apparent contradiction with experimental data on the masses of the a_1 and a_2 mesons. The physical case corresponds, therefore, to

$$a_0\cong a_2, \tag{35}$$

i.e., the mass degeneracy of the scalar and tensor meson nonets in the $I=1/2$ channel, $K_0^*\cong K_2^*$, implies a similar degeneracy also in the $I=1$ channel. Note that this relation is a general feature of the nonrelativistic quark model for the P-wave mesons we are considering here. Even in the presence of an extra term in (21)–(24) corresponding to the quark binding energy which we have ignored by absorbing into the constituent masses, Eqs. (29) and (30) will remain the same and again lead, through Eq. (33), to the relation (35).

With $K_0^*=K_2^*$ and $a_0=a_2$, Eqs. (31) and (32) may be rewritten as

$$(a_0-a_1+K_0^*-K_{1A})(a_1+b_1+2a_0)=2(K_0^*-K_{1A})(K_{1A}+K_{1B}+2K_0^*), \tag{36}$$

$$(K_{1A}-K_{1B})(a_0-a_1)=(K_0^*-K_{1A})(a_1-b_1). \tag{37}$$

One now has to determine the values of a_1, K_{1A} and K_{1B}. The remaining equation is obtained from the mixing of the K_{1A} and K_{1B} states which results in the physical K_1 and K_1' mesons; independent of the mixing angle,

$$K_{1A}^2+K_{1B}^2=K_1^2+K_1'^2. \tag{38}$$

One sees that, as follows from Eq. (37), the mass degeneracy of the $^3P_1$ and $^1P_1$ nonets in the $I=1/2$ channel, $K_{1A}=K_{1B}$, implies a similar degeneracy in the $I=1$ channel too, $a_1=b_1$, and vice versa, so that the model we are considering provides the consistent possibility:

$$a_1=b_1, \quad K_{1A}=K_{1B}. \tag{39}$$

We now check how this possibility agrees with experimental data on the meson masses. It follows from Eq. (38) and $K_1=1273\pm 7$ MeV, $K_1'=1402\pm 7$ MeV that in this case

$$K_{1A}=K_{1B}=1339\pm 7 \ \text{MeV}. \tag{40}$$

With $a_1=b_1$, $K_{1A}=K_{1B}$, Eq. (36) now reduces to

$$a_0^2-a_1^2+(a_0+a_1)(K_0^*-K_{1A})=2(K_0^{*2}-K_{1A}^2), \tag{41}$$

which for $a_0=a_2=1318$ MeV, $K_0^*=1429$ MeV, and K_{1A}, K_{1B} given in Eq. (40) has the solution

$$a_1=b_1=1211\pm 8 \ \text{MeV}, \tag{42}$$

which is only a two-standard-deviation inconsistency with the experimentally established b_1 meson mass 1231 ± 10 MeV. We also consider another solution of Eqs. (36)–(38) determined by adjusting b_1 to the experimental value 1231 MeV. It then follows that in this case the solution to (36)–(38) is

$$a_1=1191 \ \text{MeV}, \quad K_{1A}=1322 \ \text{MeV}, \quad K_{1B}=1356 \ \text{MeV}, \tag{43}$$

with small deviations from these values for possible deviations in the input parameters; e.g., with (in MeV) $b_1=1231\pm 10$, the actual solution is $a_1=1191\mp 10$, $K_{1A}=1322\mp 9$, $K_{1B}=1356\pm 9$, or with $K_1=1273\pm 7$, $K_1'=1402\pm 7$, the solution is $a_1=1191\pm 17$, $K_{1A}=1322\pm 14$, and K_{1B} remains the same. For the solution (43), we observe the sum rule

$$K_{1A}^2-a_1^2=0.329 \ \text{GeV}^2\simeq K_{1B}^2-b_1^2=0.323 \ \text{GeV}^2, \tag{44}$$

which is accurate to 2% and also holds for deviations from Eq. (43) due to uncertainties in the input parameters. Relations of the type (44) may be anticipated on the basis of the formulas

$$K^{*2}-\rho^2=K^2-\pi^2,\quad K_2^{*2}-a_2^2=K^2-\pi^2,\ \text{etc.},$$

provided by either the algebraic approach to QCD [27] or phenomenological formulas

$$m_1^2=2Bn+C,\quad m_{1/2}^2=B(n+s)+C$$

(where B is related to the quark condensate, and C is a constant within a given meson nonet) motivated by the linear mass spectrum of a nonet and the collinearity of Regge trajectories of the corresponding $I=1$ and $I=1/2$ states, as discussed in Ref. [28].

Thus, the nonrelativistic constituent quark model we are considering provides two possibilities for the mass spectra of the axial-vector and pseudovector meson nonets:

(1) $a_1=b_1\simeq 1210$ MeV, $K_{1A}=K_{1B}\simeq 1340$ MeV,

(2) $a_1\neq b_1$, $K_{1A}\neq K_{1B}$, $K_{1A}^2-a_1^2\simeq K_{1B}^2-b_1^2$.

The second case is obviously favored by current experimental data on $K\pi\pi$ production in τ decay, which do not support $\theta_K\approx 45°$ and, therefore, mass degeneracy of the K_{1A} and K_{1B}, as discussed above in the text. In this case, assume that the $K_1(1270)$ belongs to the axial-vector nonet, while the $K_1(1400)$ belongs to the pseudovector nonet, in accord with the recent suggestion by Suzuki [29], on the basis of the analysis of the τ-decay mode $\tau\rightarrow\nu_\tau K_1$, for the values [in MeV, as follows from the discussion below Eq. (43)] $K_1=1273\pm 7$, $K_1'=1402\pm 7$, $K_{1A}=1322\pm 14$, $K_{1B}=1356$. One then obtains, with the help of the formula [9]

$$\tan^2(2\theta_K)=\left(\frac{K_1^2(1400)-K_1^2(1270)}{K_{1B}^2-K_{1A}^2}\right)^2-1,$$

$$\theta_K=(37.3\pm 3.2)°, \tag{45}$$

in good qualitative agreement with the values $\approx 33°$ suggested by Suzuki [9], and $\simeq 34°$ found by Godfrey and Isgur [8] in a relativized quark model.

The parameters of the spin-spin, spin-orbit, and tensor interaction in our model may be calculated from Eqs. (28)–(30) with the meson mass values obtained above. In the isodoublet channel, e.g., one obtains

$$\langle V_{SS}\rangle=\frac{a}{ns}\simeq 37\ \text{MeV}, \tag{46}$$

$$\langle V_{LS}^+\rangle=\frac{b}{ns}\simeq 27\ \text{MeV}, \tag{47}$$

$$\langle V_T\rangle=\frac{c}{ns}\simeq 89\ \text{MeV}. \tag{48}$$

The expectation value $\langle V_{LS}^-\rangle$ may be obtained from Eqs. (20),(43):

$$\begin{aligned}\sqrt{2}\langle V_{LS}^-\rangle&\simeq K_1(1270)-K_{1A}=(1273-1322)\ \text{MeV}\\&\simeq K_{1B}-K_1(1400)=(1356-1402)\ \text{MeV}\\&\simeq -47.5\ \text{MeV},\end{aligned}$$

and therefore

$$\langle V_{LS}^-\rangle\simeq -33.5\ \text{MeV}, \tag{49}$$

so that both $\langle V_{LS}^+\rangle$ and $\langle V_{LS}^-\rangle$ are of very similar magnitude (but opposite in sign).

Using the obtained values of $\langle V_{LS}^+\rangle$ and $\langle V_{LS}^-\rangle$, along with the values of n and s given in Eq. (27), in Eqs. (8),(9), one may establish the following relation among the expectation values of the derivatives of the potentials:

$$\left\langle\frac{1}{r}\frac{dV_S(r)}{dr}\right\rangle\simeq 2.8\left\langle\frac{1}{r}\frac{dV_V(r)}{dr}\right\rangle. \tag{50}$$

In the case of the QCD-motivated Cornell potential [30]

$$V(r)=-\frac{4}{3}\frac{\alpha_s}{r}+ar, \tag{51}$$

with a spin structure $V=V_V+V_s$, $V_V=-\frac{4}{3}(\alpha_s/r)$, $V_S=ar$, the relation (50) reduces to

$$a\langle r^{-1}\rangle\simeq 3.7\alpha_s\langle r^{-3}\rangle. \tag{52}$$

Consider now the ratio [17]

$$\rho=\frac{M(^3P_2)-M(^3P_1)}{M(^3P_1)-M(^3P_0)}. \tag{53}$$

Since the measured masses of the K_2^* and K_0^* coincide (as also do those of the a_2 and a_0, as established in Sec. III), the value of this ratio is

$$\rho=-1. \tag{54}$$

By equating this value of ρ with that given in [17] for the Cornell case,

$$\rho=\frac{1}{5}\ \frac{8\alpha_s\langle r^{-3}\rangle-\frac{5}{2}a\langle r^{-1}\rangle}{2\alpha_s\langle r^{-3}\rangle-\frac{1}{4}a\langle r^{-1}\rangle}, \tag{55}$$

we obtain

$$a\langle r^{-1}\rangle=4.8\alpha_s\langle r^{-3}\rangle. \tag{56}$$

Comparison of the relation (56) with (52) shows that the nonrelativistic constituent quark model considered in this paper is completely consistent, at the 25% level, with the Cornell potential with the spin structure of a vector-scalar mixing type. We consider this to be a completely satisfactory agreement, since the expectation values and α_s are all purely determined in this region for light quark systems.[5]

One may now estimate the masses of the isoscalar mesons of the four nonets assuming that they are pure $s\bar{s}$ states, by the application of Eqs. (21)–(24) with $m_1=m_2=s$; it then follows that

$$h_1'\simeq f_1'\cong 1435 \text{ MeV}, \quad f_0'\simeq f_2'\cong 1525 \text{ MeV}. \quad (57)$$

Hence, the model we are considering suggests that 1^{++} $s\bar{s}$ state is the $f_1(1420)$ (with mass 1427 ± 2 MeV [3]) rather than $f_1(1510)$ (1512 ± 4 MeV [3]) meson, in accord with the arguments of Aihara *et al.* [31] and Close and Kirk [32]. The value 1435 given by Eq. (57) is within 4% of the h_1' meson mass, 1380 ± 20 [3]. Also, the value 1525 given by Eq. (57) agrees with the experimentally established mass of the f_2' meson, 1525 ± 5 MeV [3].

At this point we call that the nonrelativistic quark model predictions on the masses of the isoscalar states are reliable for all P-wave nonets except the scalar nonet. Indeed, as shown by 't Hooft in his study on the $U_A(1)$ problem [33], an expansion of the (euclidian) action around the one-instanton solutions of the gauge fields assuming dominance of the zero modes of the fermion fields leads to an effective $2N_f$-fermion interaction (N_f being the number of fermion flavors) not covered by perturbative gluon exchange, which gives an additional contribution to the ordinary confining quark-antiquark interaction. As shown in Ref. [34], due to its point-like nature and specific spin structure, the instanton-induced interaction in the formulation of 't Hooft acts on the states with spin zero only. The masses of the other mesons with non-vanishing spin are therefore dominantly determined by the confining interaction, leading to the conventional splitting and an ideal mixing of the $q\bar{q}$ nonets which are well reproduced by constituent quark models. The only two nonets whose mass spectra turn out to be affected by an instanton-induced interaction are spin zero pseudoscalar and scalar nonets. Quantitatively, an instanton-induced interaction for the scalar mesons is of the same magnitude but opposite in sign to that for the pseudoscalars [12]. It, therefore, lowers the mass of the scalar isoscalar singlet state, in contrast to the case of the pseudoscalar isoscalar singlet (η_0) state the mass of which is pushed up by the instanton-induced interaction before it mixes with the pseudoscalar isoscalar octet (η_8) state to form the physical η and η' states.

Thus, the only nonrelativistic quark model prediction of the masses of the isoscalar states of the scalar nonet which may be trustworthy is that on the mass of the mostly isoscalar octet state (which has a dominantly $s\bar{s}$ component): $f_0\simeq 1525$ MeV, as given in Eq. (57), in good agreement with the measured mass of the $f_0(1500)$. The second isoscalar state of the scalar nonet should be mostly $n\bar{n}$ but also contain a non-negligible $s\bar{s}$ component. Its mass may be determined from the sum rule established in Refs. [13] and [14]:

$$m^2(f_0)+m^2(f_0')+m^2(\eta)+m^2(\eta')=2(m^2(K)+m^2(K_0^*)),$$

with $f_0=1503\pm 11$ MeV [3]:

$$f_0'=1048\mp 16 \text{ MeV}. \quad (58)$$

Hence, one of the two, $f_0(980)$ [3] or $f_0(1000)$ [35], may be associated with the remaining isoscalar, which is difficult to decide (that is, on the basis of the constituent quark model we are discussing). However, two observations support the interpretation of the $f_0(980)$ as a $q\bar{q}$ state. First, the t dependence of the $f_0(980)$ and the broad background produced in $\pi^- p\to\pi^0\pi^0 n$ differ substantially [37]. The $f_0(1000)$ is produced in peripheral collisions only, while the $f_0(980)$ shows a strong t dependence, as expected for a $q\bar{q}$ state. Second, as remarked above, although the isoscalar mostly SU(3) singlet state should have a dominant $n\bar{n}$ component, its $s\bar{s}$ component should be appreciable. The $f_0(980)$ is seen strongly in $J/\Psi\to\phi f_0(980)$, but at most weakly in $J/\Psi\to\omega f_0(980)$. On the basis of quark diagrams, one must conclude that the $f_0(980)$ has a very large $s\bar{s}$ component; its decay into $\pi\pi$ with the corresponding branching ratio 78% [3] underlines an appreciable $n\bar{n}$ component.

Thus, the constituent quark model discussed in this paper supports the $q\bar{q}$ assignment for the scalar meson nonet

$$a_0(1320),\ K_0^*(1430),\quad f_0(1525),\quad f_0'(980), \quad (59)$$

found by one of the authors in Ref. [14], which is also consistent with the $q\bar{q}$ assignments for this nonet suggested in Refs. [12,13,36]. For this assignment, the f_0-f_0' mixing angle, as calculated with the help of the Gell-Mann–Okubo mass formula

$$\tan^2\theta_S=\frac{4K_0^{*2}-a_0^2-3f_0^2}{3f_0'^2+a_0^2-4K_0^{*2}}, \quad (60)$$

for $f_0=1525\pm 5$ MeV [41] and $f_0'=980\pm 10$ MeV [3], is

$$\theta_S=(21.4\pm 1.0)^\circ,$$

[5]For $a=1/(2\pi\alpha')\simeq 0.18$ GeV^2, where $\alpha'\simeq 0.9$ GeV^{-2} is the universal Regge slope, it follows from the relation [17]

$$\Delta M^2\equiv M^2(^3S_1)-M^2(^1S_0)\simeq\tfrac{32}{9}\alpha_s a\approx 0.56 \text{ GeV}^2$$

that $\alpha_s\simeq 0.9$. With these values of a and α_s, and in the approximation

$$\langle r^{-3}\rangle\sim\frac{\langle r^{-1}\rangle}{\langle r\rangle^2},$$

it then follows from (52) that $\langle r\rangle\simeq 0.9$ fm.

in reasonbly good agreement with the value predicted by Ritter *et al.* [38], $\theta_S \approx 25°$, for which the partial widths of the $f_0(1500)$ calculated in their paper are in excellent agreement with those observed experimentally [39].

IV. CONCLUDING REMARKS

As we have shown, a nonrelativistic constituent quark model confirms a simultaneous mass degeneracy of the scalar and tensor nonets in the isovector and isodoublet channels, and suggests a nearly mass degeneracy of the corresponding isoscalar mostly octet states. The mass of the remaining 0^{++} isoscalar mostly singlet state is probably shifted down to ~ 1 GeV due to instanton effects, as discussed in Refs. [12,13], thus leaving two, the $f_0(980)$ and $f_0(1000)$, mesons as candidates for this state. Out of these two, preference should be given to the $f_0(980)$, as discussed above in the text. Let us note that, if one ignores instanton or any other effects which may cause a shift in the mass of the f_0', one would arrive at a $q\bar{q}$ assignment for the scalar nonet which would be nearly mass degenerate in all isospin channels [e.g., $f_0(1300)$ in (59) in place of $f_0(980)$, as compared to $f_2(1270)$]. In this case, one would have the scalar nonet almost ideally mixed, just like the tensor one is. Then, as shown by Törnqvist [40], flavor symmetry (which should be good in the case of such an ideally mixed nonet) would predict the total width of the $a_0(1320)$ (using the experimental K_0^* width as normalization) $\Gamma > 400$ MeV, and a similar ~ 400 MeV width of the $f_0(1525)$, much larger than 130 MeV found by GAMS for the $a_0(1320)$ [42] and ≈ 90 MeV found by LASS for the $f_0(1525)$ [41]. Therefore, this case should be considered as unphysical.

Although the possibility of a simultaneous mass degeneracy of the axial-vector and pseudovector nonets in the $I=1$ and $I=1/2$ channels is not excluded in the model considered here, it is disfavored by current experimental data. By adjusting the mass of the b_1 meson to the experimentally established value, the masses of the a_1, K_{1A}, and K_{1B} mesons were calculated, leading to the predictions $m(a_1) \simeq 1190$ MeV, and $\theta_K \simeq (37 \pm 3)°$. While the former number naturally interpolates between various predictions and current experimental data (e.g., it is at the upper limit of the range (1150 ± 40) MeV established in [43] from QCD sum rules, and at the lower limit of that provided by data, (1230 ± 40) MeV [3]), the latter one is in quantitative agreement with the predictions $\theta_K \simeq 34^o$ by Godfrey and Isgur [8] and $\approx 33°$ by Suzuki [9]. The results of the work suggest that the mostly $s\bar{s}$ state of the axial-vector nonet should be associated with the $f_1(1420)$ rather than $f_1(1510)$ meson, which supports conclusions of Aihara *et al.* [31] and Close and Kirk [32]. We did not calculate the decay widths and branching ratios for this case, since that was done in Ref. [31]. We wish to give yet another argument in support of this prediction. As remarked above, instanton effects are essential for spin-0 mesons only, and for any other $q\bar{q}$ nonet with nonvakishing spin, the confining interaction leads to an almost ideal mixing. Thus, we would expect the mixing angle of the axial-vector nonet to be in close proximity to the ideal one, $\theta_{id} \approx 35.3°$, just as it is the case for the vector, tensor and 3^{--} mesons [3]. The value of this mixing angle, as calculated from Eq. (60) for a_1, K_{1A} given in Eq. (43), with deviations due to the input parameters K, K', and $f_1 = 1427$ MeV, is

$$\theta_A = (42.4 \pm 5.3)°,$$

consistent with 35.3°, while for $f_1 = 1512$ MeV and the same a_1 and K_{1A}, Eq. (60) gives

$$\theta_A = (54.8 \pm 3.4)°,$$

which is somehow farther from θ_{id} than a previous value.

The values of the a_1 and K_{1A} masses calculated in this work fix the mass of the K_{1B} to be 1356 MeV. The mass of the isoscalar octet state of the $1\,{}^1P_1$ nonet is then determined by the Gell-Mann–Okubo formula

$$h_8^2 = \frac{4K_{1B}^2 - b_1^2}{3},$$

$h_8 = 1395 \mp 3$ MeV (for $b_1 = 1231 \pm 10$ MeV). Since for the pseudovector nonet Eq. (60) may be rewritten as

$$\tan^2\theta = \frac{h_8^2 - h_1'^2}{h_1^2 - h_8^2},$$

it is clear that h_1 and h_1' cannot both be less than h_8 (since it would otherwise lead to a complex mixing angle). Therefore, h_1' should be greater than h_8, and with the PDG value $h_1' = 1380 \pm 20$ MeV, one is left with $h_1' \approx 1400$ MeV. In this case, since the h_1' lies slightly above the h_8, the pseudovector nonet has a small positive mixing angle (just opposite to the case of the pseudoscalar nonet for which the η lies slightly below the $\eta_8 = 566$ MeV leading to a small negative mixing angle). The above conclusion would change if one (or both) of the h_1, h_1' appeared to have a mass higher than the value currently adopted by PDG.

We close with a short summary of the findings of this work.

(1) The nonrelativistic constituent quark model shows a simultaneous mass degeneracy of the scalar and tensor meson nonets in the $I = 1,1/2$, and nearly mass degeneracy in the $I=0$, $s\bar{s}$ channels.

(2) Simultaneous mass degeneracy of the axial-vector and pseudovector nonets in the $I = 1,1/2$ channels is not excluded in this model, but is disfavored by current experimental data.

(3) The $q\bar{q}$ assignments for the P-wave nonets obtained on the basis of the results of the work, are

$1\,{}^1P_1$ $J^{PC} = 1^{+-}$, $b_1(1235)$, $h_1(1170)$, $h_1(1400)$, K_{1B}
$1\,{}^3P_0$ $J^{PC} = 0^{++}$, $a_0(1320)$, $f_0(980)$, $f_0(1500)$, $K_0^*(1430)$
$1\,{}^3P_1$ $J^{PC} = 1^{++}$, $a_1(1190)$, $f_1(1285)$, $f_1(1420)$, K_{1A}
$1\,{}^3P_2$ $J^{PC} = 2^{++}$, $a_2(1320)$, $f_2(1270)$, $f_2'(1525)$ $K_2^*(1430)$

ACKNOWLEDGMENTS

Correspondence of one of the authors (L.B.) with L.P. Horwitz and P. Page during the preparation of this work is greatly acknowledged.

[1] For a review, see S. Meshkov, in Proceedings of the Aspen Winter Physics Conference, Aspen, CO, Jan 5-18, 1986; C.A. Heusch, in Proceedings of the 27th International Symposium on Multiparticle Dynamics, Seewinkel, Austria, June 16–20, 1986; S. Cooper, in Proceedings of the 23rd International Conference on High Energy Physics, Berkeley, CA, July 16–23, 1986; F.E. Close, Rep. Prog. Phys. **51**, 833 (1988).

[2] M.S. Chanowitz, Phys. Lett. B **187**, 409 (1987).

[3] Particle Data Group, R.M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).

[4] G.W. Brandenburg *et al.*, Phys. Rev. Lett. **36**, 703 (1976); R.K. Carnegie *et al.*, Nucl. Phys. **B127**, 509 (1977); M.G. Bowler, J. Phys. G **3**, 775 (1977).

[5] R.K. Carnegie *et al.*, Phys. Lett. **68B**, 289 (1977).

[6] E.W. Colglazier and J.L. Rosner, Nucl. Phys. **B27**, 349 (1971).

[7] H.J. Lipkin, Phys. Lett. **72B**, 249 (1977); Phys. Lett. B **303**, 119 (1993).

[8] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).

[9] M. Suzuki, Phys. Rev. D **47**, 1252 (1993).

[10] H.G. Blundell, S. Godfrey, and B. Phelps, Phys. Rev. D **53**, 3712 (1996).

[11] L. Burakovsky and T. Goldman, Phys. Rev. D **56**, R1368 (1997).

[12] E. Klempt, B.C. Metsch, C.R. Münz, and H.R. Petry, Phys. Lett. B **361**, 160 (1995).

[13] V. Dmitrasinovic, Phys. Rev. C **53**, 1383 (1996).

[14] L. Burakovsky, Found. Phys. **27**, 315 (1997).

[15] M.R. Pennington, in *The Hadron Mass Spectrum*, Proceedings of the Conference, St. Goar, Germany, edited by E. Klempt and K. Peters [Nucl. Phys. B (Proc. Suppl.) **21**, 37 (1991)].

[16] S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986).

[17] W. Lucha, F.F. Schöberl, and D. Gromes, Phys. Rep. **200**, 127 (1991).

[18] W. Lucha, H. Rupprecht, and F.F. Schöberl, Phys. Rev. D **46**, 1088 (1992).

[19] V. Elias, M. Tong, and M.D. Scadron, Phys. Rev. D **40**, 3670 (1989).

[20] Bing An Li, Masses of spin-1 mesons and dynamical chiral symmetry breaking, hep-ph/9509368; L. Burakovsky, L.P. Horwitz, and T. Goldman, Hadron masses from the linear spectrum of a multiplet, in preparation.

[21] H.J. Lipkin and A. Tavkhelidze, Phys. Lett. **17**, 331 (1965).

[22] T. DeGrand, R.L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D **12**, 2060 (1975).

[23] I. Cohen and H.J. Lipkin, Phys. Lett. **93B**, 56 (1980).

[24] T. Goldman and S. Yankielowicz, Phys. Rev. D **12**, 2910 (1975).

[25] J.L. Basdevant and S. Boukraa, Z. Phys. C **28**, 413 (1985); Ann. Phys. (Paris) **10**, 475 (1985).

[26] Yu.A. Simonov, Phys. Lett. B **226**, 151 (1989); **228**, 413 (1989); **249**, 514 (1990).

[27] S. Oneda and K. Terasaki, Prog. Theor. Phys. Suppl. **82**, 1 (1985).

[28] L. Burakovsky and L.P. Horwitz, Found. Phys. Lett. **9**, 561 (1996); **10**, 61 (1997); Nucl. Phys. **A609**, 585 (1996); **A614**, 373 (1997).

[29] M. Suzuki, Phys. Rev. D **50**, 4708 (1994).

[30] E. Eichten *et al.*, Phys. Rev. Lett. **34**, 369 (1975); Phys. Rev. D **17**, 3090 (1978); **21**, 313(E) (1980); **21**, 203 (1980).

[31] H. Aihara *et al.*, Phys. Rev. D **38**, 1 (1988).

[32] F.E. Close and A. Kirk, Z. Phys. C **76**, 469 (1997).

[33] G. 't Hooft, Phys. Rev. D **14**, 3432 (1976).

[34] C.R. Münz, J. Resag, B.C. Metsch, and H.R. Petry, Nucl. Phys. **A578**, 418 (1994).

[35] D. Morgan and M.R. Pennington, Phys. Rev. D **48**, 1185 (1993); **48**, 5422 (1993).

[36] L. Burakovsky and T. Goldman, Towards resolution of the scalar meson nonet enigma: Gell-Mann–Okubo revisited, LA-UR-97-3738, hep-ph/9709305 [Nucl. Phys. A (to be published)].

[37] D. Alde *et al.*, Z. Phys. C **66**, 375 (1995).

[38] C. Ritter, B.C. Metsch, C.R. Münz, and H.R. Petry, Phys. Lett. B **380**, 431 (1996).

[39] C. Amsler and F.E. Close, Phys. Lett. B **353**, 385 (1995).

[40] N.A. Törnqvist, in *The Hadron Mass Spectrum* [15], p. 196.

[41] D. Aston *et al.*, Nucl. Phys. **B301**, 525 (1988).

[42] M. Boutemeur, in *Hadron '89*, Proceedings of the 3rd International Conference on Hadron Spectroscopy, Ajaccio, France, 1989, edited by F. Binon *et al.* (Editions Frontières, Gif-sur-Yvette, 1989), Vol. C29, p. 119.

[43] L.J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985).